

## [Uncertainties in Measuring Planets]

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### 1. Introduction/Motivation

The study of exoplanets continues to be a popular topic amongst astronomers as new data from our land-based and orbital telescopes comes in daily. Using two methods, astronomers can calculate an exoplanet's mass and radius to find the density of the planet discovered. The measurement of a planet's radial velocity can be used to find its mass while the measurements taken using the transit method can be used to find a planet's radius. Finally, using these two values, the density can be calculated. This can tell us about the planet's inner makeup. For example, Mercury is much smaller than Earth, but is just as dense. That could mean Mercury has a larger iron-rich core.

This can lead to some issues, however. The use of radial velocity and transit methods can lead to some uncertainties in the accuracy of the result. Here we explore the amount of uncertainties each method presents and how this affects the results of a specific planet.

### 2. Methods

Using radial velocity (RV) and transit data from the exoplanet system HD209458 b we are able to come to some conclusions regarding the mass, radius, and density of the exoplanet.

#### 2a. Planet Mass and Uncertainty

The expression for radial velocity is,

$$K = \frac{m_p}{m_*} \cdot \sqrt{\frac{Gm_*}{a}} \cdot \sin(i)$$

For an exoplanet system,  $m_*$  is the mass of the Star and  $m_p$  is the mass of the exoplanet.  $a$  is the semi-major axis of the system.

Rearranging this formula gives us,

$$m_p = Km_* \cdot \sqrt{\frac{a}{Gm_*}}$$

We assume circular orbit,  $e = 0.01$ , to find the minimum mass of the planet in terms of  $m_*$ .

Thus,  $m_* \approx 1.069 m_\odot$  (solar masses). We take the semi-major axis from Rosenthal et al. 2021,

$$a = 6.93 \times 10^9 \text{ meters} = 0.04634 \text{ AU}.$$

To ensure the data is accurate, we use MCMC (Markov chain Monte Carlo) methods to estimate parameters, and take errors into account.

From this, we find  $m_p = 0.7006 M_J$  (Jupiter masses).

An equation for uncertainty of mass is,

$$\left(\frac{\sigma_{m_p}}{m_p}\right)^2 \approx \frac{4}{9} \left(\frac{\sigma_{m_*}}{m_*}\right)^2 + \left(\frac{\sigma_{K_*}}{K_*}\right)^2$$

Using  $\sigma_{K_*} = 0.00826$ ,  $K_* = 84.7 \frac{m}{s}$ , and  $\sigma_{m_*} = 1119.8544 M_J$  as

$$\sigma_{m_p} \approx 7.0847 \times 10^{-5} = 7.0847 \times 10^{-3} \%$$

## 2b. Planet Radius and Uncertainty

The expression for radius of a planet is,

$$\delta = \left(\frac{R_p}{R_*}\right)^2$$

Rearranging we get,

$$R_p = R_* \sqrt{\delta}$$

Where  $\delta$  is the transit depth and  $R_*$  is the radius of the star. The transit depth is assumed to be 1.50% and radius of the star to be  $11.677 R_J$ .

From this we calculate  $R_p = 1.429 R_J$  (Jupiter radiuses).

An equation for uncertainty of radius is,

$$\left(\frac{\sigma_{R_p}}{R_p}\right)^2 \approx \left(\frac{\sigma_{R_*}}{R_*}\right)^2 + \frac{1}{4} \left(\frac{\sigma_{\delta}}{\delta}\right)^2$$

Using  $\sigma_{R_*} = 0.017$  and  $\sigma_{\delta} = 0.0016$ , we get that  $\sigma_{R_p} \approx 0.00222 = 0.222\%$

## 2c. Planet Density and Uncertainty

To find the density of an exoplanet, we use the equation,

$$\rho = \frac{m_p}{\frac{4}{3}\pi(R_p)^3}$$

Using  $m_p$  and  $R_p$  found previously, we find  $\rho = 318.429 \frac{kg}{m^3} = 0.0573 \frac{M_J}{R_J^3}$ .

With this equation for uncertainty,

$$\left(\frac{\sigma_{\rho}}{\rho}\right)^2 \approx \left(\frac{\sigma_{m_p}}{m_p}\right)^2 + 9\left(\frac{\sigma_{R_*}}{R_*}\right)^2 + \frac{9}{4} \left(\frac{\sigma_{\delta}}{\delta}\right)^2$$

Using  $\sigma_{R_p} = 0.00159$  and  $\sigma_{m_p} = 0.0548$ , we get that  $\sigma_{\rho} \approx 0.0398 = 3.98\%$

### 3. Results

The mass of exoplanet HD209458 b was calculated to be  $0.701 \pm 0.00005$  Jupiter Masses and the observed mass is  $0.690 \pm 0.024$  Jupiter Masses (Torres). Our calculated value for the mass falls within the observed uncertainty and therefore is considered to be an accurately performed calculation.

The radius of the exoplanet was calculated to be  $1.429 \pm 0.003$  Jupiter Radii and the observed radius is  $1.359 \pm 0.015$  Jupiter Radii (Torres). The calculated value for the radius falls just short of being within the observed uncertainty, but is still fairly accurate.

The density of the exoplanet was calculated to be  $0.636 \pm .0253 \text{ kg/m}^3$ . This level of uncertainty is higher than that of the mass and radius, this is due to the fact that we used both mass and radius to calculate density, which both have a level of uncertainty themselves.

Observed:  $0.338 \text{ g/cm}^3$  (Torres)

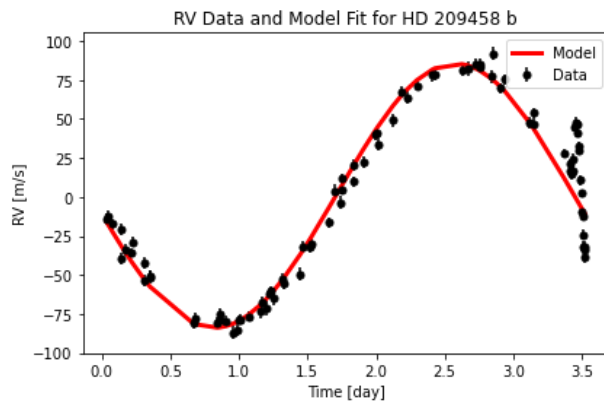


Figure A - Radial Velocity Data and Model Fit

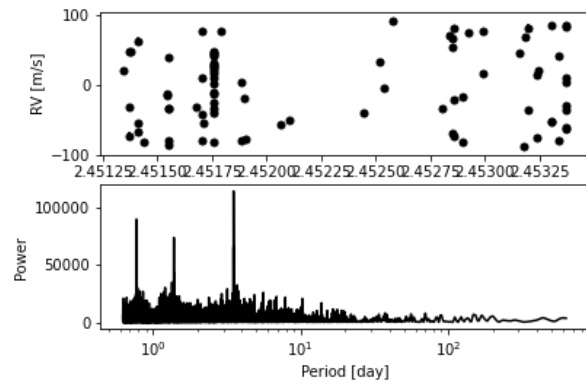


Figure B - Radial Velocity, Power and Period relation

### MCMC

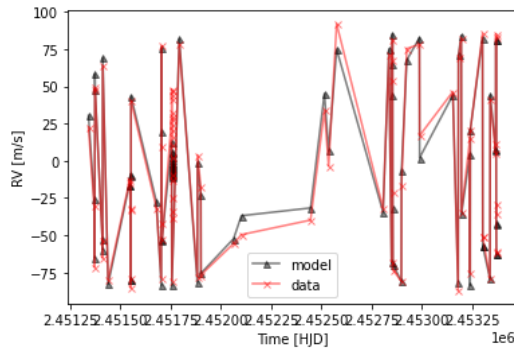


Figure C

RV data for HD 209458 b w/ uncertainty and likelihood function (MCMC)

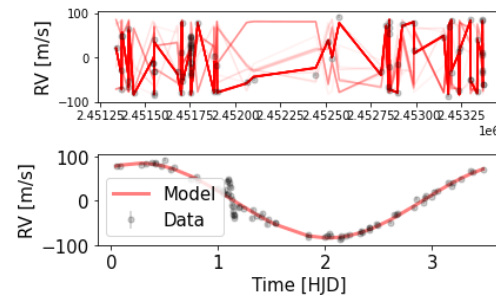


Figure D

MCMC took 408.1 seconds

Flat chain shape: (40000, 5)

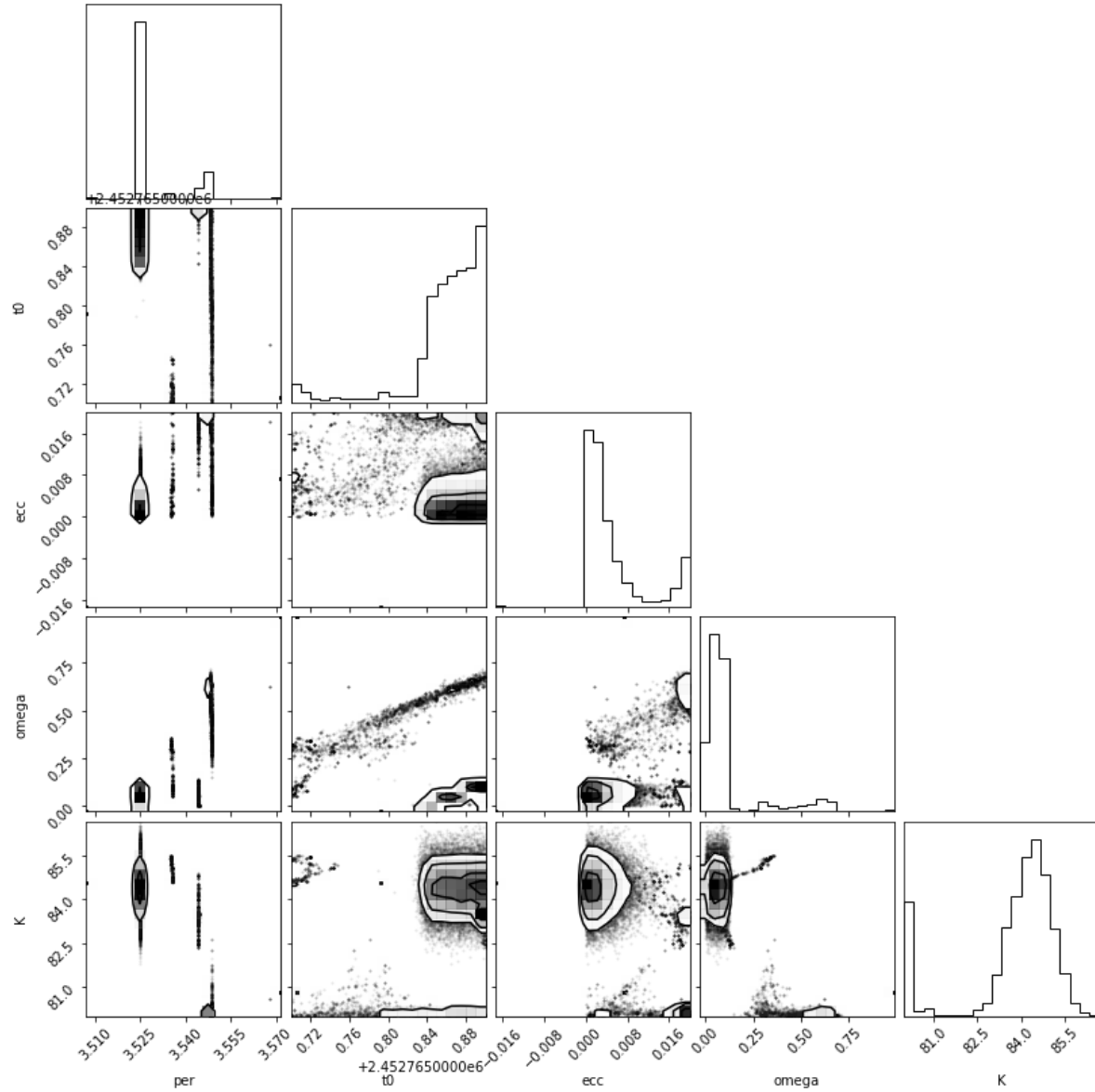


Figure E

Corner plots are used to visualize multidimensional samples using a scatterplot matrix. This plot for the parameters represents the distribution of variables for the MCMC computation of the parameters. Concentric circles show whether the data is a good fit or not.

#### 4. Discussion/Conclusions

The Jovian planet HD209458 b has a mass of  $0.701 \pm 0.00005$  Jupiter Masses, a radius of  $1.429 \pm 0.0031$  Jupiter Radii, and a density of  $318.429 \pm .12.67 \text{ kg/m}^3$ . The exoplanet has a maximum period of 3.52 days (derived from Figure B) which would classify it as a Hot Jupiter. As is shown in Figures F and G, the radius vs mass and the density vs mass are very similar compared to other exoplanets with similar masses and radii. When compared to the M-R relation from Chen & Kipping (2016), it is exactly on trend with the other Jovian worlds in the plot shown in figure H.

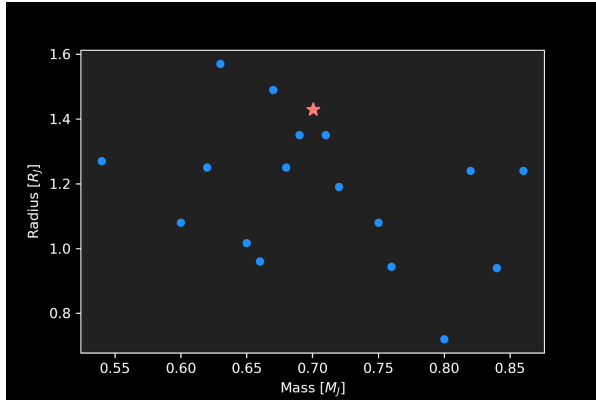


Figure F - Radius vs Mass

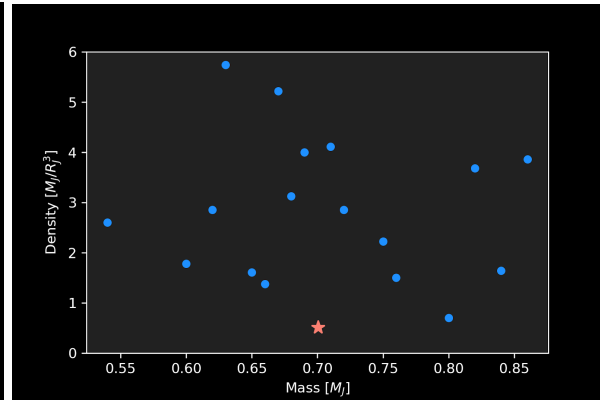


Figure G - Density vs Mass

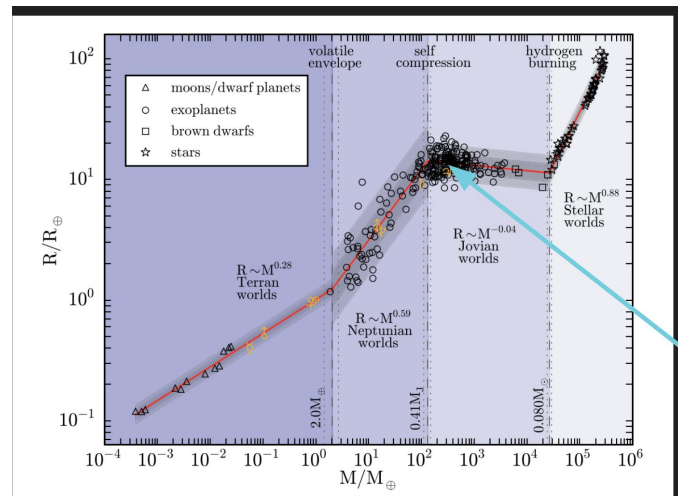


Figure H - Comparison to Chen & Kipping (2016)

#### 5. References

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doi:10.1086/529429. <https://iopscience.iop.org/article/10.1086/529429/pdf>

## 6. Contributions

Payton Cassel - Powerpoint, write-up, coding, calculations (code 2b in GitHub), presentation

Shannon McKinney - Powerpoint and write-up

Bailee Wolfe - Coding and calculations (code 2a in GitHub)

Maddie Englerth - Powerpoint, write-up, presentation

Richard Kane - Powerpoint and write-up