Finding an Earth Analog
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1. Introduction/Motivation
The study of exo-planets has become a huge topic of research lately due to recent advancements in technology; data is being received from both our land-based and orbital telescopes daily. However, the data alone doesn't automatically inform us of an exo-planet's presence, astronomers have had to develop several methods to detect these planets outside our own solar system. These detection methods can tell us a lot about the exo-planets we find: orbital speeds, radius, etc. It brings a familiar question to mind of 'could we find another Earth out there?' Is it possible to find a planet orbiting a star, not unlike our own? A planet in a similar orbit? Similar size?

We do run into the unfortunate case of sample bias, with our data being limited to the furthest reaches of our instruments' capabilities. We are then limited even more by the individual capabilities of each detection method available. The different detection methods all have their own advantages and setbacks. For instance, in order to use the Transit method, the system in question is required to be at a certain inclination with respect to the Earth; this is to make sure the possible exo-planets are able to pass in front of their star from our point of view. Fortunately, this is not the only method available or else we would have to discard countless systems that don't line up nearly perfectly edge-on. We use the others to fill in most of the gaps left behind by any one of the methods. In this project, we will be looking for the limitations that each technique presents.

2. Methods
The observation methods that were tested consisted of radial velocity, transit photometry, astrometry, and direct imaging. Each of these subsections derives the equations for each method and rationalizes the various assumptions made for the variables used in those equations.

2a. Radial Velocity = 8.944 x 10^-2 m/s
The expression for the Radial Velocity signal of a planet can be found below.

$$K = \frac{m_p}{m_*} \cdot \sqrt{\frac{Gm_*}{a}} \cdot \sin i$$

With K being the amplitude, G being the known gravitational constant, a as the semi-major axis (orbit radius), and the masses m_* and m_p being the mass of the star and planet respectively. The $\sin i$ at the end represents the inclination. Here, we assume the inclination is edge-on, or equal to 1, this is done in order to maximize the RV signal. After assuming $\sin i = 1$, we can rearrange the formula to solve for the mass of the planet as seen below.

$$m_p = K \cdot m_* \cdot \sqrt{\frac{a}{Gm_*}}$$

We will also assume K to equal $0.5ms^{-1}$ (per Dr. Ji wang) and the mass of the star to be $0.5m_{sun}$. These assumptions are due to the fact that we are looking for sun-like stars to better find planets in similar situations as to our Earth.

2b. Transit Depth = 8.389 x 10^-3 kilometers
For the transit method there are two important things needed to be taken into account: the transit depth and the transit duration. Below is a simplified expression for transit depth.

$$f = \left(\frac{R_p}{R_*}\right)^2$$

Where f is the fractional depth and the radii R_p and R_* represent the radius of the planet in question and its star respectively.
Transit Duration = 4.252 x 10^3 seconds
Then there is a simplified expression for transit duration below.

$$\text{Transit duration: } t_{\text{transit}} \approx \frac{2R_*}{v_K}$$

With v_K being the planet's average velocity.
We will go on to assume that there is an orbital period of about 90 days, which is accounted for in the planet's velocity due to Kelper's laws: $P^2 \propto a^3$, almost a third of our own orbital period. This method also requires us to assume the system is in roughly the same plane as the Earth so that we are able to detect that the planet does pass in front of its star from our point of view. If the system was, say, in a top down view in our point of view, then we wouldn't be able to detect any exo-planets using this method because we will never see them pass in front of their stars with a dip in light.
The state-of-the-art detections use a method called the Transit Least Squares Algorithm (TLS) (Hippke, Heller 2019). According to Hippke's paper, this algorithm has been optimised to search for smaller planets (although, they are still able to detect larger planets using this algorithm).

2c. Astrometry = 3.639 x 10-14 radians
Astrometric detection signals can be found through the angular separation θ below.

$$\theta = \frac{m_p}{m_\star} \frac{a}{d} = \left(\frac{G}{4\pi^2}\right)^{1/3} \frac{m_p}{m_\star^{2/3}} \frac{P^{2/3}}{d}$$

Where m_p and $m_\star$ represent the masses of the planet and star respectively, a is the semi-major axis, d is the distance the star system is from Earth (typically measured in pc), G is the Gravitational Constant, and P is the planet's orbital period. GRAVITY, an instrument of the European Southern Observatory Very Large Telescope Interferometer (Abuter et al., 2017), has a precision of $100\mu as$.

2d. Direct Imaging

    f = 8.389e-05

Direct imaging uses an equation for a star-planet contrast as well as one for angular separation.

$$f = \left(\frac{R_p}{R_*}\right)^2 \frac{\exp(h\nu/k_B T_*) - 1}{\exp(h\nu/k_B T) - 1} \sim 10^{-6}$$

Here, f is the contrast ratio, R_p and R_* are the radii of the planet and star respectively, h is the known Planck Constant, ν is the frequency of light, k_B is the Boltzmann constant, T is the planet temperature, and T_* is the temperature of the star.

$$\theta \sim 1.22 \frac{\lambda}{D}$$

Where θ is the angular separation, D is the telescope diameter, and λ is the observation wavelength.

For Direct Imaging, one of the best instruments we have available to us for locating exo-planets is the Gemini Planet Locator at the Gemini South Telescope in Chile. This instrument can resolve down to about 0.7 arcsec (McBride et al., 2011) and detect $f > 10^{-7}$ (Wallace et al,. 2010).

3. Results

Current techniques and technologies have done an excellent job so far in regards to finding exo-planets. However, they are best at finding large planets and planets with small semi-major axes; neither of these are indicators of an Earth analog.

When using the Radial Velocity method, we can see a star moving ever so slightly and can use the red and blue shifts to detect exo-planets. Our calculated semi-amplitude for an Earth analog is K = 8.944 x 10^-2 m/s (~0.09m/s). When it comes to current state-of-the-art techniques for detecting planets through the Radial Velocity method, there are as of now, two of them. The first technique EXPRESSO has the K value being between: K=0.25m/s to 0.5 m/s. The second

technique EXPRES has an ideal value of K being: K >0.1m/s. Both of these techniques aren't sensitive enough to detect an Earth analog yet.

As mentioned before, the Transit method looks for a dip in the spectra of light we receive from a star. Regular, periodic drops in luminosity typically mean there is a body in orbit around that star that passes in between the star and Earth. One method we looked at in particular is the Transit Least Squares algorithm. This method operates with a minimum transit duration of 0.00125 times the orbital period (Hippke), where the Transit Depth is: 8.389 x 10^-3 kilometers, and the Transit Duration is: 4.252 x 10^3 seconds. The conversion would be about: 0.00125 (90 days) which is 9,720 seconds. Therefore, an Earth-like planet would not be able to be detected using the transit method.

The method of Astrometry measures a star's motion and position on the sky. The signal for an Earth Analog has an angular separation of $\Theta = 3.639$ x 10^-14 Radians. As of now, State of the art detection, GRAVITY as mentioned before, can measure an angular separation of $\Theta = 100 \mu as$ which is 4.8481368 x 10^-10 Radians. Our instruments aren't yet sensitive enough to measure an angular separation as small as what we expect an Earth-like planet to have. So, the Earth analog cannot be detected using Astrometry.

And finally, we have Direct Imaging; this method is true to its name as we are able to directly capture an image of an exoplanet around another star. Our calculated star-planet contrast for an Earth-like planet is f = 8.389 x 10^-5. A small number so one would assume that planets as small as Earth would be very difficult to discern through Direct Imaging. Well, as mentioned earlier, Gemini is currently able to detect objects that small being more than 10^{-7}. So, our calculated value doesn't make sense and a mistake must have been made somewhere along the way. Another factor to consider with Direct Imaging is angular resolution. Here, we assumed a distance around 400pc from us and that the planet is also 1AU from its star like Earth is from the Sun. This would put the angular resolution at around 0.1 arcsec which is much less than the current best sensitivity as mentioned before at 0.7 arcsec. This alone tells us we shouldn't be able to find any Earth analog using this method.

4. Discussion/Conclusions

We set out to find out if it was possible to detect Earth-like planets around a Sun-like star with our current exoplanet detection methods. Unfortunately, at this time it isn't possible. The planets are too small and too close to their stars. This leads to bias in our current data and demographics of exoplanet populations and really highlights just how limited even our state-of-the-art technology and techniques are. These limitations should be taken into consideration when holistically evaluating current confirmed exoplanet populations.

5. References

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[1]Hippke, M. and Heller, R., "Optimized transit detection algorithm to search for periodic transits of small planets", <i>Astronomy and Astrophysics</i>, vol. 623, 2019. doi:10.1051/0004-6361/201834672.
https://ui.adsabs.harvard.edu/abs/2019A%26A...623A..39H/abstract

First light for GRAVITY: Phase referencing optical interferometry for the Very Large Telescope Interferometer
GRAVITY Collaboration, R. Abuter, M. Accardo, A. Amorim, N. Anugu, G. Ávila, N. Azouaoui, M. Benisty, J. P. Berger, N. Blind, H. Bonnet, P. Bourget, W. Brandner, R. Brast, A. Buron, L. Burtscher, F. Cassaing, F. Chapron, É. Choquet, Y. Clénet, C. Collin, V. Coudé du Foresto, W. de Wit, P. T. de Zeeuw, C. Deen, F. Delplancke-Ströbele, R. Dembet, F. Derie, J. Dexter, G. Duvert, M. Ebert, A. Eckart, F. Eisenhauer, M. Esselborn, P. Fédou, G. Finger, P. Garcia, C. E. Garcia Dabo, R. Garcia Lopez, E. Gendron, R. Genzel, S. Gillessen, F. Gonte, P. Gordo, M. Grould, U. Grözinger, S. Guieu, P. Haguenauer, O. Hans, X. Haubois, M. Haug, F. Haussmann, Th. Henning, S. Hippler, M. Horrobin, A. Huber, Z. Hubert, N. Hubin, C. A. Hummel, G. Jakob, A. Janssen, L. Jochum, L. Jocou, A. Kaufer, S. Kellner, S. Kendrew, L. Kern, P. Kervella, M. Kiekebusch, R. Klein, Y. Kok, J. Kolb, M. Kulas, S. Lacour, V. Lapeyrère, B. Lazareff, J.-B. Le Bouquin, P. Lèna, R. Lenzen, S. Lévêque, M. Lippa, Y. Magnard, L. Mehrgan, M. Mellein, A. Mérand, J. Moreno-Ventas, T. Moulin, E. Müller, F. Müller, U. Neumann, S. Oberti, T. Ott, L. Pallanca, J. Panduro, L. Pasquini, T. Paumard, I. Percheron, K. Perraut, G. Perrin, A. Pflüger, O. Pfuhl, T. Phan Duc, P. M. Plewa, D. Popovic, S. Rabien, A. Ramírez, J. Ramos, C. Rau, M. Riquelme, R.-R. Rohloff, G. Rousset, J. Sanchez-Bermudez, S. Scheithauer, M. Schöller, N. Schuhler, J. Spyromilio, C. Straubmeier, E. Sturm, M. Suarez, K. R. W. Tristram, N. Ventura, F. Vincent, I. Waisberg, I. Wank, J. Weber, E. Wieprecht, M. Wiest, E. Wiezorrek, M. Wittkowski, J. Woillez, B. Wolff, S. Yazici, D. Ziegler, G. Zins
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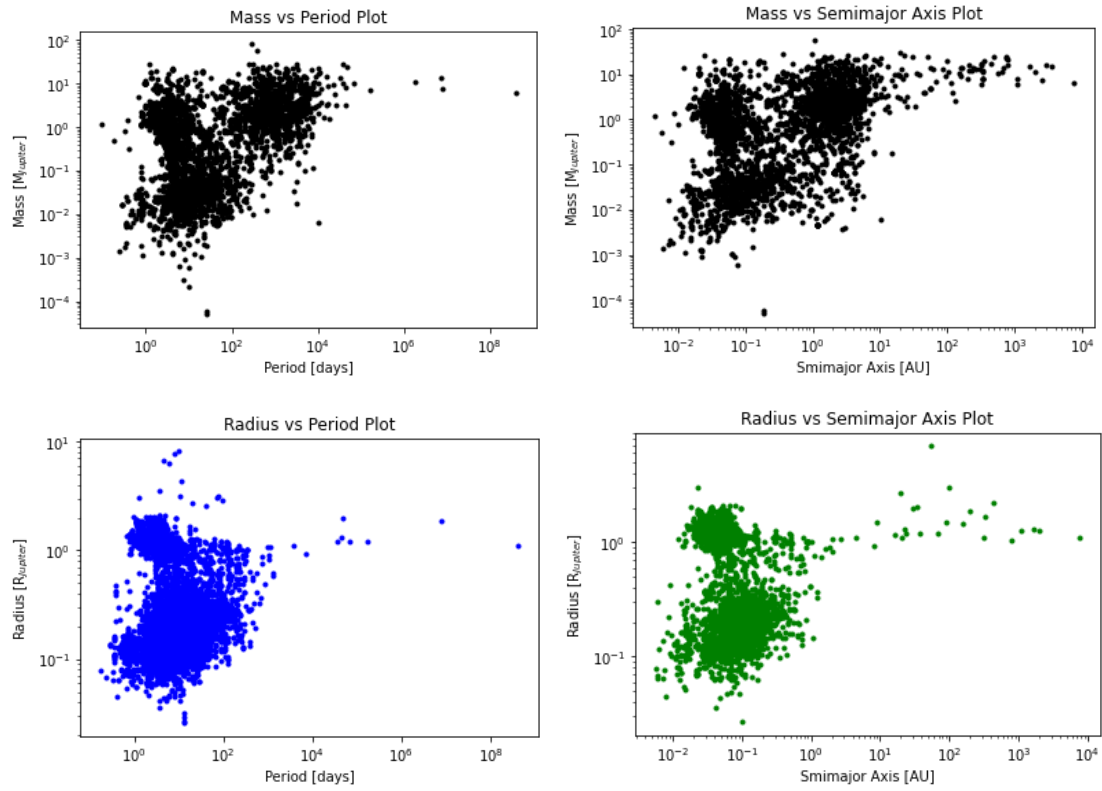DOI: 10.1051/0004-6361/201730838

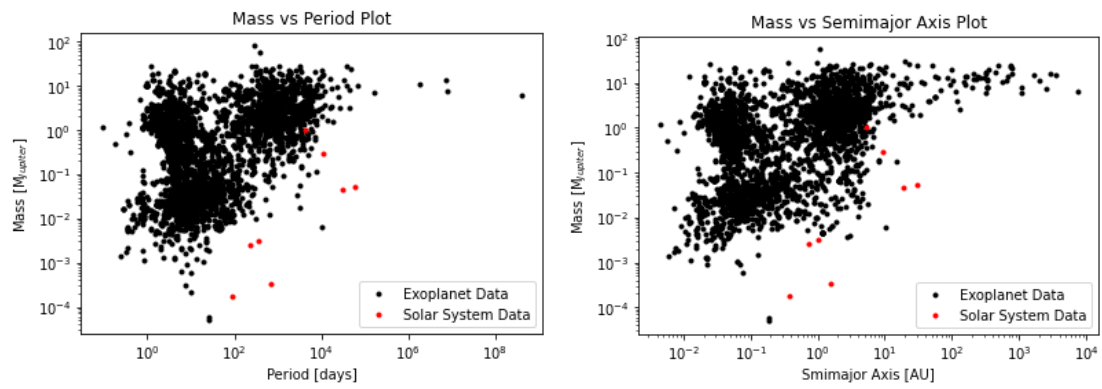Figure 1: Mass and Radius vs. Period and Semimajor Axis Plots.

Figure 2: Mass vs. Period and Semimajor Axis overplot with the Solar System.
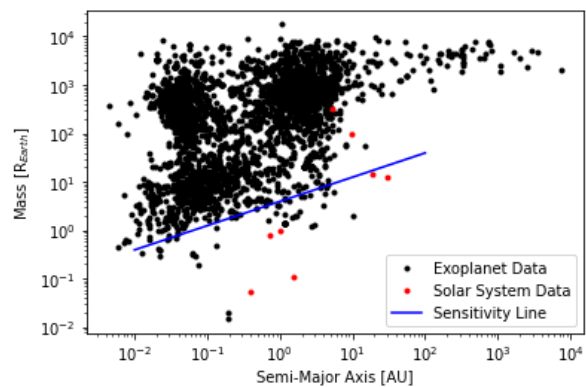
Figure 3: Mass vs. Semimajor Axis overplot with sensitivity line.

6. Contributions