Bachelor Thesis

Finding an Optimal Regularisation Parameter for the Inversion of Transient Electromagnetic Data Using the L-Curve Method

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Abstract

1 Introduction

2 Materials and Methods

2.1 State of the Art

2.2 Experimental Set up

In order to investigate the applicability of automatically determining an optimal lambda value to TEM data, we conducted a field survey in October of 2024 at the Martenhofer Lacke in the Nationalpark Neusiedlersee - Seewinkel. The Geophysics Research Unit at TU Wien kindly provided data from May 2024 for the same location but with a differing measuring configuration, which enables a comparison between varying setups.

Furthermore a python package was developed to read, filter, and invert the TEM data. The visualisation of an L-curve as well as several methods of automatically finding an optimal lambda were also implemented.

2.2.1 Measuring Device

Both survey were conducted with the TEM-Fast 48 HPC system by Applied Electromagnetic Research (AEMR). It is compact device allowing the use of a single-loop configuration. By connecting an external 12 / 24 V battery a current either 1 / 4 A can be put through the connected transmitter loop. It records up to 48 logarithmically spaced time gates, which results in a time range between $4-16000\,\mu s$. The specific number of time gates can be chosen through a time-key. Table 1 shows which time-key leads to which recording time range. To provide an optimal signal-to-noise-ratio the device automatically stacks multiple pulses (Barsukov et al. 2015). The number of stacks are given by the formula $P_{tot} = 13 \times n_s \times n_{as}$ (Aigner et al. 2021), where n_s $(1-20)$ is the chosen stacking-key and n_{as} is the number of analogue stacks depending on the chosen time-key and can be found in Table 1. More detailed infromation on the device can be found in the manual provided on the website `http://www.aemr.net/tem-fast.htm`.

Tab. 1: Parameters relating to the time-key of the TEM-FAST 48 HPC system (Excerpt from the manual).

Key	Max Time (μs)	Time Gates	Analogue Stacks
1	64	16	1024
2	128	20	512
3	256	24	256
4	512	28	128
5	1024	32	64
6	2048	36	32
7	4096	40	16
8	8192	44	8
9	16384	48	4

2.2.2 Field Survey

The field measurements were carried out at the Martenhofer Lacke in the *Nationalpark Neusiedlersee - Seewinkel* (16° 51' 23.058" N, 47° 45' 8.4348" E), which is located on the east side of the Neusiedler See, Austria. Being part of the Seewinkel, which are intermittent alkaline soda waters, this water cycle of this lake is fueled by deep saline groundwater and evaporation leading to its high salinity and shallow water depth, which also varies throughout the year (Boros et al. 2025). This location was chosen due to having sparse man-made structures in the Nationalpark, which reduces noise in the gathered data to a minimum (Aigner et al. 2024).

The first survey, consisting of 45 soundings as shown in Figure 1 with a 12.5 × 12.5 m loop, was carried out on the 22nd May 2024 and for the second survey 66 soundings, visualised in Figure 2 with a 6.25 × 6.25 m loop were measured on 8th October 2024. For both surveys a Voltage of 24V was used and Table 2 shows the parameters for each sounding. Based upon a first visual inspection of the data some soundings were marked as "anomalous" as seen in Figures 1 and 2.

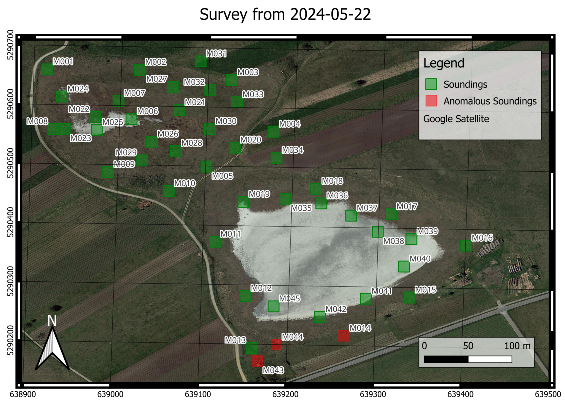

Fig. 1: Locations of all the TEM soundings measured in the first survey (22nd May 2024), where soundings are marked as anomalies, which fell out of order in a first visual inspection.

2.2.3 Python Package

In order to process the gathered data, we developed a python package mainly based on open-source python libraries. For the inversion routine we built upon the work of Aigner et al. (2021), which combines the electromagnetic wave modelling capabilities of `empymod` (Werthmüller 2017) with the inversion algorithm from `PyGIMLi` (Rücker et al. 2017).

The capabilities of this package include the reading of TEM data, adding coordinates to each sounding, the filtering based upon a time intervall, and the visualisation of the raw with the filtered data. The inversion routine requires certain starting parameters like the lambda value, a layer distribution, a start model, and the relative error of the measured signal. If not specified otherwise a homogeneous model with the median apparent resistivity of the sounding is used as the starting model and the relative error is computed based of the error output of the measuring device. In case of particularly noisy data it is possible to set a minimum value for the relative error. This inversion algorithm

Tab. 2: Device settings used as well as the resulting measured time ranges and total number of pulses stacked for each sounding of both surveys.

Sounding	Current	Time Range	Time Key	Stacking Key	Total Stacks
22nd May 2024					
M001, M002	4.1 A	$4-480\,\mu$s	4	3	4992
M003 – M014	1.0 A	$4-480\,\mu$s	4	3	4992
M015 – M045	1.0 A	$4-240\,\mu$s	3	3	9984
8th October 2024					
M001 – M066	4.1 A	$4-240\,\mu$s	3	5	16640

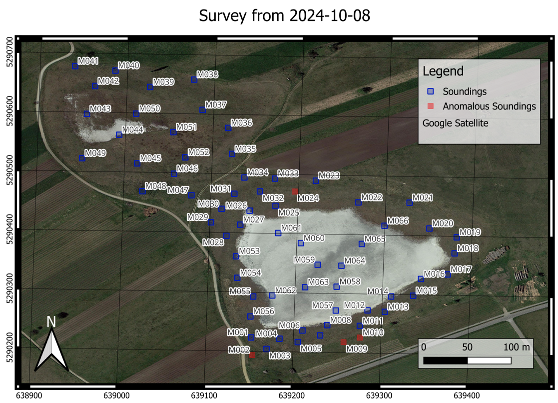

Fig. 2: Locations of all the TEM soundings measured in the second survey (8th October 2024), where soundings are marked as anomalies, which fell out of order in a first visual inspection.

works with a model of the subsurface, where a resistivity value is assigned to a layer with a certain thickness, and only modifies the resistivity value of every layer while keeping the thicknesses fixed. This makes the choice of an appropriate layer distribution (specifying the number and thicknesses of layers) vital (Welkens 2025).

We also implemented the functionality to compute and visualise an L-Curve for a TEM sounding. For this we run the inversion for various logarithmically spaced lambda values, specified by the lower bound, the upper bound, and the number of values. For each inversion, we compute the root-mean-square (RMS) misfit of the data with the model as well as the roughness of the model and use these two values as the coordinates of a point corresponding to each inversion, which should result in an L-Curve (Cultrera et al. 2020; Hansen 1999).

To find an optimal lambda value for the inversion we implemented several search algorithms, which all try to find the point (corresponding to a lambda value) with the highest curvature on the L-Curve (Lloyd et al. 1997; Cultrera et al. 2020). We implemented the method used by Lloyd et al. (1997), which fits a cubic spline function to the L-Curve and computes the first and second derivative of this function, which are used to compute the curvature of the function at each point. We also implemented a similar approach, where we used the `numpy.gradient` function (`https://numpy.org/doc/1.26/reference/generated/numpy.gradient.html`) to compute the first and second derivative for the curvature. Lastly we implemented the iterative golden section search algorithm as described by Cultrera et al. (2020), where a lower and upper bound is defined for the lambda value and by comparing two curvatures within the interval and discarding the lower one, this method contracts the interval towards the optimal lambda. The advantage compared to the other two methods is that it is not bound to the predifined list of logarithmically spaced lambda values, which in theory allows a more precise determination of the optimal lambda.

3 Results

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4 Conclusion

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this thesis.

Data availability

The python package for this thesis was developed in cooperation with Jakob Welkens and is building upon the routine from Aigner et al. (2021). It is available open-source and can be accesed on github (`https://github.com/pb-tuwien/Bsc_TEM_tools.git`). To facilitate full reproducibility of the results all data and python routines used throughout this work can also be accessed on github (`https://github.com/pb-tuwien/BSc_Soda_Lakes_Balogh.git`)

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