IBM3103 – Mathematical Methods for Biological and Medical Engineering
Fall 2021
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Assignment #3

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Due on July 9, 2021

Problem #1: Digital communications (100pts)

In communications we often encode messages using orthogonal signals. In this problem, we will find such orthogonal set from a collection of symbols to later decode a message.

i. Read the file `symbols.npz` and plot the signals $s_1, s_2, s_3, s_4 \in \mathbb{R}^N$. What is the value of N?

ii. What is the number M of orthogonal signals needed to represent s_1, s_2, s_3 and s_4? Justify your answer.

iii. Use the `qr` function from `numpy.linalg` to find an orthonormal set of signals $\{\phi_i\}_{i=1}^M$ from the signals s_1, s_2, s_3. **Hint:** Create a matrix

$$S = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \end{bmatrix}$$

and then apply the QR decomposition to the matrix.

iv. Plot the signals s_1, s_2, s_3 and s_4 in the space defined by the base $\{\phi_i\}_{i=1}^M$. This is also called the constellation diagram. **Hint:** In the basis $\{\phi_i\}_{i=1}^M$ the signals $s_i \in \mathbb{R}$ can be represented as a vector in $\mathbb{R}^M$. Use what we have seen about orthogonal projections.

v. Read the file `message.npz`. This file has 164 noisy symbols sent through the air and then received by an antenna. Plot the received messages in the constellation diagram. Determine regions of the constellation diagram that correspond to each symbol s_i.

vi. Decode the message using Table 1. To convert from binary to text use an ASCII converter[1].

Symbol	Binary
s_1	00
s_2	01
s_3	10
s_4	11

TABLE 1. Symbol to binary.

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[1]`https://www.convertbinary.com/to-text/`