

Final Paper

An Investigation into Multi-Pivot Quicksort Algorithms

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1 Introduction

Test

2 Analysis

Test

References

A Quicksort Code

A.1 Base Quicksort

```
--author-- = 'paymahnmoghadasian'

class BaseQuicksort():
    '''
    Base class for the quicksorts that we will implement
    '''

    def __init__(self, data, doInsertionSort = False, insertionSortThreshold=13,
        pivotSelection=1, numPivots=1):
        self.__numSwaps = 0
        self.__numComparisons = 0
        self.__data = data
        self.__doInsertionSort = doInsertionSort
        self.__insertionSortThreshold = insertionSortThreshold
        self.__pivotSelection = pivotSelection
        self.__numPivots = numPivots

    def __getPivotSelection(self):
        return self.__pivotSelection
    pivotSelection = property(__getPivotSelection)

    def __getNumPivots(self):
        return self.__numPivots
    numPivots = property(__getNumPivots)

    def __getInsertionSortThreshold(self):
        return self.__insertionSortThreshold
    insertionSortThreshold = property(__getInsertionSortThreshold)

    def __getDoInsertionSort(self):
        return self.__doInsertionSort
    doInsertionSort = property(__getDoInsertionSort)

    def getNumSwaps(self):
        return self.__numSwaps
    def setNumSwaps(self, value):
        self.__numSwaps = value
    def delNumSwaps(self):
```

```

    del self.__numSwaps
numSwaps = property(getNumSwaps, setNumSwaps, delNumSwaps, "The number of
    swaps done by the sort algorithm")

def getNumComparisons(self):
    return self.__numComparisons
def setNumComparisons(self, value):
    self.__numComparisons = value
def delNumComparisons(self):
    del self.__numComparisons
numComparisons = property(getNumComparisons, setNumComparisons,
    delNumComparisons, "The number of comparisons done by the sorting
    algorithm")

def getData(self):
    return self.__data
def setData(self, value):
    self.__data = value
def delData(self):
    del self.__data
data = property(getData, setData, delData, "The data to be sorted")

def sort(self):
    raise NotImplementedError("Cannot sort on base quicksort")

def _insertionSort(self, data, lower, upper):
    '''
    Sorts self.data from [lower, upper) using insertion sort
    Lower is an inclusive bound
    Upper is an exclusive bound
    '''
    for i in xrange(lower + 1, upper):
        j = i
        while j > lower and self.lessThan(data[j], data[j-1]):
            data[j], data[j-1] = data[j-1], data[j]
            self.numSwaps += 1
            j -= 1

    return data

def lessThan(self, a, b):

```

```
'''
    determines whether  $a < b$ 
'''
    self.numComparisons += 1
    return a < b

def lessThanEqual(self, a, b):
    '''
    determines whether  $a \leq b$ 
    '''
    self.numComparisons += 1
    return a <= b

def greaterThan(self, a, b):
    '''
    determines whether  $a > b$ 
    '''
    self.numComparisons += 1
    return a > b

def greaterThanEqual(self, a, b):
    '''
    determines whether  $a \geq b$ 
    '''
    self.numComparisons += 1
    return a >= b

def equal(self, a, b):
    '''
    determines whether  $a == b$ 
    '''
    self.numComparisons += 1
    return a == b

def swap(self, index1, index2):
    '''
    swaps the data[index1] and data[index2]
    '''
    self.numSwaps += 1
    self.data[index1], self.data[index2] = self.data[index2], self.data[
        index1]
```

A.2 Classic Quicksort

```

from BaseQuicksort import BaseQuicksort

__author__ = 'paymahnmoghadasian'

class ClassicQuicksort(BaseQuicksort):
    def __init__(self, data, doInsertionSort=False, insertionSortThreshold=10,
                  pivotSelection=1):
        """
        :param pivotSelection: Determines how the pivot should be chosen. 1 = 1
                               st in range. 2 = last in range. 3 = median of first, middle and last
                               in range.
        """
        if pivotSelection != 1 and pivotSelection != 2 and pivotSelection != 3:
            raise ValueError("The value of the pivot selection (%d) is invalid.
                               Must be 1 or 2." % self.pivotSelection)

        BaseQuicksort.__init__(self, data, doInsertionSort,
                                insertionSortThreshold, pivotSelection, 1)

    def sort(self):
        self._sort(0, len(self.data))
        return self.data

    def _sort(self, lower, upper):
        #check whether we should do an insertion sort instead of quicksort
        if self.doInsertionSort and upper - lower <= self.insertionSortThreshold:
            :
            self._insertionSort(self.data, lower, upper)
            return

        #base cases
        if upper - lower <= 1:
            return
        if upper - lower == 2:
            if self.lessThan(self.data[upper-1], self.data[lower]):
                self.swap(lower, upper-1)
            return

```

```

    # recursive case
    pivot = self.__selectPivot(lower, upper)
    i = lower+1
    for j in xrange(lower+1, upper):
        if self.lessThan(self.data[j], pivot):
            self.swap(i, j)
            i += 1

    #do the recursive calls
    self.swap(lower, i-1)
    self.__sort(lower, i-1)
    self.__sort(i, upper)

def __selectPivot(self, lower, upper):
    if self.pivotSelection == 2:
        self.swap(lower, upper-1)
    elif self.pivotSelection == 3:
        middle = lower + (upper - lower) / 2
        pivots = self._insertionSort([(self.data[lower], lower), (self.data[
            middle], middle), (self.data[upper-1], upper-1)], 0, 3)
        self.swap(lower, pivots[1][1])

    return self.data[lower]

```

A.3 Dual Pivot Quicksort

```

__author__ = 'paymahnmoghadasian'

from BaseQuicksort import BaseQuicksort

class DualPivotQuicksort(BaseQuicksort):

    def __init__(self, data, doInsertionSort=False, insertionSortThreshold=10,
        pivotSelection=1, behaveOptimally=False):
        '''
        :param pivotSelection: This determines how pivots are chosen. There are
            exactly 2 valid values: 1 and 2. 1 is the default where the first
            and last values in the range are chosen. 2 is where tertiles are
            chosen.
        '''

```

```

    if pivotSelection != 1 and pivotSelection != 2:
        raise ValueError("The value of the pivot selection (%d) is invalid.
            Must be 1 or 2." % pivotSelection)

BaseQuicksort.__init__(self, data, doInsertionSort,
    insertionSortThreshold, pivotSelection, 2)

self.__behaveOptimally = behaveOptimally

def sort(self):
    self.__sort(0, len(self.data))

def __partitionOptimally(self, largePivot, lower, smallPivot, upper):
    lowerSwap = i = lower + 1
    upperSwap = upper - 2
    smallCount = largeCount = 0 # number of elements smaller than the small
        pivot and larger than the large pivot

    while i <= upperSwap:
        if smallCount >= largeCount:
            if self.lessThan(self.data[i], smallPivot):
                self.swap(i, lowerSwap)
                lowerSwap += 1
                smallCount += 1
            elif self.greaterThan(self.data[i], largePivot):
                #don't want to swap stuff that's bigger than the largePivot
                while i < upperSwap and self.greaterThan(self.data[upperSwap
                    ], largePivot):
                    upperSwap -= 1

                self.swap(i, upperSwap)
                upperSwap -= 1
                largeCount += 1

            if self.lessThan(self.data[i], smallPivot):
                self.swap(i, lowerSwap)
                lowerSwap += 1
                smallCount += 1
        else:
            if self.greaterThan(self.data[i], largePivot):
                #don't want to swap stuff that's bigger than the largePivot

```

```

        while i < upperSwap and self.greaterThan(self.data[upperSwap
            ], largePivot):
            upperSwap -= 1

        self.swap(i, upperSwap)
        upperSwap -= 1
        largeCount += 1

        if self.lessThan(self.data[i], smallPivot):
            self.swap(i, lowerSwap)
            lowerSwap += 1
            smallCount += 1

        elif self.lessThan(self.data[i], smallPivot):
            self.swap(i, lowerSwap)
            lowerSwap += 1
            smallCount += 1

    i += 1

    return lowerSwap, upperSwap

def __partition(self, largePivot, lower, smallPivot, upper):

    lowerSwap = i = lower + 1
    upperSwap = upper - 2

    if self.__behaveOptimally:
        lowerSwap, upperSwap = self.__partitionOptimally(largePivot, lower,
            smallPivot, upper)
    else:
        while i <= upperSwap:
            if self.lessThan(self.data[i], smallPivot):
                self.swap(i, lowerSwap)
                lowerSwap += 1
            elif self.greaterThan(self.data[i], largePivot):
                #don't want to swap stuff that's bigger than the largePivot
                while i < upperSwap and self.greaterThan(self.data[upperSwap
                    ], largePivot):
                    upperSwap -= 1
                self.swap(i, upperSwap)

```

```

        upperSwap -= 1

        if self.lessThan(self.data[i], smallPivot):
            self.swap(i, lowerSwap)
            lowerSwap += 1

        i += 1

    return lowerSwap, upperSwap

def __sort(self, lower, upper):

    #check whether we should do an insertion sort instead of quicksort
    if self.doInsertionSort and upper - lower <= self.insertionSortThreshold
        :
        self._insertionSort(self.data, lower, upper)
        return

    #base cases
    if upper - lower <= 1:
        return
    if upper - lower == 2:
        if self.lessThan(self.data[upper-1], self.data[lower]):
            self.swap(lower, upper-1)
        return

    # select the pivots. If they're the same, this section is sorted
    smallPivot, largePivot = self._selectPivots(lower, upper)
    # if self.equal(smallPivot, largePivot):
    #     return

    lowerSwap, upperSwap = self._partition(largePivot, lower, smallPivot,
        upper)

    self.swap(lower, lowerSwap - 1)
    self.swap(upper - 1, upperSwap + 1)

    self._sort(lower, lowerSwap-1)
    self._sort(lowerSwap, upperSwap+1)
    self._sort(upperSwap+2, upper)

def _selectPivots(self, lower, upper):

```

```

'''
Default pivot selection guarantees that if the pivots can be different,
they will be
Tertiles does NOT make this guarantee
'''

# to do tertiles pivot selection, the range must be big enough
if upper - lower >= 5 and self.pivotSelection == 2:
    middle = lower + (upper - lower) / 2
    left = lower + (middle - lower) / 2
    right = middle + (upper - middle) / 2

    # sort the potential pivot values and keep track of their index
    pivots = [(self.data[lower], lower), (self.data[left], left), (self.
        data[middle], middle),
        (self.data[right], right), (self.data[upper - 1], upper -
            1)]
    pivots = self._insertionSort(pivots, 0, len(pivots))

    # put the desired pivots at the beginning and end of the range
    self.swap(lower, pivots[1][1])
    if self.equal(lower, pivots[3][1]):
        self.swap(upper - 1, pivots[1][1])
    else:
        self.swap(upper - 1, pivots[3][1])

else:
    i = lower
    while self.equal(self.data[i], self.data[upper - 1]) and i < upper:
        i += 1

    self.swap(i, lower)

    if self.lessThan(self.data[upper - 1], self.data[lower]):
        self.swap(lower, upper - 1)

return self.data[lower], self.data[upper - 1]

```

A.4 M Pivot Quicksort

```
__author__ = 'paymahn'
```

```

from BaseQuicksort import BaseQuicksort

class MPivotQuicksort(BaseQuicksort):

    INSERTION_SORT_THRESHOLD = 13

    def __init__(self, data, numPivots, minHeapOptimization=False):
        if numPivots <= 0 or (data is not None and 2*numPivots > len(data) and
            2*numPivots > MPivotQuicksort.INSERTION_SORT_THRESHOLD):
            raise ValueError("Invalid value for the number of pivots. Must be
                greater than 0 and less than half the length of the data to be
                sorted")

        BaseQuicksort.__init__(self, data, True, MPivotQuicksort.
            INSERTION_SORT_THRESHOLD, 1, numPivots)
        self.__minHeapOptimization = minHeapOptimization

    def sort(self):
        self.__sort(0, len(self.data) - 1)

    def __minHeapify(self, first, last):
        '''
        make self.data[first] to self.data[last] satisfy the min heap property
        :param first: inclusive lower bound on what part of the data should be
            heapified
        :param last: inclusive upper bound on what part of the data should be
            heapified
        '''
        diff = last - first + 1
        offset = first
        for i in range(diff):
            leftChildIndex = 2 * i + 1
            rightChildIndex = 2 * i + 2

            hasLeftChild = leftChildIndex < diff
            hasRightChild = rightChildIndex < diff

            if not hasLeftChild:

```

```

        # current element has no "children". No further elements will
        either
        break

    minChildIndex = leftChildIndex
    if hasRightChild and self.lessThan(self.data[rightChildIndex +
        offset], self.data[leftChildIndex + offset]):
        minChildIndex = rightChildIndex

    if self.greaterThan(self.data[i + offset], self.data[minChildIndex +
        offset]):
        self.swap(i + offset, minChildIndex + offset)

def __sort(self, first, last):
    if first >= last or first < 0:
        return

    if last - first < MPivotQuicksort.INSERTION_SORT_THRESHOLD:
        self._insertionSort(self.data, first, last + 1)
        return

    if self.__minHeapOptimization:
        self.__minHeapify(first, last)

    pivots = self.__choosePivots(first, last)

    pivots = self._insertionSort(pivots, 0, len(pivots))
    self._insertionSort(self.data, pivots[0]-1, last + 1)

    nextStart = first
    for i, currPivot in enumerate(pivots):
        nextGreater = nextStart
        nextGreater = self._partition(nextStart, nextGreater, currPivot)
        self.swap(nextGreater, currPivot)
        pivots[i] = nextGreater
        self.swap(nextGreater + 1, currPivot + 1)

    if self.equal(nextStart, first) and self.greaterThan(pivots[i],
        nextStart + 1):

```

```
        self.__sort(nextStart, pivots[i] - 1)

    if not self.equal(nextStart, first) and self.greaterThan(pivots[i],
        pivots[i-1] + 2):
        self.__sort(pivots[i-1]+1, pivots[i]+1)

    nextStart = nextGreater + 2
    if self.greaterThan(last, pivots[-1] + 1):
        self.__sort(pivots[-1]+1, last)

def __choosePivots(self, first, last):
    pivots = range(self.numPivots)

    size = last - first + 1
    segments = self.numPivots + 1
    candidate = size / segments - 1

    next = 2
    if candidate >= 2:
        next = candidate + 1

    candidate += first
    for i in range(self.numPivots):
        pivots[i] = candidate
        candidate += next

    for i in reversed(range(self.numPivots)):
        self.swap(pivots[i]+1, last)
        last -= 1
        self.swap(pivots[i], last)
        pivots[i] = last
        last -= 1

    return pivots

def __partition(self, nextStart, nextGreater, curPivot):
    for curUnknown in range(nextStart, curPivot):
        if self.lessThan(self.data[curUnknown], self.data[curPivot]):
            self.swap(curUnknown, nextGreater)
```



```
        nextGreater += 1
    return nextGreater
```

A.5 Three Pivot Quicksort

```
--author-- = 'paymahn'

from BaseQuicksort import BaseQuicksort

class ThreePivotQuicksort(BaseQuicksort):

    def __init__(self, data, insertionSortThreshold=13):
        BaseQuicksort.__init__(self, data, True, insertionSortThreshold, 1, 3)

    def sort(self):
        self.__sort(0, len(self.data)-1)

    def __sort(self, left, right):

        if self.doInsertionSort and right - left < self.insertionSortThreshold:
            self.data = self.__insertionSort(self.data, left, right+1)
            return

        smallPivot, middlePivot, largePivot = self.__selectPivots(left, right)

        a,b,c,d = self.__partition(smallPivot, middlePivot, largePivot, left,
                                   right)

        self.__sort(left, a-1)
        self.__sort(a+1, b-1)
        self.__sort(b+1, d-1)
        self.__sort(d+1, right)

    def __partition(self, smallPivot, middlePivot, largePivot, left, right):
        a = b = left + 2
        c = d = right - 1

        while self.lessThanEqual(b,c):
            while self.lessThan(self.data[b], middlePivot) and self.
                lessThanEqual(b, c):
```

```

        if self.lessThan(self.data[b], smallPivot):
            self.swap(a,b)
            a += 1
        b += 1
    while self.greaterThan(self.data[c], middlePivot) and self.
lessThanEqual(b,c):
        if self.greaterThan(self.data[c], largePivot):
            self.swap(c,d)
            d -= 1
        c -= 1
    if self.lessThanEqual(b, c):
        if self.greaterThan(self.data[b], largePivot):
            if self.lessThan(self.data[c], smallPivot):
                self.swap(b,a)
                self.swap(a,c)
                a += 1
            else:
                self.swap(b,c)
            self.swap(c,d)
            b += 1
            c -= 1
            d -= 1
        else:
            if self.lessThan(self.data[c], smallPivot):
                self.swap(b,a)
                self.swap(a,c)
                a += 1
            else:
                self.swap(b,c)
            b += 1
            c -= 1

    a -= 1
    b -= 1
    c += 1
    d += 1
    self.swap(left + 1, a)
    self.swap(a,b)
    a -= 1
    self.swap(left, a)
    self.swap(right,d)

```

```

    return a,b,c,d

def __movePivots(self, pivots, lower, upper):
    """
    both bounds are inclusive
    moves pivots[0] to lower, pivots[1] to lower + 1 and pivots[2] to upper

    """
    pivotIndices = [pivot[1] for pivot in pivots]

    currPivot = pivotIndices[0]
    self.swap(lower, currPivot)

    # if one the remaining pivots was previously located at lower, it's now
    # located at currPivot
    pivotIndices = [currPivot if index == lower else index for index in
                    pivotIndices[1:]]

    currPivot = pivotIndices[0]
    self.swap(lower + 1, currPivot)

    # if one the remaining pivots was previously located at lower+1, it's
    # now located at currPivot
    pivotIndices = [currPivot if index == lower+1 else index for index in
                    pivotIndices[1:]]

    currPivot = pivotIndices[0]
    self.swap(currPivot, upper)


def __selectPivots(self, lower, upper):
    """
    returns the indices of the 3 pivots we want to choose

    """
    diff = upper - lower
    jump = diff / 6
    if jump < 1:
        raise ValueError("Cannot select pivots on a range less than 7 values
                           wide")

```

```

    if diff % 6 != 0:
        upper -= diff % 6

    potentialPivots = []

    for i in range(lower, upper+1, jump):
        potentialPivots.append((self.data[i], i))

    if len(potentialPivots) != 7:
        raise RuntimeError("Shit broke. We expected 7 potential pivots")

    potentialPivots = self._insertionSort(potentialPivots, 0, 7)

    pivots = [potentialPivots[1], potentialPivots[3], potentialPivots[5]]
    self._movePivots(pivots, lower, upper + diff % 6) # + diff % 6 because
        we subtract that about 10 lines higher

    return [pivot[0] for pivot in pivots] #return the actual pivot values

```

A.6 Yaroslavskiy Quicksort

```

__author__ = 'paymahnmoghadasian'

# http://yaroslavski.narod.ru/quicksort/DualPivotQuicksort.pdf

from BaseQuicksort import BaseQuicksort

class YaroslavskiyQuicksort(BaseQuicksort):

    INSERTION_SORT_THRESHOLD = 17
    DIST_SIZE = 13

    def __init__(self, data):
        BaseQuicksort.__init__(self, data, True, YaroslavskiyQuicksort.
                                INSERTION_SORT_THRESHOLD, 1, 2)

    def sort(self):
        self._sort(0, len(self.data) - 1)

    def _sort(self, left, right):

```

```

'''
Note that the upper bound here is INCLUSIVE
unlike other sort implementations
'''

len = right - left

if len < YaroslavskiyQuicksort.INSERTION_SORT_THRESHOLD:
    self._insertionSort(self.data, left, right + 1) #plus 1 because
    insertion sort has an exclusive upper bound
    return

sixth = len / 6
m1 = left + sixth
m2 = m1 + sixth
m3 = m2 + sixth
m4 = m3 + sixth
m5 = m4 + sixth

if self.greaterThan(self.data[m1], self.data[m2]):
    self.swap(m1, m2)
if self.greaterThan(self.data[m4], self.data[m5]):
    self.swap(m4, m5)
if self.greaterThan(self.data[m1], self.data[m3]):
    self.swap(m1, m3)
if self.greaterThan(self.data[m2], self.data[m3]):
    self.swap(m2, m3)
if self.greaterThan(self.data[m1], self.data[m4]):
    self.swap(m1, m4)
if self.greaterThan(self.data[m3], self.data[m4]):
    self.swap(m3, m4)
if self.greaterThan(self.data[m2], self.data[m5]):
    self.swap(m2, m5)
if self.greaterThan(self.data[m2], self.data[m3]):
    self.swap(m2, m3)
if self.greaterThan(self.data[m4], self.data[m5]):
    self.swap(m4, m5)

self.swap(m2, left)
self.swap(m4, right)
pivot1 = self.data[left]
pivot2 = self.data[right]

```

```
diffPivots = pivot1 != pivot2

less = left + 1
great = right - 1

if diffPivots:
    k = less
    while k <= great:
        x = self.data[k]

        if self.lessThan(x, pivot1):
            self.swap(k, less)
            less += 1
        elif self.greaterThan(x, pivot2):
            while self.greaterThan(self.data[great], pivot2) and k <
                great:
                    great -= 1
            self.swap(k, great)
            great -= 1
            x = self.data[k]

        if self.lessThan(x, pivot1):
            self.swap(k, less)
            less += 1

        k += 1

else:
    k = less
    while k <= great:

        x = self.data[k]

        if self.equal(x, pivot1):
            continue
        if self.lessThan(x, pivot1):
            self.swap(k, less)
            less += 1
        elif self.greaterThan(x, pivot2):
            while self.greaterThan(self.data[great], pivot2) and k <
                great:
```

```
        great -= 1
        self.swap(k, great)
        great -= 1
        x = self.data[k]

        if self.lessThan(x, pivot1):
            self.swap(k, less)
            less += 1

    k += 1

# swap
    self.swap(left, less - 1)
    self.swap(right, great + 1)

#recursive calls
    self.__sort(left, less - 2)
    self.__sort(great + 2, right)

if great - less > len - YaroslavskiyQuicksort.DIST_SIZE and diffPivots:
    k = less
    while k <= great:

        x = self.data[k]

        if self.equal(x, pivot1):
            self.swap(k, less)
            less += 1
        elif self.equal(x, pivot2):
            self.swap(k, great)
            great -= 1
            x = self.data[k]

        if self.equal(x, pivot1):
            self.swap(k, less)
            less += 1

    k += 1
if diffPivots:
    self.__sort(less, great)
```

B Data Analysis Code

```
from matplotlib import pyplot as plt
import numpy as np
from scipy.optimize import curve_fit

fileName = 'alldata.csv'

dataAbsPath = '../quicksorts/' + fileName

DEBUG = False

def getData(filePath):

    flink = open(filePath, 'r')

    data = {}

    header = "Name,Length,Median Selection,Num Pivots,Used Insertion Sort,
              Insertion Sort Threshold,Time,Comparisons,Swaps\n"

    isFirst = True

    for line in flink:
        if line not in header and len(line) > 0 and line!=header:

            isNoError = True

            temp = line.strip().split(',')

            name = temp[0]

            try:
                length = int(temp[1])
            except:
                print "Error:",temp[1]
                isNoError = False

            try:
                medianSelection = int(temp[2])
            except:
                print "Error:",temp[2]
```



```
        isNoError = False

    try:
        numPivots = int(temp[3])
    except:
        print "Error:",temp[3]
        isNoError = False

    try:
        usedInsertionSort = bool(temp[4])
    except:
        print "Error:",temp[4]
        isNoError = False

    try:
        insertionSortThreshold = int(temp[5]) # will not use
    except:
        print "Error:",temp[5]
        isNoError = False

    try:
        time = float(temp[6])
    except:
        print "Error:",temp[6]
        isNoError = False

    try:
        comparisons = int(temp[7])
    except:
        print "Error:",temp[7]
        isNoError = False

    try:
        swaps = int(temp[8])
    except:
        print "Error:",temp[8]
        isNoError = False

    if isNoError:
        label = ( name,medianSelection,numPivots,usedInsertionSort)

        if label in data :
```

```
        sizeList,timeList,complist,swapList = data[label]
    else:
        sizeList = []
        timeList = []
        complist = []
        swapList = []

        sizeList.append(length)
        timeList.append(time)
        complist.append(comparisons)
        swapList.append(swaps)

    data[label] = sizeList,timeList,complist,swapList

for label in data.keys() :
    sizeList,timeList,complist,swapList = data[label]

    tempList = zip(sizeList,timeList,complist,swapList)

    tempList.sort()

    sizeList = [ item[0] for item in tempList]
    timeList = [ item[1] for item in tempList]
    complist = [ item[2] for item in tempList]
    swapList = [ item[3] for item in tempList]

    sizeList = np.array(sizeList, dtype = np.int64)
    timeList = np.array(timeList, dtype = np.float64)
    complist = np.array(complist, dtype = np.int64)
    swapList = np.array(swapList, dtype = np.int64)

    data[label] = sizeList,timeList,complist,swapList

return averageData(data)

def averageData(data):
    """
    This function will take in the data dictionary and average all the data
    points with the same size value
    """
```

```

for label in data.keys() :
    sizeList,timeList,compList,swapList = data[label]

    sizeListTemp = []
    timeListTemp = []
    compListTemp = []
    swapListTemp = []

    for size in np.nditer(sizeList):

        if size not in sizeListTemp:
            sizeListTemp.append(size)

            indexArray = size == sizeList
            numValues = len(indexArray) * 1.0

            # The averages
            timeTemp = np.sum(timeList[ indexArray ])/numValues
            compTemp = np.sum(compList[ indexArray ])/numValues
            swapTemp = np.sum(swapList[ indexArray ])/numValues

            timeListTemp.append(timeTemp)
            compListTemp.append(compTemp)
            swapListTemp.append(swapTemp)

    sizeList = np.array(sizeListTemp,dtype = np.int64)
    timeList = np.array(timeListTemp,dtype = np.float128)
    compList = np.array(compListTemp,dtype = np.int64)
    swapList = np.array(swapListTemp,dtype = np.int64)

    data[label] = sizeList,timeList,compList,swapList

return data

def markerGenerator(index,withSymbol= True, withColor = True):
    '''
    This function was created so that we can generate lines
    with varying symbols and colors without creating them by
    hand.
    '''

```

```

    colors = 'rbgmcky'
    numColors = len(colors)
    markers = 'ox^spdvX'
    numMarkers = len(markers)

    return withSymbol*markers[index%numMarkers]+withColor*colors[index%numColors
        ]

def convertLabelToStr(label):
    name,medianSelection,numPivots,usedInsertionSort = label
    return "%s — %s — %s" %(name,medianSelection,numPivots)

def plotData(data, plotTime = False,
              plotComp = True,
              plotSwap = True,
              goodFunction = lambda x:True ,
              badFunction = lambda x:False,
              makeLegend = True,
              legendSize = 10,
              plotTitle = None,
              fontsize = 12,
              plotter = plt.plot,
              connectDataPoints = False,
              xlim = None,
              specialFlag = False) :
    '''

```

Plot data will take the data dictionary and create plots according to the key word argunments.

Note that labels are defined as follows (name,medianSelection,numPivots,usedInsertionSort)

:param data: the dictionary that contains data to be plotted
:param plotTime: boolean to control if the size vs time plots will actually be rendered
:param plotComp: boolean to control if the size vs comparisons plots will actually be rendered
:param plotSwap: boolean to control if the size vs swaps plots will actually be rendered
:param goodFunction: a function that will take in the label tuple and determine if it will plot that label

```

:param badFunction: a function that will take in the label tuple and
                    determine if it will not plot that label
:param makeLegend: boolean to control if the legend should be rendered
:param legendSize: size of the legend
:param plotTitle: string of title of the plot. Will be placed on the top of
                  the figure.
:param fontsize: The font size of the title of the figure
:param plotter: function that will plot the data ( plt.plot, plt.semilogx,
            plt.semilogy, plt.loglog )
:param connectDataPoints: boolean to control if the data points will be
                          connected with a solid line
'''

```

```

keyList = list(data.keys())
keyList.sort()

```

```

#print keyList

```

```

if plotTime :
    timeFigure = plt.figure()
if plotComp :
    compFigure = plt.figure()
if plotSwap :
    swapFigure = plt.figure()

if xlim and xlim[0] > xlim[1]:

    xlim[1],xlim[0] = xlim[0],xlim[1]

for count,label in enumerate(keyList):

    marker = connectDataPoints*'—' + markerGenerator(count)

    if goodFunction(label) and not badFunction(label) :
        sizeList,timeList,compList,swapList = data[label]

        if specialFlag :
            timeList = timeList/( sizeList*np.log2(sizeList) )
            compList = compList/( sizeList*np.log2(sizeList) )
            swapList = swapList/( sizeList*np.log2(sizeList) )
            sizeList = np.log2(sizeList)

```

```
    if plotTime :
        plt.figure(timeFigure.number)
        plotter(sizeList,timeList,marker,label=convertLabelToStr(label))
    if plotComp :
        plt.figure(compFigure.number)
        plotter(sizeList,compList,marker,label=convertLabelToStr(label))
    if plotSwap :
        plt.figure(swapFigure.number)
        plotter(sizeList,swapList,marker,label=convertLabelToStr(label))

    if DEBUG:
        index = ( xlim[0] <= sizeList) * (sizeList <= xlim[1] )
        print ""
        print convertLabelToStr(label)
        print sizeList[index]
        print compList[index]
        print swapList[index]

returnList = []

legendProp = {'size':legendSize}

if xlim:
    timeYLim,compYLim,swapYLim = extractYLim(data, goodFunction, badFunction
        , xlim)

if plotTime :
    returnList.append(timeFigure)
    plt.figure(timeFigure.number)
    plt.xlabel('Size')
    plt.ylabel('Time')

    if makeLegend :
        plt.legend(loc = "upper left",prop = legendProp)

    if plotTitle:
        plt.title(plotTitle + ' (Time)',fontsize = fontsize)

    if xlim:
        plt.xlim(xlim[0],xlim[1])
        plt.ylim(timeYLim[0],timeYLim[1])
```

```
if plotComp :
    returnList.append(compFigure)
    plt.figure(compFigure.number)
    plt.xlabel('Size')
    plt.ylabel('Comparisons')

if makeLegend :
    plt.legend(loc = "upper left",prop = legendProp)

if plotTitle:
    plt.title(plotTitle + ' (Comparisons)',fontsize = fontsize)

if xlim:
    plt.xlim(xlim[0],xlim[1])
    plt.ylim(compYLim[0],compYLim[1])

if plotSwap:
    returnList.append(swapFigure)
    plt.figure(swapFigure.number)
    plt.xlabel('Size')
    plt.ylabel('Swaps')

if makeLegend :
    plt.legend(loc = "upper left",prop = legendProp)

if plotTitle:
    plt.title(plotTitle + ' (Swaps)',fontsize = fontsize)

if xlim:
    plt.xlim(xlim[0],xlim[1])
    plt.ylim(swapYLim[0],swapYLim[1])

if specialFlag and plotTime:
    plt.figure(timeFigure.number)
    plt.xlabel('log(Size)')
    plt.ylabel('Time / (Size log(Size) )')

if specialFlag and plotComp:
    plt.figure(compFigure.number)
    plt.xlabel('log(Size)')
    plt.ylabel('Comparisons / (Size log(Size) )')
```

```

    if specialFlag and plotSwap:
        plt.figure(swapFigure.number)
        plt.xlabel('log(Size)')
        plt.ylabel('Swaps / (Size log(Size) )')

    return tuple(returnList)

def plotPolynomialFit(data, fitParameters, figureList,
                      plotComp = True,
                      plotSwap = True,
                      goodFunction = lambda x: True,
                      badFunction = lambda x: False,
                      makeLegend = True,
                      legendSize = 10,
                      plotter = plt.plot,
                      xlim = None,
                      linewidth = 1.5,
                      numPoints = 10**3,
                      specialFlag = False) :

    keyList = list(data.keys())
    keyList.sort()

    #print keyList
    count = 0
    if plotComp :
        compFigure = figureList[count]
        count+=1
    if plotSwap :
        swapFigure = figureList[count]
        count+=1

    if xlim and xlim[0] > xlim[1]:

        xlim[1],xlim[0] = xlim[0],xlim[1]

    for count,label in enumerate(keyList):

        if count < 7 :
            marker = '—' + markerGenerator(count,withSymbol=False)
        else:
            marker = '—' + markerGenerator(count,withSymbol=False)

```



```
if goodFunction(label) and not badFunction(label) :
    sizeList,timeList,compList,swapList = data[label]
    compCoef,compCov,swapCoef,swapCov = fitParameters[label]

    if xlim :
        xMin = min( min(sizeList),xlim[0])
        xMax = max( max(sizeList),xlim[1])
    else :
        xMin = min(sizeList)
        xMax = max(sizeList)

    xVals = np.linspace(xMin,xMax,numPoints)

    if plotComp :
        compFitFunc = lambda xx:fitFunction(xx,*tuple(compCoef))

        if specialFlag :
            # Plot something special
            yVals = compFitFunc(xVals) / ( xVals * np.log2(xVals) )
            xVals = np.log2(xVals)
        else:
            yVals = compFitFunc(xVals)

        plt.figure(compFigure.number)
        plotter(xVals,yVals,marker,label=convertLabelToStr(label)+" Fit"
            ,linewidth = linewidth)

    if plotSwap :
        swapFitFunc = lambda xx:fitFunction(xx,*tuple(swapCoef))

        if specialFlag :
            # Plot something special
            yVals = swapFitFunc(xVals) / ( xVals * np.log2(xVals) )
            xVals = np.log2(xVals)
            print yVals
        else:
            yVals = swapFitFunc(xVals)

        plt.figure(swapFigure.number)
        plotter(xVals,yVals,marker,label=convertLabelToStr(label)+" Fit"
            ,linewidth = linewidth)
```

```
legendProp = {'size':legendSize}

if xlim:
    timeYLim,compYLim,swapYLim = extractYLim(data, goodFunction, badFunction
        , xlim)

if plotComp and makeLegend:
    plt.figure(compFigure.number)
    plt.legend(loc = "upper left",prop = legendProp)

if plotSwap and xlim:
    plt.xlim(xlim[0],xlim[1])
    plt.ylim(compYLim[0],compYLim[1])

if plotSwap and makeLegend:
    plt.figure(swapFigure.number)
    plt.legend(loc = "upper left",prop = legendProp)

if plotSwap and xlim:
    plt.xlim(xlim[0],xlim[1])
    plt.ylim(swapYLim[0],swapYLim[1])

def extractYLim(data,goodFunction,badFunction,xlim):
    keyList = list(data.keys())
    keyList.sort()

    if xlim[0] > xlim[1]:

        xlim[1],xlim[0] = xlim[0],xlim[1]

    yMinTime = None
    yMaxTime = -1

    yMinComp = None
    yMaxComp = -1

    yMinSwap = None
    yMaxSwap = -1

    for label in keyList:
        if goodFunction(label) and not badFunction(label) :
```

```
sizeList,timeList,compList,swapList = data[label]

sizeIndex = (xlim[0] <= sizeList) * ( sizeList <= xlim[1] )

yMinTimeTemp = np.min(timeList[sizeIndex])
yMaxTimeTemp = np.max(timeList[sizeIndex])

yMinCompTemp = np.min(compList[sizeIndex])
yMaxCompTemp = np.max(compList[sizeIndex])

yMinSwapTemp = np.min(swapList[sizeIndex])
yMaxSwapTemp = np.max(swapList[sizeIndex])

if not bool(yMinTime) or yMinTime > yMinTimeTemp:
    yMinTime = yMinTimeTemp

if yMaxTime < yMaxTimeTemp:
    yMaxTime = yMaxTimeTemp

if not bool(yMinComp) or yMinComp > yMinCompTemp:
    yMinComp = yMinCompTemp

if yMaxComp < yMaxCompTemp:
    yMaxComp = yMaxCompTemp

if not bool(yMinSwap) or yMinSwap > yMinSwapTemp:
    yMinSwap = yMinSwapTemp

if yMaxSwap < yMaxSwapTemp:
    yMaxSwap = yMaxSwapTemp

if not bool(yMinTime):
    yMinTime = 0

if not bool(yMinComp):
    yMinComp = 0

if not bool(yMinSwap):
    yMinSwap = 0

return [yMinTime,yMaxTime],[yMinComp,yMaxComp],[yMinSwap,yMaxSwap]
```

```

def calcLeastSquaresOnData(data):
    '''
    We want to fit the data to :
         $y = A x \log(x) + B x + C \log(x)$ 

    We use a non-linear curve fitter.

    Other considerations:

    Method 2 :
         $y = A x \log(x) + B$ 

    So we make a transformation so that :
         $X = x \log(x) = x \ln(x)/\ln(2)$ 
         $Y = y$ 

    Method 3:
         $y = A x \log(x) + Bx = x ( A \log(x) + B )$ 

    So we make a transformation so that :
         $X = \log(x) = \ln(x)/\ln(2)$ 
         $Y = y/x$ 

    Method 3:
         $y = A x \log(x) + B \log(x) = \log(x) ( A x + B )$ 

    So we make a transformation so that :
         $X = x$ 
         $Y = y/\log(x)$ 
    '''

    keyList = list(data.keys())
    keyList.sort()

    fitParameters = {}

    for label in keyList :
        sizeList, timeList, compList, swapList = data[label]

        sizeList = np.array(sizeList, dtype = np.float64)
        timeList = np.array(timeList, dtype = np.float64)

```

```

    compList = np.array(compList, dtype = np.float64)
    swapList = np.array(swapList, dtype = np.float64)

    compCoef, compCov = curve_fit(fitFunction, sizeList, compList)
    swapCoef, swapCov = curve_fit(fitFunction, sizeList, swapList)

    fitParameters[label] = compCoef, compCov, swapCoef, swapCov

printSpecifier = "%40s | %9.5f +- %9.5f | %9.5f +- %9.5f | %10.5f +- %9.5f "

# xxxCov = The estimated covariance of optimal values.
#          The diagonals provide the variance of the parameter estimate.

# http://stats.stackexchange.com/questions/50830/can-i-convert-a-covariance-matrix-into-uncertainties-for-variables

print ""
print "COMPARISON COEFFICIENTS"
for label in keyList :
    compCoef, compCov, swapCoef, swapCov = fitParameters[label]

    compCov[compCov>=0] = np.sqrt(compCov[compCov>=0])

    print printSpecifier%(convertLabelToStr(label), compCoef[0], compCov[0,0],
        compCoef[1], compCov[1,1], compCoef[2], compCov[2,2])

    compCov[compCov>=0] *= compCov[compCov>=0]

print ""
print "SWAP COEFFICIENTS"
for label in keyList :
    compCoef, compCov, swapCoef, swapCov = fitParameters[label]

    swapCov[swapCov>=0] = np.sqrt(swapCov[swapCov>=0])

    print printSpecifier%(convertLabelToStr(label), swapCoef[0], swapCov[0,0],
        swapCoef[1], swapCov[1,1], swapCoef[2], swapCov[2,2])

    swapCov[swapCov>=0] *= swapCov[swapCov>=0]

return fitParameters

```

```
def fitFunction(xx,AA,BB,CC):
    return AA*xx*np.log2(xx)+BB*xx+CC*np.log2(xx)

def saveFigure(figure,fileName,fileExtention = '.png',dpi = 600):
    fullFileName = fileName + fileExtention
    plt.figure(figure.number)
    plt.savefig(fullFileName,
                dpi = dpi,
                facecolor = 'w',
                edgecolor = 'w',
                orientation = 'portrait',
                papertype = None,
                format = None,
                transparent = True,
                bbox_inches = None,
                pad_inches = 0.15,
                frameon = None)

def plotDataAndFit(data,fitParameters,
                  plotComp = True,
                  plotSwap = True,
                  goodFunction = lambda x:True ,
                  badFunction = lambda x:False,
                  makeLegend = True,
                  legendSize = 10,
                  plotTitle = None,
                  fontsize = 12,
                  plotter = plt.plot,
                  xlim = None,
                  connectDataPoints = False,
                  linewidth = 1.5,
                  numPoints = 10*3,
                  savePlot = False,
                  dpi = 600,
                  specialFlag = False) :

    figureList = plotData(data,
                        plotComp = plotComp,
                        plotSwap = plotSwap,
                        goodFunction = goodFunction ,
                        badFunction = badFunction,
                        makeLegend = makeLegend,
```

```

        legendSize = legendSize,
        plotTitle = plotTitle,
        xlim = xlim,
        fontsize = fontsize,
        plotter = plotter,
        connectDataPoints = connectDataPoints,
        specialFlag = specialFlag)

if not connectDataPoints :
    plotPolynomialFit(data, fitParameters, figureList,
        plotComp = plotComp,
        plotSwap = plotSwap,
        goodFunction = goodFunction ,
        badFunction = badFunction,
        makeLegend = makeLegend,
        legendSize = legendSize,
        plotter = plotter,
        xlim = xlim,
        linewidth = linewidth,
        numPoints = numPoints,
        specialFlag = specialFlag)

if savePlot :
    fileName = "".join(plotTitle.split() )
    compFigure, swapFigure = figureList
    saveFigure(compFigure, fileName+"_comp", fileExtention = '.png', dpi = dpi)
    saveFigure(swapFigure, fileName+"_swap", fileExtention = '.png', dpi = dpi)

return figureList

def main():
    # Note that labels are defined as follows
    # (name, medianSelection, numPivots, usedInsertionSort)
    #
    # List of all labels as of April 4
    #
    # ('ClassicQuicksort', 1, 1, True)
    # ('ClassicQuicksort', 2, 1, True)
    # ('ClassicQuicksort', 3, 1, True)
    # ('DualPivotQuicksort', 1, 2, True)
    # ('DualPivotQuicksort', 2, 2, True)
    # ('HeapOptimizedMPivotQuicksort', 1, 3, True)

```

```

# ('HeapOptimizedMPivotQuicksort', 1, 4, True)
# ('HeapOptimizedMPivotQuicksort', 1, 5, True)
# ('HeapOptimizedMPivotQuicksort', 1, 6, True)
# ('MPivotQuicksort', 1, 3, True)
# ('MPivotQuicksort', 1, 4, True)
# ('MPivotQuicksort', 1, 5, True)
# ('MPivotQuicksort', 1, 6, True)
# ('OptimalDualPivotQuicksort', 1, 2, True)
# ('OptimalDualPivotQuicksort', 2, 2, True)
# ('ThreePivotQuicksort', 1, 3, True)
# ('YaroslavskiyQuicksort', 1, 2, True)

classicQuickSortOnly          = lambda x: x[0] == 'ClassicQuicksort'
dualPivotQuicksortOnly        = lambda x: x[0] == 'DualPivotQuicksort'
heapOptimizedMPivotQuicksortOnly = lambda x: x[0] == '
    HeapOptimizedMPivotQuicksort'
mPivotQuicksortOnly           = lambda x: x[0] == 'MPivotQuicksort'
optimalDualPivotQuicksortOnly = lambda x: x[0] == '
    OptimalDualPivotQuicksort'
threePivotQuicksortOnly       = lambda x: x[0] == 'ThreePivotQuicksort'
yaroslavskiyQuicksortOnly     = lambda x: x[0] == 'YaroslavskiyQuicksort'

onePivot = lambda x: x[2] == 1
twoPivot = lambda x: x[2] == 2
threePivot = lambda x: x[2] == 3

usedInsertionSort = lambda x: x[3]

mPivotQuicksortOnly3 = lambda x: mPivotQuicksortOnly(x) and threePivot(x)

customPlot = lambda x: classicQuickSortOnly(x) or dualPivotQuicksortOnly(x)
    or threePivotQuicksortOnly(x) or mPivotQuicksortOnly3(x)

allMPivotQuicksortKinds = lambda x: mPivotQuicksortOnly(x) or
    heapOptimizedMPivotQuicksortOnly(x)

data = getData(dataAbsPath)

fitParameters = calcLeastSquaresOnData(data)

```



```

maskFunctionList = [ classicQuickSortOnly, dualPivotQuicksortOnly,
                    heapOptimizedMPivotQuicksortOnly,
                    mPivotQuicksortOnly, optimalDualPivotQuicksortOnly,
                    threePivotQuicksortOnly,
                    yaroslavskiyQuicksortOnly, onePivot, twoPivot, threePivot,
                    allMPivotQuicksortKinds]

maskFunctionTitleList = ['Classic QuickSorts', 'Dual Pivot Quicksorts', 'Heap
                        Optimized M-Pivot Quicksorts',
                        'Non Optimized M-Pivot Quicksorts', 'Optimal Dual
                        Pivot Quicksorts', 'Three Pivot Quicksorts',
                        'Yaroslavskiy Quicksorts', 'One Pivots', 'Two Pivots',
                        'Three Pivots', 'M-Pivot Quicksorts']

smallScaleLimits = [100, 1000]

plotDataAndFit(data, fitParameters, plotTitle = 'Legend Plot',
               connectDataPoints = True, legendSize=15, savePlot=True)
plotDataAndFit(data, fitParameters, plotTitle = 'All the Plots Small Scale',
               xlim = smallScaleLimits, connectDataPoints = True, makeLegend=False,
               savePlot=True)
plotDataAndFit(data, fitParameters, plotTitle = 'All the Plots Large Scale',
               connectDataPoints = True, makeLegend=False, savePlot = True)
plotDataAndFit(data, fitParameters, plotTitle = 'Semilogx All Plots Large
Scale ', connectDataPoints = True, makeLegend=False, savePlot = True,
               plotter = plt.semilogx)

for maskFunc, plotTitle in zip(maskFunctionList, maskFunctionTitleList):
    plotDataAndFit(data, fitParameters, goodFunction = maskFunc, plotTitle =
        plotTitle+" Large Scale", savePlot = True)

for maskFunc, plotTitle in zip(maskFunctionList, maskFunctionTitleList):
    plotDataAndFit(data, fitParameters, goodFunction = maskFunc, plotTitle =
        plotTitle+" Small Scale", xlim = smallScaleLimits, connectDataPoints =
        True, savePlot = True)

plotDataAndFit(data, fitParameters, goodFunction = allMPivotQuicksortKinds,
               plotTitle = "M-Pivot Quicksorts Large Scale", savePlot = True, legendSize
               =7)
plotDataAndFit(data, fitParameters, goodFunction = allMPivotQuicksortKinds,
               plotTitle = "M-Pivot Quicksorts Small Scale", xlim = smallScaleLimits,
               connectDataPoints = True, savePlot = True, legendSize=7)

```

```
# The special plot
plotDataAndFit(data, fitParameters, plotTitle = 'All Plots Large Scale logn
vs y_OVER_nlogn', connectDataPoints = True, makeLegend=False, savePlot =
True, specialFlag = True)

#plotDataAndFit(data, fitParameters, goodFunction = customPlot, plotTitle =
plotTitle+" Large Scale", savePlot = True)
#plotDataAndFit(data, fitParameters, goodFunction = customPlot, plotTitle = '
customPlot', xlim = smallScaleLimits, connectDataPoints = True, savePlot =
True)

#plt.show()

if __name__ == '__main__':
    main()
```
