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A new dynamic programming procedure for three-staged cutting patterns

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Abstract Three-staged patterns are often used to solve the 2D cutting stock problem of rectangular items. They can be divided into items in three stages: Vertical cuts divide the plate into segments; then horizontal cuts divide the segments into strips, and finally vertical cuts divide the strips into items. An algorithm for unconstrained three-staged patterns is presented, where a set of rectangular item types are packed into the plate so as to maximize the pattern value, and there is no constraint on the frequencies of each item type. It can be used jointly with the linear programming approach to solve the cutting stock problem. The algorithm solves three large knapsack problems to obtain the optimal pattern: One for the item layout on the widest strip, one for the strip layout on the longest segment, and the third for the segment layout on the plate. The computational results indicate that the algorithm is efficient.

Keywords Cutting stock · Two-dimensional cutting · Three-staged patterns · Knapsack problems

1 Introduction

Cutting problems appear in many industrial areas, such as the cutting of metal plate, plate glass and wood panel into rectangular items. Good algorithms for generating cutting patterns are useful for better material utilization.

This paper discusses the unconstrained two-dimensional cutting (UTDC) problem: m types of rectangular items are cut from plate $L \times W$ (length $\times$ width) so as to maximize the pattern value. The cuts must be guillotine ones. The piece length, width, and value of the ith ($i = 1, \ldots, m$) type are l_i, w_i and c_i respectively. The sizes of the plate and items are integers. Assume that pattern R contains z_i pieces of type i; $\boldsymbol{N}$ is the set of non-negative integers. The formulation of the UTDC is:

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$$z = \max \sum_{i=1}^{m} c_i z_i; \ \boldsymbol{R} \text{ is a pattern with specified features; } z_i \in \boldsymbol{N}, i = 1, \ldots, m$$

It is assumed that the direction of each item type is fixed. This does not affect the applicability of the UTDC algorithm. If an item type of size $l \times w$ can be rotated, it can be treated as two item types of fixed direction: one of size $l \times w$, and the other of size $w \times l$.

The UTDC is formally referred to as SLOPP (Single Large Object Placement Problem [1]). It is closely related with the two-dimensional cutting stock problem (TDCS). The TDCS is the problem of cutting rectangular items from stock plates so as to minimize the plate cost, where the demand of each item type must be fulfilled. A solution of the TDCS is a cutting plan that contains a group of distinct cutting patterns with given frequencies. The linear programming (LP) approach is widely used to solve the TDCS [2]. It considers a large number of patterns to obtain the cutting plan. These patterns are generated using an UTDC algorithm. The UTDC algorithm must be executed many times before the LP approach finds a solution close to optimal. Fast UTDC algorithms are useful to reduce computation time.

The pattern type must be determined to solve the UTDC. Staged patterns have been widely used [3–6]. A k-staged pattern can be divided into items in k stages. The cuts made at the same stage have the same direction. The cut directions of two adjacent stages are perpendicular to each other. When the number of stages increases, both material utilization and cutting complexity increase. Three-staged patterns make a good balance between material utilization and cutting complexity, subsequently they and their variants have been often used in solving cutting and cutting stock problems [2,4,7–10].

This paper presents a dynamic programming algorithm for three-staged patterns. A three-staged pattern consists of several segments. Each segment contains a set of parallel strips. The algorithm considers the segments jointly; subsequently the number of strips considered is reduced drastically. The computational results indicate that the algorithm is time efficient.

The contents are organized as follows. Three-staged patterns and the literature review are described in Sect. 2, followed by the presentation of the algorithm in Sect. 3, the computational results in Sect. 4, and the conclusions in Sect. 5.

2 Three-staged patterns and literature review

2.1 Geometric structure of three-staged patterns

A three-staged pattern can be divided into items in three stages. It is an X-pattern if the first stage cuts are vertical; it is a Y-pattern otherwise. Figure 1 shows the cutting process of a three-staged X-pattern, where the numbers denote the item types and the arrows denote the cuts. Vertical cuts divide the plate into three segments at the first stage (Fig. 1a). Each segment consists of only horizontal strips. Although by intuition the first segment contains one vertical strip, it can be seen as consisting of nine horizontal strips, each of which contains only one piece of type 1. Horizontal cuts divide the segments into strips at the second stage (Fig. 1b), and vertical cuts divide the strips into items at the third stage (Fig. 1c). Figure 2 shows a three-staged Y-pattern. It contains three segments arranged vertically from bottom to top. Each segment can be seen as consisting of only vertical strips.

2.2 Literature review

Algorithms for k-staged patterns [3–6] can be used for three-staged ($k = 3$) patterns. Gilmore and Geomory [3] presented a dynamic programming algorithm. This algorithm was