Course Code	**:**	**BCS-054**
Course Title	**:**	**Computer Oriented Numerical Techniques**
Assignment Number	**:**	**BCA(V)/054/Assignment/2025-26**
Maximum Marks	**:**	**100**
Weightage	**:**	**25%**
Last Dates for Submission	**:**	**31stOctober,2025(For July, Session)**
		30thApril, 2026(For January, Session)

This assignment has seven questions of total 80 marks. Answer all the questions. 20 marks are for viva voce. You may use illustrations and diagrams to enhance explanations. Please go through the guidelines regarding assignments given in the Programme Guide for the format of presentation. Illustrations/ examples, where-ever required, should be different from those given in the course material. You must use only simple calculator to perform the calculations.

Q1. (a) Find floating point representation, if possible normalized, in the 4-digit mantissa, two digit exponent, if necessary use approximation for each of the following numbers: **(8 Marks)**

(i) 27.94 (ii) -0.00943 (iii) -6781014 (iv) 0.0644321

Also, find absolute error, if any, in each ca

(b) Convert the decimal integer -465 to binary using both the methods (as shown in Pg No:16 of Block-1) . Show all the steps. **(4 Marks)**

(c) Convert the number given as binary fraction $-(0.101110101)_2$ to decimal. **(3 Marks)**

(d) Find the sum of the two floating numbers x1=0.1364X101 and x2=0.7342X10-1. Further express the result in normal form, using (i) Chopping (ii) Rounding. Also, find the absolute error.

(5 Marks)

Q2. (a) Solve the system of equations **(5 Marks)**

$$2x + y + z = 3$$
$$x + 3y + 3z = 4$$
$$x - 4y + 2z = 9$$

using Gauss elimination method with **partial pivoting**. Show all thesteps.

(b) Perform four iterations (rounded to four decimal places) using **(5 Marks)**

(i) Jacobi Method and

(ii) Gauss-Seidel method ,

for the following system of equations.

$$\begin{bmatrix} 5 & 4 & -3 \\ 4 & -4 & 3 \\ -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ -4 \end{bmatrix}$$

With $\mathbf{x}^{(0)} = (0, 0, 0)^T$. The exact solution is $(1,- 4, - 5)^T$.

Which method gives better approximation to the exact solution?

Q3. Determine the smallest positive root of the following equation: **(10 Marks)**

$$f(x) \equiv x^3 - 9x^2 - x + 9 = 0$$

to three significant digits using

(a) Regula-falsi method (b) Newton-Raphson method

(c) Bisectionmethod (d) Secant method

Q4. (a) Find Lagrange's interpolating polynomial for the following data. Hence obtain the value of f(4). **(5 Marks)**

x	0	2	3	5
f(x)	2	11	21	121

(b) Using the inverse Lagrange's interpolation, find the value of x when y=3 for the following data: **(5 Marks)**

x	25	35	55	75
y=f(x)	-2	-1	1	5

Q5. (a) The population of a country for the last 25 years is given in the following table:. **(3+2+3=8 Marks)**

Year (x)	: 1995	2000	2005	2010	2015
Population in lakhs (y)	: 678	1205	1855	2745	3403

(i) Using Stirling's central difference formula, estimate the populationfor the year 2007
(ii) Using Newton's forward formula, estimate the population for theyear 1998.
(iii) Using Newton's backward formula, estimate the population for theyear 2013.

(b) Derive the relationship for the operators δ in terms of E. **(2 Marks)**

Q6. (a) Find the values of the first and second derivatives of y = f(x) for x=2.1 using the following table. Use forward difference method. Also, find Truncation Error (TE) and actual errors. **(5 Marks)**

x	:	2	2.5	3	3.5
y	:	8.7	12.7	16.8	20.9

(b) Find the values of the first and second derivatives of $y = f(x)$ for $x=2.1$ from the following table using Lagrange's interpolation formula. Compare the results with (a) part above.

(5 Marks)

x	:	2	2.5	3	3.5
y	:	8.7	12.7	16.8	20.9

Q7. Compute the value of the integral **(10 Marks)**

$$\int_0^8 (4x^4 + 5x^3 + 6x + 5)\, dx$$

By taking 8 equal subintervals using (a) Trapezoidal Rule and then (b) Simpson's 1/3 Rule. Compare the result with the actual value.

BCS-54

Q1. (a) Floating Point Representation

The representation format is a 4-digit mantissa and a 2-digit exponent. A number `N` is represented as `± 0.d1d2d3d4 × 10^±e1e2`.

(i) 27.94

- **Normalized Form:** `0.2794 × 10^2`
- **Representation:** Mantissa = `2794`, Exponent = `02`
- **Floating Point Representation:** `0.2794E02`
- **Absolute Error:** The number is represented exactly. Absolute Error = 0.

(ii) -0.00943

- **Normalized Form:** `-0.9430 × 10^-2`
- **Representation:** Mantissa = `9430`, Exponent = `-02`
- **Floating Point Representation:** `-0.9430E-02`
- **Absolute Error:** The number is represented exactly. Absolute Error = 0.

(iii) -6781014

- **Normalized Form:** `-0.6781014 × 10^7`

- **Approximation (Rounding to 4 digits):** `-0.6781 × 10^7`
- **Representation:** Mantissa = `6781`, Exponent = `07`
- **Floating Point Representation:** `-0.6781E07`
- **Absolute Error:**
    - Original Number: -6781014
    - Approximated Number: -6781000
    - Absolute Error = | -6781014 - (-6781000) | = |-14| = **14**

(iv) 0.0644321

- **Normalized Form:** `0.644321 × 10^-1`
- **Approximation (Rounding to 4 digits):** `0.6443 × 10^-1`
- **Representation:** Mantissa = `6443`, Exponent = `-01`
- **Floating Point Representation:** `0.6443E-01`
- **Absolute Error:**
    - Original Number: 0.0644321
    - Approximated Number: 0.06443
    - Absolute Error = | 0.0644321 - 0.06443 | = **0.0000021**

Q1. (b) Decimal to Binary Conversion

To convert the decimal integer -465 to binary:

Method 1: Conversion of Magnitude
First, convert the absolute value (465) to binary using repeated division by 2.

- 465 / 2 = 232 R 1
- 232 / 2 = 116 R 0
- 116 / 2 = 58 R 0
- 58 / 2 = 29 R 0
- 29 / 2 = 14 R 1

- 14 / 2 = 7 R 0
- 7 / 2 = 3 R 1
- 3 / 2 = 1 R 1
- 1 / 2 = 0 R 1

Reading the remainders from bottom to top, $(465)_{10} = (111010001)_2$. The negative sign is handled by representation schemes like sign-magnitude or two's complement.

Method 2: Two's Complement Representation (assuming a 16-bit system)
This is the standard method for representing negative integers in computers.

1. **Binary of +465:** Write the binary of 465, padded with leading zeros to fit 16 bits.
   `0000 0001 1101 0001`
2. **One's Complement:** Invert all the bits (0 becomes 1, 1 becomes 0).
   `1111 1110 0010 1110`
3. **Two's Complement:** Add 1 to the one's complement.
   `1111 1110 0010 1110 + 1 = 1111 1110 0010 1111`

So, $(-465)_{10}$ in 16-bit two's complement is $(1111111000101111)_2$.

Q1. (c) Binary Fraction to Decimal

To convert $-(0.101110101)_2$ to decimal, we convert the fractional part by summing the powers of 2.

$-(1\times2^{-1} + 0\times2^{-2} + 1\times2^{-3} + 1\times2^{-4} + 1\times2^{-5} + 0\times2^{-6} + 1\times2^{-7} + 0\times2^{-8} + 1\times2^{-9})$

$= -(0.5 + 0 + 0.125 + 0.0625 + 0.03125 + 0 + 0.0078125 + 0 + 0.001953125)$

$= -(0.728515625)$

So, $-(0.101110101)_2 = -0.728515625_{10}$.

Q1. (d) Floating Point Addition

Given $x_1 = 0.1364 \times 10^1$ and $x_2 = 0.7342 \times 10^{-1}$.

1. **Equalize Exponents:** Rewrite x_2 to have the same exponent as x_1 (the larger exponent).
   $x_2 = 0.007342 \times 10^1$

2. **Add Mantissas:**
   $0.1364 + 0.007342 = 0.143742$

3. **Sum:** The sum is 0.143742×10^1. The exact sum is 1.43742.

Now, we express the result in the 4-digit mantissa format.

(i) Chopping: Truncate the mantissa to 4 digits.

- Mantissa: 0.1437
- Result: $0.1437 \times 10^1 = 1.437$
- Absolute Error = |1.43742 - 1.437| = **0.00042**

(ii) Rounding: Examine the 5th digit of the mantissa (4). Since it's less than 5, we keep the 4th digit as is.

- Mantissa: 0.1437
- Result: $0.1437 \times 10^1 = 1.437$
- Absolute Error = |1.43742 - 1.437| = **0.00042**

Q2. (a) Gauss Elimination with Partial Pivoting

System of equations:

$2x + y + z = 3$

$x + 3y + 3z = 4$

$x - 4y + 2z = 9$

Augmented Matrix:

```
[ 2 1 1 | 3 ]
[ 1 3 3 | 4 ]
[ 1 -4 2 | 9 ]
```

Step 1: Forward Elimination

- **Pivot for Column 1:** The pivot element is $a_{11} = 2$, which is the largest in the first column. No row interchange is needed.
- **Eliminate x from Row 2 and Row 3:**
  - $R_2 \to R_2 - (1/2)R_1$ ⇒ [0 2.5 2.5 | 2.5]

- $R_3 \to R_3 - (1/2)R_1 \Rightarrow$ `[ 0 -4.5 1.5 | 7.5 ]`

- The matrix becomes:
  `[ 2 1 1 | 3 ]`
  `[ 0 2.5 2.5 | 2.5 ]`
  `[ 0 -4.5 1.5 | 7.5 ]`

- **Pivot for Column 2:** We look at elements from a_{22} downwards: $|2.5|$ and $|-4.5|$. Since $|-4.5| > |2.5|$, we perform partial pivoting by swapping Row 2 and Row 3.

- The matrix becomes:
  `[ 2 1 1 | 3 ]`
  `[ 0 -4.5 1.5 | 7.5 ]`
  `[ 0 2.5 2.5 | 2.5 ]`

- **Eliminate y from Row 3:**
  - $R_3 \to R_3 - (2.5 / -4.5)R_2 = R_3 + (5/9)R_2$
  - New R_3: `[ 0, 0, 2.5 + (5/9)(1.5), | 2.5 + (5/9)(7.5) ]`
  - New R_3: `[ 0, 0, 10/3, | 20/3 ]` which is `[ 0, 0, 3.33, | 6.67 ]`

- Final Upper Triangular Matrix:
  `[ 2 1 1 | 3 ]`
  `[ 0 -4.5 1.5 | 7.5 ]`
  `[ 0 0 10/3 | 20/3 ]`

Step 2: Back Substitution

1. From R_3: $(10/3)z = 20/3 \Rightarrow z = 2$
2. From R_2: $-4.5y + 1.5z = 7.5 \Rightarrow -4.5y + 1.5(2) = 7.5 \Rightarrow -4.5y = 4.5 \Rightarrow y = -1$
3. From R_1: $2x + y + z = 3 \Rightarrow 2x + (-1) + 2 = 3 \Rightarrow 2x = 2 \Rightarrow x = 1$

The solution is **(x, y, z) = (1, -1, 2)**.

Q2. (b) Jacobi and Gauss-Seidel Methods

The system is:

$5x + 4y - 3z = 4$

`4x - 4y + 3z = 5`

`-x + 2y - z = -4`

Iteration formulas:

`x(k+1) = (1/5) * (4 - 4y(k) + 3z(k))`

`y(k+1) = (1/4) * (4x(k) + 3z(k) - 5)`

`z(k+1) = 4 - x(k) + 2y(k)`

Initial guess `x(0) = (0, 0, 0)`. Exact solution is `(1, -4, -5)`.

Note: The system is not diagonally dominant, so convergence is not guaranteed.

(i) Jacobi Method (all values from previous iteration `k`)

Iteration	x	y	z
0	0	0	0
1	0.8000	-1.2500	4.0000
2	4.2000	2.5500	0.7000
3	-0.8200	3.4750	4.9000
4	0.9600	1.6050	11.7700

(ii) Gauss-Seidel Method (uses most up-to-date values in the same iteration)

Iteration	x	y	z
0	0	0	0
1	0.8000	-0.4500	2.3000
2	2.5400	3.0150	7.4900
3	2.8820	7.2495	15.6170
4	4.3706	14.8334	29.2962

Comparison:

Neither method is converging towards the exact solution `(1, -4, -5)`. Both are diverging. To determine which gives a "better" (less divergent) approximation after 4 iterations, we can compare their distance (error) from the exact solution.

- **Jacobi Error:** `|| (1, -4, -5) - (0.96, 1.605, 11.77) || ≈ || (0.04, -5.6, -16.77) || ≈ 17.68`
- **Gauss-Seidel Error:** `|| (1, -4, -5) - (4.37, 14.83, 29.30) || ≈ || (-3.37, -18.83, -34.30) || ≈ 39.27`

The error for the Jacobi method is smaller. Therefore, after 4 iterations, the **Jacobi method gives a better approximation** to the exact solution, although both methods are ultimately failing to converge.

Q3. Smallest Positive Root of $f(x) = x^3 - 9x^2 - x + 9$

By factoring, $f(x) = x^2(x-9) - 1(x-9) = (x^2-1)(x-9) = (x-1)(x+1)(x-9)$.

The roots are x = -1, 1, and 9. The smallest positive root is **x = 1**.

We will find this root using an initial interval ``, since `f(0) = 9` and `f(2) = -21`.

(a) Regula-Falsi Method

- **Iter 1:** $x_1 = (0*(-21) - 2*9) / (-21 - 9) = 0.6$. `f(0.6) = 5.376`. New interval `[0.6, 2]`.
- **Iter 2:** $x_2 = (0.6*(-21) - 2*5.376) / (-21 - 5.376) \approx 0.885$. `f(0.885) ≈ 1.761`. New interval `[0.885, 2]`.
- **Iter 3:** $x_3 = (0.885*(-21) - 2*1.761) / (-21 - 1.761) \approx 0.971$. `f(0.971) ≈ 0.458`.
- **Iter 4:** $x_4 = (0.971*(-21) - 2*0.458) / (-21 - 0.458) \approx 0.993$.

  The method is converging to 1. To three significant digits, the root is **1.00**.

(b) Newton-Raphson Method

$f(x) = x^3 - 9x^2 - x + 9$, $f'(x) = 3x^2 - 18x - 1$. Let's start with $x_0 = 0.5$.

- **Iter 1:** $x_1 = 0.5 - f(0.5)/f'(0.5) = 0.5 - (6.375 / -9.25) \approx 1.189$.
- **Iter 2:** $x_2 = 1.189 - f(1.189)/f'(1.189) = 1.189 - (-1.458 / -18.16) \approx 1.109$.
- **Iter 3:** $x_3 = 1.109 - f(1.109)/f'(1.109) = 1.109 - (-0.803 / -17.27) \approx 1.062$.

  The method is converging to 1. To three significant digits, the root is **1.00**.

(c) Bisection Method

Using interval ``.

- **Iter 1:** $x_1 = (0 + 2) / 2 = 1$.

- $f(1) = 1^3 - 9(1)^2 - 1 + 9 = 0$.

  The method found the exact root **1.00** in a single iteration. This happens because the root was exactly at the midpoint of the initial interval.

(d) Secant Method

Using initial points $x_0 = 0$ and $x_1 = 2$.

- **Iter 1:** $x_2 = 2 - f(2) * (2 - 0) / (f(2) - f(0)) = 2 - (-21 * 2) / (-21 - 9) = 0.6$.
- **Iter 2:** Use $x_1=2,\ x_2=0.6$. $x_3 = 0.6 - f(0.6)*(0.6-2)/(f(0.6)-f(2)) \approx 0.885$.
- **Iter 3:** Use $x_2=0.6,\ x_3=0.885$. $x_4 = 0.885 - f(0.885)*(0.885-0.6)/(f(0.885)-f(0.6)) \approx 1.024$.
- **Iter 4:** Use $x_3=0.885,\ x_4=1.024$. $x_5 \approx 0.999$.

  The method is converging to 1. To three significant digits, the root is **1.00**.

Q4. (a) Lagrange's Interpolating Polynomial

Data: $(0, 2), (2, 11), (3, 21), (5, 121)$.

The Lagrange polynomial $P(x)$ is given by $\Sigma\ y_i L_i(x)$. To find $f(4)$, we can compute $P(4) = \Sigma\ y_i L_i(4)$.

- $L_0(4) = (4-2)(4-3)(4-5) / (0-2)(0-3)(0-5) = -2 / -30 = 1/15$
- $L_1(4) = (4-0)(4-3)(4-5) / (2-0)(2-3)(2-5) = -4 / 6 = -2/3$
- $L_2(4) = (4-0)(4-2)(4-5) / (3-0)(3-2)(3-5) = -8 / -6 = 4/3$
- $L_3(4) = (4-0)(4-2)(4-3) / (5-0)(5-2)(5-3) = 8 / 30 = 4/15$

Now, calculate $f(4)$:

$f(4) = y_0L_0(4) + y_1L_1(4) + y_2L_2(4) + y_3L_3(4)$

$f(4) = 2(1/15) + 11(-2/3) + 21(4/3) + 121(4/15)$

$f(4) = 2/15 - 22/3 + 84/3 + 484/15$

$f(4) = (2 + 484)/15 + (84 - 22)/3 = 486/15 + 62/3$

$f(4) = 486/15 + 310/15 = 796/15$

So, $f(4) = 796/15 \approx 53.07$.

The full polynomial is $P(x) = (796/15)x^3 - (2338/15)x^2 + (3022/15)x + 2$. (This step is not required by the question but confirms the complexity).

Q4. (b) Inverse Lagrange's Interpolation

Data: `(x, y) = (25, -2), (35, -1), (55, 1), (75, 5)`. Find `x` when `y=3`.

We treat `x` as a function of `y`, `x(y)`.

- $L_0(3) = (3-(-1))(3-1)(3-5) / (-2-(-1))(-2-1)(-2-5) = -16 / -21 = 16/21$
- $L_1(3) = (3-(-2))(3-1)(3-5) / (-1-(-2))(-1-1)(-1-5) = -20 / 12 = -5/3$
- $L_2(3) = (3-(-2))(3-(-1))(3-5) / (1-(-2))(1-(-1))(1-5) = -40 / -24 = 5/3$
- $L_3(3) = (3-(-2))(3-(-1))(3-1) / (5-(-2))(5-(-1))(5-1) = 40 / 168 = 5/21$

Now, calculate `x(3)`:

$x(3) = x_0L_0(3) + x_1L_1(3) + x_2L_2(3) + x_3L_3(3)$

$x(3) = 25(16/21) + 35(-5/3) + 55(5/3) + 75(5/21)$

$x(3) = 400/21 - 175/3 + 275/3 + 375/21$

$x(3) = (400+375)/21 + (275-175)/3 = 775/21 + 100/3$

$x(3) = 775/21 + 700/21 = 1475/21$

The value of `x` when `y=3` is $1475/21 \approx 70.24$.

Q5. (a) Population Estimation

First, we create the difference table:

Year (x)	Pop (y)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1995	678				
		527			
2000	1205		123		
		650		117	
2005	1855		240		-589
		890		-472	
2010	2745		-232		
		658			
2015	3403				

(i) Stirling's Formula for 2007:

- Center: $x_0 = 2005, y_0 = 1855$. $h = 5$.
- $p = (2007 - 2005) / 5 = 0.4$.
- $y(2007) = y_0 + p(\Delta y_0+\Delta y_{-1})/2 + p^2/2!\ \Delta^2 y_{-1} + ...$
- $y(2007) = 1855 + 0.4(890+650)/2 + (0.4)^2/2\ (240) + ...$
- $y(2007) = 1855 + 0.4(770) + 0.08(240) = 1855 + 308 + 19.2 = 2182.2$
- Estimated population for 2007 is **2182.2 lakhs**.

(ii) Newton's Forward Formula for 1998:

- Start: $x_0 = 1995, y_0 = 678$. $h = 5$.
- $p = (1998 - 1995) / 5 = 0.6$.
- $y(1998) = y_0 + p\Delta y_0 + p(p-1)/2!\ \Delta^2 y_0 + ...$
- $y(1998) = 678 + 0.6(527) + 0.6(-0.4)/2\ (123) + ...$
- $y(1998) = 678 + 316.2 - 0.12(123) = 678 + 316.2 - 14.76 = 979.44$
- Estimated population for 1998 is **979.44 lakhs**.

(iii) Newton's Backward Formula for 2013:

- End: $x_n = 2015, y_n = 3403$. $h = 5$.
- $p = (2013 - 2015) / 5 = -0.4$.
- $y(2013) = y_n + p\nabla y_n + p(p+1)/2!\ \nabla^2 y_n + ...$
- $y(2013) = 3403 + (-0.4)(658) + (-0.4)(0.6)/2\ (-232) + ...$
- $y(2013) = 3403 - 263.2 + (-0.12)(-232) = 3403 - 263.2 + 27.84 = 3167.64$
- Estimated population for 2013 is **3167.64 lakhs**.

Q5. (b) Operator Relationship

- The shift operator E is defined as $E\ f(x) = f(x+h)$.
- The central difference operator δ is defined as $\delta\ f(x) = f(x + h/2) - f(x - h/2)$.
- We can write $f(x + h/2)$ as $E^{1/2}\ f(x)$ and $f(x - h/2)$ as $E^{-1/2}\ f(x)$.

- Substituting these into the definition of δ:

  $\delta f(x) = E^{1/2} f(x) - E^{-1/2} f(x) = (E^{1/2} - E^{-1/2}) f(x)$

- Therefore, the relationship is $\boldsymbol{\delta = E^{1/2} - E^{-1/2}}$.

Q6. Numerical Differentiation

Difference table for the data:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
2.0	8.7			
		4.0		
2.5	12.7		0.1	
		4.1		0.0
3.0	16.8		0.1	
		4.1		
3.5	20.9			

(a) Forward Difference Method

- $x = 2.1,\ x_0 = 2.0,\ h = 0.5$. So, $p = (2.1 - 2.0) / 0.5 = 0.2$.

- $y'(x) = (1/h) [\Delta y_0 + (2p-1)/2\ \Delta^2 y_0 + \dots]$

  $y'(2.1) = (1/0.5) [4.0 + (2*0.2 - 1)/2 * 0.1] = 2 [4.0 - 0.3 * 0.1] = 2[3.97] = 7.94$

- $y''(x) = (1/h^2) [\Delta^2 y_0 + (p-1)\Delta^3 y_0 + \dots]$

  $y''(2.1) = (1/0.25) [0.1 + (0.2-1)*0] = 4 [0.1] = 0.4$

- **Truncation/Actual Errors:** Since $\Delta^3 y$ is zero, the data perfectly fits a quadratic polynomial $P(x) = 0.2x^2 + 7.1x - 6.3$. The derivatives from this polynomial are $P'(x) = 0.4x + 7.1$ and $P''(x) = 0.4$.
  - Actual $y'(2.1) = 0.4(2.1) + 7.1 = 7.94$.
  - Actual $y''(2.1) = 0.4$.

- The computed values are exact for the interpolating polynomial, so the **Truncation Error and Actual Error are both 0**.

(b) Lagrange's Interpolation Formula

The interpolating polynomial `P(x)` for a set of points is unique. As found in part (a), the quadratic Lagrange polynomial passing through the first three points is `P(x) = 0.2x² + 7.1x - 6.3`.

- First Derivative: `P'(x) = 0.4x + 7.1`
  `P'(2.1) = 0.4(2.1) + 7.1 = 7.94`
- Second Derivative: `P''(x) = 0.4`
  `P''(2.1) = 0.4`
- **Comparison:** The results from the forward difference method (a) and the Lagrange polynomial method (b) are **identical.** This is because the finite difference formulas are themselves derived from the unique Newton-Gregory interpolating polynomial, which is equivalent to the Lagrange polynomial for the same set of points.

Q7. Numerical Integration

`I = ∫[0 to 8] (4x⁴ + 5x³ + 6x + 5) dx` with `n=8` subintervals.
`h = (8-0)/8 = 1`. The points are `x = 0, 1, 2, ..., 8`.

Values of `y = f(x)`:

- $y_0 = f(0) = 5$
- $y_1 = f(1) = 20$
- $y_2 = f(2) = 121$
- $y_3 = f(3) = 482$
- $y_4 = f(4) = 1373$
- $y_5 = f(5) = 3160$
- $y_6 = f(6) = 6305$
- $y_7 = f(7) = 11366$
- $y_8 = f(8) = 18997$

(a) Trapezoidal Rule

$I\_T = (h/2) [y_0 + y_8 + 2(y_1 + y_2 + ... + y_7)]$

$I\_T = (1/2) [5 + 18997 + 2(20+121+482+1373+3160+6305+11366)]$

$I\_T = (1/2) [19002 + 2(22827)] = (1/2) [19002 + 45654] = 32328$

Result: **32,328**

(b) Simpson's 1/3 Rule

$I\_S = (h/3) [y_0 + y_8 + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)]$

$I\_S = (1/3) [5 + 18997 + 4(20+482+3160+11366) + 2(121+1373+6305)]$

$I\_S = (1/3) [19002 + 4(15028) + 2(7799)]$

$I\_S = (1/3) [19002 + 60112 + 15598] = (1/3) [94712] \approx 31570.67$

Result: **31,570.67**

Comparison with Actual Value

- **Actual Value:**
  $I = [(4/5)x^5 + (5/4)x^4 + 3x^2 + 5x]$ from 0 to 8
  $I = (4/5)(8)^5 + (5/4)(8)^4 + 3(8)^2 + 5(8) = 26214.4 + 5120 + 192 + 40 = 31566.4$
- **Comparison:**
  - Trapezoidal Error = |32328 - 31566.4| = **761.6**
  - Simpson's Error = |31570.67 - 31566.4| = **4.27**
- **Conclusion:** Simpson's 1/3 Rule provides a significantly more accurate result than the Trapezoidal Rule. This is expected, as it is a higher-order method that accounts for the curvature of the function more effectively.