ISTANBUL TECHNICAL UNIVERSITY

COMPUTER ENGINEERING DEPARTMENT

BLG 368E

OPERATIONS RESEARCH
ASSIGNMENT 1 REPORT

DATE : 17.05.2021

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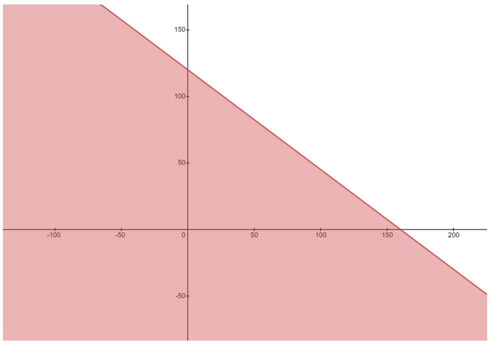
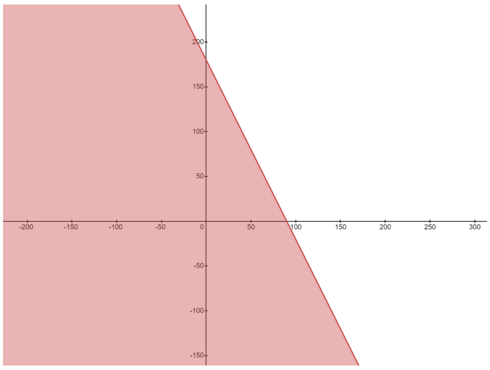
SPRING 2021

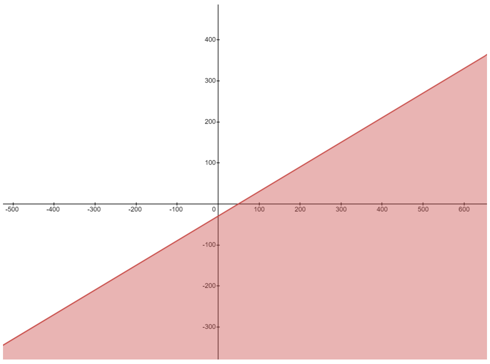
Question 1



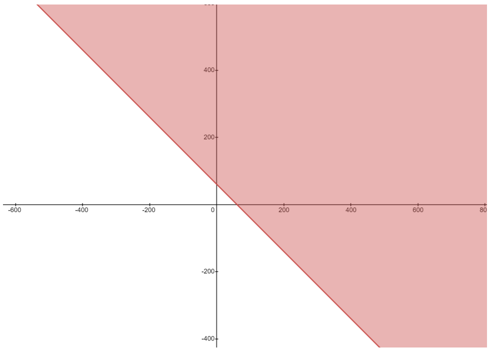
$3X + 4Y \leq 480$



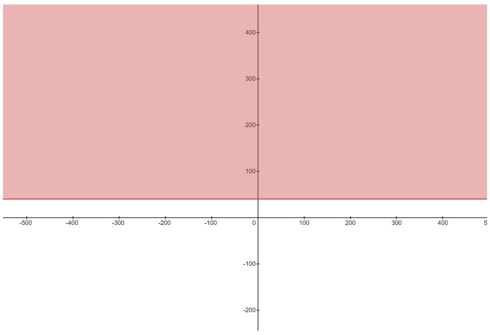
$4X + 2Y \leq 360$



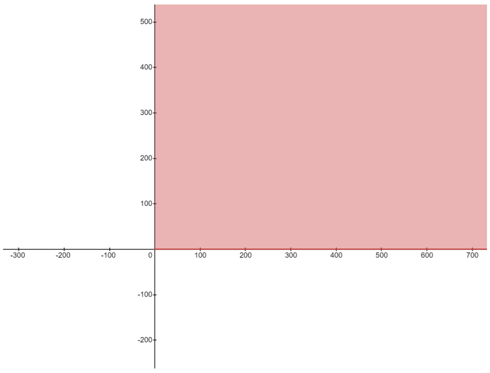
$3X - 5Y \geq 150$



$X + Y \geq 60$



$Y \geq 40$



$X,Y \geq 0$

Intersection of these regions does not produce a feasible region. So there is no solution.

Question 2

Part a)

Maximize

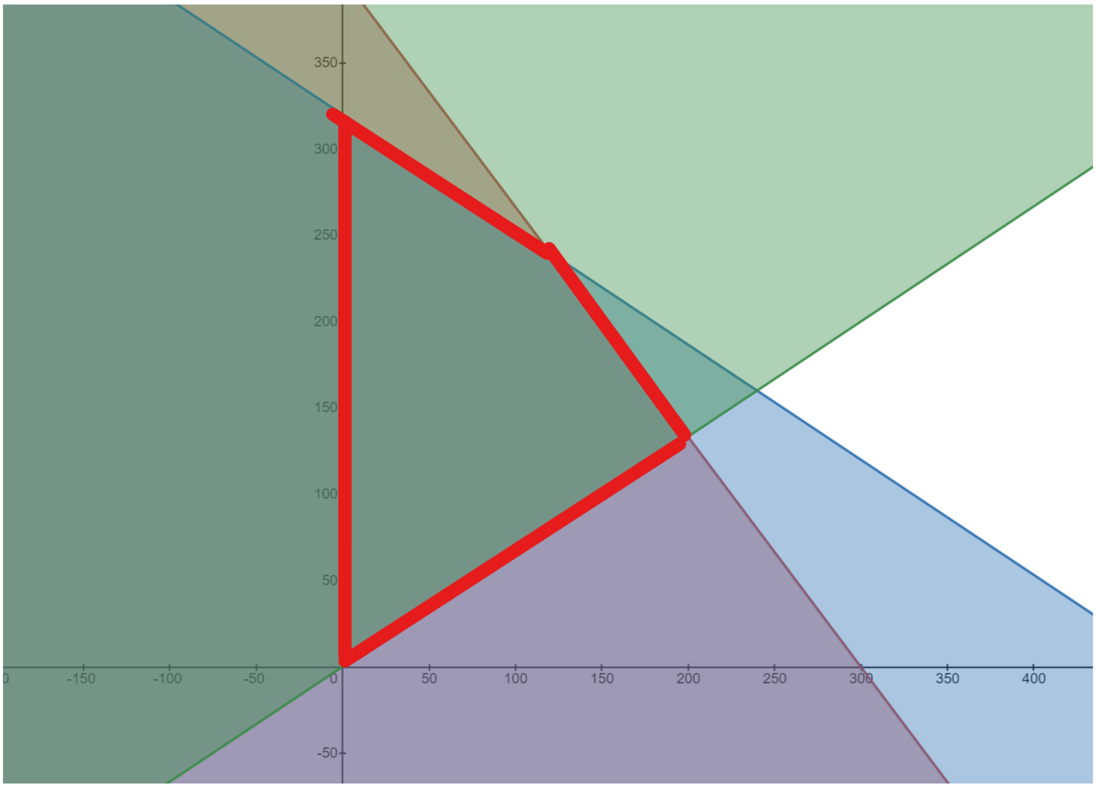
$$12D + 10S$$

Subject to:

$$\frac{1}{3}D + \frac{1}{4}S \leq 100$$
$$\frac{1}{6}D + \frac{1}{4}S \leq 80$$
$$2D \leq 3S$$

Part b)

Since there are 2 variables we can find a graphical solution after finding feasible region and trying the corner points.



Feasible Region

Corner Points and corresponding profits:

$$S = 133.\overline{3}, D = 200 \Rightarrow 3733.\overline{3}$$

$$S = 240, D = 120 \Rightarrow 3840$$

$$S = 320, D = 0 \Rightarrow 3200$$

$$S = 0, D = 0 \Rightarrow 0$$

The maximum profit occurs at $S = 240$ and $D = 120$ as 3840.

Part c)

Sensitivity Reports.

Değişken Hücreleri

Hücre	Ad	Son Değer	Azaltılmış Maliyet	Hedef Katsayı	İzin Verilen Artış	İzin Verilen Azalış
B3	Count Deluxe	120	0	12	1.333333333	5.333333333
C3	Count Special	240	0	10	8	1

Kısıtlamalar

Hücre	Ad	Son Değer	Gölge Ücret	Kısıtlama Sağ Taraf	İzin Verilen Artış	İzin Verilen Azalış
B8	C3 Deluxe	240	0	0	1E+30	480
D6	C1	100	32	100	20	20
D7	C2	80	8	80	20	13.33333333

Sensitivity Reports Produced by Excel

From the sensitivity report we observe that we can make the Special Model more expensive. In the allowable increase column of the variable cells we can see that it is possible to increase the price of the special by $8 until the optimal solution changes. It is also possible to make the Deluxe model cheaper.

By analyzing the constraints section we observe that it is sensible to increase the construction stage hours since every hour we get until 20 hours increases our profit by $32.

We can also decrease the finishing and inspection times by 13 hours so we can move those hours to construction to make more profit.

Question 3

For the decision variables of the LP problem we will use g_{ij} to denote the amount of crude type i in barrels we use to produce gas type j. Example:

$$g_{23} = \text{the amount of crude type 2 in barrels we use to produce gas type 3.}$$

Creating the objective function:

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\begin{aligned}\text{Profit} = {} & 70(g_{11} + g_{21} + g_{31}) + 60(g_{12} + g_{22} + g_{32}) + 50(g_{13} + g_{23} + g_{33}) \\ & - 45(g_{11} + g_{12} + g_{13}) - 35(g_{21} + g_{22} + g_{23}) - 25(g_{31} + g_{32} + g_{33}) \\ & - 4(g_{11} + g_{21} + g_{31} + g_{12} + g_{22} + g_{32} + g_{13} + g_{23} + g_{33}) - (ad_1 + ad_2 + ad_3)\end{aligned}$$

$$\text{Profit} = 21g_{11}+31g_{21}+41g_{31}+11g_{12}+21g_{22}+31g_{32}+g_{13}+11g_{23}+21g_{33}-ad_1-ad_2-ad_3$$

Sunco can purchase up to 5,000 barrels of each type of crude oil daily.

$$g_{11} + g_{12} + g_{13} \leq 5000$$

$$g_{21} + g_{22} + g_{23} \leq 5000$$

$$g_{31} + g_{32} + g_{33} \leq 5000$$

Sunco's refinery can produce up to 14,000 barrels of gasoline daily.

$$g_{11} + g_{21} + g_{31} + g_{12} + g_{22} + g_{32} + g_{13} + g_{23} + g_{33} \leq 14000$$

Demands can be formulated as:

$$g_{11} + g_{21} + g_{31} \geq 3000 + 10ad_1$$

$$g_{12} + g_{22} + g_{32} \geq 2000 + 10ad_2$$

$$g_{13} + g_{23} + g_{33} \geq 1000 + 10ad_3$$

Octane constraints can be formulated as:

$$12\frac{g_{11}}{g_{11} + g_{21} + g_{31}} + 6\frac{g_{21}}{g_{11} + g_{21} + g_{31}} + 8\frac{g_{31}}{g_{11} + g_{21} + g_{31}} \geq 10$$

$$12\frac{g_{12}}{g_{12} + g_{22} + g_{32}} + 6\frac{g_{22}}{g_{12} + g_{22} + g_{32}} + 8\frac{g_{32}}{g_{12} + g_{22} + g_{32}} \geq 8$$

$$12\frac{g_{13}}{g_{13} + g_{23} + g_{33}} + 6\frac{g_{23}}{g_{13} + g_{23} + g_{33}} + 8\frac{g_{33}}{g_{13} + g_{23} + g_{33}} \geq 6$$

These inequalities simplify to following inequalities:

$$2g_{11} - 4g_{21} - 2g_{31} \geq 0$$

$$4g_{12} - 2g_{22} \geq 0$$

$$6g_{13} + 2g_{33} \geq 0$$

Sulfur constraints can be formulated as:

$$0.005\frac{g_{11}}{g_{11}+g_{21}+g_{31}} + 0.02\frac{g_{21}}{g_{11}+g_{21}+g_{31}} + 0.03\frac{g_{31}}{g_{11}+g_{21}+g_{31}} \leq 0.01$$

$$0.005\frac{g_{12}}{g_{12}+g_{22}+g_{32}} + 0.02\frac{g_{22}}{g_{12}+g_{22}+g_{32}} + 0.03\frac{g_{32}}{g_{12}+g_{22}+g_{32}} \leq 0.02$$

$$0.005\frac{g_{13}}{g_{13}+g_{23}+g_{33}} + 0.02\frac{g_{23}}{g_{13}+g_{23}+g_{33}} + 0.03\frac{g_{33}}{g_{13}+g_{23}+g_{33}} \leq 0.01$$

These inequalities simplify to following inequalities:

$$0.005g_{11} - 0.01g_{21} - 0.02g_{31} \geq 0$$

$$0.015g_{12} - 0.01g_{32} \geq 0$$

$$0.005g_{13} - 0.01g_{23} - 0.02g_{33} \geq 0$$

Finally the LP can be written as:

Maximize: $21g_{11}+31g_{21}+41g_{31}+11g_{12}+21g_{22}+31g_{32}+g_{13}+11g_{23}+21g_{33}-ad_1-ad_2-ad_3$

Subject to:

$$g_{11} + g_{21} + g_{31} + g_{12} + g_{22} + g_{32} + g_{13} + g_{23} + g_{33} \leq 14000$$

$$g_{11} + g_{12} + g_{13} \leq 5000$$

$$g_{21} + g_{22} + g_{23} \leq 5000$$

$$g_{31} + g_{32} + g_{33} \leq 5000$$

$$g_{11} + g_{21} + g_{31} \geq 3000 + 10ad_1$$

$$g_{12} + g_{22} + g_{32} \geq 2000 + 10ad_2$$

$$g_{13} + g_{23} + g_{33} \geq 1000 + 10ad_3$$

$$2g_{11} - 4g_{21} - 2g_{31} \geq 0$$

$$4g_{12} - 2g_{22} \geq 0$$

$$6g_{13} + 2g_{33} \geq 0$$

$$0.005g_{11} - 0.01g_{21} - 0.02g_{31} \geq 0$$

$$0.015g_{12} - 0.01g_{32} \geq 0$$

$$0.005g_{13} - 0.01g_{23} - 0.02g_{33} \geq 0$$

$$g_{11}, g_{12}, g_{13}, g_{21}, g_{22}, g_{23}, g_{31}, g_{32}, g_{33}, ad_1, ad_2, ad_3 \geq 0$$