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All Sorts of Quicksorts!

An investigation into multi-pivot quicksorts

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University of Manitoba

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- $O(n \log(n))$ Average Case Run Time
- In place algorithm
- Pick Pivots

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- Partitioning Data

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- Recurse to a smaller sub-array

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- In place algorithm
- Pick Pivots
- Partitioning Data
- Recurse to a smaller sub-array
- Use Insertion Sort for a small sub-array

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- $2n \log n - 1.51n + O(\log(n))$ Comparisons
- $0.33n \log n - 0.58n + O(\log(n))$ Swaps

[Wild and Nebel(2012)]

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[Wild and Nebel(2012)]

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 - First Element

[Wild and Nebel(2012)]

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 - First Element
 - Last Element

[Wild and Nebel(2012)]

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- One Pivot
 - First Element
 - Last Element
 - Median of Three

[Wild and Nebel(2012)]

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 - Median of Three
- Simple Partitioning

[Wild and Nebel(2012)]

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- Two Recursive Calls

[Wild and Nebel(2012)]

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■ $2.13n \log n - 2.57n + O(\log(n))$ Comparisons

[Wild and Nebel(2012)]

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- $2.13n \log n - 2.57n + O(\log(n))$ Comparisons
- $0.8n \log n - 0.3n + O(\log(n))$ Swaps

[Wild and Nebel(2012)]

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[Wild and Nebel(2012)]

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 - First and Last Element

[Wild and Nebel(2012)]

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 - Middle 2 of 5 elements (Evenly Spaced Out)

[Wild and Nebel(2012)]

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- Partitions Smalls then Bigs (Middle is automatic)

[Wild and Nebel(2012)]

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- Partitions Smalls then Bigs (Middle is automatic)
- Three Recursive Calls

[Wild and Nebel(2012)]

Dual Pivot Quicksort with Optimal Partitioning

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- $1.8n \log n + O(n)$ Comparisons

[Aumüller and Dietzfelbinger(2013)]

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[Aumüller and Dietzfelbinger(2013)]

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- $1.8n \log n + O(n)$ Comparisons
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- Two Pivots
 - Keeps Tracks of smalls and bigs

[Aumüller and Dietzfelbinger(2013)]

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- $1.8n \log n + O(n)$ Comparisons
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- Two Pivots
 - Keeps Tracks of smalls and bigs
 - Uses the information to see who to compare to first

[Aumüller and Dietzfelbinger(2013)]

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- $0.33n \log n + O(n)$ Swaps
- Two Pivots
 - Keeps Tracks of smalls and bigs
 - Uses the information to see who to compare to first
- Otherwise very similar to standard Dual Pivot Quicksort

[Aumüller and Dietzfelbinger(2013)]

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- $1.9n \log n - 2.46n + O(\log(n))$ Comparisons

[Wild and Nebel(2012)]

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- $1.9n \log n - 2.46n + O(\log(n))$ Comparisons
- $0.6n \log n + 0.08n + O(\log(n))$ Swaps

[Wild and Nebel(2012)]

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[Wild and Nebel(2012)]

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- $0.6n \log n + 0.08n + O(\log(n))$ Swaps
- Two Pivots
 - Middle 2 of 5 elements (Evenly Spaced Out)

[Wild and Nebel(2012)]

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- Two Pivots
 - Middle 2 of 5 elements (Evenly Spaced Out)
 - Uses 5-element sorting network

[Wild and Nebel(2012)]

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 - Middle 2 of 5 elements (Evenly Spaced Out)
 - Uses 5-element sorting network
- Simultaneous Partition algorithm

[Wild and Nebel(2012)]

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- Two Pivots
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 - Uses 5-element sorting network
- Simultaneous Partition algorithm
- Two Recursive Calls

[Wild and Nebel(2012)]

Kushagra-Ortiz-Qiao-Munro Tri-Pivot Quicksort

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- $1.846n \log n + O(n)$ Comparisons

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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- $1.846n \log n + O(n)$ Comparisons
- $0.615n \log n + O(n)$ Swaps

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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- $1.846n \log n + O(n)$ Comparisons
- $0.615n \log n + O(n)$ Swaps
- Three Pivots
 - Middle 3 of 7 elements (Evenly Spaced Out)

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

Kushagra-Ortiz-Qiao-Munro Tri-Pivot Quicksort

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- Three Pivots
 - Middle 3 of 7 elements (Evenly Spaced Out)
- Simultaneous Partition algorithm

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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- Three Pivots
 - Middle 3 of 7 elements (Evenly Spaced Out)
- Simultaneous Partition algorithm
- Four Recursive Calls

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

M-Pivot Quicksort

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Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

Two Pivots

Three Pivots

M Pivots

Curve Fit

The End

- $O(n \log n)$ Comparisons

[Edmondson(2007)]

M-Pivot Quicksort

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

Two Pivots

Three Pivots

M Pivots

Curve Fit

The End

- $O(n \log n)$ Comparisons
- $O(n \log n)$ Swaps

[Edmondson(2007)]

M-Pivot Quicksort

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

Two Pivots

Three Pivots

M Pivots

Curve Fit

The End

- $O(n \log n)$ Comparisons
- $O(n \log n)$ Swaps
- M Pivots

[Edmondson(2007)]

M-Pivot Quicksort

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

Two Pivots

Three Pivots

M Pivots

Curve Fit

The End

- $O(n \log n)$ Comparisons
- $O(n \log n)$ Swaps
- M Pivots
 - Sort $2M$ elements then pick every other element

[Edmondson(2007)]

M-Pivot Quicksort

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

Two Pivots

Three Pivots

M Pivots

Curve Fit

The End

- $O(n \log n)$ Comparisons
- $O(n \log n)$ Swaps
- M Pivots
 - Sort $2M$ elements then pick every other element
- Partition each segment one at a time

[Edmondson(2007)]

M-Pivot Quicksort

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

Two Pivots

Three Pivots

M Pivots

Curve Fit

The End

- $O(n \log n)$ Comparisons
- $O(n \log n)$ Swaps
- M Pivots
 - Sort $2M$ elements then pick every other element
- Partition each segment one at a time
- $M + 1$ Recursive Calls

[Edmondson(2007)]

Example M-Pivot Selection and Partitioning

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

Two Pivots

Three Pivots

M Pivots

Curve Fit

The End

Pivot Candidate Selection with 3 Pivots (25 elements)



[Edmondson(2007)]

Example M-Pivot Selection and Partitioning

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's
Quicksort

Three Pivot
Quicksort

M-Pivot
Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

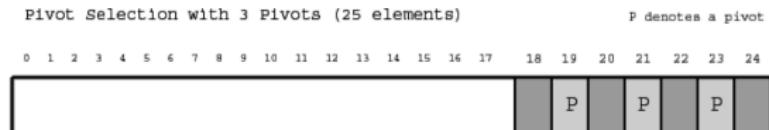
Two Pivots

Three Pivots

M Pivots

Curve Fit

The End



[Edmondson(2007)]

Example M-Pivot Selection and Partitioning

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort
Dual Pivot

Quicksort
Yaroslavskiy's
Quicksort

Three Pivot
Quicksort

M-Pivot
Quicksort

Summary

Legend

Results

Mass
Comparison

One Pivot

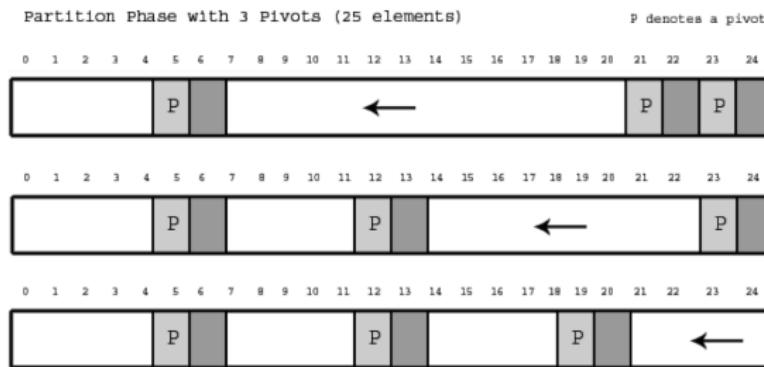
Two Pivots

Three Pivots

M Pivots

Curve Fit

The End



[Edmondson(2007)]

Heap Optimized M-Pivot Quicksort

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

Two Pivots

Three Pivots

M Pivots

Curve Fit

The End

- Enforce the Heap Property to the array before any computation

[Edmondson(2007)]

Heap Optimized M-Pivot Quicksort

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

Two Pivots

Three Pivots

M Pivots

Curve Fit

The End

- Enforce the Heap Property to the array before any computation
 - Adds only $O(n)$ run time at each call

[Edmondson(2007)]

Heap Optimized M-Pivot Quicksort

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort
Dual Pivot
Quicksort

Yaroslavskiy's
Quicksort

Three Pivot
Quicksort

M-Pivot
Quicksort

Summary

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Results

Mass
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One Pivot
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Three Pivots
M Pivots

Curve Fit

The End

- Enforce the Heap Property to the array before any computation
 - Adds only $O(n)$ run time at each call
 - Improves the sort to optimize pivot selection

[Edmondson(2007)]

Example M-Pivot Heap Optimized Pivot Selection

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's
Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

Two Pivots

Three Pivots

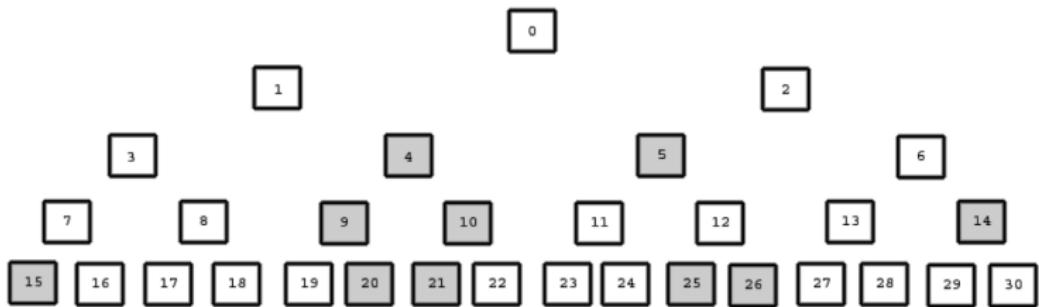
M Pivots

Curve Fit

The End

Candidate Selection from Min Heap (100 elements)

Base Candidate Pair = (31 / 6) - 1 = 4
Next Candidate Pair = 4 + 1 = every 5 elements



[Edmondson(2007)]

Theoretical Average Case Run Time

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort
Dual Pivot

Quicksort
Yaroslavskiy's

Quicksort
Three Pivot

Quicksort
M-Pivot

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M Pivots

Curve Fit

The End

Sort Method	Comparisons
Classic	$2n \log n - 1.51n + O(\log(n))$
Dual Pivot	$2.13n \log n - 2.57n + O(\log(n))$
Optimal Dual Pivot	$1.8n \log n + O(n)$
Three Pivot	$1.846n \log n + O(n)$
Yaroslavskiy	$1.9n \log n - 2.46n + O(\log(n))$
M Pivot	$O(n \log n)$

[Aumüller and Dietzfelbinger(2013)]

[Wild and Nebel(2012)]

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

Theoretical Average Case Run Time

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

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The End

Sort Method	Swaps
Classic	$0.33n \log n - 0.58n + O(\log(n))$
Dual Pivot	$0.8n \log n - 0.3n + O(\log(n))$
Optimal Dual Pivot	$0.33n \log n + O(n)$
Three Pivot	$0.615n \log n + O(n)$
Yaroslavskiy	$0.6n \log n + 0.08n + O(\log(n))$
M Pivot	$O(n \log n)$

[Aumüller and Dietzfelbinger(2013)]

[Wild and Nebel(2012)]

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

Legend

- ClassicQuicksort - 1 - 1 - True
- ✗—✗ ClassicQuicksort - 2 - 1 - True
- ▲—▲ ClassicQuicksort - 3 - 1 - True
- DualPivotQuicksort - 1 - 2 - True
- DualPivotQuicksort - 2 - 2 - True
- ◀—▶ HeapOptimizedMPivotQuicksort - 1 - 3 - True
- ▼—▼ HeapOptimizedMPivotQuicksort - 1 - 4 - True
- HeapOptimizedMPivotQuicksort - 1 - 5 - True
- ◀—▶ HeapOptimizedMPivotQuicksort - 1 - 6 - True
- MPivotQuicksort - 1 - 3 - True
- ✗—✗ MPivotQuicksort - 1 - 4 - True
- ▲—▲ MPivotQuicksort - 1 - 5 - True
- MPivotQuicksort - 1 - 6 - True
- OptimalDualPivotQuicksort - 1 - 2 - True
- OptimalDualPivotQuicksort - 2 - 2 - True
- ▼—▼ ThreePivotQuicksort - 1 - 3 - True
- YaroslavskiyQuicksort - 1 - 2 - True

Mass Comparison Small Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

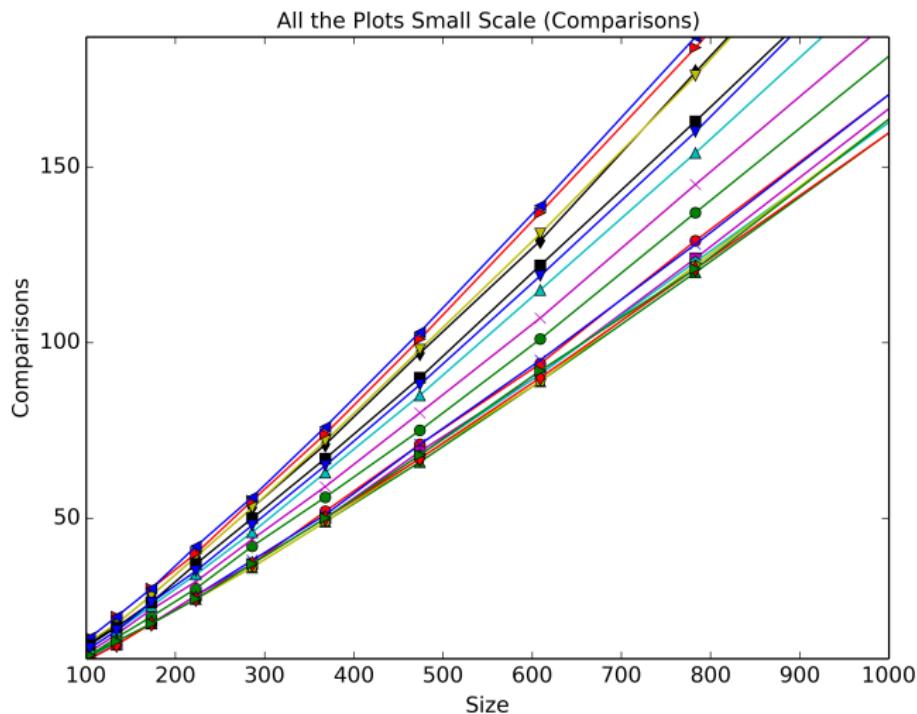
Classic Quicksort
Dual Pivot
Quicksort
Yaroslavskiy's
Quicksort
Three Pivot
Quicksort
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Quicksort
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M Pivots
Curve Fit

The End



Mass Comparison Small Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

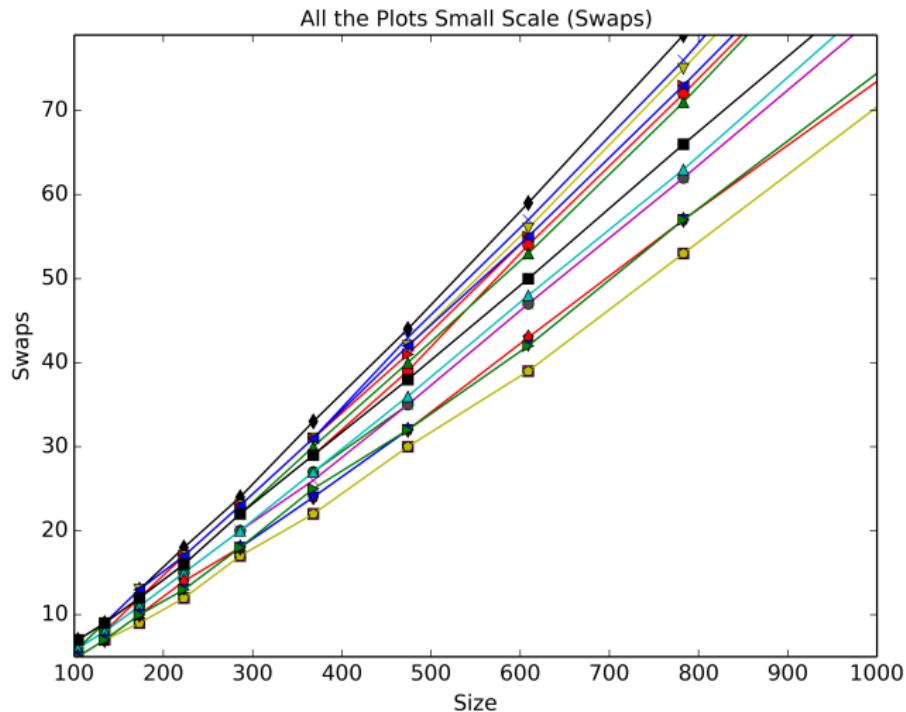
Classic Quicksort
Dual Pivot
Quicksort
Yaroslavskiy's
Quicksort
Three Pivot
Quicksort
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Quicksort
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Mass
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Mass Comparison Large Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

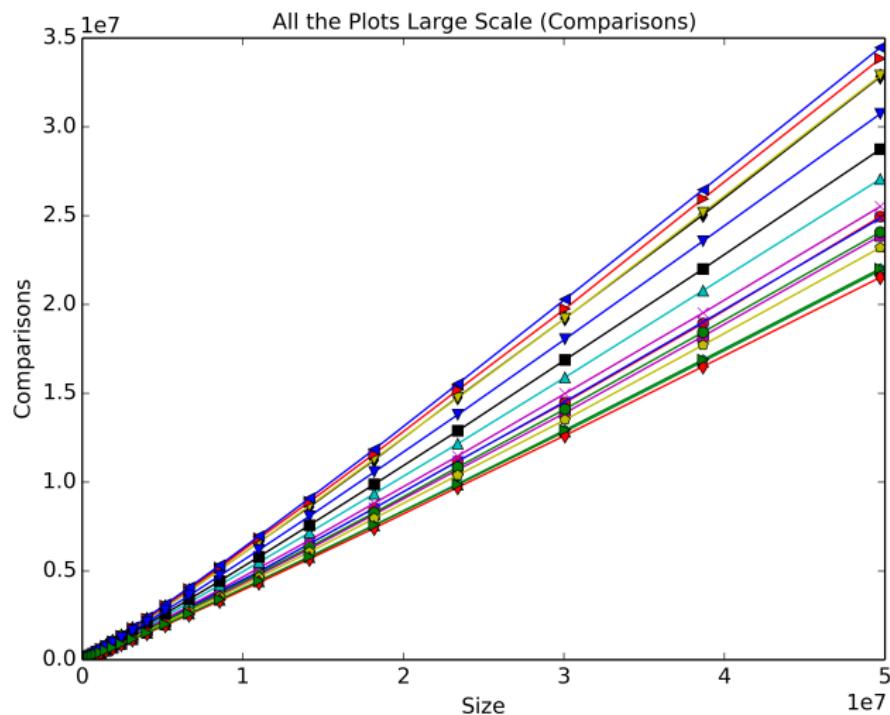
Two Pivots

Three Pivots

M Pivots

Curve Fit

The End



Mass Comparison Large Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

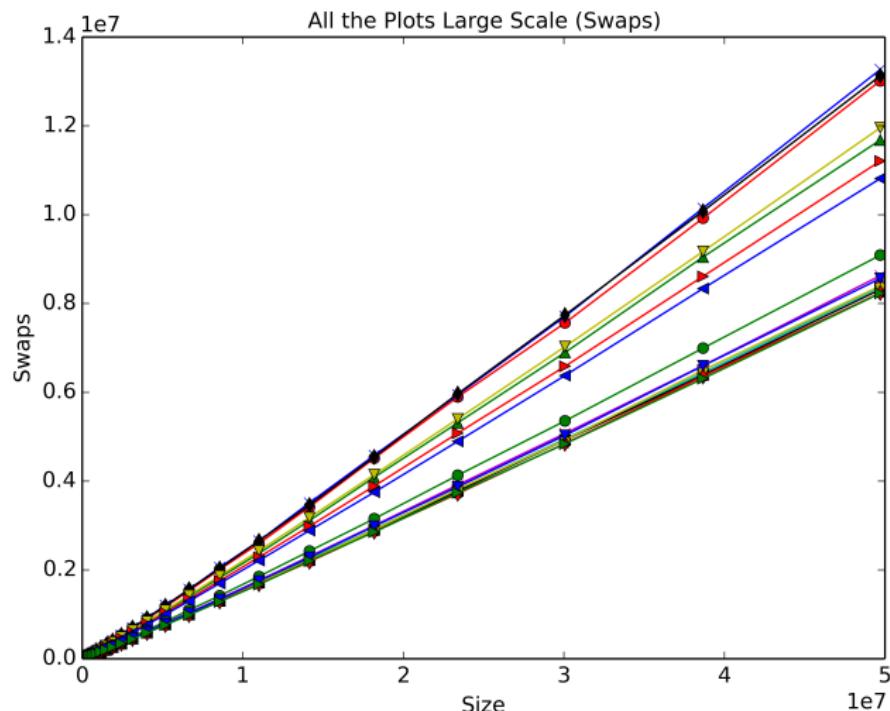
Two Pivots

Three Pivots

M Pivots

Curve Fit

The End



Mass Comparison $\log(n)$ vs $\frac{y}{n \log(n)}$

QuickSort

Moghadasian,
Hernandez

Quicksorts

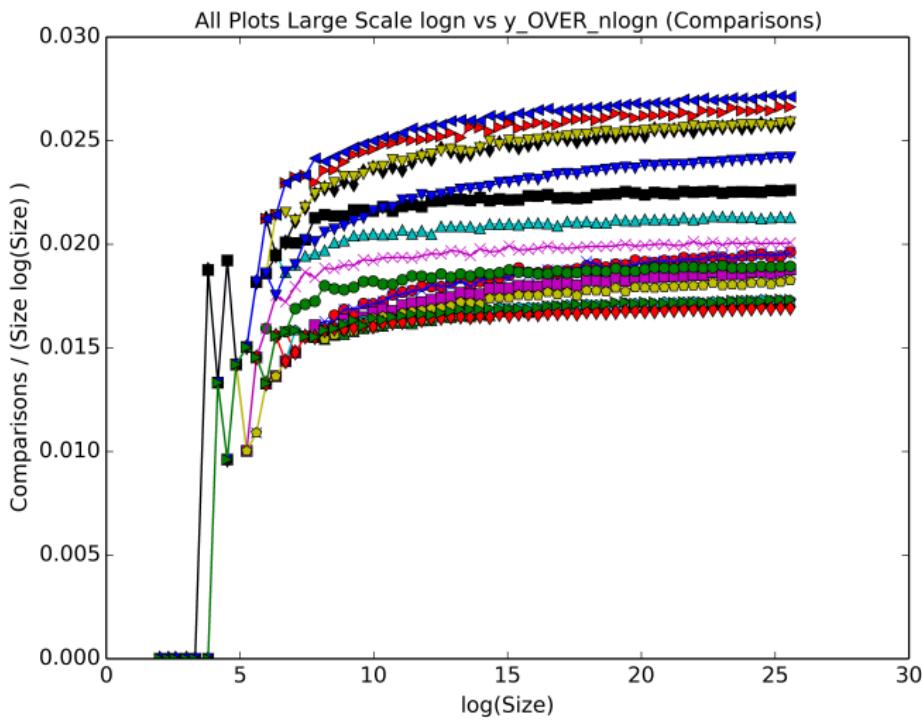
Classic Quicksort
Dual Pivot
Quicksort
Yaroslavskiy's
Quicksort
Three Pivot
Quicksort
M-Pivot
Quicksort
Summary

Legend

Results

Mass
Comparison
One Pivot
Two Pivots
Three Pivots
M Pivots
Curve Fit

The End



Mass Comparison $\log(n)$ vs $\frac{y}{n \log(n)}$

QuickSort

Moghadasian,
Hernandez

Quicksorts

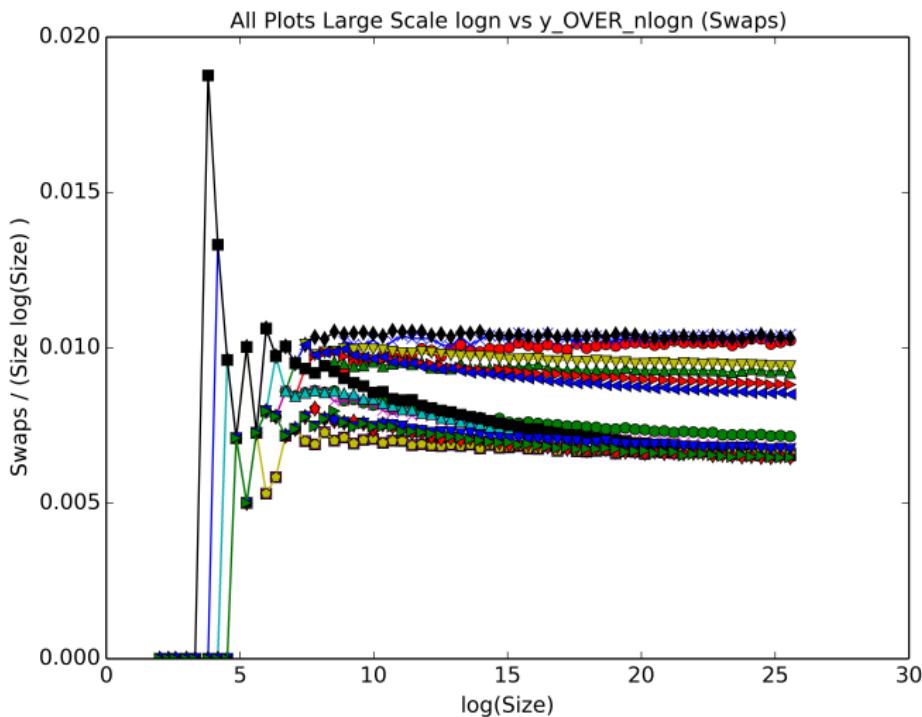
Classic Quicksort
Dual Pivot
Quicksort
Yaroslavskiy's
Quicksort
Three Pivot
Quicksort
M-Pivot
Quicksort
Summary

Legend

Results

Mass
Comparison
One Pivot
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Three Pivots
M Pivots
Curve Fit

The End



One Pivot Comparison Small Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

Legend

Results

Mass

Comparison

One Pivot

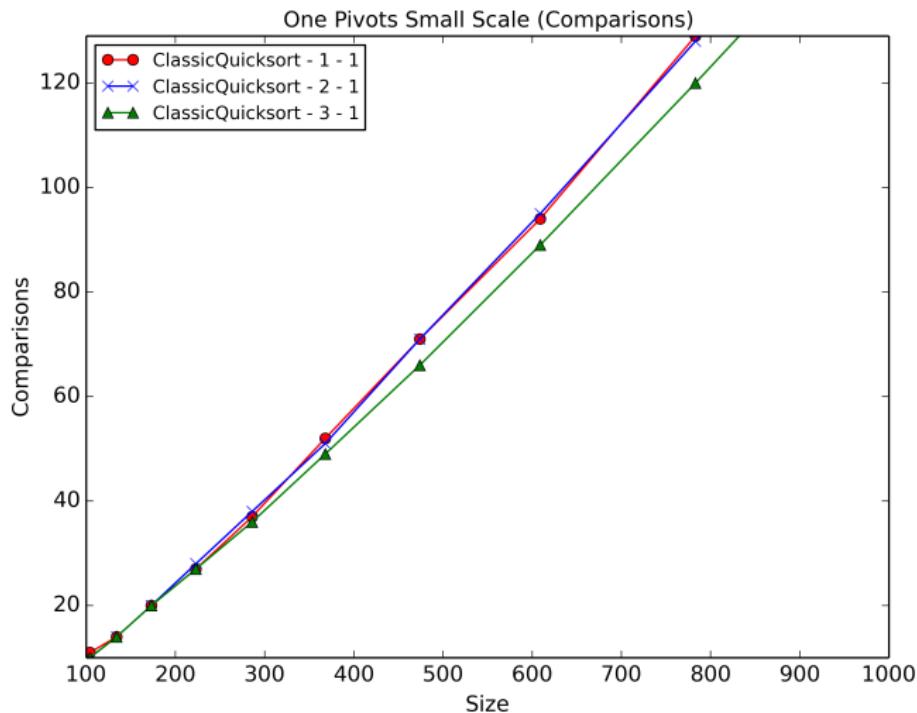
Two Pivots

Three Pivots

M Pivots

Curve Fit

The End



One Pivot Comparison Small Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort
Dual Pivot
Quicksort
Yaroslavskiy's
Quicksort

Three Pivot
Quicksort

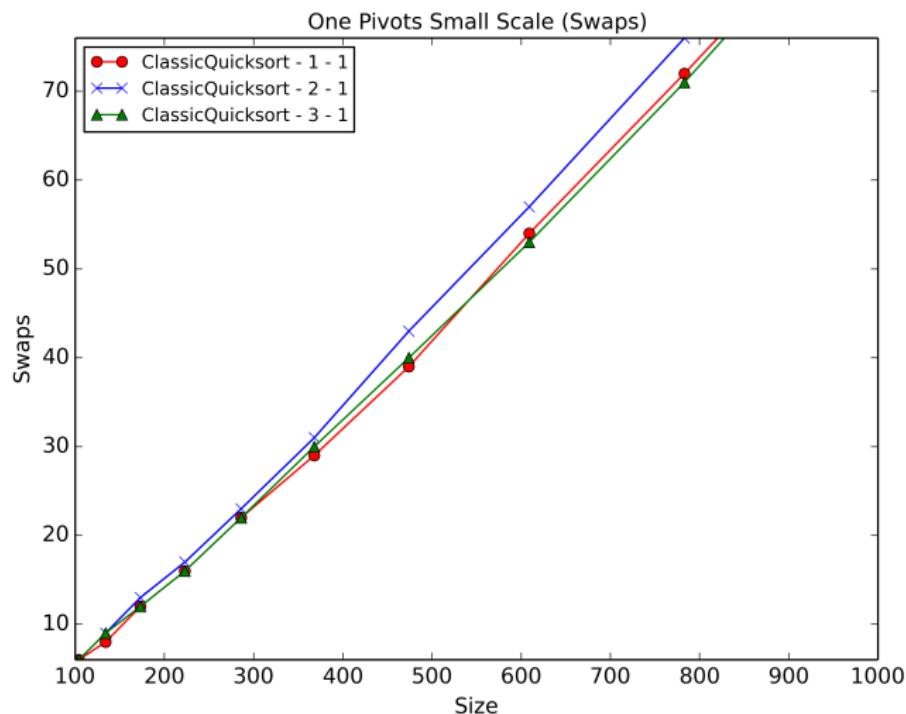
M-Pivot
Quicksort
Summary

Legend

Results

Mass
Comparison
One Pivot
Two Pivots
Three Pivots
M Pivots
Curve Fit

The End



One Pivot Comparison Large Scale

QuickSort

Moghadasiyan,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

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Mass

Comparison

One Pivot

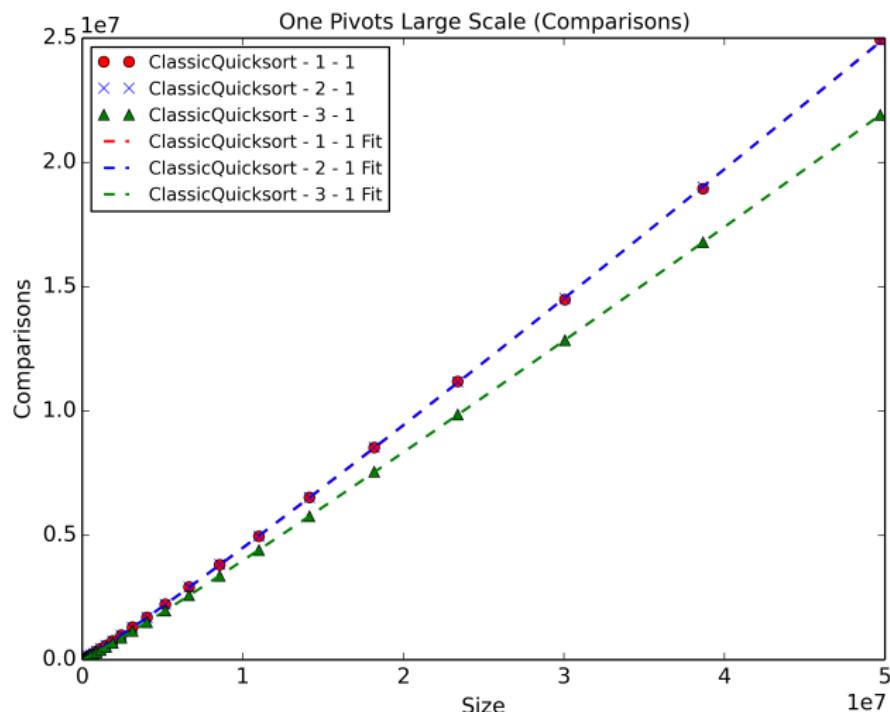
Two Pivots

Three Pivots

M Pivots

Curve Fit

The End



One Pivot Comparison Large Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort
Dual Pivot
Quicksort
Yaroslavskiy's
Quicksort

Three Pivot
Quicksort

M-Pivot
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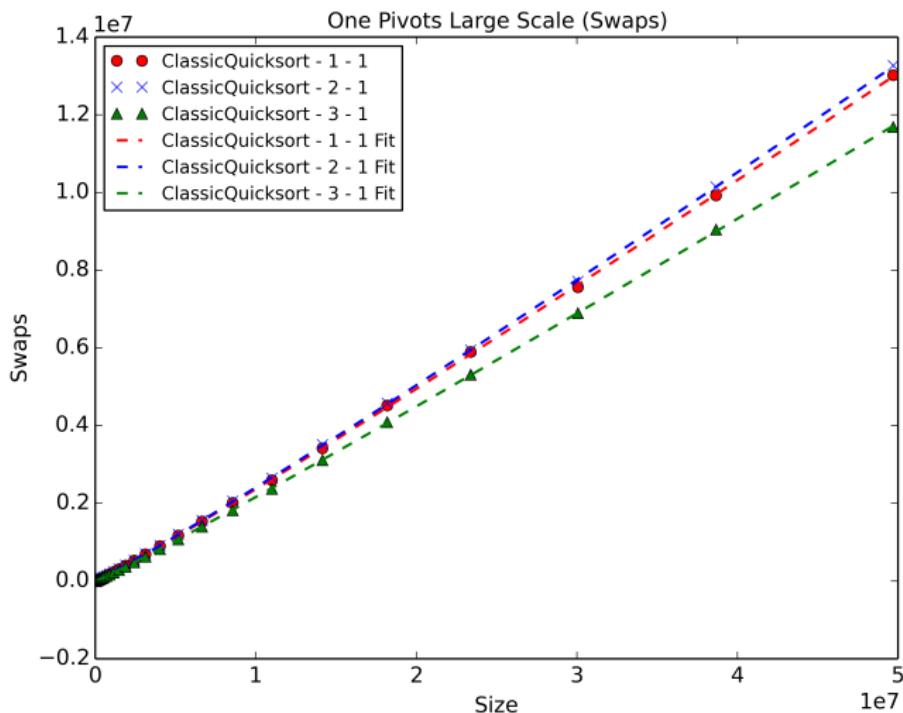
Mass
Comparison
One Pivot

Two Pivots

Three Pivots

M Pivots
Curve Fit

The End



Two Pivot Comparison Small Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

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Comparison

One Pivot

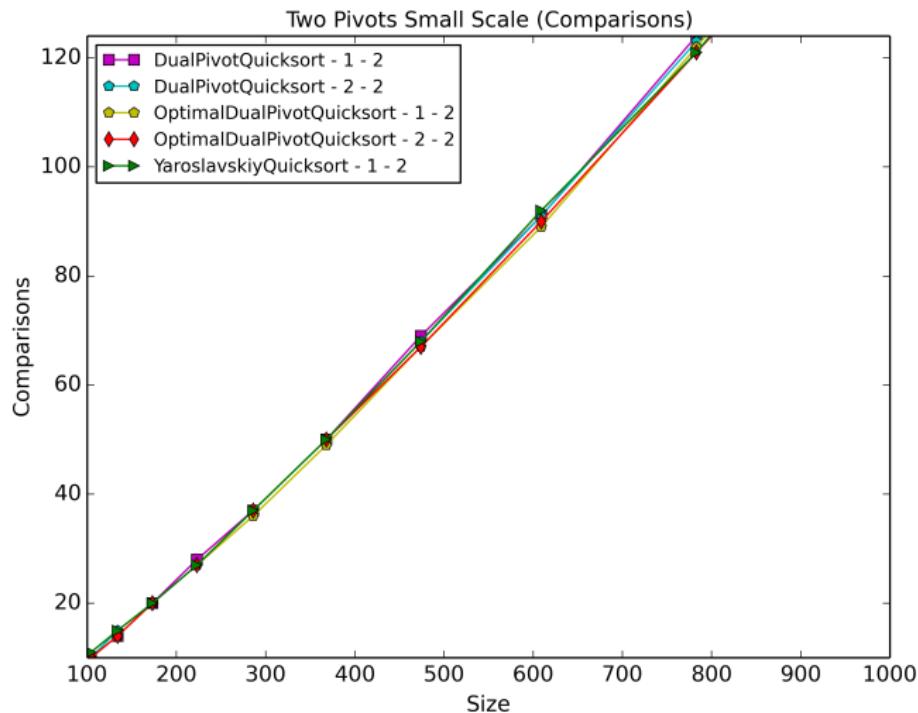
Two Pivots

Three Pivots

M Pivots

Curve Fit

The End



Two Pivot Comparison Small Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

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Quicksort

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One Pivot

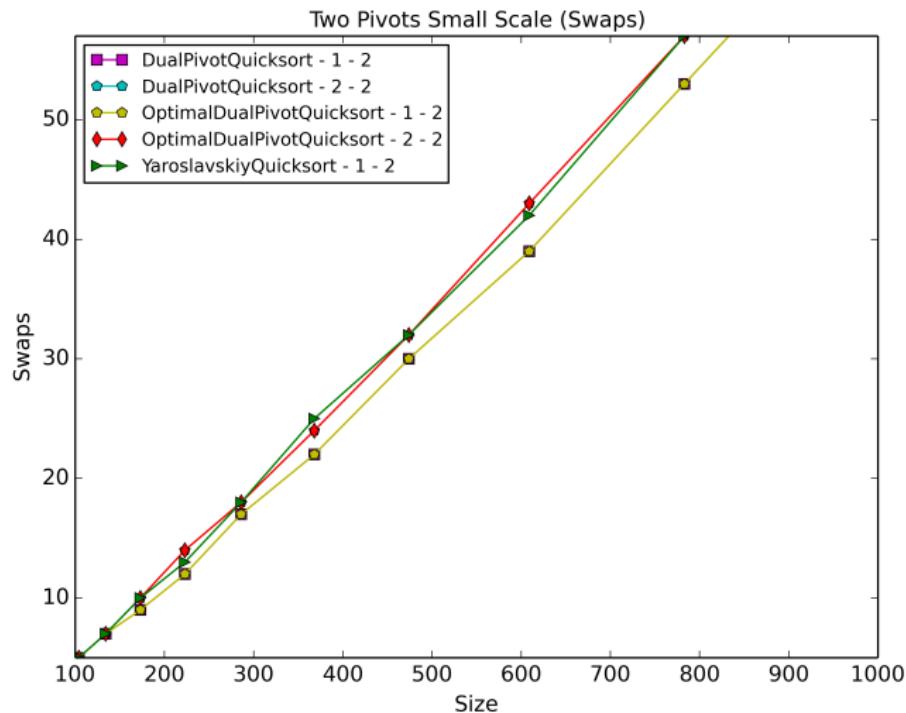
Two Pivots

Three Pivots

M Pivots

Curve Fit

The End



Two Pivot Comparison Large Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

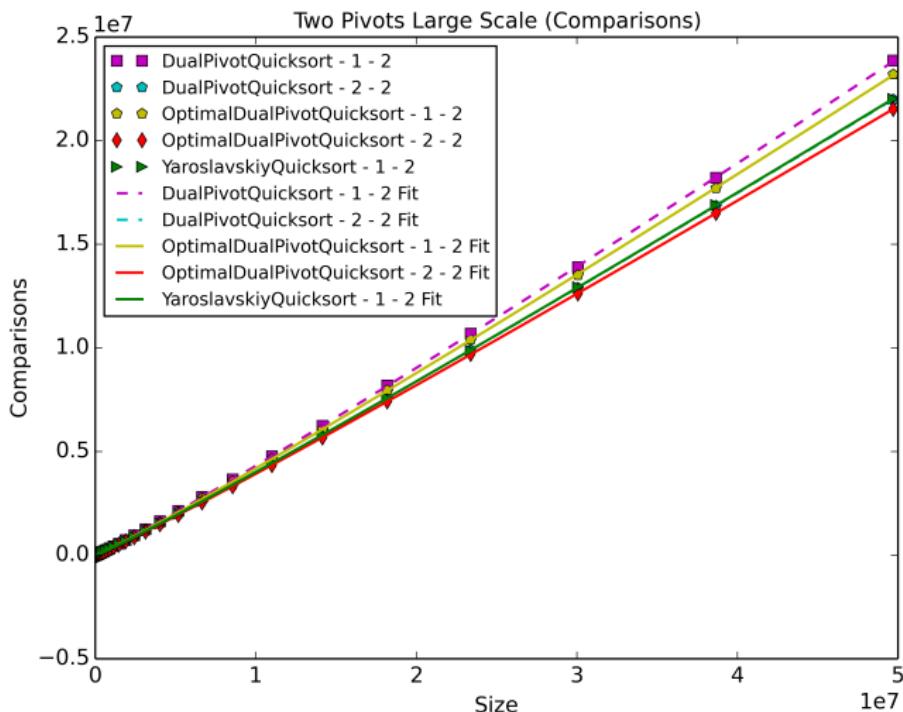
Classic Quicksort
Dual Pivot
Quicksort
Yaroslavskiy's
Quicksort
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Quicksort
M-Pivot
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The End



Two Pivot Comparison Large Scale

QuickSort

Moghadasian,
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Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

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Mass

Comparison

One Pivot

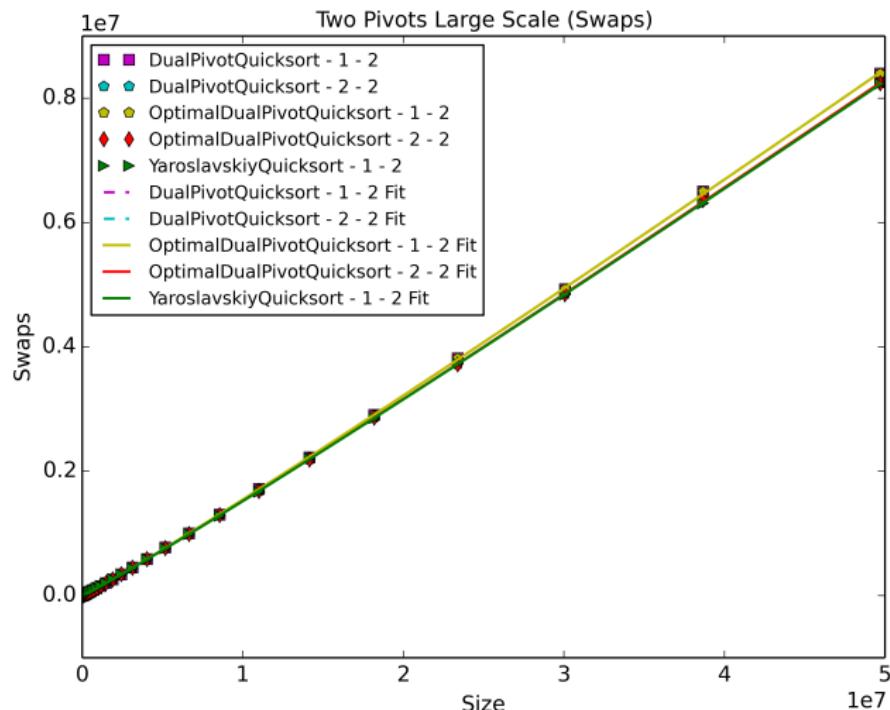
Two Pivots

Three Pivots

M Pivots

Curve Fit

The End



Three Pivot Comparison Small Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

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Comparison

One Pivot

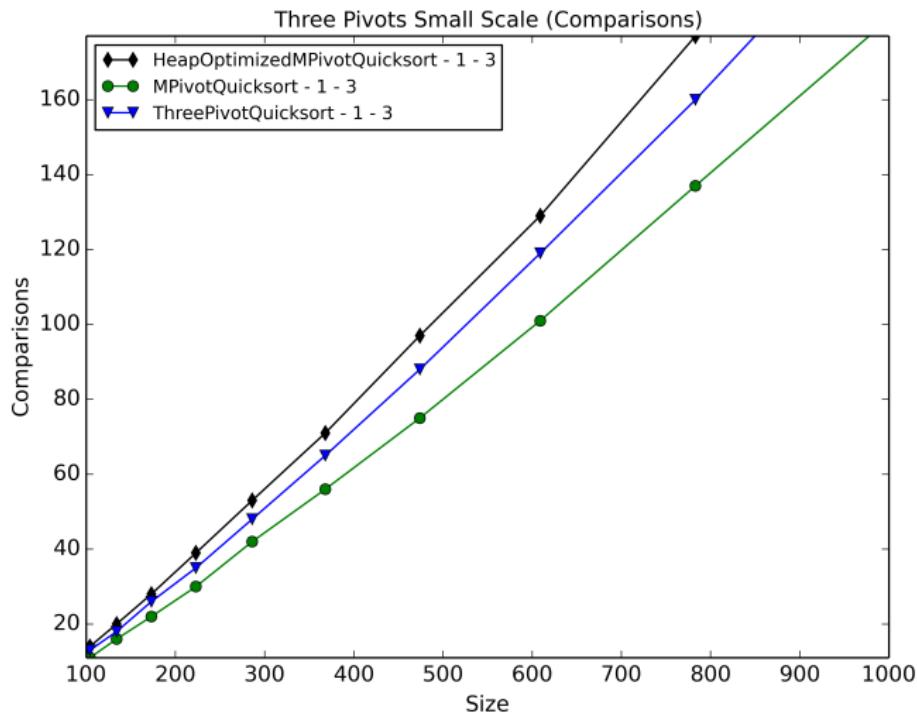
Two Pivots

Three Pivots

M Pivots

Curve Fit

The End



Three Pivot Comparison Small Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

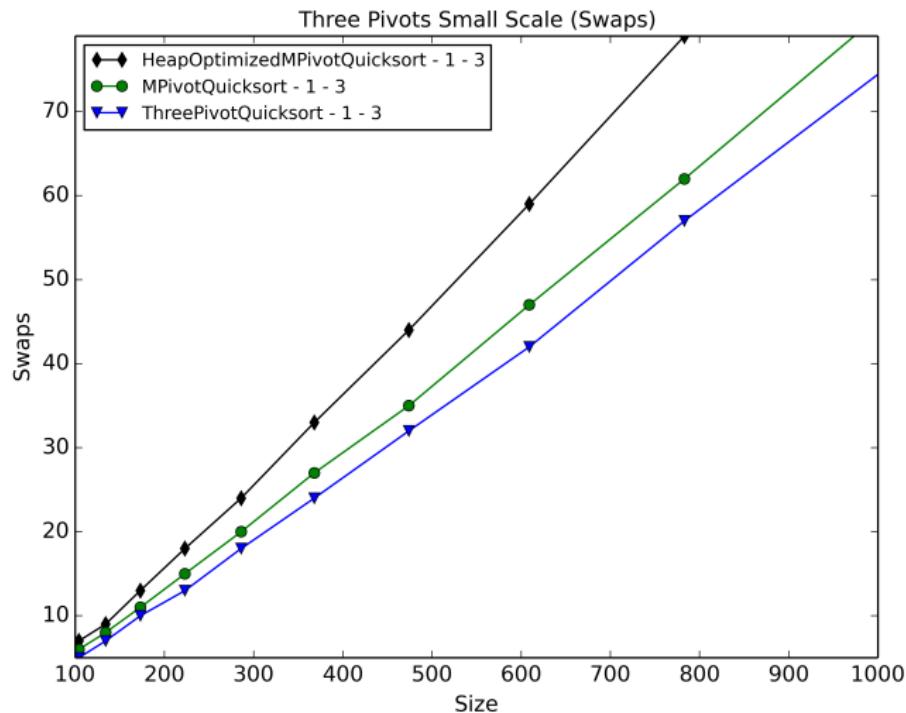
Classic Quicksort
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Quicksort
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Quicksort
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Quicksort
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The End



Three Pivot Comparison Large Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort
Dual Pivot
Quicksort
Yaroslavskiy's
Quicksort

Three Pivot
Quicksort

M-Pivot
Quicksort
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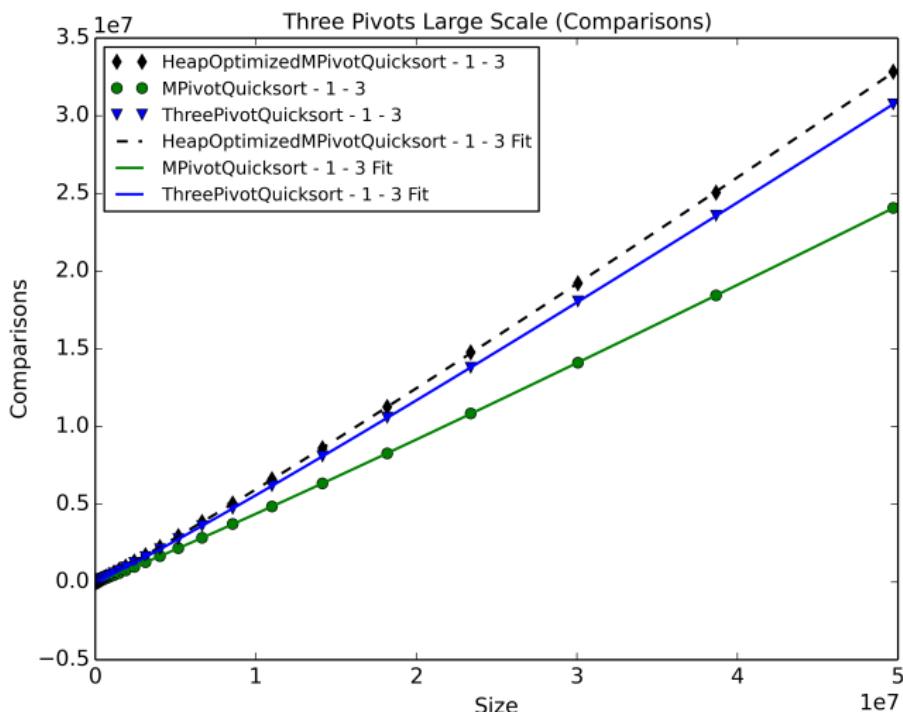
Results

Mass
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One Pivot
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The End



Three Pivot Comparison Large Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

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Yaroslavskiy's

Quicksort

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One Pivot

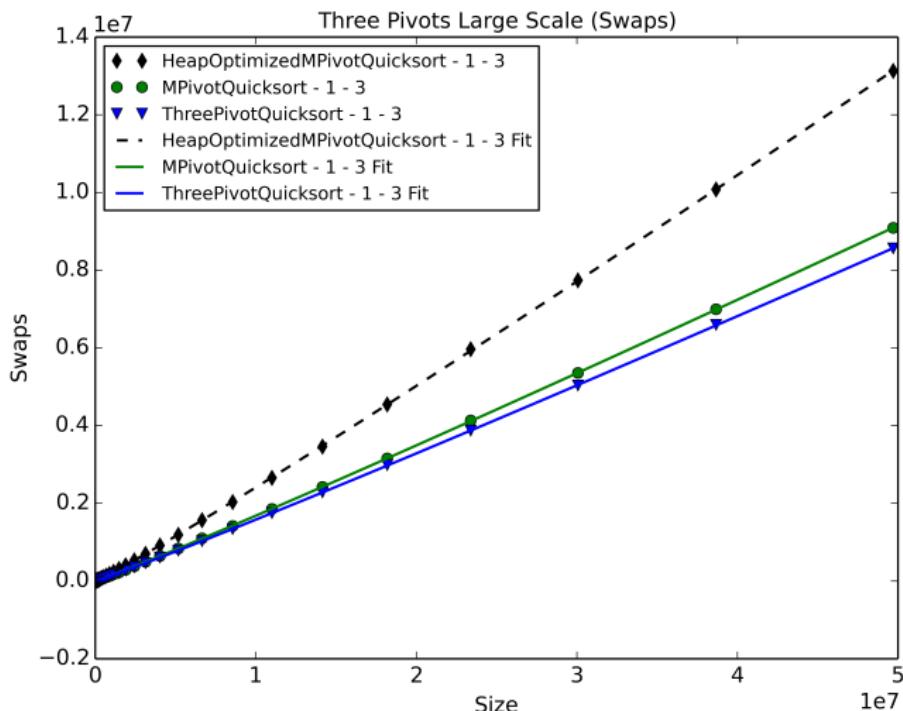
Two Pivots

Three Pivots

M Pivots

Curve Fit

The End



M Pivot Comparison Small Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

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One Pivot

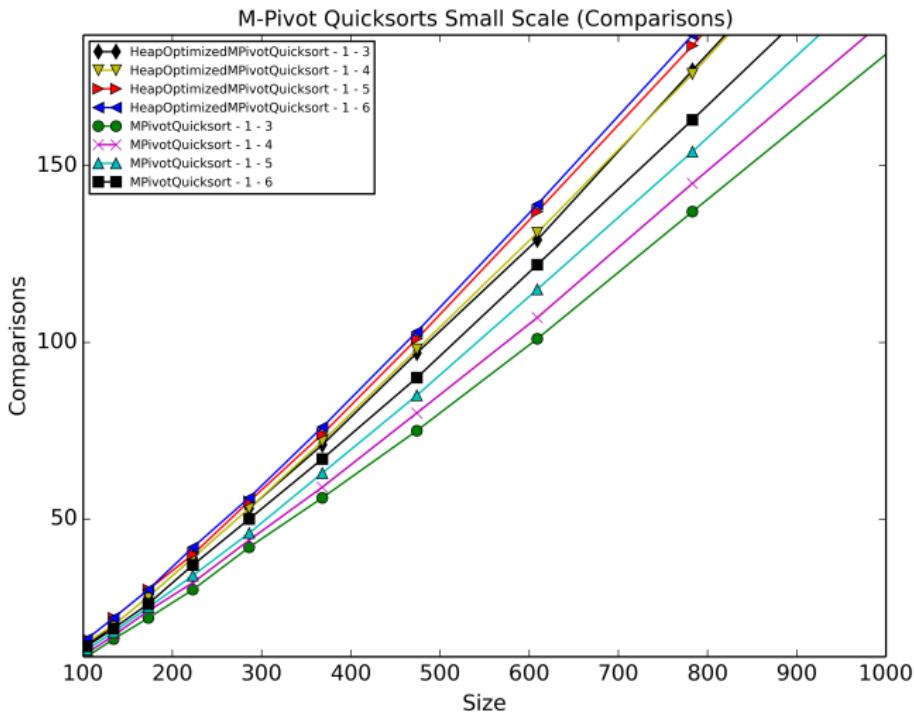
Two Pivots

Three Pivots

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Curve Fit

The End



M Pivot Comparison Small Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

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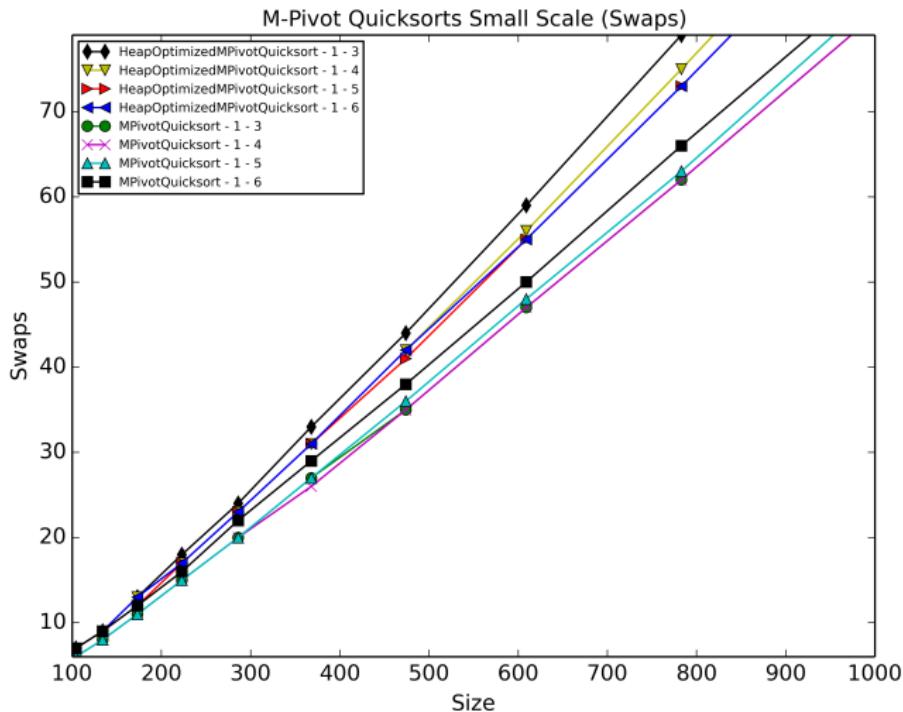
Two Pivots

Three Pivots

M Pivots

Curve Fit

The End



M Pivot Comparison Large Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

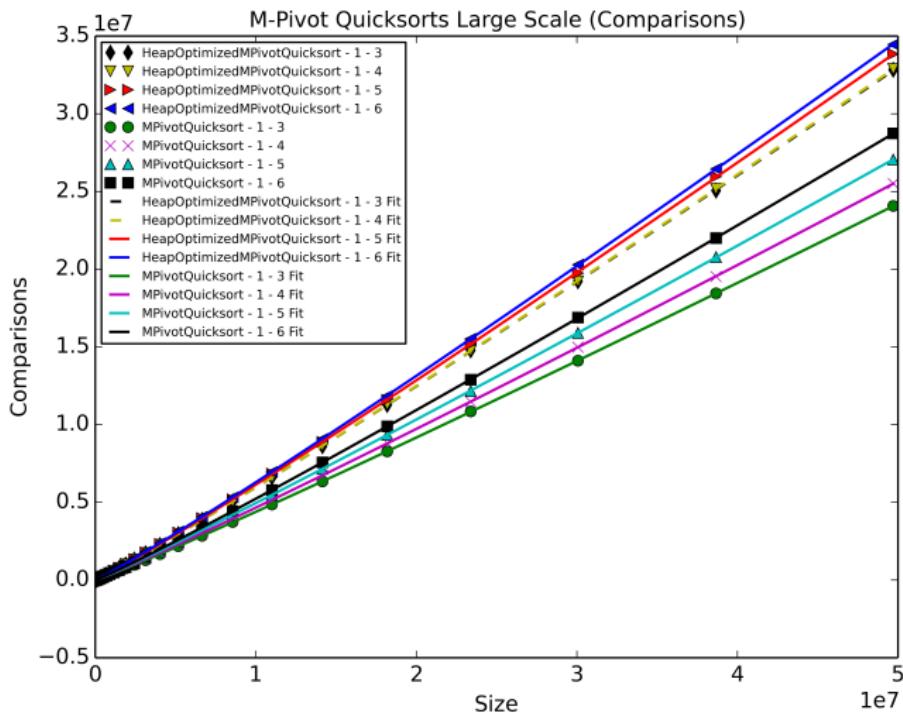
Classic Quicksort
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M Pivot Comparison Large Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

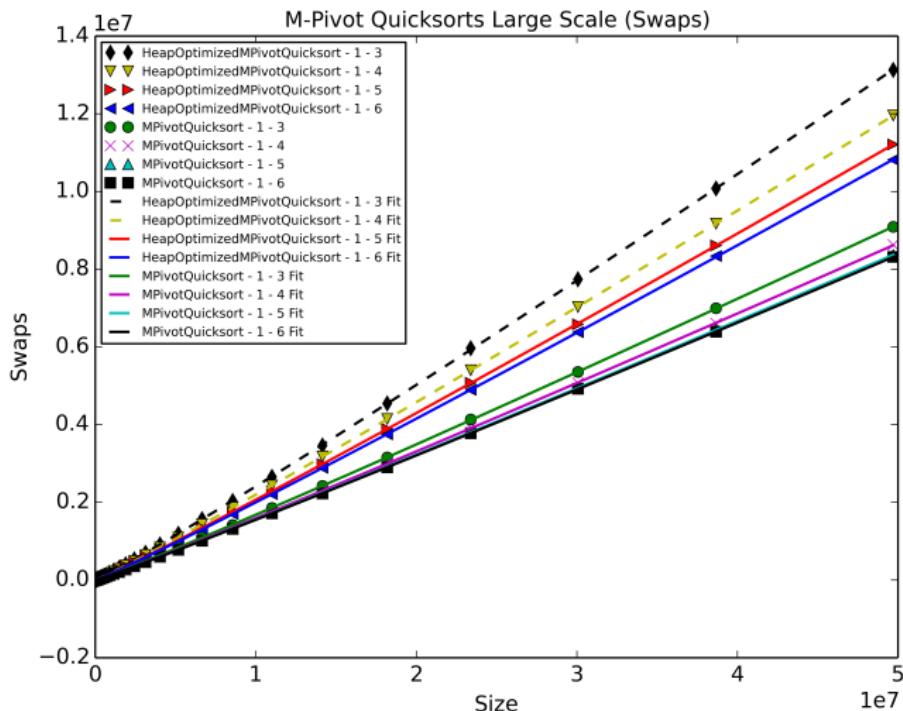
Classic Quicksort
Dual Pivot
Quicksort
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The End



Fit Coefficients of $A \cdot n \log(n) + B \cdot n + C \log(n)$

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort
Dual Pivot

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Yaroslavskiy's
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Sort Method	Comparisons	Swaps
Classic - 1 - 1	0.02219	0.01060
Classic - 2 - 1	0.02126	0.01110
Classic - 3 - 1	0.01799	0.00828
Dual Pivot - 1 - 2	0.02109	0.00636
Dual Pivot - 2 - 2	0.01787	0.00603
Optimal Dual Pivot - 1 - 2	0.02044	0.00636
Optimal Dual Pivot - 2 - 2	0.01754	0.00603
Three Pivot - 1 - 3	0.02595	0.00616
Yaroslavskiy - 1 - 2	0.01811	0.00584

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Sort Method	Comparisons	Swaps
Heap M Pivot - 1 - 3	0.02755	0.00999
Heap M Pivot - 1 - 4	0.02782	0.00885
Heap M Pivot - 1 - 5	0.02903	0.00809
Heap M Pivot - 1 - 6	0.02801	0.00769
M Pivot - 1 - 3	0.01955	0.00640
M Pivot - 1 - 4	0.02039	0.00594
M Pivot - 1 - 5	0.02136	0.00532
M Pivot - 1 - 6	0.02369	0.00524

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Questions?