Incisos a y b punto 8

a) $\Omega = \{(x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2))\}$

$$P(x) = f(x_0)\cdot l_0(x) + f(x_1)\cdot l_1 + f(x_2)\cdot l_2(x)$$

$$l_0 = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \quad \Big| \quad l_1 = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \quad \Big| \quad l_2 = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$P(x) = \underbrace{f(x_0)\left[\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\right] + f(x_1)\left[\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\right] + f(x_2)\left[\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\right]}_{y}$$

$$p'(x) = \frac{dy}{dx} \longrightarrow \text{Derivada progresiva}$$

(a) $x = x_0 \to f'(x_0) \simeq p'(x_0)$

$$f'(x_0) \approx p'(x_0) = \frac{1}{2h}\left(-3f(x_0) + 4f(x_1) - f(x_2)\right)$$

$$\hookrightarrow f'(x) \cong \frac{1}{2h}\left(-3f(x) + 4f(x+h - f(x+2h)\right)$$

b) (1) $$\frac{d}{dx}\left[f(x)\cdot\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\right] = f(x_0)\left[\frac{(x-x_1)\cdot\frac{d(x-x_0)}{dx} + (x-x_2)\cdot\frac{d(x-x_1)}{dx}}{(x_0-x_1)(x_0-x_2)}\right]$$

$$\boxed{\frac{d}{dx}(x-x_1) = 1 \quad \wedge \quad \frac{d}{dx}(x-x_2) = 1} \quad = f(x_0)\cdot\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

$$= f(x_0)\left[\frac{2x-(x_1+x_2)}{(x_0-x_1)(x_0-x_2)}\right]$$

(2) $$\frac{d}{dx}\left[f(x)\left[\frac{(x-x_1)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\right]\right] = f(x_0)\left[\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\right]$$

(3) $$\frac{d}{dx}\left[f(x)\left[\frac{(x-x_1)(x-x_2)}{(x_2-x_0)(x_2-x_1)}\right]\right] = f(x)\left[\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\right]$$

$$P'(x_0) = \left(f(x_0) \left[\frac{2x_0 - (x_1 + x_2)}{(x_0 - x_1)(x_0 - x_2)} \right] \right) + \left(f(x_1) \left[\frac{\cancel{(x_0 - x_0)}(x_0 - x_2)}{(x_1 - x_0)(x_1 - x_2)} \right] \right) + \dots$$

$$\dots \left(F(x_2) \left[\frac{\cancel{(x_0 - x_0)}(x_0 - x_1)}{(x_2 - x_0)(x_2 - x_1)} \right] \right.$$

$$P'(x_0) = \frac{f(x_1)(x_0 - x_2)}{(x_1 - x_0)(x_1 - x_2)} + \frac{f(x_2)(x_0 - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

Punto 2

2)

a)

$$L_i(x) = \prod_{\substack{0<k<N \\ k\neq i}} \frac{x - x_k}{x_i - x_k}$$

es 1 en el nodo x_i y 0 en todos los demás nodos x_j con $j \neq i$

Si $i = j$

$$L_i(x_i) = \prod_{\substack{0<k<N \\ k\neq i}} \frac{x_i - x_k}{x_i - x_k} = 1$$

$L_i(x_j)$, $i = j$

todos los terminos $\frac{x_j - x_k}{x_i - x_k}$ se vuelven 1

Se cumple $L_i(x_j) = \delta_{ij}$ para $i = j$

b) si $i \neq j$

$$L_i(x_j) = \prod_{\substack{0<k<N \\ k\neq i}} \frac{x_j - x_k}{x_i - x_k} = 0$$

todos los $\frac{x_j - x_k}{x_i - x_k}$ se vuelven 0

se cumple $L_i(x_j) = \delta_{ij}$ para $i \neq j$

Ya que, $L_i(x_j) = \delta_{ij}$ para todos los i, j en el conjunto $\{0, \dots, N\}$, Todas las funciones cardinales forman una base del espacio de polinomios de grado N.