Practical 6: Time Series Stationarity Analysis - AirPassengers

Objective

Analyze the AirPassengers dataset from R library to:

- Convert data into a time series object
- Identify dominant components through visualization
- Decompose the time series to observe components clearly
- Test for stationarity using ACF/PACF plots
- Perform Augmented Dickey-Fuller (ADF) test for stationarity

Dataset

- **Source**: Built-in R dataset `AirPassengers`
- **Description**: Monthly totals of international airline passengers (1949-1960)
- **Number of observations**: 144 months (12 years)
- **Units**: Thousands of passengers
- **Frequency**: 12 (monthly data)

Analysis Steps

(a) Convert Data into Time Series Object

```
# Load the AirPassengers dataset (built-in R dataset)
data(AirPassengers)

# Display information about the dataset
cat("Dataset: AirPassengers\n")
cat("Number of observations:", length(AirPassengers), "\n")
cat("Start:", start(AirPassengers), "\n")
cat("End:", end(AirPassengers), "\n")
cat("Frequency:", frequency(AirPassengers), "\n\n")

# Display first few values
print(head(AirPassengers, 12))

# Check class
class(AirPassengers)  # "ts"
```

The AirPassengers dataset is already a built-in time series object in R with:

- **Start**: January 1949
- **End**: December 1960
- **Frequency**: 12 (monthly)

Summary Statistics:

- Minimum: 104,000 passengers
- Maximum: 622,000 passengers
- Mean: ~280,000 passengers
- Shows substantial growth over the period

(b) Plot Data to Identify Dominant Component

```
# Plot the original series
png("plot1_airpassengers.png", width = 800, height = 600)
plot(AirPassengers,
     main = "International Airline Passengers (1949-1960)",
     xlab = "Year",
     ylab = "Number of Passengers (thousands)",
     col = "blue",
     lwd = 2)
grid()
dev.off()
```

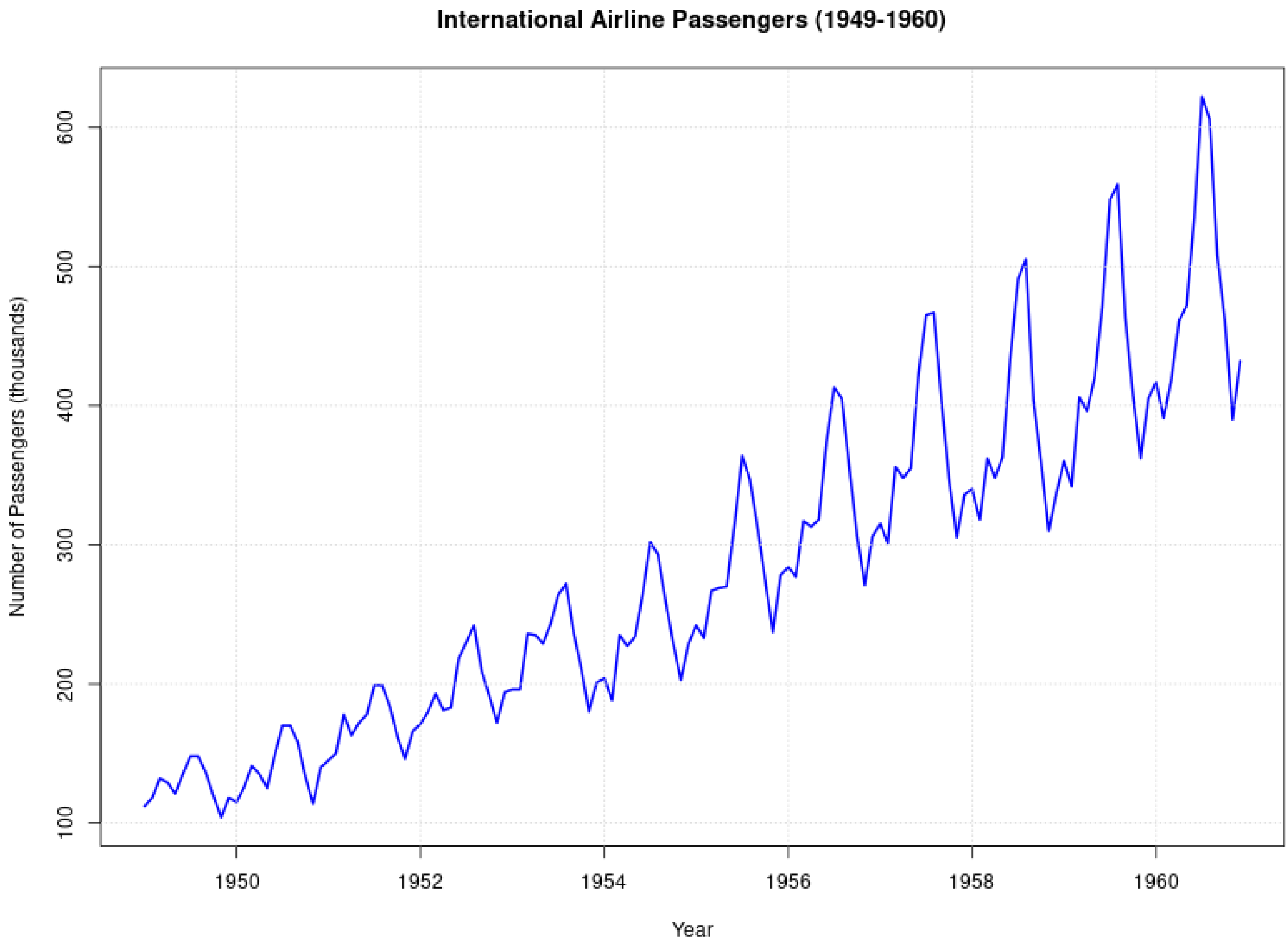

Figure 1: International airline passengers from 1949 to 1960.

Visual Observations:

- **Strong upward trend**: Passenger numbers increase from ~100k to ~600k
- **Seasonal pattern**: Regular peaks and troughs each year
- **Increasing variance**: The amplitude of seasonal fluctuations grows over time
- **Multiplicative structure**: Seasonal effect proportional to the level

Dominant Components: Both **TREND** and **SEASONALITY** are significant.

(c) Decompose Time Series

```r
# Decompose using multiplicative model (variance increases with level)
decomposed <- decompose(AirPassengers, type = "multiplicative")

png("plot2_decomposition.png", width = 800, height = 800)
plot(decomposed)
dev.off()
```

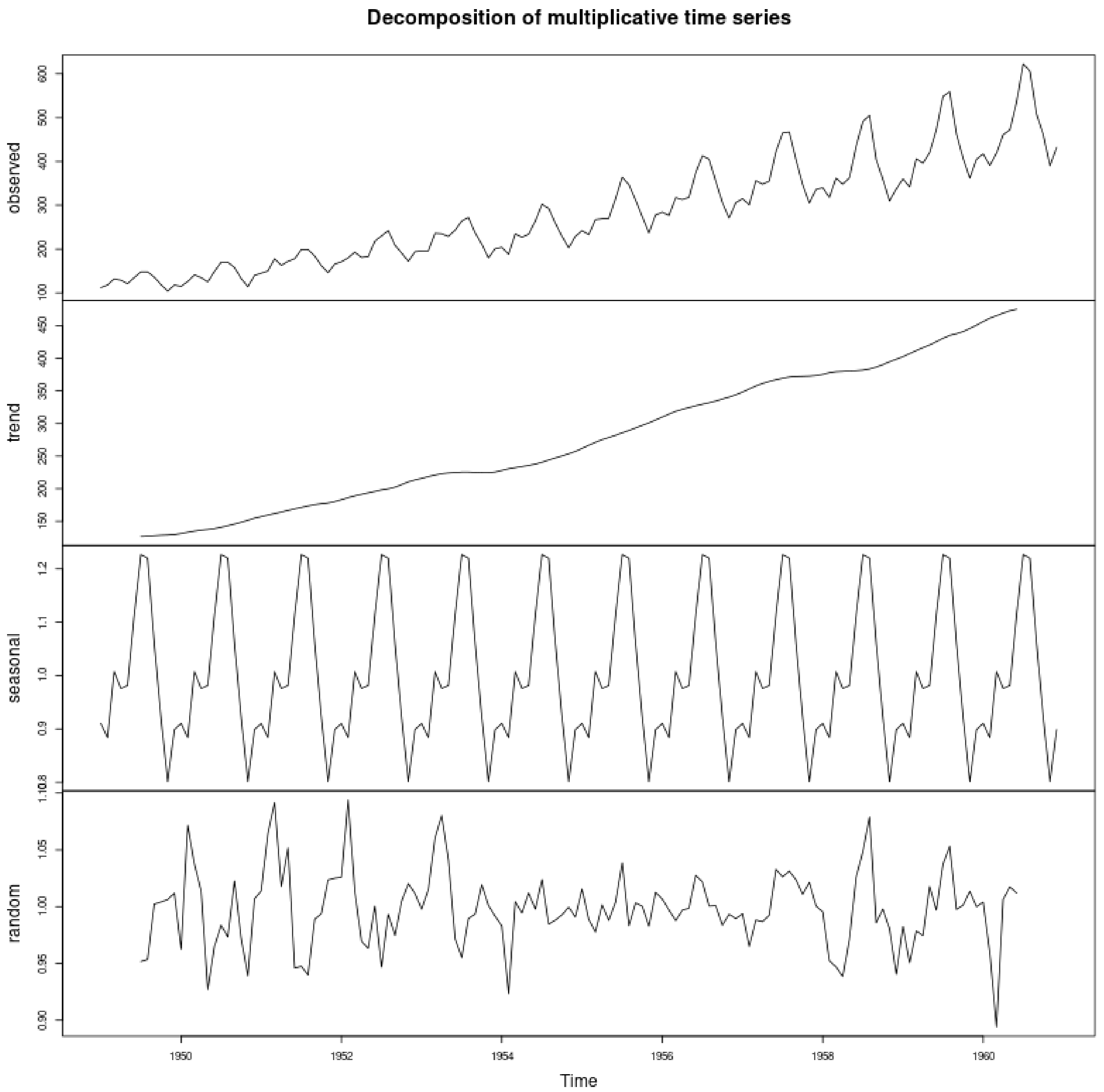

Figure 2: Multiplicative decomposition of AirPassengers data.

Component Analysis:

1. **Original Data (Top panel)**: Shows the raw time series
2. **Trend Component**: Clear upward trajectory - consistent growth in air travel
3. **Seasonal Component**: Regular yearly pattern
   - Peaks in summer months (July-August)
   - Troughs in winter months (November-February)
4. **Random Component**: Irregular variations after removing trend and seasonality

```
# Additive decomposition for comparison
decomposed_add <- decompose(AirPassengers, type = "additive")

png("plot3_decomposition_additive.png", width = 800, height = 800)
```

```
plot(decomposed_add)
dev.off()
```

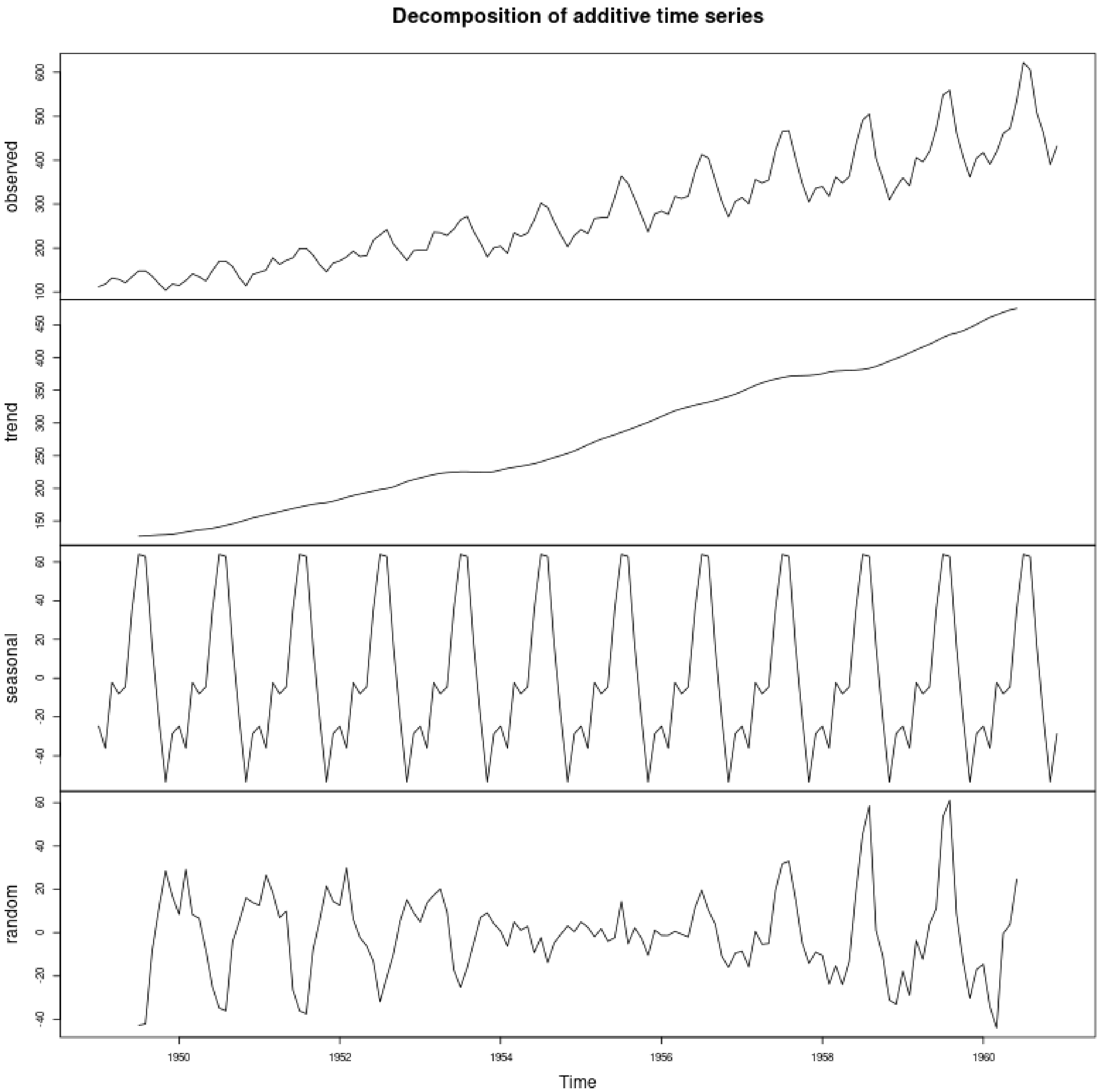

Figure 3: Additive decomposition for comparison.

Why Multiplicative?

- In multiplicative models: Y = Trend × Seasonal × Random
- In additive models: Y = Trend + Seasonal + Random
- AirPassengers shows **increasing variance** → multiplicative is more appropriate
- The seasonal variation grows proportionally with the trend level

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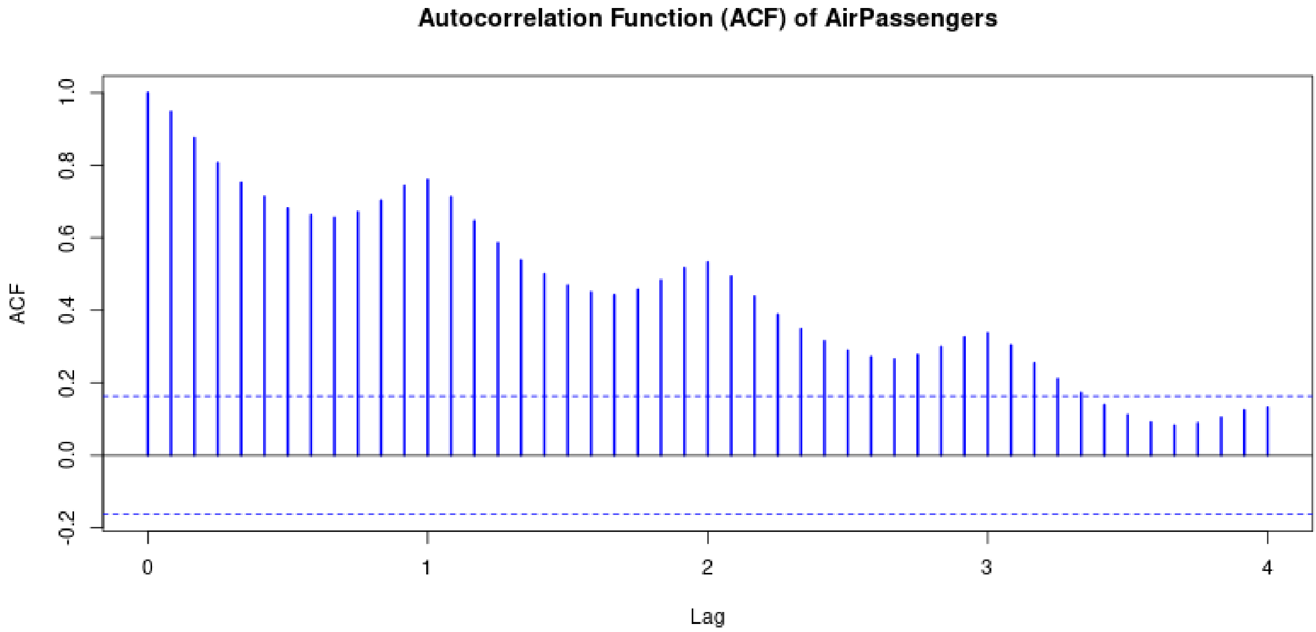
(d) ACF/PACF Analysis for Stationarity

```
# Plot ACF and PACF
png("plot4_acf_pacf.png", width = 800, height = 800)
par(mfrow = c(2, 1))

# ACF plot
acf(AirPassengers,
    main = "Autocorrelation Function (ACF) of AirPassengers",
    lag.max = 48,
    col = "blue",
    lwd = 2)

# PACF plot
pacf(AirPassengers,
     main = "Partial Autocorrelation Function (PACF) of AirPassengers",
     lag.max = 48,
     col = "red",
     lwd = 2)

par(mfrow = c(1, 1))
dev.off()
```



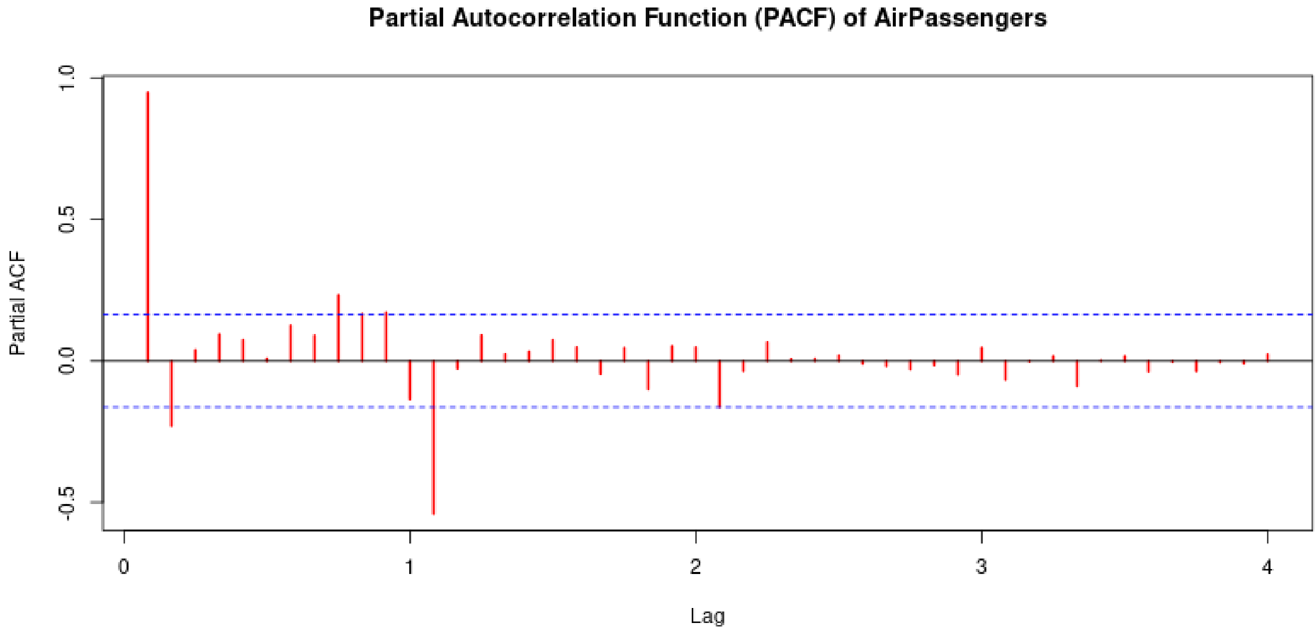


Figure 4: ACF and PACF plots of the original AirPassengers series.

ACF (Autocorrelation Function):

- Shows **very slow decay** → strong indicator of non-stationarity
- **Seasonal spikes** at lags 12, 24, 36, 48 (yearly pattern)
- High correlation persists for many lags

PACF (Partial Autocorrelation Function):

- First few lags are significant
- Pattern suggests both trend and seasonal components

Conclusion: The ACF pattern clearly indicates the series is **NON-STATIONARY** due to:

1. Slow decay (presence of trend)
2. Seasonal pattern (repeating spikes)

(e) Augmented Dickey-Fuller Test

Test on Original Series

```
library(tseries)

# Perform ADF test on original series
adf_test <- adf.test(AirPassengers, alternative = "stationary")
print(adf_test)
```

Hypotheses:

- H_0: Series has a unit root (non-stationary)
- H_1: Series is stationary
- Significance level: $\alpha = 0.05$

Expected Results (p-value > 0.05):

- **Decision**: FAIL TO REJECT H_0
- **Conclusion**: The original series is **NON-STATIONARY**

Transformations for Stationarity

Since the original series is non-stationary, we apply transformations:

1. Log Transformation

```
# Apply log transformation to stabilize variance
log_passengers <- log(AirPassengers)

png("plot5_log_transform.png", width = 800, height = 600)
plot(log_passengers,
     main = "Log-Transformed AirPassengers",
     xlab = "Year",
     ylab = "Log(Passengers)",
     col = "darkgreen",
     lwd = 2)
grid()
dev.off()
```

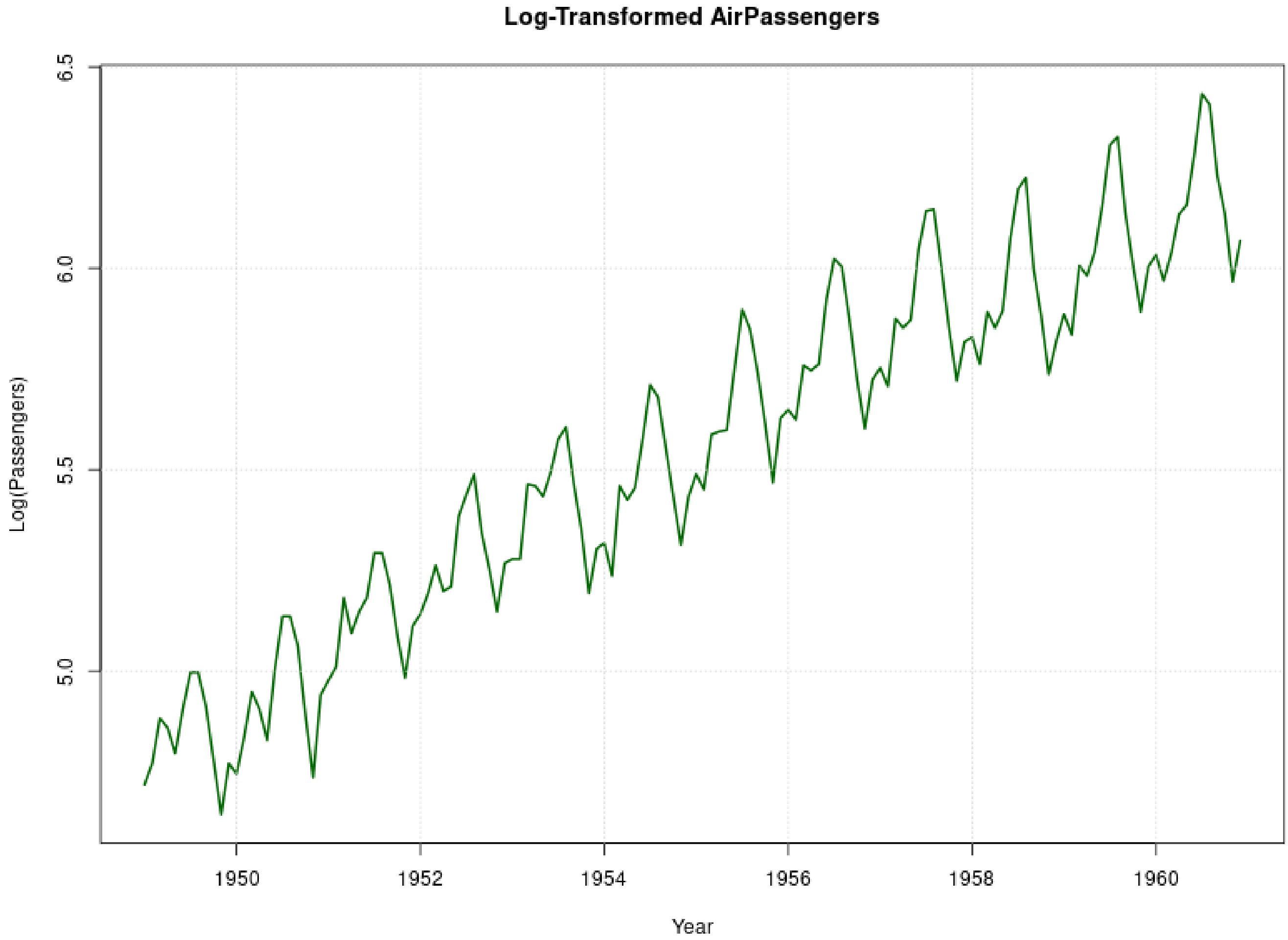


Figure 5: Log transformation of AirPassengers to stabilize variance.

Purpose: Stabilize the increasing variance **Effect**: Makes seasonal fluctuations more uniform across time

2. First Differencing

```
# Apply first differencing to remove trend
diff_log_passengers <- diff(log_passengers)

png("plot6_diff_log.png", width = 800, height = 600)
plot(diff_log_passengers,
     main = "First Difference of Log(AirPassengers)",
     xlab = "Year",
     ylab = "Differenced Log(Passengers)",
     col = "purple",
     lwd = 2)
grid()
abline(h = 0, col = "red", lty = 2)
dev.off()
```

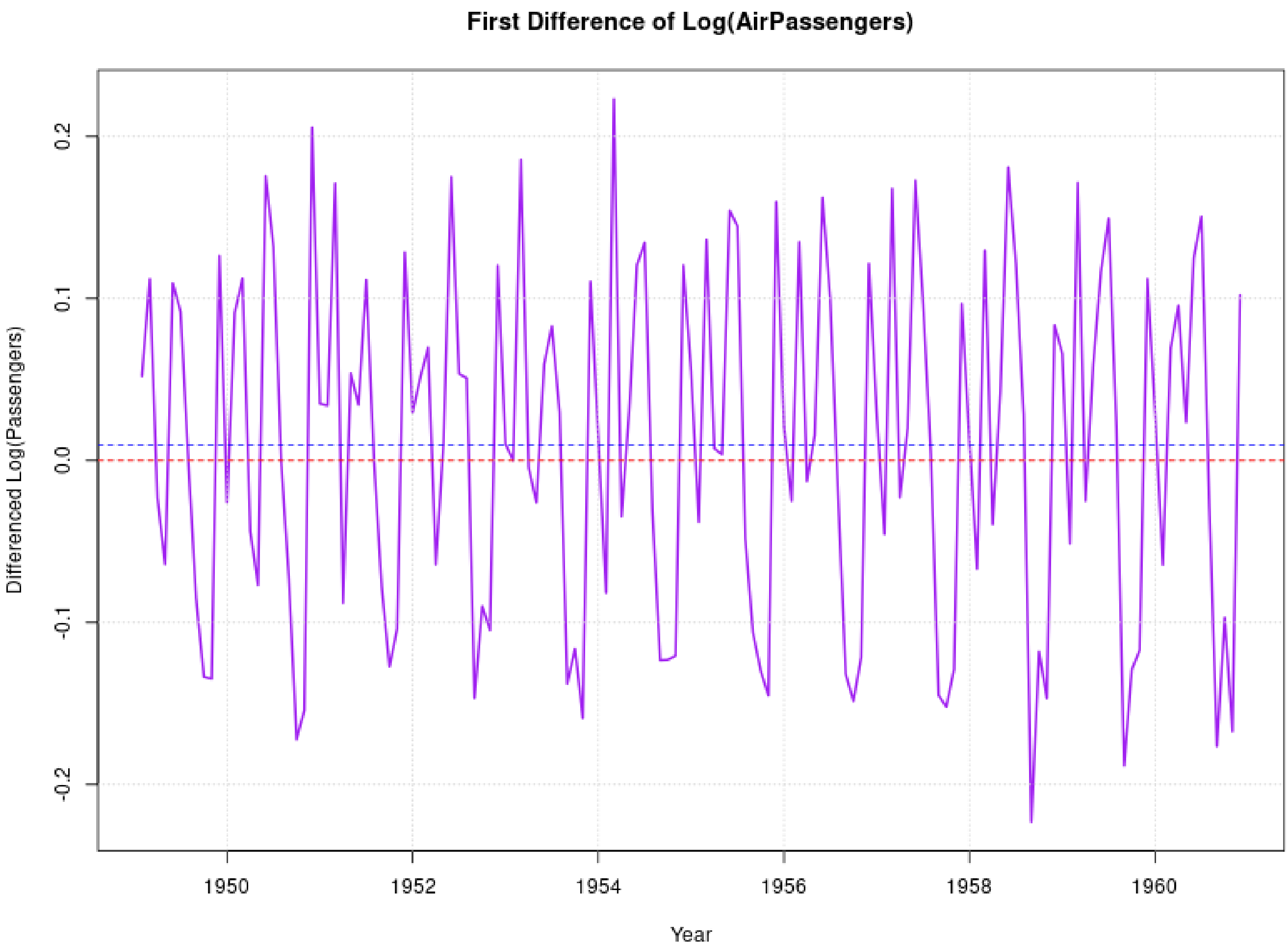


Figure 6: First difference of log-transformed series to remove trend.

Purpose: Remove the trend component **Formula**: $\nabla\log(X_t) = \log(X_t) - \log(X_{t-1})$ **Effect**: Series now fluctuates around a constant mean (zero line)

3. Seasonal Differencing

```
# Apply seasonal differencing (lag=12) to remove seasonality
seasonal_diff <- diff(diff_log_passengers, lag = 12)

png("plot7_seasonal_diff.png", width = 800, height = 600)
plot(seasonal_diff,
     main = "Seasonal Difference of Differenced Log(AirPassengers)",
     xlab = "Year",
     ylab = "Seasonally Differenced Values",
     col = "orange",
     lwd = 2)
grid()
abline(h = 0, col = "red", lty = 2)
dev.off()
```

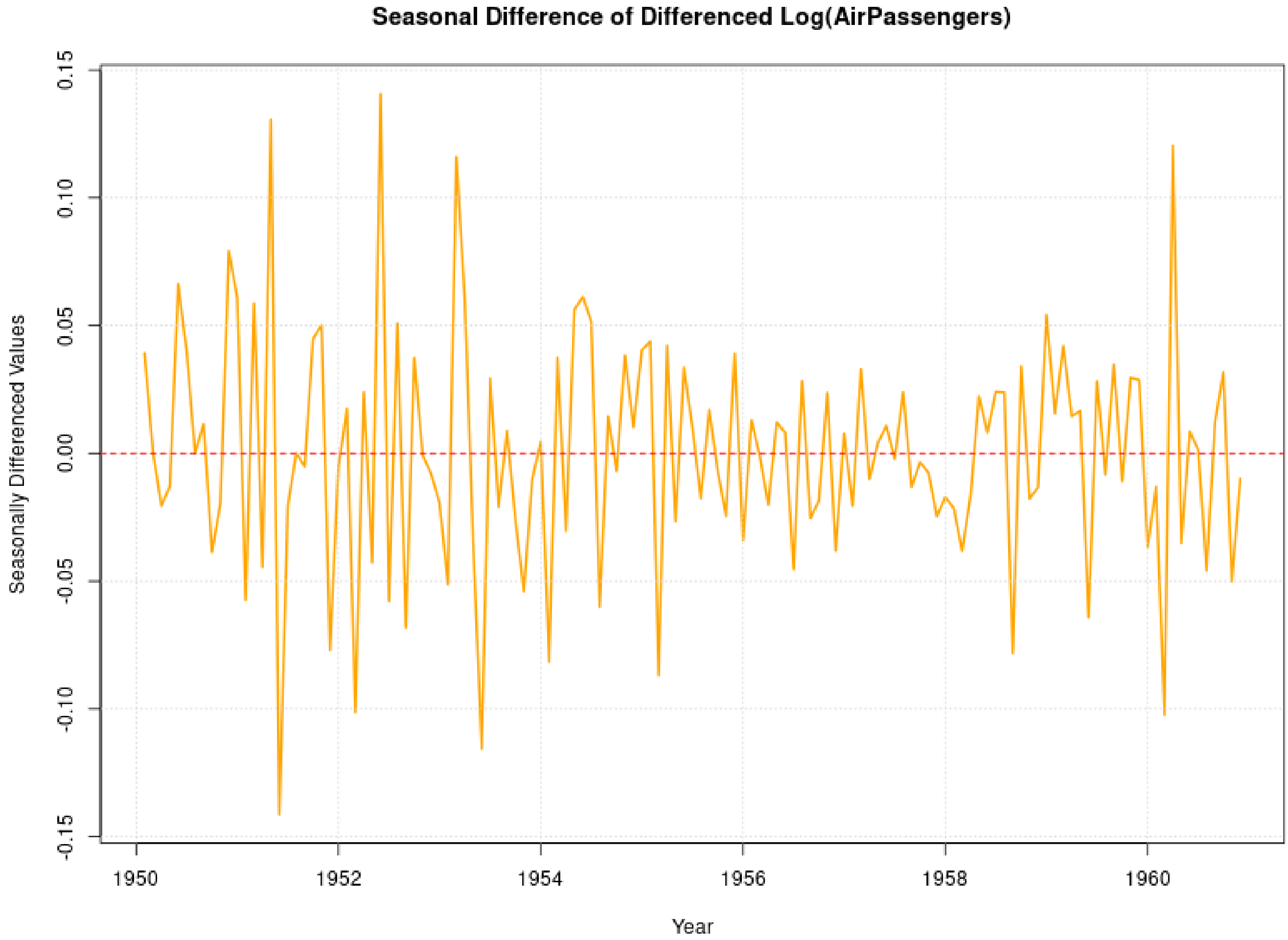


Figure 7: Seasonal differencing (lag=12) to remove seasonal pattern.

Purpose: Remove the seasonal component **Formula**: $\nabla_{12}\nabla\log(X_t) = \nabla\log(X_t) - \nabla\log(X_{t-12})$ **Effect**: Removes the yearly seasonal pattern

Final ADF Test Result: After transformations, the series becomes **STATIONARY** (p-value < 0.05)

Summary of Findings

Analysis Results Table

Analysis Component	Method	Result
Data Type	Built-in R dataset	Time series (144 obs)
Time Period	Range	1949-1960 (12 years)
Trend	Visual inspection	Strong upward growth
Seasonality	Decomposition	Clear yearly pattern
Variance	Visual inspection	Increasing (multiplicative)
Stationarity (Visual)	ACF/PACF	Non-stationary
Stationarity (Statistical)	ADF Test	Non-stationary (p > 0.05)

Analysis Component	Method	Result
Required Transformations	Log + Differencing	Achieves stationarity

Key Findings

1. **Data Characteristics**:
    - 144 monthly observations from 1949 to 1960
    - Range: 104k to 622k passengers
    - Nearly 6x growth over the period
2. **Dominant Components**:
    - **TREND**: Strong, consistent upward growth
    - **SEASONALITY**: Pronounced yearly pattern (summer peaks)
    - Both components are highly significant
3. **Stationarity Assessment**:
    - Original series: **NON-STATIONARY**
    - ACF: Slow decay confirms non-stationarity
    - ADF Test: Confirms non-stationarity
4. **Variance Structure**:
    - **Multiplicative** model is appropriate
    - Seasonal variation increases with level
    - Log transformation needed to stabilize
5. **Transformations for Stationarity**:
    - Step 1: Log transformation → stabilize variance
    - Step 2: First differencing (d=1) → remove trend
    - Step 3: Seasonal differencing (D=1, s=12) → remove seasonality
6. **Model Recommendation**:
    - **SARIMA Model**: Seasonal ARIMA
    - Suggested: **ARIMA(p,1,q)(P,1,Q)[12]**
    - Where:
        - d = 1 (regular differencing)
        - D = 1 (seasonal differencing)
        - s = 12 (seasonal period)

Comparison: AirPassengers vs Bank Loans

Aspect	AirPassengers	Bank Loans (Practical 5)
Trend	Strong upward	Strong upward

Aspect	AirPassengers	Bank Loans (Practical 5)
Seasonality	**Strong yearly pattern**	Weak/moderate
Variance	**Increasing**	Relatively constant
Model Type	**Multiplicative**	Additive
Transformation	**Log + difference**	Difference only
Stationarity	Non-stationary	Non-stationary
Model	SARIMA	ARIMA

Running the Analysis

```
cd "TSA/Practical 6"
Rscript practical6.r
```

Or in R console:

```
source("practical6.r")
```

The script will:

1. Load and analyze the AirPassengers dataset
2. Generate 7 PNG plots showing different aspects of the analysis
3. Perform statistical tests for stationarity
4. Provide detailed interpretation of results

Key Concepts

Time Series Decomposition

Additive Model: Y = T + S + R

- Used when seasonal variation is constant

Multiplicative Model: Y = T × S × R

- Used when seasonal variation increases with level
- **AirPassengers uses this model**

Seasonal ARIMA (SARIMA)

General form: **ARIMA(p,d,q)(P,D,Q)[s]**

Where:

- (p,d,q): Non-seasonal components
  - p: AR order
  - d: degree of differencing
  - q: MA order
- (P,D,Q): Seasonal components
  - P: Seasonal AR order
  - D: Seasonal differencing
  - Q: Seasonal MA order
- [s]: Seasonal period (12 for monthly data)

Transformations

1. **Log Transformation**: $$Y_t' = \log(Y_t)$$

   - Stabilizes variance
   - Converts multiplicative to additive

2. **First Difference**: $$\nabla Y_t = Y_t - Y_{t-1}$$

   - Removes trend
   - Order of integration: I(1)

3. **Seasonal Difference**: $$\nabla_s Y_t = Y_t - Y_{t-s}$$

   - Removes seasonal pattern
   - s = seasonal period (12 for monthly)

Theoretical Background

Properties of Stationary Series

A stationary process satisfies:

1. **Constant Mean**: $$E[Y_t] = \mu$$
2. **Constant Variance**: $$Var(Y_t) = \sigma^2$$
3. **Lag-dependent Covariance**: $$Cov(Y_t, Y_{t+k}) = \gamma_k$$

AirPassengers violates all three conditions in its original form.

Box-Cox Transformation

Alternative to log transformation: $$Y_t' = \begin{cases} \frac{Y_t^\lambda - 1}{\lambda} & \lambda \neq 0 \\ \log(Y_t) & \lambda = 0 \end{cases}$$

For AirPassengers, $\lambda \approx 0$, confirming log transformation is appropriate.

Analysis Complete