Simulation of a Theoretical Photonic Neuron under Noisy Conditions

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Introduction

The purpose of this paper is to explain the design of an experimental model simulating a photonic, computational device and present results from simulations of this device under noisy conditions. The device modeled is a theoretical device, which uses a Vertical Cavity Surface Emitting Laser (VCSEL) to behave similarly to a biological neuron but which could pick up noise on its input channels. A similar computational device, known popularly as The Photonic Neuron, already exists in a physical, working form. The workings of The Photonic Neuron will be described in detail to establish an idea of the type of computation being carried out. The VCSEL approach, which serves as an alternative design to The Photonic Neuron, possesses several attractive features. The model studies the isolated effects of noise on the performance of the VCSEL. First, the workings of The Photonic Neuron will be described, and then the VCSEL design will be laid out. The model of the VCSEL system will be explained, and the causes of noise will be described. Then the predicted results of the tests will be stated, followed by details of testing and findings.

1 Princeton's Photonic Neuron Project

The Photonic Neuron Project is an experiment exploring cutting edge computational methods and devices. A team at Princeton's Lightwave Communications Laboratory, under the direction of Professor Paul Prucnal, have successfully built and tested an optic device modeled after an organic neuron's structure. Similar projects have been undertaken using electronic circuits [1]. However, the speed of electronic devices is naturally much slower than that of a photonic system [1]. The photonic neuron structure holds promise in a wide variety of applications and is scalable to create complex neural networks for a wide variety of uses [1].

The general structure of the system combines analog and digital methods of computation to simulate the workings of an organic neuron. The Leaky Integrate and Fire Neuron (LIF) is a well established model that neuroscientists use to describe the behavior of neurons [1]. The LIF system combines advantages of both analog and digital computing to yield quick and accurate results. The Photonic Neuron operates in a fashion similar to the LIF's structure and consists of two major sections, an integrator and a thresholder: each is described below [1].

1.1 Structure of the Neuron

In the LIF model of a biological neuron, a tree of dendrites accepts signals and attaches weights and delays to these signals [1]. In a biological neuron, electro-chemical spikes are transferred across synapses [1]. This information is carried across the synapse via diffusing chemicals [1]. Depending on how a system should react to an input, the synapse can carry a different delay and different strength attached to each signal [1]. The popular phrase "nerves that fire together, wire together" reflects that neural connections' weights and delays vary across inputs. To simulate this effect, each input of the photonic neuron has a delay and weight attached to it. Typically, these signals are transmitted and arrive as electronic pulses. In the photonic neuron, the pulses are carried as pulses of light.

1.1.1 The Integrator

Each of the weighted and delayed pulses enters the next section of the photonic neuron, the integrator. In a biological neuron, signals are integrated by the soma [1]. In the leaky integrate-and-fire model of a neuron, the behavior of each cell's soma can be modeled by the following equation:

$$\frac{dV_m}{dt} = \frac{V_{rest}}{\tau_m} - \frac{V_m(t)}{\tau_m} + \frac{1}{C_m} V_m(t)\sigma(t)$$

Where C is a capacitance associated within the cell, V is the voltage of the cell at the axon hillock, $\tau_m$is a time constant associated with the cell's ability to reset, and $\sigma(t)$ is a function representing the inputs to the cell [4]. Prucnal's team simulates the soma with a semiconductor optical amplifier (SOA) [4]. This device integrates the input signals but has a leaking pattern governed by the following rate equation:

$$\frac{N^{'}(t)}{dt} = \frac{N'_{rest}}{\tau_e} - \frac{N^{'}(t)}{\tau_e} - \frac{\Gamma \mathrm{a}}{\mathrm{E_p}} \mathrm{N}^{'}(\mathrm{t})\mathrm{I}(\mathrm{t})$$

Where *N* represents the carrier concentration or the number of excited electrons within the semiconductor. The $\frac{\Gamma a}{E_p} N^{'}(t)I(t)$ term represents the input pulses gathered by the integrator. N'_{rest} term represents a resting carrier concentration to which the neuron will return. The τ_c term is a time constant which determines how quickly power leaks from the integrator [3].

One can easily recognize the striking similarities between the equations governing the SOA's behavior and the soma's behavior. The two function in a very similar fashion: if either neuron is hit with a single pulse, then after a time, the neuron will "reset," returning to the resting potential, N'_{rest} , at a rate determined by τ_c. If we solve the rate equation, we realize that this decay takes place in an exponential fashion. If the neuron receives several pulses close together, the pulses, depending on timing and magnitude, interact together to stimulate the neuron to higher potentials. Some pulses may also be "inhibiting pulses" that have the effect of lowering the potential of the neuron [4]. We can think of these signals as having a negative weight attached. In the photonic neuron, these pulses are carried on a different wavelength [4].

Since there are several inputs to each neuron, the $\frac{\Gamma a}{E_p} N'(t) I(t)$ term can be rewritten as $\sum_{i=1}^{k} N'_i(t)$ where k is the number in inputs and the $N'_i(t)$ term represents the spaced pulses of each input. Therefore the final term becomes a sum of each input's pulses. We can now rewrite the transfer function as

$$\frac{N'(t)}{dt} = \frac{N'_{rest}}{\tau_e} - \frac{N'(t)}{\tau_e} + \sum_{i=1}^{k} N'_i(t)$$

Since the neuron is integrating over a large time frame, in early simulation, the pulses received by the integrator can be simplified to delta functions, simplifying the nature of the last term. As the pulses arrive, Prucnal's team uses a mode-locked laser to sample the SOA and send a pulse from the integrator to the thresholder [4].

1.1.2 The Thresholder

The Thresholder receives the pulses sent from the integrator and compares these pulses to a threshold concentration of photons. Ideally, the transfer function of the thresholder would be a step function – after a certain threshold input, a signal of uniform magnitude appears on the output. In this way, the thresholder would act similar to a comparator, producing a signal which is either "on" or "off" like in digital methods of computation. This method also serves to reduce noise buildup across the system, which may appear if only analog processing were used. In reality, the input – output plot cannot be a perfect step function, and follows a pattern similar to figure (2) [2].

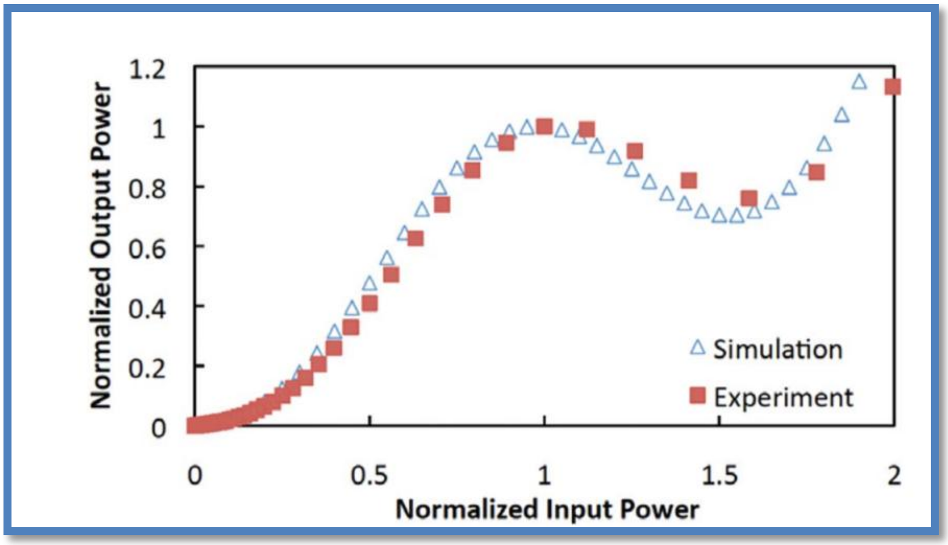


Figure (2): the actual transfer function of the thresholder as drawn from data taken in Princeton's Light wave Communications Laboratory [2].

The thresholder processes signals sent from the integrator and releases pulses if they are of appropriate magnitude. Presumably, this signal is then attached as an input to other neurons or an actuator which is triggered.

2 The Excitable Laser Neuron

The Excitable Laser Neuron is an idea incepted by Mitchell Nahmias, a student working under Professor Prucnal in the Princeton Lightwave Communications Laboratory. The idea is an alternative approach to creating the functionality of a neuron in the photonic system using a VCSEL cavity [4]. This approach has a number of attractive features, notably that it can be more easily scaled than the system described above [4]. The theoretical workings of the device will be described, followed by the advantages offered by this model.

2.1 The New Integrator

Fundamentally, the laser cavity approach to neural computing still contains the two main components in the model discussed above; that is, the leaky integrator and the thresholder [4]. The device itself is, as mentioned before, is a Vertical Cavity Surface Emitting Laser (VCSEL). In the laser, a gain medium is pumped by a constant current to keep the gain medium at a certain carrier concentration [4]. The gain medium acts as an integrator, just like the SOA in the previous model. In fact, this gain medium and the SOA belong to the same class of device and function by the same dynamical equations [4]. The gain medium is also pumped by an input signal, which in this application would be the pulses from other neurons in the network [4]. Within the medium there is a resting carrier potential to which the device returns by a similar time constant to that of the SOA [4]. Excitatory and inhibitory pulses travel on different wavelengths and increase or decrease the carrier concentration in the gain medium [4].

2.2 The New Thresholder

The Thresholder takes the form of a saturable absorber; this device absorbs photons when photon concentration is low and becomes transparent at higher concentrations [4]. The saturable absorber's ability to switch when carrier concentration increases allows it to function as a thresholder in this application. When the carrier concentration within the gain medium is high enough, the concentration of photons is great enough to "bleach" the saturable absorber [4]. When the saturable absorber is bleached, it is transparent and allows photons to exit the laser, thus creating a pulse [4]. After this pulse is released, the carrier concentration of the medium is depleted, and must return to the resting concentration in a time determined by τ_m [4]. During this time, the saturable absorber absorbs photons once again since the carrier concentration is low [4].

3 Differences in the Excitable Laser Neuron

The Excitable Laser neuron possesses several attractive features. The device is small and, given its design, could be fabricated on semiconductor material, making the design scalable for the design of larger, more complex systems. The thresholding characteristic of the saturable absorber also possesses a transfer function very similar to a step function. Additionally, the refractory period after the laser fires allows for a robust pulsing mechanism.

3.1 Refractory Period

When the neuron fires, we desire a single pulse and no other emission from the device; otherwise pulses could have unpredictable duration and power. The saturable absorber serves this purpose well since it becomes bleached at high photon intensities and, after the high intensity burst of photons is released from the laser, the saturable absorber again absorbs photons as a lower density. However, the saturable absorber must undergo a recovery period after bleaching, during which the threshold which will trigger bleaching is lower [4]. This forces a constraint on the recovery period of the gain medium. Having been depleted by the outgoing pulse, the gain medium's recovery time must be relatively long compared to the recovery period of the absorber, otherwise false pulses will be sent through the network. The long recovery period can be engineered into the device [4]. This creates a condition in which outgoing spikes will not occur too closely together and will maintain a consistent power and shape. We will see this refractory period illustrated in simulation.

3.2 Thresholding Characteristic

The saturable absorber has a transfer function similar to a step function; however, the transfer function is still not flawless. From figure (3), we can see the saturable absorber responds in a linear fashion to higher power inputs. This would cause significant deviation in the magnitude of output spikes, which is undesirable.

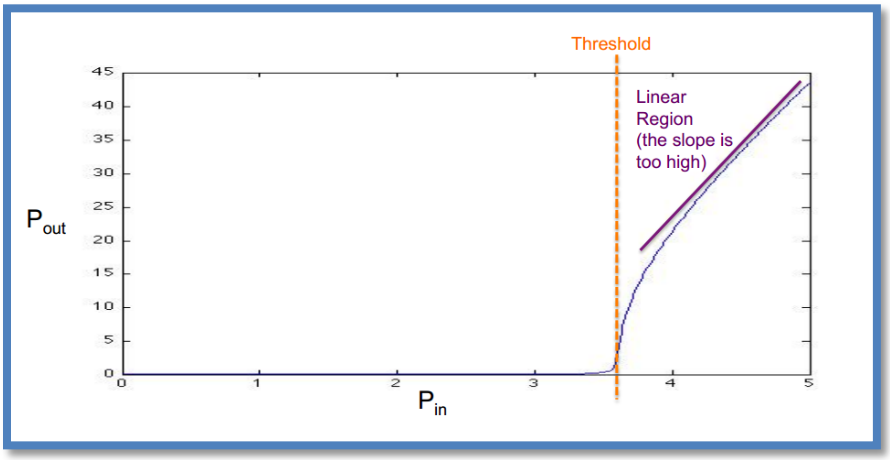


Figure (3): the transfer function of the saturable absorber [4].

However, as Nahmias points out, this problem can be solved by designing the device to operate close to the threshold point of the saturable absorber [4]. The new transfer function can be seen in figure (4). This operating region yields a response much closer to a step function, and higher magnitude inputs would not result in significant variance in magnitude on the outputs.

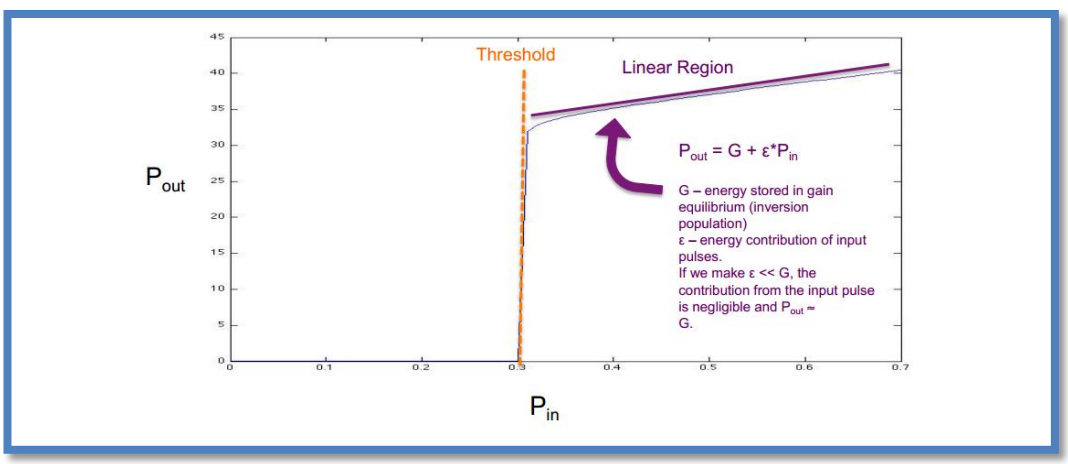


Figure (4): the response of a saturable absorber when inputs are close to the threshold of the device [4].

4 Modeling the Excitable Laser Neuron

The motivation for modeling the network is to predict what errors will arise at different noise levels. The current plan for constructing the neuron connects a photo detector to the laser itself to feed in the signals from the inputs [4]. However, the current generated by such a photo detector may be too small to have the proper effect on the laser medium. Therefore, the signal traveling out of the detector may need to be amplified. However, Shot noise and Thermal noise picked up in the receiver would be amplified with the input signal. There are also many other sources of noise in the system which could cause similar problems to this amplified noise. We simulated Gaussian noise to get an idea of general noise levels that would affect performance. These general levels would give us an estimate for how much noise can be present on the input entering the gain medium. Adding Shot noise would be a different task, since shot noise will show up more significantly on the peaks due to higher photon concentration.

4.1 Procedure

The original model, created by Nahmias, uses a set of modified Yamada equations shown below:

$$G = \gamma_1(A - G + G(\sigma(t) - I))$$

$$Q = \gamma_2(B - Q - aQI)$$

$$I = (G - Q - 1 -)I + \varepsilon f(G)$$

G represents the gain, Q represents the loss, I is the photon concentration, A is the bias current of the gain, B represents absorption, and a is the differential absorption relative to differential gain [4]. γ_1 is the relaxation rate of the gain, and γ_2 is the relaxation rate of the absorber [4]. $\sigma(t)$ is the resulting increase in optical intensity delivered by the inputs [4]. $\varepsilon f(G)$ represents the spontaneous emission dependent on G; however, this value is small [4]. The model solves these equations given realistic parameters and a given $\sigma(t)$ and produces an analysis of the gain’s photon concentration, the output pulses, and the state of the saturable absorber [4]. Using the results, we can observe the theoretical behavior of the excitable laser device.

4.2 Functionality under Ideal Conditions

With no noise addition, the device theoretically functions as follows:

- Pulses from various inputs are collected into photodetectors and summed by the photodetectors to act as one continuous input of current into the gain medium. In figure (5), the Input Spikes represent this continuous signal traveling into the gain medium.
- The carrier density within the gain medium is increased or decreased depending on weather the pulse is excitatory or inhibitory. The carrier density always tends toward the equilibrium carrier density by the functions described earlier.
- When the carrier density of the gain medium crosses the threshold level, the saturable absorber bleaches and pulse is released, exhausting the carrier concentration in the gain medium to low amplitude.
- As the threshold of the saturable absorber recovers, the carrier density in the medium recovers as well, but at a much slower rate than that of the saturable absorber. This prevents misfires quickly after an output spike. Each of these effects can be observed in figure (5).

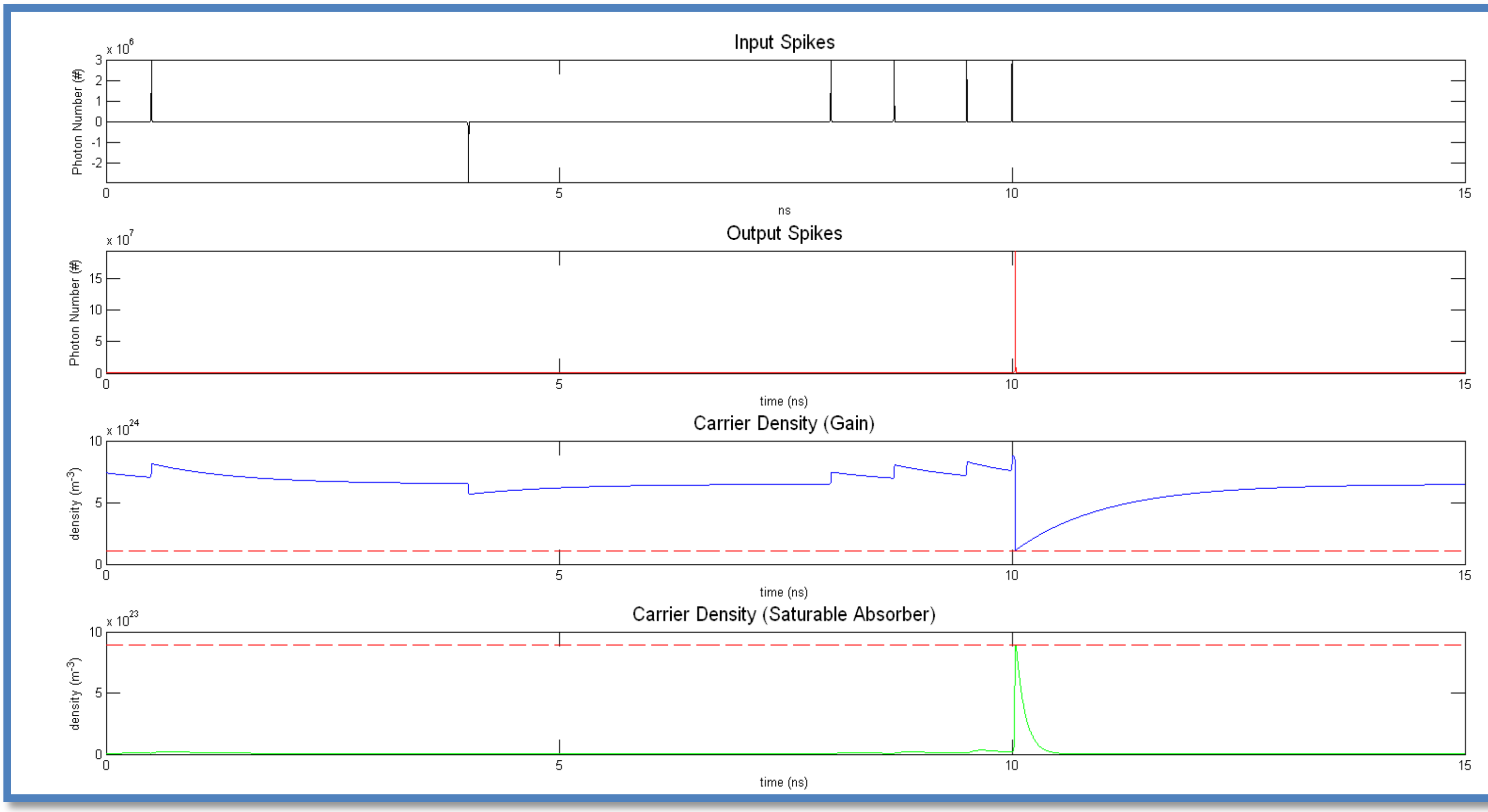


Figure (5) shows the ideal behavior of the neuron without noisy conditions. Key features include the summed excitatory and inhibitory pulses, output spikes, carrier density in the gain medium, and recovery times of both the gain medium and the saturable absorber.

4.3 Noise Addition

To observe non-ideal conditions, a function was added to create Gaussian noise on both the excitatory input and inhibitory input. We assume that noise appears on both inhibitory and excitatory channels equally.

5 Predictions

We can easily predict results by viewing noise on either input channel as a low power pulse which has been falsely sent. Noise on the excitatory channel would therefore excite the gain medium and possibly cause the device to accidently pass the saturable absorber's threshold and output a pulse. This kind of error is more likely if the device is operating close to the saturable absorber's threshold, especially if the gain medium has been excited by previous pulses. On the other hand, noise on the inhibitory channel would cause decrease in carrier concentration, perhaps inhibiting the saturable absorber from bleaching and preventing a pulse from being sent. Either of these mistakes could be costly since losing all information attached to a pulse or gaining extra information attached to a false pulse could cause highly undesired results elsewhere in the network.

Other problems that could occur would affect the timing and magnitude of output pulses. Noise may cause the saturable absorber to bleach too soon or too late, therefore causing timing jitter on output pulses. Since pulses in this system carry a great deal of information with the time at which they arrive, timing jitter can cause undesired results elsewhere in the network. Also, given that the transfer function of the saturable absorber is not a perfect step function, noise on input channels could increase the power of the input and thus cause variation in the power of the output pulses. This could result in stronger pulses being sent and over exciting other neurons due to their larger power.

Finally, given the nature of the saturable absorber, the device could become partially bleached but not completely cross the threshold. This would result in a malformed pulse – a pulse of this nature would yield unpredictable results.

6 Results

Test inputs consisted of six pulses: one inhibitory and five excitatory. Two pulses were spaced apart from the rest, and four were placed close together to emit a pulse. Each trial was simulated for 15 ns, and the carrier concentration began at a low density. In each of the tests, the signal to noise ratio was increased until a consistent failure rate was achieved. Naturally, each test takes longer to run depending on how much noise is present on the input channel. Each SNR was repeated 200 times for accurate results that could be achieved in a timely manner (given the computing power available).

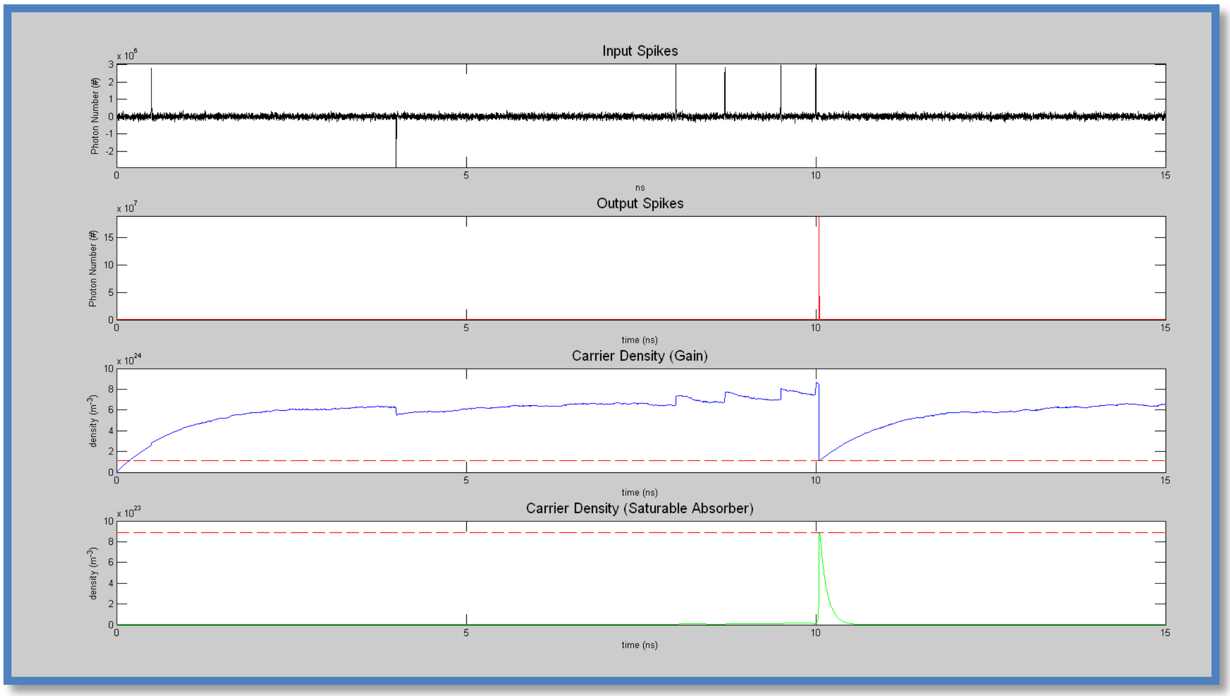
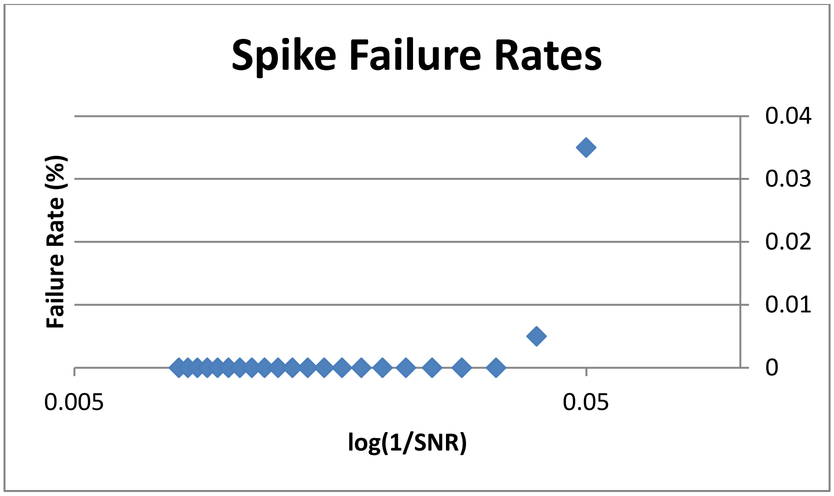


Figure (6): the neuron's function under a SNR of 20 dB

6.1 Pulse Failure

The critical test was a situation in which a pulse would be emitted under ideal conditions. If fewer input spikes were on the channel, the SNR would have to be higher to yield a false pulse, and therefore these scenarios would not arise before the critical test experienced failure. Therefore, we tested a number of pulses which, under ideal conditions, would yield a single output pulse. If this pulse were missing, we marked that trial as a failure. Figure (7) shows the failure rates for various magnitudes of noise.



Figure(7)

We can see that at an SNR of 25 dB, a lone neuron experiences failure rates around 0.5%, and at 20 dB, the failure rate jumps to 3.5%. Presumably, the failure rate increases in an exponential fashion after this (these tests have not yet been run). These noise levels are actually quite high, and given that neural networks can be designed with redundancy for additional robustness, these error rates may be acceptable depending on the system's design. These results give us a good estimation point at which redundant design would become a necessity to deal with noise.

6.2 Timing Jitter

For the same tests, the variance in output spike timing was measured. As explained before, the timing of a spike carries the majority of its information and thus is crucial in determining how it affects neurons. Figure (8) shows the results of noise on timing jitter of output pulses.

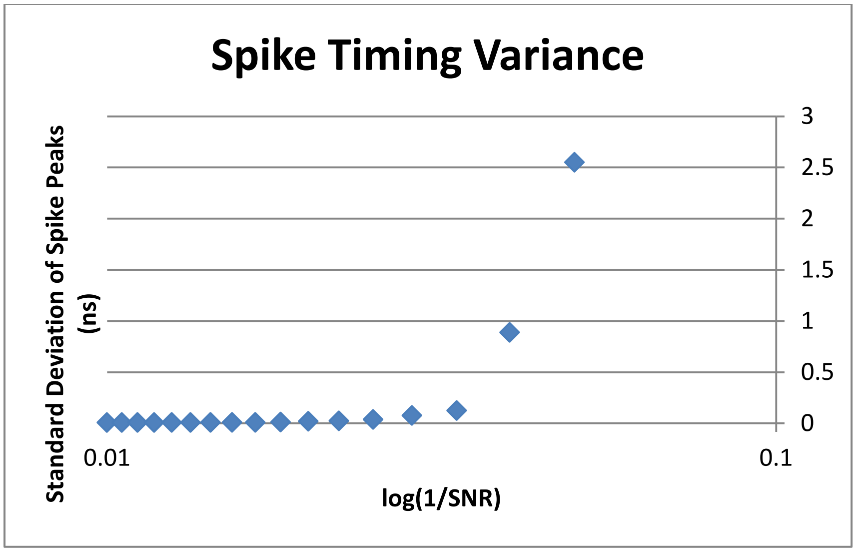


Figure (8)

We can see that the variance of the timing of the peaks does not change significantly until about the same noise levels at which peaks failed to appear. Similarly, we can also guess that this failure rate increases exponentially at lower SNR's.

6.3 Pulse Magnitude

In our tests, the magnitude of pulses did not vary significantly with noise ratios. The variance had no clear correlation with SNR and, therefore, conclusions cannot be drawn about SNR's effects on pulse magnitude. It is possible that the pulse magnitudes will display more variance at higher noise levels, but these tests have not yet been run.

6.4 Malformed Pulses

Malformed pulses were not an observed issue in these tests. One controlled example was created under ideal conditions; the inputs stimulated the neuron very close to the threshold without firing. Figure (9) illustrates the results of the simulation. The first three peaks on the output are expected and are small enough that they can be thrown out. The fourth peak on the output is a malformed pulse; however, the magnitude of the pulse is very small even in comparison to the noise levels previously tested. This pulse would therefore be lost in noise on the input channels to other neurons.

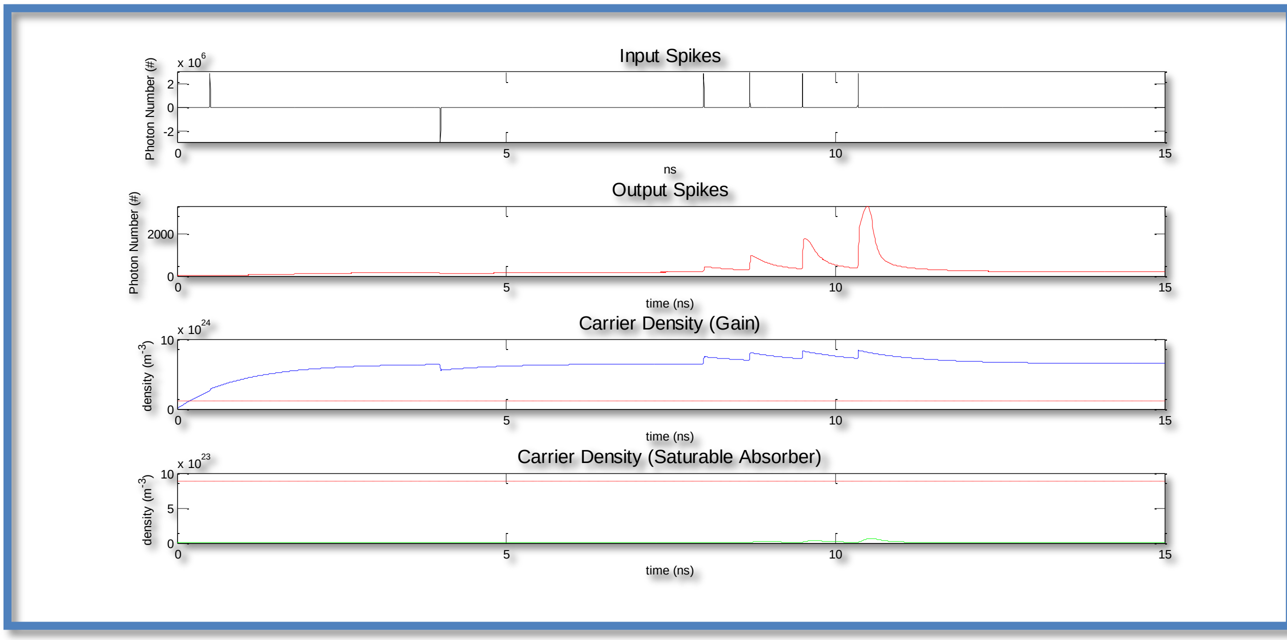


Figure (9): illustration of malformed pulses emitted by the neuron.

7 Conclusion

Overall, the VCSEL approach to photonic computing can withstand significant noise levels and remain functional. When the system actually begins the fail, the error rates are low and controllable. However, it is important to note that these failure rates are highly dependent on how close the carrier concentration in the gain medium is to the threshold and how long the carrier concentrations remain at these levels. Therefore, depending on the specific system, error rates may vary with noise levels. These tests supply a sample set of noise levels at which a system of this kind can function. If noise vastly exceeds this level when the device is built, alternative models may have to be considered. If noise levels are manageable or result in low error rates, building redundancy within networks may provide a solution. These kinds of redundancies would determine what error rates are acceptable within the network while retaining reasonable accuracy. Overall, the project provides much promise in the field of optical computing; the speed and scalability of the system make it unique and attractive.

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