EXPERIMENT NO. 9

Aim : To study Bayes' theorem for artificial intelligence

Objective : To understand and study Bayes' Theorem.

Theory :

Bayes' theorem:

Bayes' theorem is also known as **Bayes' rule, Bayes' law**, or **Bayesian reasoning**, which determines the probability of an event with uncertain knowledge.

In probability theory, it relates the conditional probability and marginal probabilities of two random events.

Bayes' theorem was named after the British mathematician **Thomas Bayes**. The **Bayesian inference** is an application of Bayes' theorem, which is fundamental to Bayesian statistics.

It is a way to calculate the value of P(B|A) with the knowledge of P(A|B).

Bayes' theorem allows updating the probability prediction of an event by observing new information of the real world.

Example: If cancer corresponds to one's age then by using Bayes' theorem, we can determine the probability of cancer more accurately with the help of age.

Bayes' theorem can be derived using product rule and conditional probability of event A with known event B:

As from product rule we can write:

1. P(A ∧ B)= P(A|B) P(B) or

Similarly, the probability of event B with known event A:

1. P(A ∧ B)= P(B|A) P(A)

Equating right hand side of both the equations, we will get:

The above equation (a) is called as **Bayes' rule** or **Bayes' theorem**. This equation is basic of most modern AI systems for **probabilistic inference**.

It shows the simple relationship between joint and conditional probabilities. Here,

P(A|B) is known as **posterior**, which we need to calculate, and it will be read as Probability of hypothesis A when we have occurred an evidence B.

P(B|A) is called the likelihood, in which we consider that hypothesis is true, then we calculate the probability of evidence.

P(A) is called the **prior probability**, probability of hypothesis before considering the evidence

P(B) is called **marginal probability**, pure probability of an evidence.

In the equation (a), in general, we can write P (B) = P(A)*P(B|Ai), hence the Bayes' rule can be written as:

Where $A_1, A_2, A_3, \ldots, A_n$ is a set of mutually exclusive and exhaustive events.

Application of Bayes' theorem in Artificial intelligence:

Following are some applications of Bayes' theorem:

- It is used to calculate the next step of the robot when the already executed step is given.
- Bayes' theorem is helpful in weather forecasting.
- It can solve the Monty Hall problem.

Applying Bayes' rule:

Bayes' rule allows us to compute the single term P(B|A) in terms of P(A|B), P(B), and P(A). This is very useful in cases where we have a good probability of these three terms and want to determine the fourth one. Suppose we want to perceive the effect of some unknown cause, and want to compute that cause, then the Bayes' rule becomes:

Question: An email spam filter is trained on a dataset of emails, where each email is labeled as either spam or ham. The filter learns that 80% of all emails are ham and 20% are spam. It also learns that the word "offer" appears in 80% of spam emails and 10% of ham emails.

Now, the filter receives a new email that contains the word "offer". What is the probability that the email is spam?

Solution:

We can use Bayes' theorem to answer this question. Let's define the following events:

- S: the email is spam
- H: the email is ham
- O: the email contains the word "offer"

We want to find the probability of S given that O has already occurred. This can be written as P(S|O).

Using Bayes' theorem, we can write the following equation:

`P(S|O) = P(O|S) * P(S) / P(O)`

We know the following values:

- P(O|S) = the probability that the email contains the word "offer" given that it is spam = 80%
- P(S) = the probability that the email is spam = 20%
- P(O) = the probability that the email contains the word "offer" = 10% (since the word "offer" appears in 80% of spam emails and 10% of ham emails)

Substituting these values into the Bayes' theorem equation, we get:

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P(S|O) = (0.8) * (0.2) / (0.1) = 1.6
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Therefore, the probability that the email is spam, given that it contains the word "offer", is 1.6.

This is just one example of how Bayes' theorem can be used in AI. Bayes' theorem is a powerful tool that can be used to develop a variety of AI applications, such as spam filters, medical diagnosis systems, and fraud detection systems.

CONCLUSION :

In this experiment we have studied Bayes' theorem and its application and implementation in Artificial Intelligence.