IBM3103 – Mathematical Methods for Biological and Medical Engineering

Fall 2021

Instructor: Carlos A. Sing Long
Teaching Assistant: Carlos Castillo

Assignment #3

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Due on June 11, 2021

Problem #1 (100pts)

In this problem we will explore some ideas related to linear maps and matrices. To do this, download the file `hw03_map.py` containing the function `A` that takes as input a vector in $\mathbb{R}^{64}$. To import this function into your script, remember to use the command

```
from hw03_map import A
```

The function `A` represents a linear map. **You do not need to show this is a linear map**.

i. What is the codomain of A? In other words, given $\boldsymbol{x} \in \mathbb{R}^{64}$, what is the number of components of the output $A(\boldsymbol{x})$?

ii. Download the file `hw03_signal_1.npz` containing the vector `x1`. Plot the vector `x1` and the vector `A(x1)`. Explain, in your own words, some features of the vector `x1` and `A(x1)`. For instance, does the vector `x1` have sharp transitions? Does `A(x1)` have sharp transitions?

iii. Download the file `hw03_signal_2.npz` containing the vector `x2`. Repeat ii. with the vector `x2`.

iv. Find the matrix representation `Amtx` of the linear map `A`. What is the size of the matrix `Amtx`? Plot the 0-th, 15-th, 31-th and 63-th columns. Do you see any relation between them? **Note:** Here we used Python's numbering convention, starting from 0.

v. Is the nullspace of `Amtx` trivial? If your answer is **yes** you need to provide a proof of this fact. If your answer is **no** it suffices to provide a non-zero vector `z` such that `A(z) = 0`. You should also plot the vector `z`.

vi. Download the file `hw03_signal_3.npz` containing the vector `x3`. Is this vector on the range of the matrix `Amtx`? **Hint:** Try to formulate this question in terms of a least-squares problem.

vii. Find the vector in the range of the matrix `Amtx` that is closest in Euclidean distance to `x3`. Plot this vector.

viii. Is the matrix `Amtx` invertible?