[Uncertainties in Measuring Planets]

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Payton Cassel, Shannon McKinney, Madison Englerth, Richard Kane, Bailee Wolfe

1. Introduction/Motivation

The study of exoplanets continues to be a popular topic amongst astronomers as new data from our land-based and orbital telescopes comes in daily. Using two methods, astronomers can calculate an exoplanet's mass and radius to find the density of the planet discovered. The measurement of a planet's radial velocity can be used to find its mass while the measurements taken using the transit method can be used to find a planet's radius. Finally, using these two values, the density can be calculated. This can tell us about the planet's inner makeup. For example, Mercury is much smaller than Earth, but is just as dense. That could mean Mercury has a larger iron-rich core.

This can lead to some issues, however. The use of radial velocity and transit methods can lead to some uncertainties in the accuracy of the result. Here we explore the amount of uncertainties each method presents and how this affects the results of a specific planet.

2. Methods

Using radial velocity (RV) and transit data from the exoplanet system HD209458 b we are able to come to some conclusions regarding the mass, radius, and density of the exoplanet.

2a. Planet Mass and Uncertainty

The expression for radial velocity is,

$$K = \frac{m_p}{m_*} \cdot \sqrt{\frac{Gm_*}{a}} \cdot sin(i)$$

For an exoplanet system, m_* is the mass of the Star and m_p is the mass of the exoplanet. a is the semi-major axis of the system.

Rearranging this formula gives us,

$$m_p = Km_* \cdot \sqrt{\frac{a}{Gm_*}}$$

We assume circular orbit, $e = 0.01$, to find the minimum mass of the planet in terms of m_*. Thus, $m_* \approx 1.069\ m_\odot$ (solar masses). We take the semi-major axis from Rosenthal et al. 2021, $a = 6.93 \times 10^9 meters = 0.04634\ AU$.

To ensure the data is accurate, we use MCMC (Markov chain Monte Carlo) methods to estimate parameters, and take errors into account.

From this, we find $m_p = 0.7006\ M_J$ (Jupiter masses).

An equation for uncertainty of mass is,

$$\left(\frac{\sigma_{m_p}}{m_p}\right)^2 \approx \frac{4}{9}\left(\frac{\sigma_{m_*}}{m_*}\right)^2 + \left(\frac{\sigma_{K_*}}{K_*}\right)^2$$

Using $\sigma_{K_*} = 0.00826$, $K_* = 84.7\frac{m}{s}$, and $\sigma_{m_*} = 1119.8544\ M_J$ as

$$\sigma_{m_p} \approx 7.0847 \times 10^{-5} = 7.0847 \times 10^{-3}\%$$

2b. Planet Radius and Uncertainty

The expression for radius of a planet is,

$$\delta = \left(\frac{R_p}{R_*}\right)^2$$

Rearranging we get,

$$R_p = R_*\sqrt{\delta}$$

Where δ is the transit depth and R_* is the radius of the star. The transit depth is assumed to be 1.50% and radius of the star to be $11.677\ R_J$.

From this we calculate $R_p = 1.429\ R_J$ (Jupiter radiuses).

An equation for uncertainty of radius is,

$$\left(\frac{\sigma_{R_p}}{R_p}\right)^2 \approx \left(\frac{\sigma_{R_*}}{R_*}\right)^2 + \frac{1}{4}\left(\frac{\sigma_\delta}{\delta}\right)^2$$

Using $\sigma_{R_*} = 0.017$ and $\sigma_\delta = 0.0016$, we get that $\sigma_{R_p} \approx 0.00222 = 0.222\%$

2c. Planet Density and Uncertainty

To find the density of an exoplanet, we use the equation,

$$p = \frac{m_p}{\frac{4}{3}\pi(R_p)^3}$$

Using m_p and R_p found previously, we find $p = 318.429\ \frac{kg}{m^3} = 0.0573\frac{M_J}{R_J^3}$.

With this equation for uncertainty,

$$\left(\frac{\sigma_p}{p}\right)^2 \approx \left(\frac{\sigma_{m_p}}{m_p}\right)^2 + 9\left(\frac{\sigma_{R_*}}{R_*}\right) + \frac{9}{4}\left(\frac{\sigma_\delta}{\delta}\right)^2$$

Using $\sigma_{R_p} = 0.00159$ and $\sigma_{m_p} = 0.0548$, we get that $\sigma_p \approx 0.0398 = 3.98\%$

3. Results

The mass of exoplanet HD209458 b was calculated to be 0.701 ± 0.00005 Jupiter Masses and the observed mass is 0.690 ± 0.024 Jupiter Masses (Torres). Our calculated value for the mass falls within the observed uncertainty and therefore is considered to be an accurately performed calculation.

The radius of the exoplanet was calculated to be 1.429 ± 0.003 Jupiter Radii and the observed radius is 1.359 ± 0.015 Jupiter Radii (Torres). The calculated value for the radius falls just short of being within the observed uncertainty, but is still fairly accurate.

The density of the exoplanet was calculated to be 0.636 ± .0253 kg/m^3. This level of uncertainty is higher than that of the mass and radius, this is due to the fact that we used both mass and radius to calculate density, which both have a level of uncertainty themselves.

Observed: 0.338 g/cm^3 (Torres)

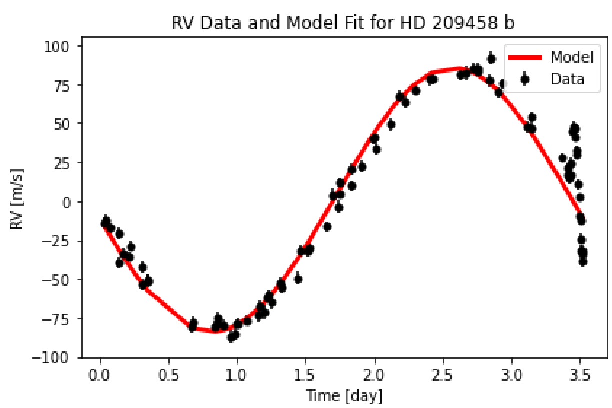

Figure A - Radial Velocity Data and Model Fit

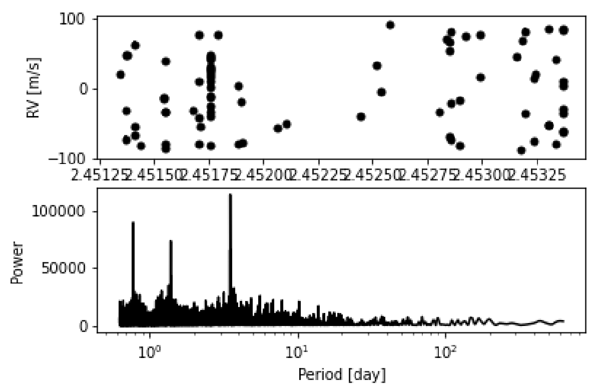

Figure B - Radial Velocity, Power and Period relation

MCMC

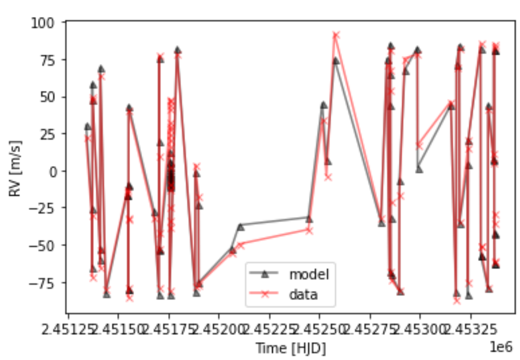

Figure C

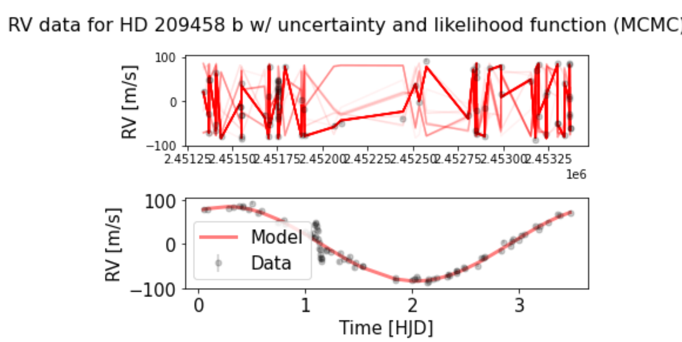

Figure D

MCMC took 408.1 seconds

Flat chain shape: (40000, 5)

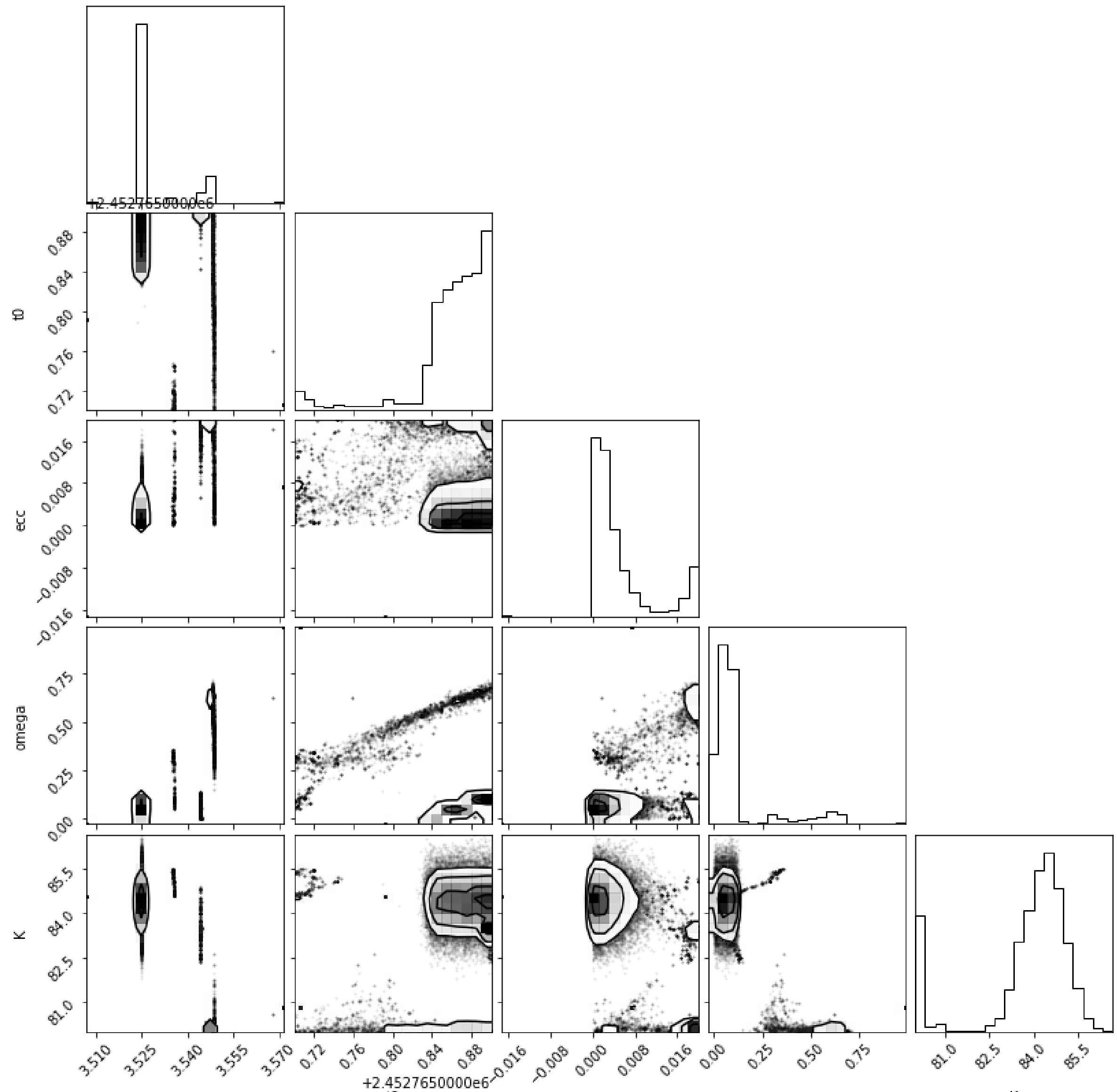

Figure E

Corner plots are used to visualize multidimensional samples using a scatterplot matrix. This plot for the parameters represents the distribution of variables for the MCMC computation of the parameters. Concentric circles show whether the data is a good fit or not.

4. Discussion/Conclusions

The Jovian planet HD209458 b has a mass of 0.701 ± 0.00005 Jupiter Masses, a radius of 1.429 ± 0.0031 Jupiter Radii, and a density of 318.429 ± .12.67 kg/m^3. The exoplanet has a maximum period of 3.52 days (derived from Figure B) which would classify it as a Hot Jupiter. As is shown in Figures F and G, the radius vs mass and the density vs mass are very similar compared to other exoplanets with similar masses and radii. When compared to the M-R relation from Chen & Kipping (2016), it is exactly on trend with the other Jovian worlds in the plot shown in figure H.

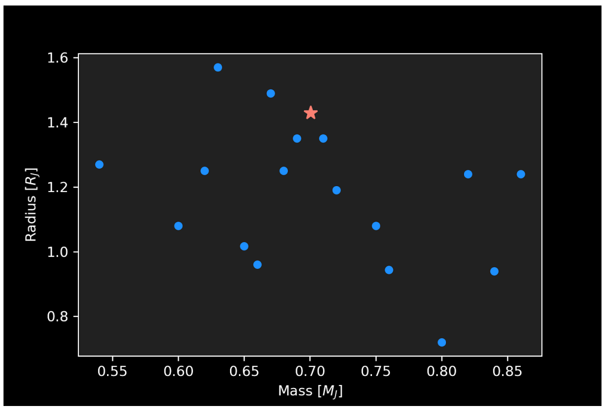

Figure F - Radius vs Mass

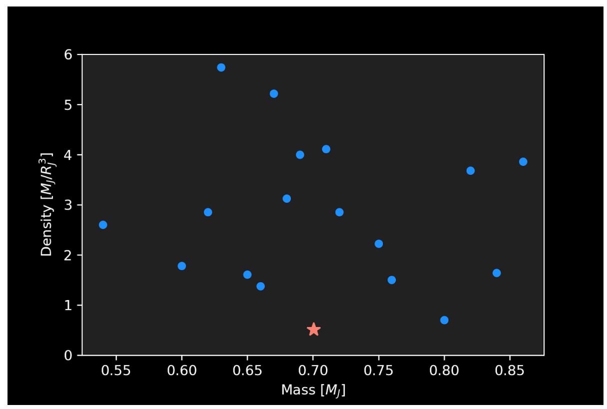

Figure G - Density vs Mass

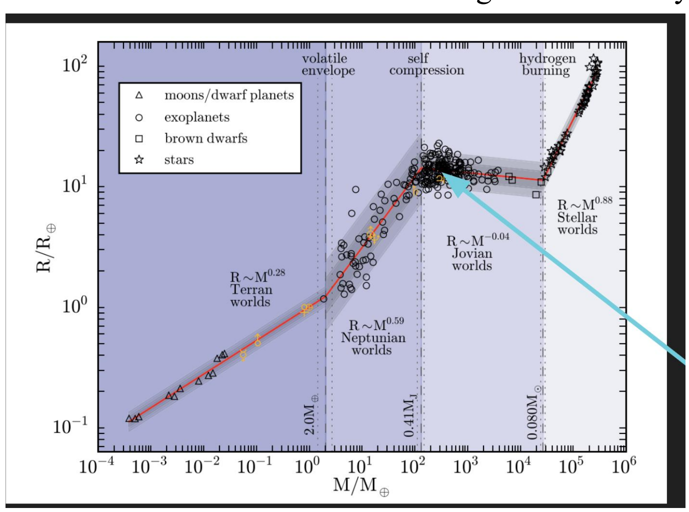

Figure H - Comparison to Chen % Kipping (2016)

5. References

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6. Contributions
Payton Cassel - Powerpoint, write-up, coding and calculations (code 2b in GitHub)
Shannon McKinney - Powerpoint and write-up
Bailee Wolfe - Coding and calculations (code 2a/2b in GitHub)
Maddie Englerth - Powerpoint and write-up
Richard Kane - Powerpoint and write-up