ISTANBUL TECHNICAL UNIVERSITY

COMPUTER ENGINEERING DEPARTMENT

BLG 354E
SIGNALS AND SYSTEMS FOR COMPUTER ENGINEERING
ASSIGNMENT 3

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Part a)

We square the values since increases and decreases are 40 dB per decade:

$$H(s) = \frac{Ks^2}{(1+\frac{s}{10})^2(1+\frac{s}{100})^2}$$

$$H(jw) = \frac{K(j\omega)^2}{(1+\frac{j\omega}{5}+100)(1+\frac{j\omega}{50}+10000)}$$

$$|H(w)| = \frac{K\omega^2}{(\frac{\omega^2}{100}+1)(\frac{\omega^2}{10000}+1)}\Big|_{\omega=1} = 0.1$$

$$\frac{K}{1.01 \cdot 1.0001} = 0.1$$

$$K = 0.1$$

$$H(s) = \frac{0.1s^2}{(1+\frac{s}{10})^2(1+\frac{s}{100})^2}$$

Part b)

The code to generate the plots are included in the zip file.

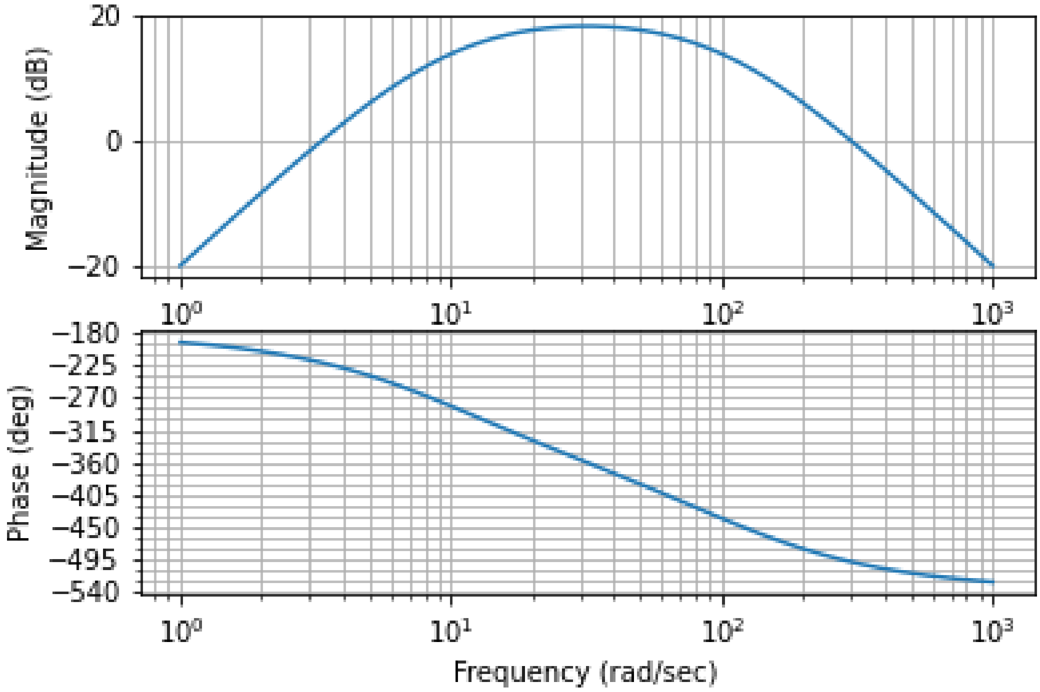

Figure 1: Magnitude and Phase response of the filter

Part c)

$$x(t) = 10(sin(2\pi t) + sin(10\pi t) + sin(100\pi t))$$
$$X(s) = 20\pi\left(\frac{5}{s^2 + (10\pi)^2} + \frac{50}{s^2 + (100\pi)^2} + \frac{1}{s^2 + (2\pi)^2}\right)$$
$$Y(s) = X(s)H(s)$$

Taking the inverse laplace transform of $Y(s)$ gives us following function (calculated by Wolfram Alpha):

$$\begin{gathered} 0.1((133.661 + 44.7887i)e^{-6.28319it}((0.798097 - 0.602529i) + (1 + 0i)e^{12.5664it}) + \\ (413.205 + 3.11833i)e^{-31.4159it}((0.0150925 + 0.999886i) - ie^{62.8319it}) \\ -(28.8943 - 35.7326i)e^{-314.159it}((1 + 0i)e^{628.319it} + (-0.20928 + 0.977856i)) \\ -2.10095 \cdot 10^{10}e^{-100.t} + 2.10095 \cdot 10^{10}e^{-100.t} - 8.037610^{10}e^{-10.t} + 8.0376 \cdot 10^{10}e^{-10.t}) \end{gathered}$$

Plot of y(t) The code to generate is included in the zip file:

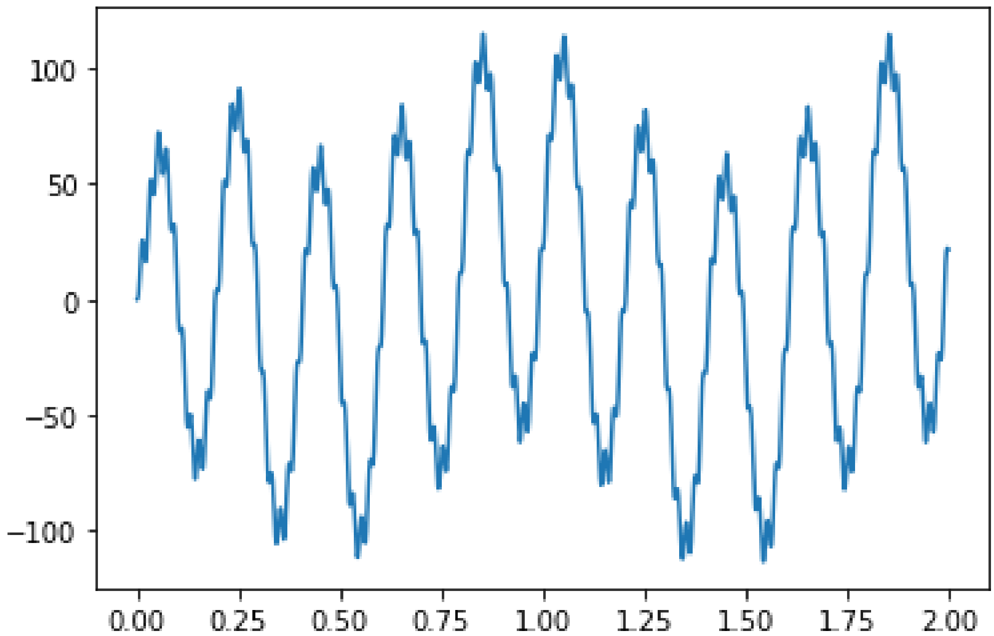

Figure 2: Plot of y(t)

Part d)

To find the canonical form we use the bilinear transform on $H(s)$:

$$s = \frac{2(1-z^{-1})}{T(1+z^{-1})}$$

Since sampling frequency $f_s = 500$ Hz, $T = 0.002$ s we can obtain $H(z)$ as follows (code is included in zip file):

$$\frac{100000z^4 - 200000z^2 + 100000}{1234321z^4 - 4439556z^3 + 5971806z^2 - 3560436z^1 + 793881}$$

The canonical form of the system is drawn below:

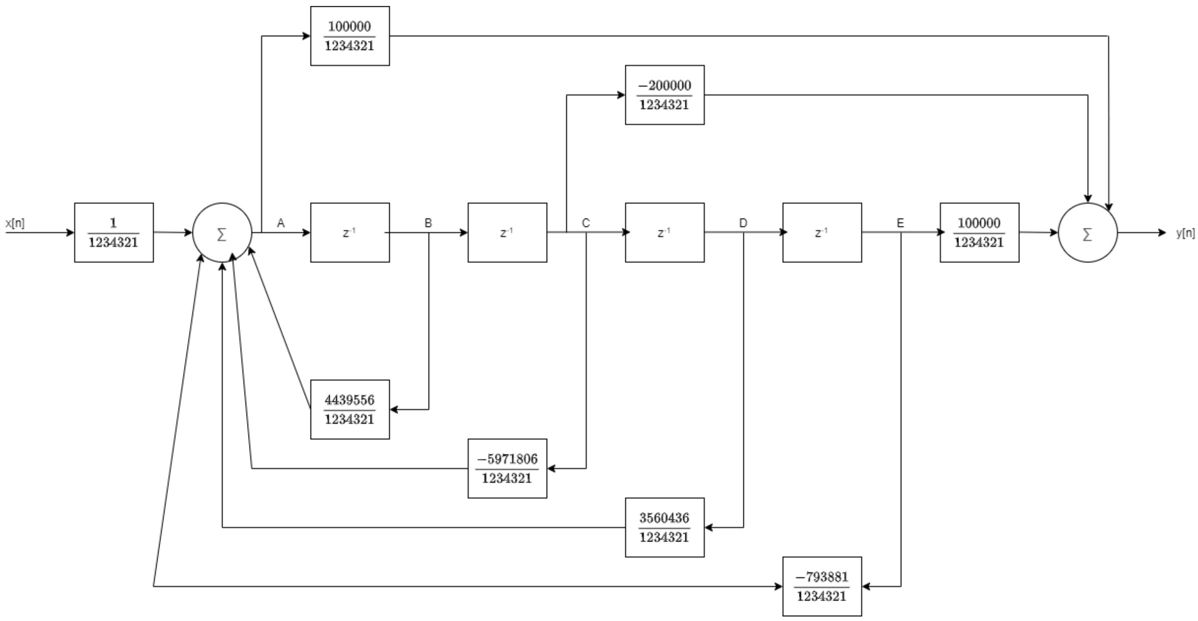

Figure 3: System Diagram

Pseudocode is written below:

```
ISR @Ts=1/fs=1/500Hz = 0.002s
X = READ(ADC)
A = (X + 4439556B − 5971806C + 3560436D − 793881E)/1234321
Y = (100000A − 200000C + 100000E)/1234321
OUTPUT Y
E = D
D = C
C = B
B = A
RETURN
```

Part e)

Inverse z transform of the Transfer function (Calculated by Wolfram Alpha):

$$400000 \cdot 1111^{-n-2} \cdot 9^{n-4} \left(1 - \mathrm{u}\left(-n\right)\right) \left(101^{n+2} n + 101^{n+2} + 100 \cdot 11^{2n} n - 11111 \cdot 11^{2n}\right) + 0.0810162 \mathrm{u}\left(-n\right)$$

Convolving $h(t)$ and $x(t)$ produces the following plot:

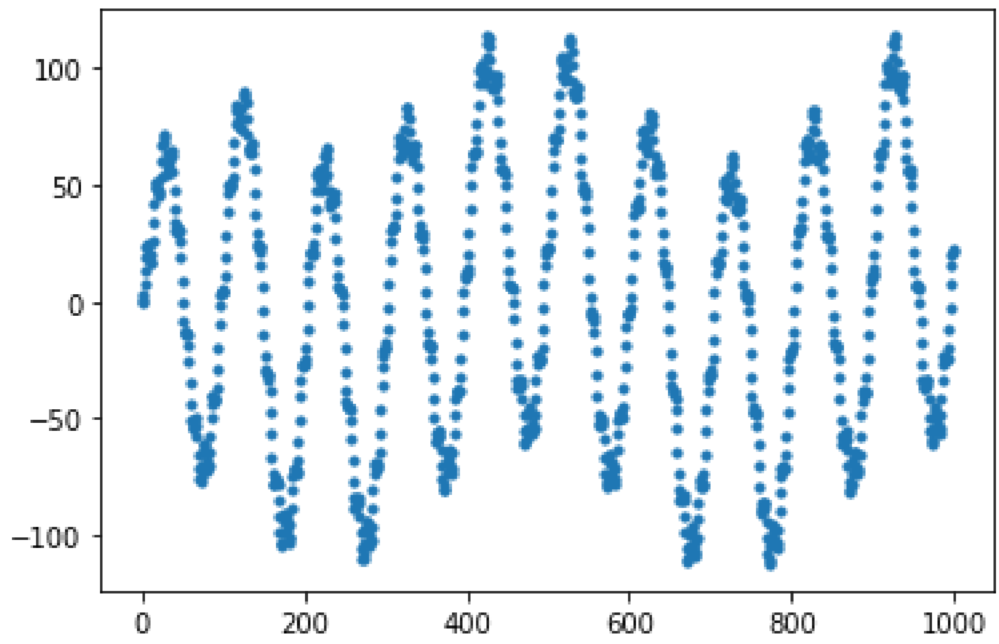

Figure 4: Plot of h(t) * x(t)

To compare the two outputs in c) and e) I calculated the absolute mean of their difference (the code is included in the zip file):

$$\frac{|\text{Part c)} - \text{Part e)}|}{1000}$$

(1000 since $\frac{2}{0.002} = 1000$) The result was 0.0007 so the results in Part c) and Part e) were almost equal.