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The End

All Sorts of Quicksorts!

An investigation into multi-pivot quicksorts

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University of Manitoba

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- $O(n \log(n))$ Average Case Run Time
- In place algorithm

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- $O(n \log(n))$ Average Case Run Time
- In place algorithm
- Picking Pivots

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- Partitioning Data

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- Recurse to a smaller sub-array

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- In place algorithm
- Picking Pivots
- Partitioning Data
- Recurse to a smaller sub-array
- Use Insertion Sort for a small sub-array

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- $2.13n \log n - 2.57n + O(\log(n))$ Comparisons

[Wild and Nebel(2012)]

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- $2.13n \log n - 2.57n + O(\log(n))$ Comparisons
- $0.33n \log n - 0.58n + O(\log(n))$ Swaps

[Wild and Nebel(2012)]

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[Wild and Nebel(2012)]

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- One Pivot
 - First Element

[Wild and Nebel(2012)]

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- One Pivot
 - First Element
 - Last Element

[Wild and Nebel(2012)]

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- One Pivot
 - First Element
 - Last Element
 - Median of Three

[Wild and Nebel(2012)]

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- One Pivot
 - First Element
 - Last Element
 - Median of Three
- Simple Partitioning

[Wild and Nebel(2012)]

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- One Pivot
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 - Median of Three
- Simple Partitioning
- Two Recursive Calls

[Wild and Nebel(2012)]

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- $2n \log n - 1.51n + O(\log(n))$ Comparisons

[Aumüller and Dietzfelbinger(2013)]

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- $2n \log n - 1.51n + O(\log(n))$ Comparisons
- $0.8n \log n - 0.3n + O(\log(n))$ Swaps

[Aumüller and Dietzfelbinger(2013)]

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[Aumüller and Dietzfelbinger(2013)]

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 - First and Last Element

[Aumüller and Dietzfelbinger(2013)]

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 - Middle 2 of 5 elements (Evenly Spaced Out)

[Aumüller and Dietzfelbinger(2013)]

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- Partitions Smalls then Bigs (Middle is automatic)

[Aumüller and Dietzfelbinger(2013)]

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- Three Recursive Calls

[Aumüller and Dietzfelbinger(2013)]

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- $1.9n \log n - 2.46n + O(\log(n))$ Comparisons

[Wild and Nebel(2012)]

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- $1.9n \log n - 2.46n + O(\log(n))$ Comparisons
- $0.6n \log n + 0.08n + O(\log(n))$ Swaps

[Wild and Nebel(2012)]

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[Wild and Nebel(2012)]

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[Wild and Nebel(2012)]

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 - Uses 5-element sorting network

[Wild and Nebel(2012)]

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 - Uses 5-element sorting network
- Simple Simultaneous Partition algorithm

[Wild and Nebel(2012)]

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 - Middle 2 of 5 elements (Evenly Spaced Out)
 - Uses 5-element sorting network
- Simple Simultaneous Partition algorithm
- Two Recursive Calls

[Wild and Nebel(2012)]

Kushagra-Ortiz-Qiao-Munro Tri-Pivot Quicksort

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- $1.846n \log n + O(n)$ Comparisons

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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- $1.846n \log n + O(n)$ Comparisons
- $0.615n \log n + O(n)$ Swaps

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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- $1.846n \log n + O(n)$ Comparisons
- $0.615n \log n + O(n)$ Swaps
- Three Pivots
 - Middle 3 of 7 elements (Evenly Spaced Out)

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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- Three Pivots
 - Middle 3 of 7 elements (Evenly Spaced Out)
- Simultaneous Partition algorithm

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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- Three Pivots
 - Middle 3 of 7 elements (Evenly Spaced Out)
- Simultaneous Partition algorithm
- Four Recursive Calls

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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- $O(n \log n)$ Comparisons

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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- $O(n \log n)$ Comparisons
- $O(n \log n)$ Swaps
- M Pivots
 - Sort $2M$ elements

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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- $O(n \log n)$ Comparisons
- $O(n \log n)$ Swaps
- M Pivots
 - Sort $2M$ elements
- Partition each segment on at a time

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

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- $O(n \log n)$ Comparisons
- $O(n \log n)$ Swaps
- M Pivots
 - Sort $2M$ elements
- Partition each segment on at a time
- $M + 1$ Recursive Calls

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

Example M-Pivot Selection and Partitioning

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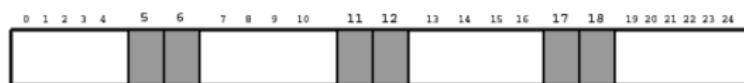
Two Pivots

Three Pivots

M Pivots

The End

Pivot Candidate Selection with 3 Pivots (25 elements)



[Kushagra et al.(2013) Kushagra, López-Ortiz, Qiao, and Munro]

Example M-Pivot Selection and Partitioning

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

Three Pivot

Quicksort

M-Pivot

Quicksort

Summary

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Comparison

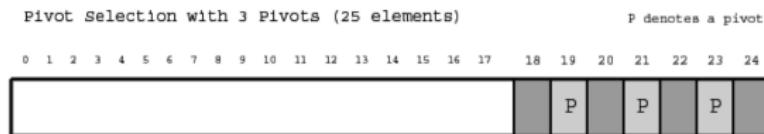
One Pivot

Two Pivots

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M Pivots

The End



[Kushagra et al.(2013) Kushagra, López-Ortiz, Qiao, and Munro]

Example M-Pivot Selection and Partitioning

QuickSort

Moghadasian,
Hernandez

Quicksorts

Classic Quicksort

Dual Pivot

Quicksort

Yaroslavskiy's

Quicksort

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One Pivot

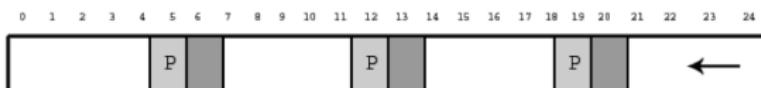
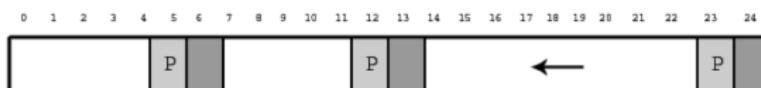
Two Pivots

Three Pivots

M Pivots

The End

Partition Phase with 3 Pivots (25 elements) P denotes a pivot



[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

Heap Optimized M-Pivot Quicksort

QuickSort

Moghadasian,
Hernandez

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The End

- Enforce the Heap Property to the array before any computation

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

Heap Optimized M-Pivot Quicksort

QuickSort

Moghadasian,
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The End

- Enforce the Heap Property to the array before any computation
 - Adds only $O(n)$ run time at each call

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

Heap Optimized M-Pivot Quicksort

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The End

- Enforce the Heap Property to the array before any computation
 - Adds only $O(n)$ run time at each call
 - Improves the sort to optimize pivot selection

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

Example M-Pivot Heap Optimized Pivot Selection

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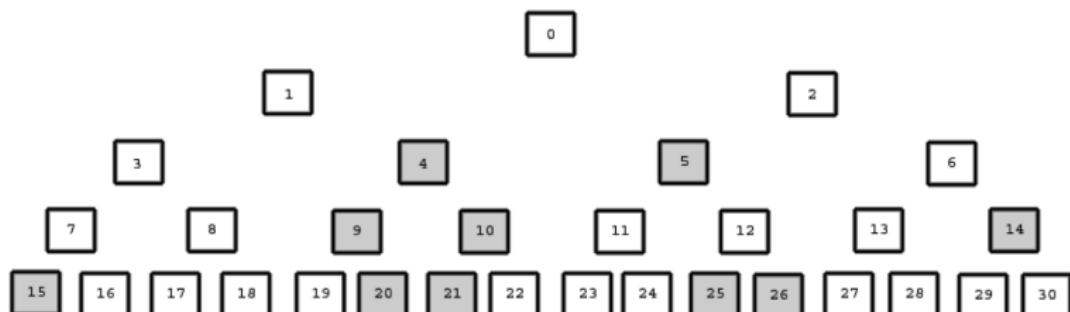
One Pivot
Two Pivots

Three Pivots
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The End

Candidate Selection from Min Heap (100 elements)

Base Candidate Pair = (31 / 6) - 1 = 4
Next Candidate Pair = 4 + 1 = every 5 elements



[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

Theoretical Average Case Run Time

QuickSort

Moghadasiyan,
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Sort Method	Comparisons
Classic	$2n \log n - 1.51n + O(\log(n))$
Dual Pivot	$2.13n \log n - 2.57n + O(\log(n))$
Optimal Dual Pivot	$1.8n \log n + O(n)$
Three Pivot	$1.846n \log n + O(n)$
Yaroslavskiy	$1.9n \log n - 2.46n + O(\log(n))$
M Pivot	$O(n \log n)$

[Aumüller and Dietzfelbinger(2013)]

[Wild and Nebel(2012)]

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

Theoretical Average Case Run Time

QuickSort

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Hernandez

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Sort Method	Swaps
Classic	$0.33n \log n - 0.58n + O(\log(n))$
Dual Pivot	$0.8n \log n - 0.3n + O(\log(n))$
Optimal Dual Pivot	$0.33n \log n + O(n)$
Three Pivot	$0.615n \log n + O(n)$
Yaroslavskiy	$0.6n \log n + 0.08n + O(\log(n))$
M Pivot	$O(n \log n)$

[Aumüller and Dietzfelbinger(2013)]

[Wild and Nebel(2012)]

[Kushagra et al.(2013)Kushagra, López-Ortiz, Qiao, and Munro]

Legend

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The End

- ClassicQuicksort - 1 - 1 - True
- ✗✗ ClassicQuicksort - 2 - 1 - True
- ▲▲ ClassicQuicksort - 3 - 1 - True
- DualPivotQuicksort - 1 - 2 - True
- DualPivotQuicksort - 2 - 2 - True
- ◀◀ HeapOptimizedMPivotQuicksort - 1 - 3 - True
- ▼▼ HeapOptimizedMPivotQuicksort - 1 - 4 - True
- HeapOptimizedMPivotQuicksort - 1 - 5 - True
- ◀◀ HeapOptimizedMPivotQuicksort - 1 - 6 - True
- MPivotQuicksort - 1 - 3 - True
- ✗✗ MPivotQuicksort - 1 - 4 - True
- ▲▲ MPivotQuicksort - 1 - 5 - True
- MPivotQuicksort - 1 - 6 - True
- OptimalDualPivotQuicksort - 1 - 2 - True
- OptimalDualPivotQuicksort - 2 - 2 - True
- ▼▼ ThreePivotQuicksort - 1 - 3 - True
- YaroslavskiyQuicksort - 1 - 2 - True

Mass Comparison Small Scale

QuickSort

Moghadasian,
Hernandez

Quicksorts

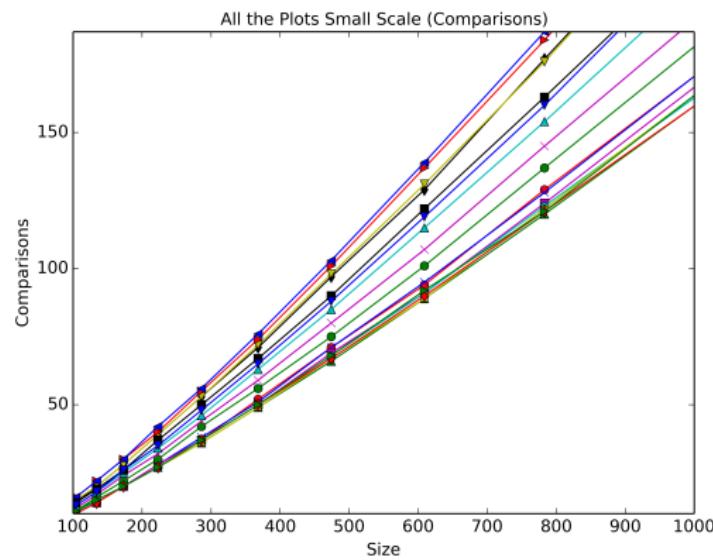
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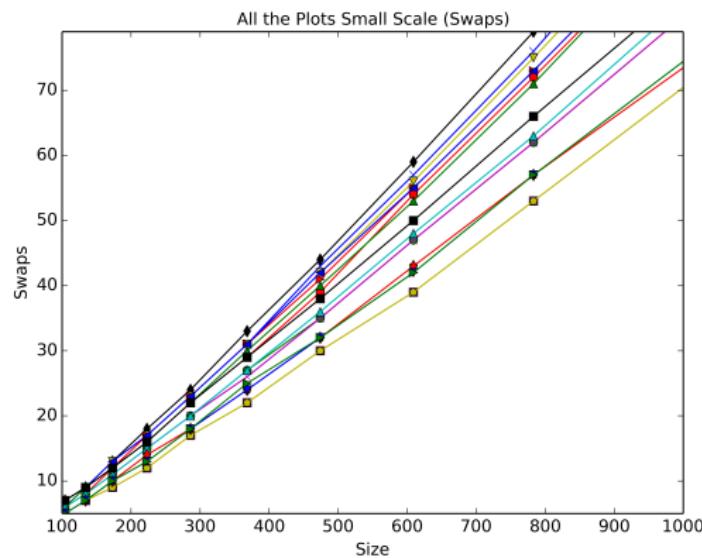
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Mass Comparison Large Scale

QuickSort

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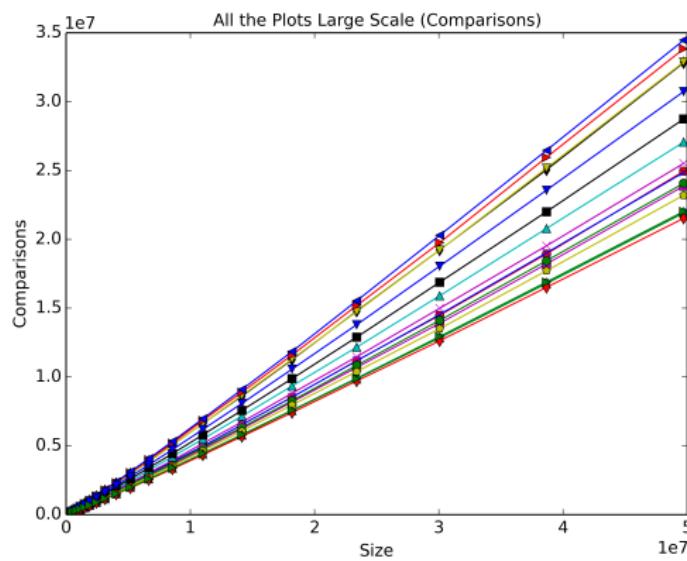
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Mass Comparison Large Scale

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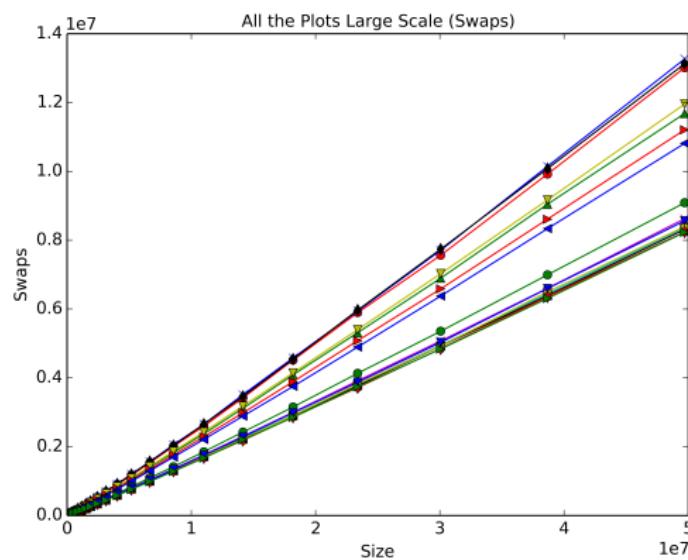
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Mass Comparison Semi Log x

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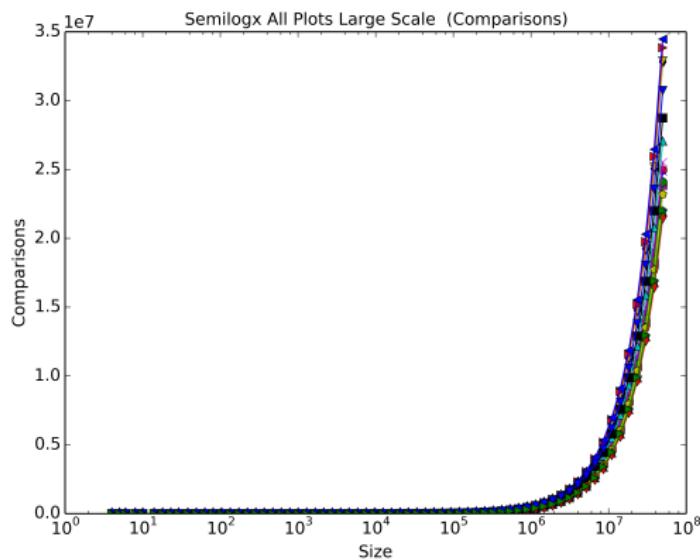
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Mass Comparison Semi Log x

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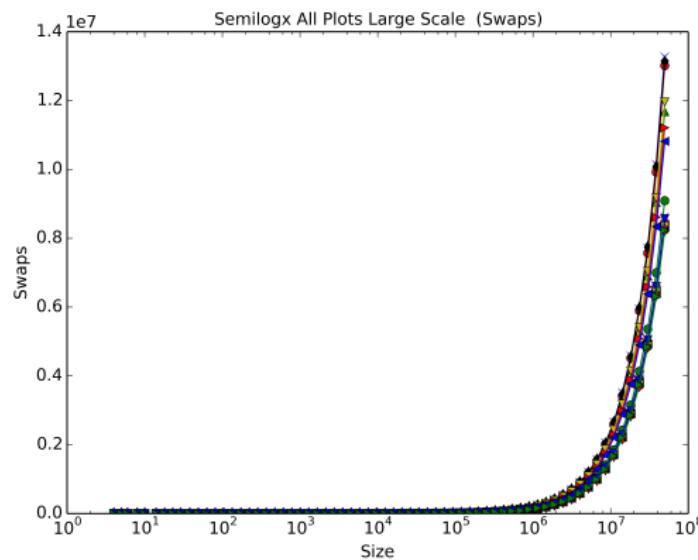
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One Pivot Comparison Small Scale

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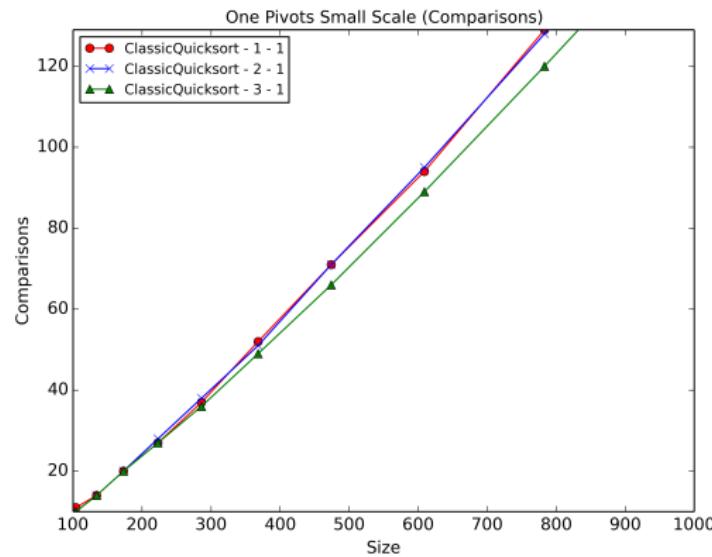
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One Pivot Comparison Small Scale

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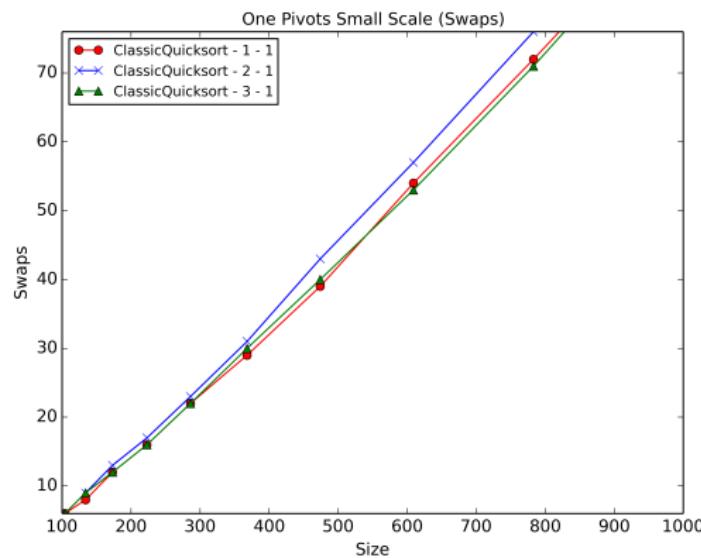
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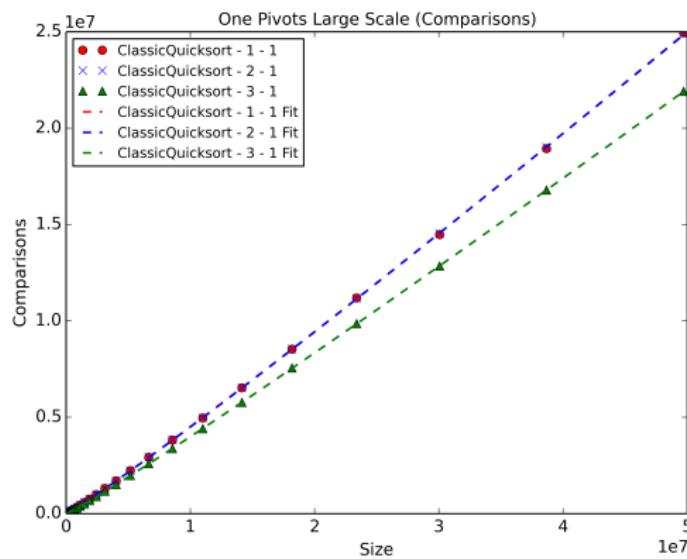
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One Pivot Comparison Large Scale

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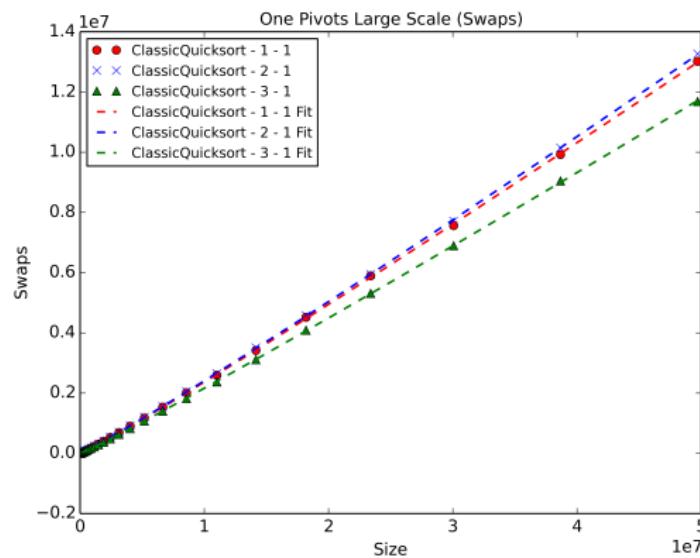
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Two Pivot Comparison Small Scale

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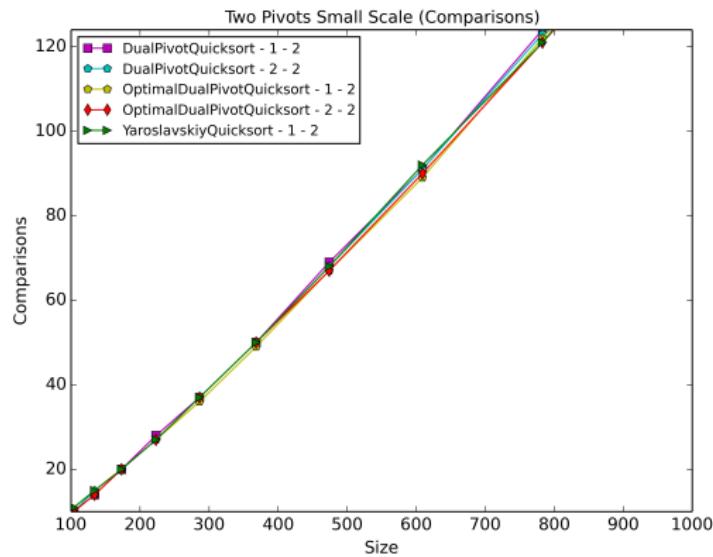
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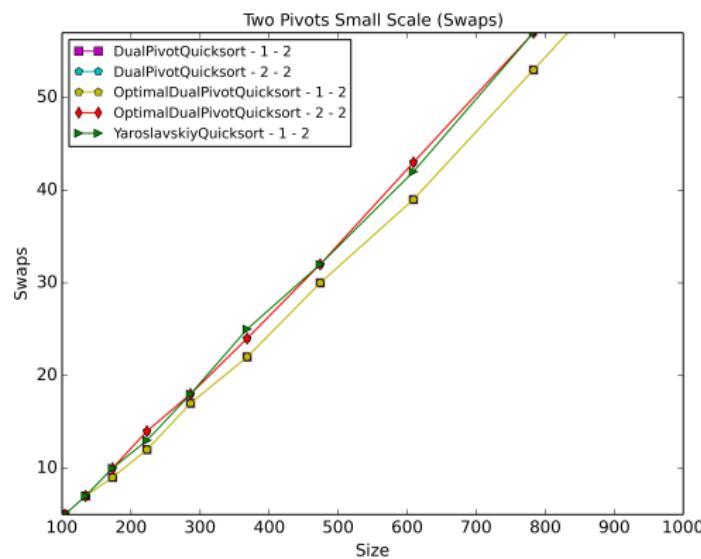
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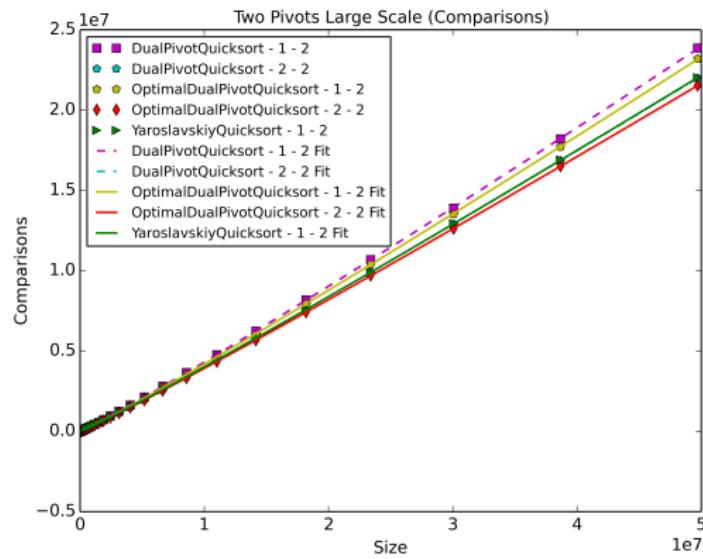
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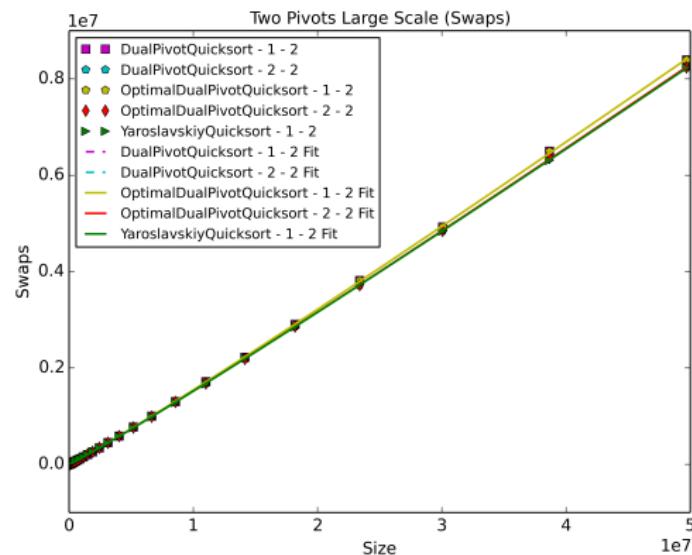
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Three Pivot Comparison Small Scale

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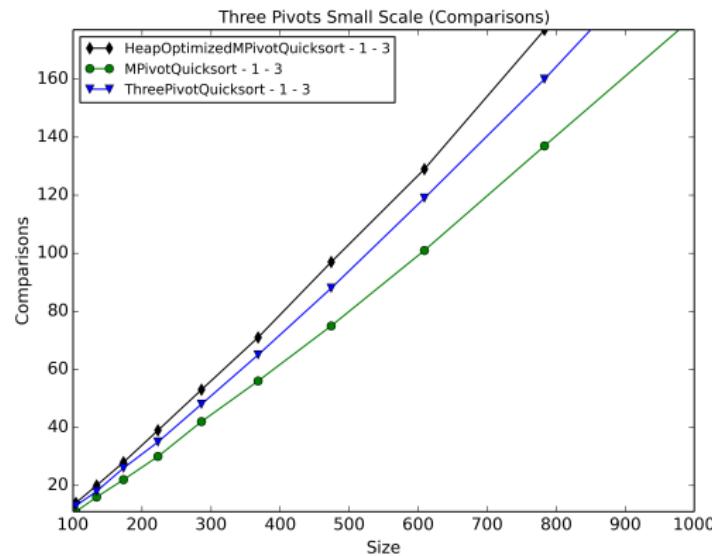
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Three Pivot Comparison Small Scale

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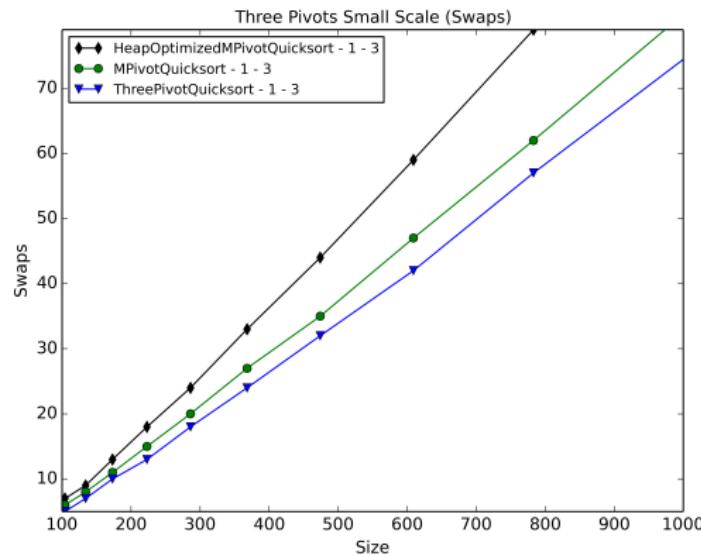
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Three Pivot Comparison Large Scale

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Moghadasi, Hernandez

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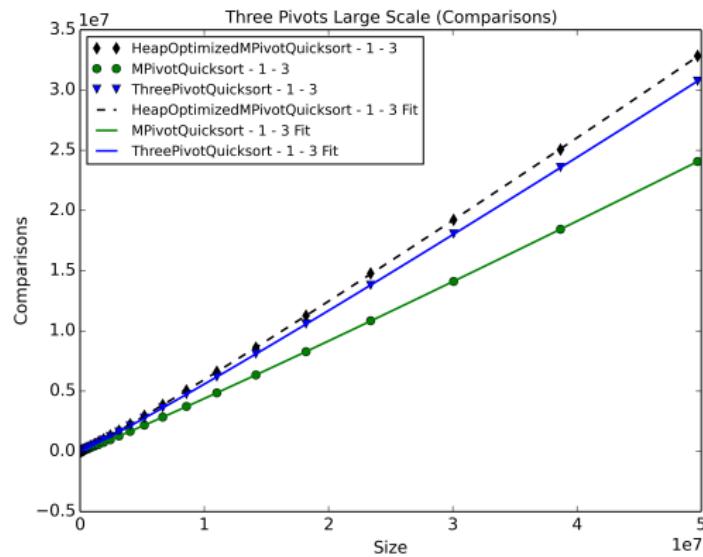
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Three Pivot Comparison Large Scale

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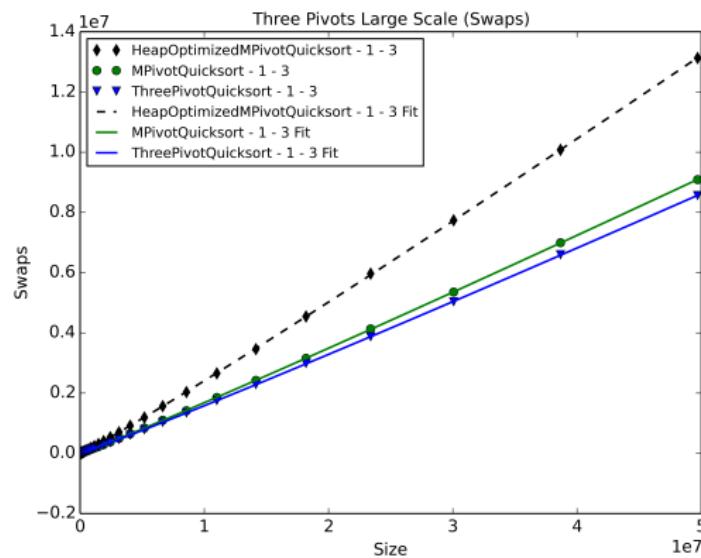
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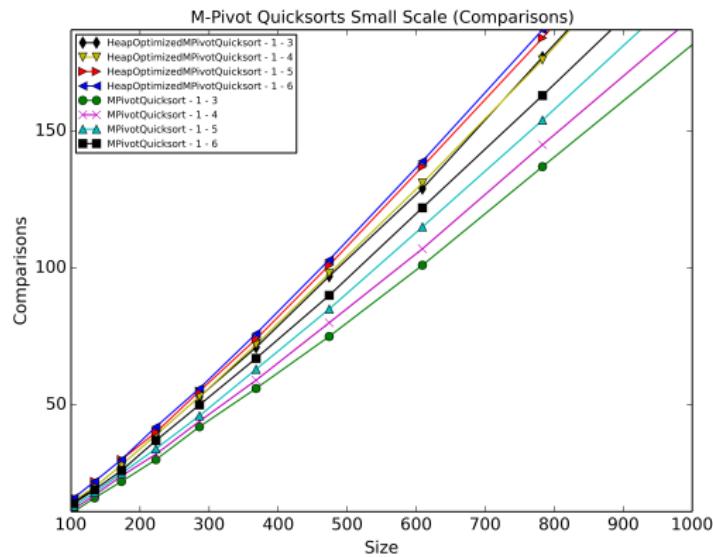
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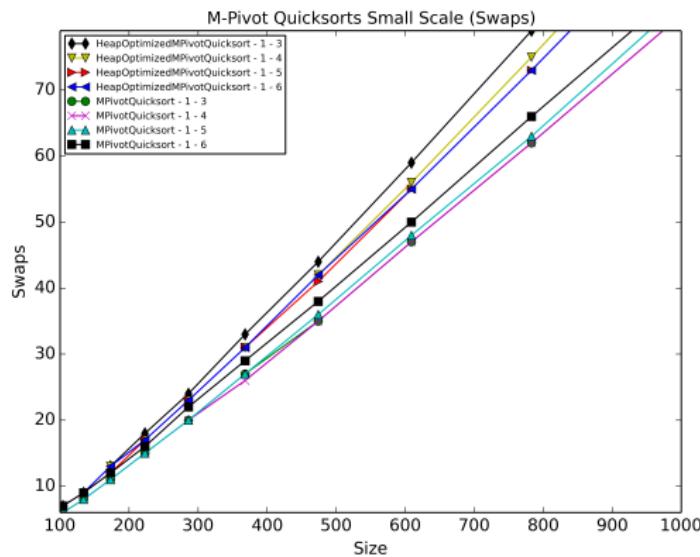
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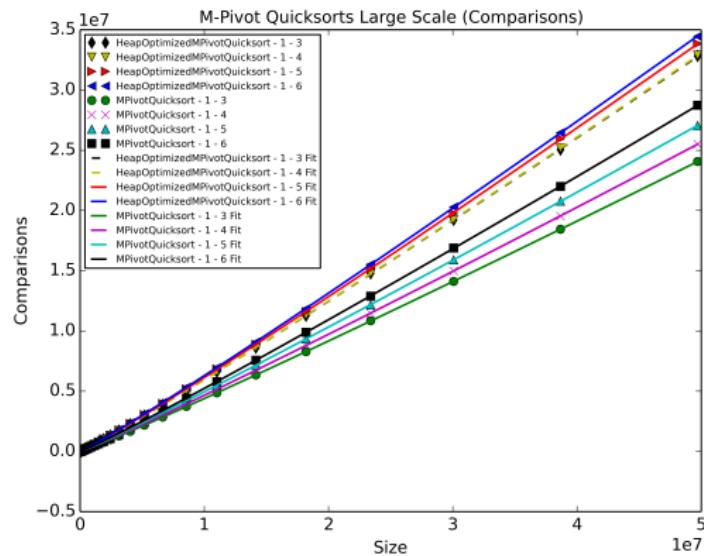
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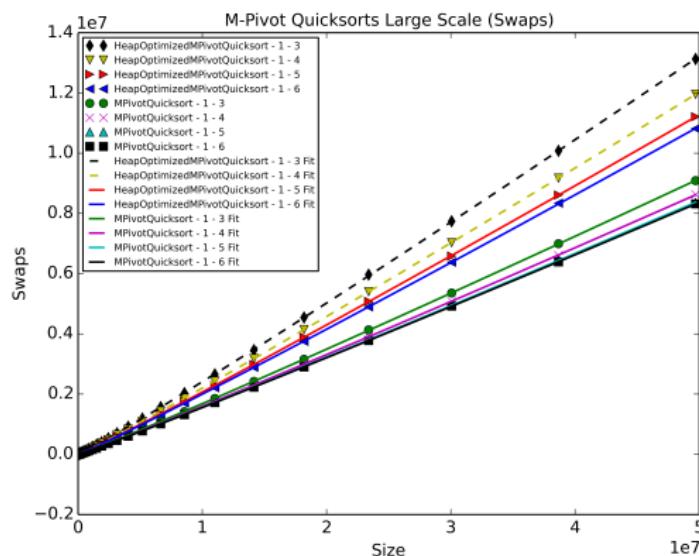
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Questions?

QuickSort

Moghadasian,
Hernandez