ISTANBUL TECHNICAL UNIVERSITY

COMPUTER ENGINEERING DEPARTMENT

BLG 354E

SIGNALS AND SYSTEMS FOR COMPUTER ENGINEERING
ASSIGNMENT 4 REPORT

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Question 1

$$x[n] = \{1, 2, 3\}$$

$$0.5n^2(u[n] - u[n-3]) = \{0, 0.5, 2\}$$

$$y_0 = 1 \cdot 0 = 0$$

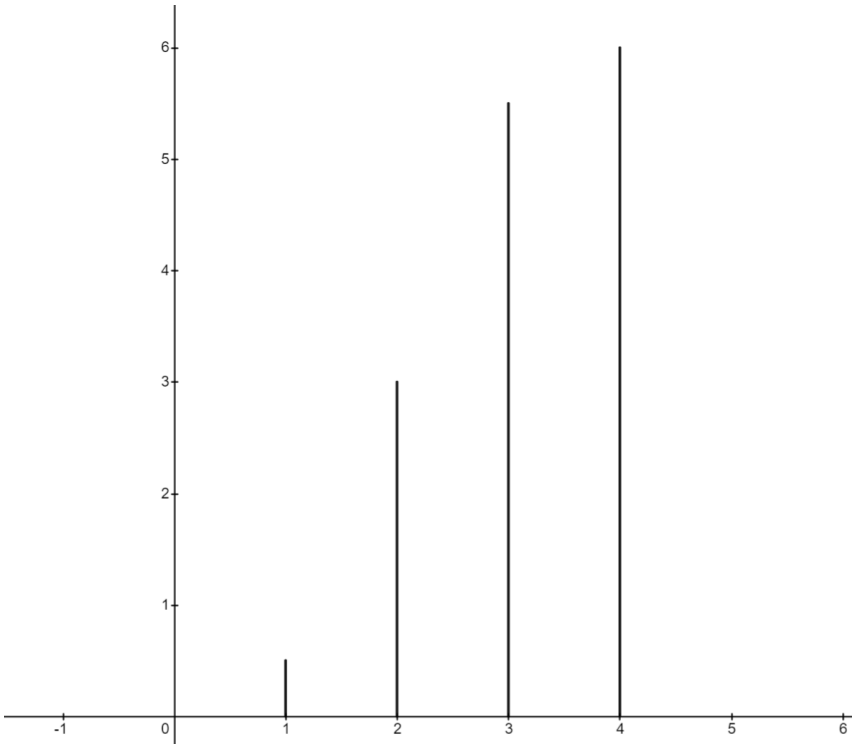
$$y_1 = 2 \cdot 0 + 1 \cdot 0.5 = 0.5$$

$$y_2 = 3 \cdot 0 + 2 \cdot 0.5 + 1 \cdot 2 = 3$$

$$y_3 = 3 \cdot 0.5 + 2 \cdot 2 = 5.5$$

$$y_4 = 3 \cdot 2 = 6$$

$$y[n] = \{0, 0.5, 3, 5.5, 6\}$$



Plot of y[n]

Question 2

$$x[402] = 10sin(314t - \frac{\pi}{4}) \quad \text{where} \quad t = 402 \cdot 5\text{ms}$$

$$x[402] = 10sin\left(\frac{314 \cdot 402}{200} - \frac{\pi}{4}\right) \approx 8.94$$

Question 3

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} x[0] = -7.07 \\ x[1] = 7.06 \\ x[2] = 7.08 \\ x[3] = -7.05 \end{bmatrix} = \begin{bmatrix} 0 \\ -14.15 - 14.15j \\ 0 \\ -14.15 + 14.15j \end{bmatrix}$$

Question 4

$$(x+1)(u(x+1)-u(x))+u(x)-u(x-1)+(-x+3)(u(x-1)-u(x-3))$$

$$xu(x+1)-xu(x)+u(x+1)-u(x)+u(x)-u(x-1)-xu(x-1)+xu(x-3)+3u(x-1)-3u(x-3)$$

$$(x+1)u(x+1)-xu(x)+(-x+2)u(x-1)+(x-3)u(x-3)$$

Question 5

DIT FFT

$$f[n] = x[2n] = \{1, -1, 1, -1\}$$

$$g[n] = x[2n+1] = \{2, 0, 2, 0\}$$

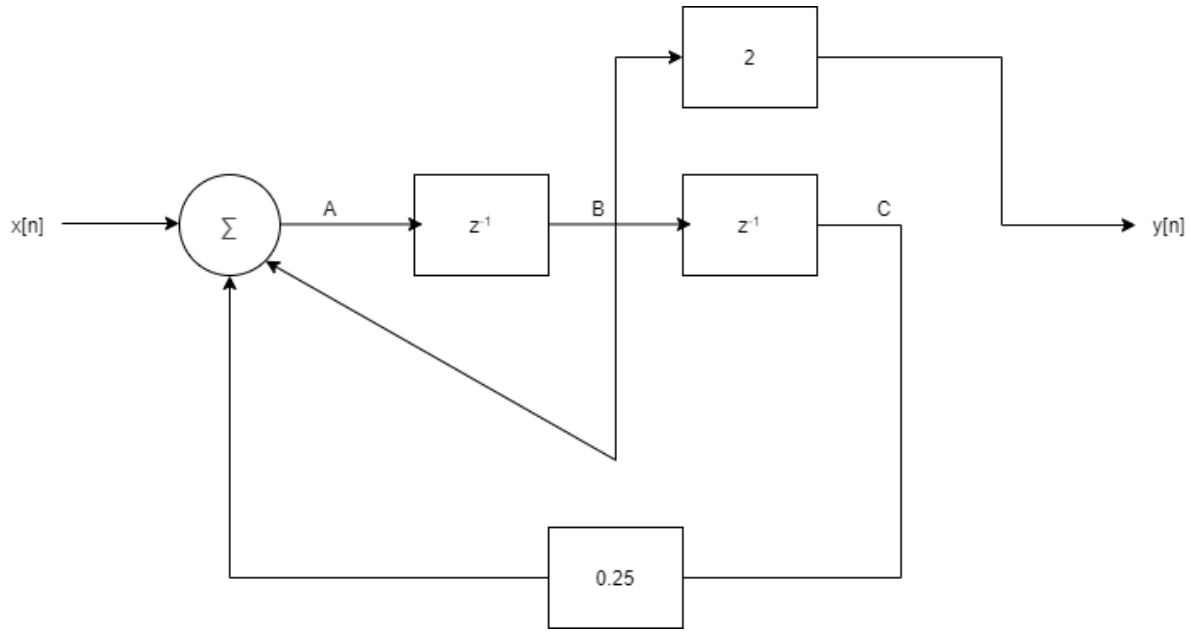
$$\begin{bmatrix} F[0] \\ F[1] \\ F[2] \\ F[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} G[0] \\ G[1] \\ G[2] \\ G[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$

$$X[0] = F[0] + W_8^0 G[0] = 0 + 1 \cdot 4 = 4$$
$$X[1] = F[1] + W_8^1 G[1] = 0$$
$$X[2] = F[2] + W_8^2 G[2] = 4 + (-j) \cdot 4 = 4 - 4j$$
$$X[3] = F[3] + W_8^3 G[3] = 0$$
$$X[4] = F[4] - W_8^0 G[4] = 0 - 1 \cdot 4 = -4$$
$$X[5] = F[5] - W_8^1 G[5] = 0$$
$$X[6] = F[6] - W_8^2 G[6] = 4 - (-j) \cdot 4 = 4 + 4j$$
$$X[7] = F[7] - W_8^3 G[7] = 0$$

$$X = \{4, 0, 4 - 4j, 0, -4, 0, 4 + 4j, 0\}$$

Question 6



DT System Diagram

n	X	A	B	C	Y
0	1	1	0	0	0
1	1	2	1	0	2
2	1	2.75	2	1	4
3	1	3.25	2.75	2	5.5

$$y[n] = \{0, 2, 4, 5.5\}$$

Question 7

X = READ(ADC)
A = X + B − 0.25C
Y = 2B
Output (Y)
C = B
B = A
RETURN

Question 8

a)

$$\frac{1000}{5s+1000} = \frac{1}{1+\frac{s}{200}}$$
$$\frac{1}{200} = \frac{1}{\omega_0} \Rightarrow \omega_0 = 200 \text{ rad/s}$$
$$f_0 = 31.83\text{Hz}$$

b)

$$H(j\omega) = \frac{1}{\frac{j\omega}{200}+1}$$
$$|H(j\omega)| = \frac{1}{\sqrt{\frac{\omega^2}{40000}+1}}$$
$$20\log|j\omega|\Big|_{\omega=x} = -3 \text{ dB}$$
$$\frac{1}{\sqrt{\frac{x^2}{40000}+1}} = 10^{-\frac{3}{20}}$$
$$x = \text{BW} = 199.526 \text{ rad/s} = 31.76 \text{ Hz}$$

Question 9

$$\mathcal{L}\{10sin(400\pi t)\} = \frac{4000\pi}{s^2+160000\pi^2}$$
$$\mathcal{L}^{-1}\left\{\frac{1}{1+\frac{s}{200}}\frac{4000\pi}{s^2+160000\pi^2}\right\} = \frac{20\pi e^{-200t}}{1+4\pi^2} + \frac{10sin(400\pi t)}{1+4\pi^2} - \frac{20\pi cos(400\pi t)}{1+4\pi^2}$$

Question 10

Formula for square wave using Fourier expansion:

$$\frac{4A}{\pi}\sum_{k=1}^{\infty}\frac{sin((2k-1)\omega t)}{(2k-1)}$$

$$A = 5 \;\; T = 0.2 \text{ ms} \;\; \omega = 10000 \text{ rad/s}$$

$$\frac{20}{\pi}\left(sin(10000\pi t) + \frac{1}{3}sin(30000\pi t)\right)$$

Question 11

$$Y = Xz^{-2} + 2aYz^{-1} - a^2Yz^{-2}$$

$$(a^2z^{-2} - 2az^{-1} + 1)Y = Xz^{-2}$$

$$T(z) = \frac{Y(z)}{X(z)} = \frac{z^{-2}}{a^2z^{-2} - 2az^{-1} + 1} = \frac{z^2}{a^2 - 2az + z^2} = \frac{z^2}{(z-a)^2} \;\; |z| > |a|$$

BIBO stable if $|a| < 1$

Question 12

$$\omega_1 = \frac{14\pi}{3}, \;\; f_1 = \frac{7}{3}, \;\; T_1 = \frac{3}{7}$$

$$\omega_2 = \frac{5\pi}{4}, \;\; f_2 = \frac{5}{8}, \;\; T_2 = \frac{8}{5}$$

$$T_0 = LCM\left(\frac{3}{7}, \frac{8}{5}\right) = \frac{LCM(3,8)}{GCD(7,5)} = 24$$