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The WILLIAM STATES LEE COLLEGE of ENGINEERING

Introduction to ML
Lecture 8: Naive Bayes Classifier

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Naive Bayes Classifier

Thomas Bayes

1702 - 1761

English statistician

Naive Bayes Classifier

Some basics

The probability of two events A and B happening, P(A∩B), is the probability of A, P(A), times the probability of B given that A has occurred, P(B|A).

$$P(A \cap B) = P(A)P(B|A) \tag{1}$$

On the other hand, the probability of A and B is also equal to the probability of B times the probability of A given B.

$$P(A \cap B) = P(B)P(A|B) \tag{2}$$

Equating the two yields: **(1) = (2)**

$$P(B)P(A|B) = P(A)P(B|A) \tag{3}$$

Bayes Theorem or Bayes Rule

$$P(A|B) = P(A)\frac{P(B|A)}{P(B)}$$

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Naive Bayes Classifier

- Bayesian classifiers use **Bayes theorem**, which says

$$\boxed{p(c_j \mid d)} = \frac{\boxed{p(d \mid c_j)}\,\boxed{p(c_j)}}{p(d)}$$

prior

posterior probability

likelihood

$$P(A|B) = P(A)\frac{P(B|A)}{P(B)}$$

feature d

- $p(c_j \mid d)$ = probability of instance d being in class c_j,
  This is what we are trying to compute
- $p(d \mid c_j)$ = probability of generating instance d given class c_j,
  We can imagine that being in class c_j, causes you to have feature d with some probability
- $p(c_j)$ = probability of occurrence of class c_j,
  This is just how frequent the class c_j, is in our database
- $p(d)$ = probability of instance d occurring

  This can actually be ignored, since it is the same for all classes
  Can be viewed as normalization factor

The following slides are adapted from Eamonn Keogh and Eric Eaton.

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Naive Bayes Classifier

- Why do we make things this complicated?
  - Often $p(d \mid \mathrm{c}_j)$ $p(c_j)$ are easier to get

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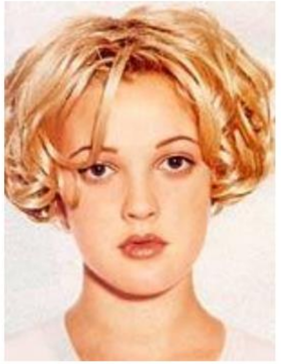
Assume that we have two classes

c_1 = male, and c_2 = female.

We have a person whose sex we do not know, say "*drew*" or *d*.

Classifying *drew* as male or female is equivalent to asking is it more probable that *drew* is male or female, I.e which is greater $p(\text{male} \mid drew)$ or $p(\text{female} \mid drew)$

(Note: "Drew can be a male or female name")



Drew Barrymore

Drew Carey

What is the probability of being called "*drew*" given that you are a male?

What is the probability of being a male?

$$p(\text{male} \mid drew) = \frac{p(drew \mid \text{male})\, p(\text{male})}{p(drew)}$$

What is the probability of being named "*drew*"?

(actually irrelevant, since it is that same for all classes)

Officer Drew

This is Officer Drew

Is Officer Drew a Male or Female?

Luckily, we have a small database with names and sex.

We can use it to apply Bayes rule…

This is our training data

Name	Sex
Drew	Male
Claudia	Female
Drew	Female
Drew	Female
Alberto	Male
Karin	Female
Nina	Female
Sergio	Male

$$p(c_j \mid d) = \frac{p(d \mid c_j)\, p(c_j)}{p(d)}$$

Officer Drew

This is Officer Drew

Is Officer Drew a Male or Female?

Luckily, we have a small database with names and sex.

We can use it to apply Bayes rule…

feature label

Name	Sex
Drew	Male
Claudia	Female
Drew	Female
Drew	Female
Alberto	Male
Karin	Female
Nina	Female
Sergio	Male

$$p(c_j \mid d) = \frac{p(d \mid c_j)\, p(c_j)}{p(d)}$$

$$p(\text{male} \mid drew) = \frac{p(drew \mid \text{male})\, p(\text{male})}{p(drew)}$$

The probability of being a male

Officer Drew

1. The probability of being a male

$$p(\text{male} \mid drew) = \frac{p(drew \mid \text{male})\, p(\text{male})}{p(drew)}$$

$p(drew \mid \text{male})$?

Name	Sex
Drew	Male
Claudia	Female
Drew	Female
Drew	Female
Alberto	Male
Karin	Female
Nina	Female
Sergio	Male

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Officer Drew

1. The probability of being a male

$$p(\text{male} \mid drew) = \frac{p(drew \mid \text{male})\, p(\text{male})}{p(drew)}$$

$p(drew \mid \text{male})$ =1/3

$\frac{1}{3}$

Name	Sex
Drew	Male
Claudia	Female
Drew	Female
Drew	Female
Alberto	Male
Karin	Female
Nina	Female
Sergio	Male

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Officer Drew

1. The probability of being a male

$$p(\text{male} \mid drew) = \frac{p(drew \mid \text{male})\, p(\text{male})}{p(drew)}$$

$p(\text{male})$ = 3/8

Name	Sex
Drew	Male
Claudia	Female
Drew	Female
Drew	Female
Alberto	Male
Karin	Female
Nina	Female
Sergio	Male

Officer Drew

1. The probability of being a male

$$p(male \mid drew) = \frac{p(drew \mid male)\, p(male)}{p(drew)}$$

$p(drew)$ = 3/8

Name	Sex
Drew	Male
Claudia	Female
Drew	Female
Drew	Female
Alberto	Male
Karin	Female
Nina	Female
Sergio	Male

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Officer Drew

1. The probability of being a male

Name	Sex
Drew	Male
Claudia	Female
Drew	Female
Drew	Female
Alberto	Male
Karin	Female
Nina	Female
Sergio	Male

$$p(\text{male} \mid drew) = \frac{p(drew \mid \text{male})\, p(\text{male})}{p(drew)}$$

$$p(\text{male} \mid drew) = \frac{1/3 \; * \; 3/8}{3/8} \qquad = \frac{0.125}{3/8}$$

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Officer Drew

Name	Sex
Drew	Male
Claudia	Female
Drew	Female
Drew	Female
Alberto	Male
Karin	Female
Nina	Female
Sergio	Male

We can follow the same procedure to compute the probability of "Drew" being a female.

$$p(\text{female} \mid drew) = \frac{\overset{p(drew|female)}{2/5} \; * \; \overset{p(female)}{5/8}}{3/8} = \frac{0.250}{3/8}$$

Officer Drew

$$p(c_j \mid d) = \frac{p(d \mid c_j)\, p(c_j)}{p(d)}$$

Name	Sex
Drew	Male
Claudia	Female
Drew	Female
Drew	Female
Alberto	Male
Karin	Female
Nina	Female
Sergio	Male

$$p(\text{male} \mid drew) = \frac{\mathbf{1/3 \;*\; 3/8}}{\mathbf{3/8}} = \frac{\mathbf{0.125}}{3/8}$$

$$\wedge$$

$$p(\text{female} \mid drew) = \frac{\mathbf{2/5 \;*\; 5/8}}{\mathbf{3/8}} = \frac{\mathbf{0.250}}{3/8}$$

Officer Drew is more likely to be a Female.

Officer Drew IS a female!

Officer Drew

$$p(\text{male} \mid drew) = \frac{1/3 * 3/8}{3/8} = \frac{0.125}{3/8}$$

$$p(\text{female} \mid drew) = \frac{2/5 * 5/8}{3/8} = \frac{0.250}{3/8}$$

Naive Bayes Classifier

So far we have only considered Bayes Classification when we have one attribute/feature (the "*name*"). But we may have many features.

How do we use all the features?

Naive Bayes Classifier

- Let's see another example

Sky	Temp	Humid	Wind	Water	Forecast	Play?
sunny	warm	normal	strong	warm	same	*yes*
sunny	warm	high	strong	warm	same	*yes*
rainy	cold	high	strong	warm	change	*no*
sunny	warm	high	strong	cool	change	*yes*

Naive Bayes Classifier

Each individual feature

Estimate $P(X_j \mid Y)$ and $P(Y)$ directly from the training data by counting!

Sky	Temp	Humid	Wind	Water	Forecast	Play?
sunny						*yes*
sunny						*yes*
rainy	cold	high	strong	warm	change	*no*
sunny						*yes*

$P(\text{play}) = 3/4$ $\qquad$ $P(\neg\text{play}) = 1/4$

$P(\text{Sky} = \text{sunny} \mid \text{play}) = 1$ $\qquad$ $P(\text{Sky} = \text{sunny} \mid \neg\text{play}) = ?$

$P(\text{Humid} = \text{high} \mid \text{play}) = ?$ $\qquad$ $P(\text{Humid} = \text{high} \mid \neg\text{play}) = ?$

... $\qquad$ ...

Naive Bayes Classifier

Sky	Temp	Humid	Wind	Water	Forecast	Play?
sunny	warm	normal	strong	warm	same	*yes*
sunny	warm	high	strong	warm	same	*yes*
rainy						*no*
sunny	warm	high	strong	cool	change	*yes*

$P(\text{play}) = 3/4$

$P(\text{Sky} = \text{sunny} \mid \text{play}) = 1$

$P(\text{Humid} = \text{high} \mid \text{play}) = ?$

…

$P(\neg\text{play}) = 1/4$

$P(\text{Sky} = \text{sunny} \mid \neg\text{play}) = 0$

$P(\text{Humid} = \text{high} \mid \neg\text{play}) = ?$

…

No such sample

Naive Bayes Classifier

Sky	Temp	Humid	Wind	Water	Forecast	Play?
sunny	warm	normal	strong	warm	same	*yes*
sunny	warm	high	strong	warm	same	*yes*
rainy	cold	high	strong	warm	change	*no*
sunny	warm	high	strong	cool	change	*yes*

$P(\text{play}) = 3/4$

$P(\neg\text{play}) = 1/4$

$P(\text{Sky} = \text{sunny} \mid \text{play}) = 1$

$P(\text{Sky} = \text{sunny} \mid \neg\text{play}) = 0$

$P(\text{Humid} = \text{high} \mid \text{play}) = 2/3$

$P(\text{Humid} = \text{high} \mid \neg\text{play}) = 1$

…

…

Naive Bayes Classifier

General form

Sky	Temp	Humid	Wind	Water	Forecast	Play?
sunny	warm	normal	strong	warm	same	*yes*
sunny	warm	high	strong	warm	same	*yes*
rainy	cold	high	strong	warm	change	*no*
sunny	warm	high	strong	cool	change	*yes*

x_1 x_2 - - - x_6

$K=2$

An instance/sample is represented by a vector $\mathbf{x} = (x_1, \dots, x_n)$

representing some *n* features (independent variables)

K possible outcomes or *classes* C_k

Goal: assigns to this instance probabilities $p(C_k \mid x_1, \dots, x_n)$

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Naive Bayes Classifier

Using Bayes' theorem, the conditional probability can be decomposed as

$$p(C_k \mid \mathbf{x}) = \frac{p(C_k)\ p(\mathbf{x} \mid C_k)}{p(\mathbf{x})} \qquad \text{posterior} = \frac{\text{prior} \times \text{likelihood}}{\text{evidence}}$$

The numerator is equivalent to the joint probability model

$$p(C_k)\ p(\mathbf{x} \mid C_k) = p(C_k, x_1, \ldots, x_n)$$

Recall

$$P(A \cap B) = P(A)P(B|A)$$

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Naive Bayes Classifier

- Chain rule

$P(A_1, A_2, \ldots, A_n) =$
$P(A_1) * P(A_2 \mid A_1) * P(A_3 \mid A_1, A_2) * \ldots * P(A_n \mid A_1, A_2 \ldots A_{n-1})$

Using the chain rule for repeated applications of the definition of conditional probability:

$$P(A \cap B) = P(A)P(B|A)$$

$$\begin{aligned}
p(C_k, x_1, \ldots, x_n) &= p(x_1, \ldots, x_n, C_k) \\
&= p(x_1 \mid x_2, \ldots, x_n, C_k)\, p(x_2, \ldots, x_n, C_k) \\
&= p(x_1 \mid x_2, \ldots, x_n, C_k)\, p(x_2 \mid x_3, \ldots, x_n, C_k)\, p(x_3, \ldots, x_n, C_k) \\
&= \ldots \\
&= p(x_1 \mid x_2, \ldots, x_n, C_k)\, p(x_2 \mid x_3, \ldots, x_n, C_k) \ldots p(x_{n-1} \mid x_n, C_k)\, p(x_n \mid C_k)\, p(C_k)
\end{aligned}$$

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Naive Bayes Classifier

- "naive" conditional independence assumptions

Assume that each feature x_i is conditionally independent of every other feature x_j for $j \neq i$, given the category C_k

That means:

$$p(x_i \mid \cancel{x_{i+1}, \ldots, x_n}, C_k) = p(x_i \mid C_k)$$

Detour … some background

Independence

- Two events A, B are independent, if (the following are equivalent)
  - $P(A, B) = P(A) * P(B)$
  - $P(A \mid B) = P(A)$
  - $P(B \mid A) = P(B)$

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Coming back - Naive Bayes Classifier

- "naive" conditional independence assumptions

Assume that each feature x_i is conditionally independent of every other feature x_j for $j \neq i$, given the category C_k

That means:

$$p(x_i \mid \cancel{x_{i+1}, \ldots, x_n}, C_k) = p(x_i \mid C_k)$$

Don't care about these anymore

Naive Bayes Classifier

- Thus, the joint model can be expressed as

$$
\begin{aligned}
p(C_k \mid x_1, \dots, x_n) \boxed{\propto} \, & p(C_k, x_1, \dots, x_n) \\
&= p(C_k) \, p(x_1 \mid C_k) \, p(x_2 \mid C_k) \, p(x_3 \mid C_k) \, \cdots \\
&= p(C_k) \prod_{i=1}^{n} p(x_i \mid C_k) \, ,
\end{aligned}
$$

Denote " proportional "

$$
p(C_k \mid \mathbf{x}) = \frac{p(C_k) \, p(\mathbf{x} \mid C_k)}{\boxed{p(\mathbf{x})}}
$$

Naive Bayes Classifier

Using the Naïve Bayes Classifier

To classify a new point $\mathbf{x}$

Find the maximum

$$\hat{y} = \underset{k \in \{1,\dots,K\}}{\operatorname{argmax}} \ p(C_k) \prod_{i=1}^{n} p(x_i \mid C_k)$$

Any potential problem ?

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Naive Bayes Classifier

Using the Naïve Bayes Classifier

$$\hat{y} = \underset{k \in \{1,\ldots,K\}}{\operatorname{argmax}}\ p(C_k) \prod_{i=1}^{n} p(x_i \mid C_k)$$

We are multiplying lots of small numbers

Danger of underflow!

- $0.5^{57} = 7$ E -18

a number of smaller absolute value than the computer can actually represent in memory on its CPU.

Solution?

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Naive Bayes Classifier

We are multiplying lots of small numbers
Danger of underflow!

- $0.5^{57} = 7$ E -18

Solution? $\boxed{\text{Use logs and add!}}$

– In practice, we use log-probabilities to prevent underflow

$$\arg\max_{y_k} \log P(Y = y_k) + \sum_{j=1}^{n} \log P(X_j = x_j \mid Y = y_k)$$

class label {1, … , K}

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Problems with Naive Bayes Classifier

- Naive Bayes assumption
  - Usually, features are not conditionally independent:

$$P(X_1 ... X_n | Y) \neq \prod_i P(X_i | Y)$$

  - The naïve Bayes assumption is often violated, yet it performs surprisingly well in many cases.

More deeper investigation: Zhang, Harry. "The optimality of naive Bayes." AAAI (2004).

Problems with Naive Bayes Classifier

Remember this case ?

Sky	Temp	Humid	Wind	Water	Forecast	Play?
sunny	warm	normal	strong	warm	same	*yes*
sunny	warm	high	strong	warm	same	*yes*
rainy						*no*
sunny	warm	high	strong	cool	change	*yes*

$P(\text{play}) = 3/4$

$P(\text{Sky} = \text{sunny} \mid \text{play}) = 1$

$P(\text{Humid} = \text{high} \mid \text{play}) = ?$

...

$P(\neg\text{play}) = 1/4$

$P(\text{Sky} = \text{sunny} \mid \neg\text{play}) = 0$

$P(\text{Humid} = \text{high} \mid \neg\text{play}) = ?$

...

Problems with Naive Bayes Classifier

x_1 x_2 - - - x_6

Sky	Temp	Humid	Wind	Water	Forecast	Play?
sunny	warm	normal	strong	warm	same	*yes*
sunny	warm	high	strong	warm	same	*yes*
rainy	cold	high	strong	warm	change	*no*
sunny	warm	high	strong	cool	change	*yes*

$P(\text{Sky} = \text{sunny} \mid \neg\text{play}) = 0$

Assume a new point: x = (x1: sky=sunny, x2, … , x6)

$$p(C_k \mid x_1, \dots, x_n) \propto p(C_k) \prod_{i=1}^{n} p(x_i \mid C_k)$$

P($\neg$play | X1 (sky=sunny), x2, … , x6)=P($\neg$play)$P(\text{Sky} = \text{sunny} \mid \neg\text{play})$ P( ) …

no matter what values X2, X3,….,X6 take

0

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Problems with Naive Bayes Classifier

- Notice that some probabilities estimated by counting might be zero

- Fix by using Laplace smoothing:
  - Likelihood:

$$P(X_i = x|Y = y) = \frac{Count(X_i = x, Y = y) \boxed{+1}}{\sum_{x'} Count(X_i = x', Y = y) \boxed{+ \ |\mathrm{values}(X_i)|}}$$

$|\mathrm{values}(X_i)|$ is the number of values X_i can take on

Naive Bayes with Laplace Smoothing

Estimate $P(X_j \mid Y)$ and $P(Y)$ directly from the training data by counting with Laplace smoothing:

Sky	Temp	Humid	Wind	Water	Forecast	Play?
sunny						*yes*
sunny						*yes*
rainy	cold	high	strong	warm	change	*no*
sunny						*yes*

Type equation here.

$$\frac{3 + (1)}{3 + (2)}$$

P(play) = 3/4

P(¬play) = 1/4

P(Sky = sunny | play) = 4/5

P(Sky = sunny | ¬play) = ?

P(Humid = high | play) = ?

P(Humid = high | ¬play) = ?

...

...

$$P(X_i = x|Y = y) = \frac{Count(X_i = x, Y = y) \boxed{+1}}{\sum_{x'} Count(X_i = x', Y = y) \boxed{+ \ |\text{values}(X_i)|}}$$

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Naive Bayes with Laplace Smoothing

Estimate $P(X_j \mid Y)$ and $P(Y)$ directly from the training data by counting with Laplace smoothing:

Sky	Temp	Humid	Wind	Water	Forecast	Play?
sunny	warm	normal	strong	warm	same	*yes*
sunny	warm	high	strong	warm	same	*yes*
rainy						*no*
sunny	warm	high	strong	cool	change	*yes*

P(play) = 3/4

P(Sky = sunny | play) = 4/5

P(Humid = high | play) = ?

...

P(¬play) = 1/4

P(Sky = sunny | ¬play) = 1/3

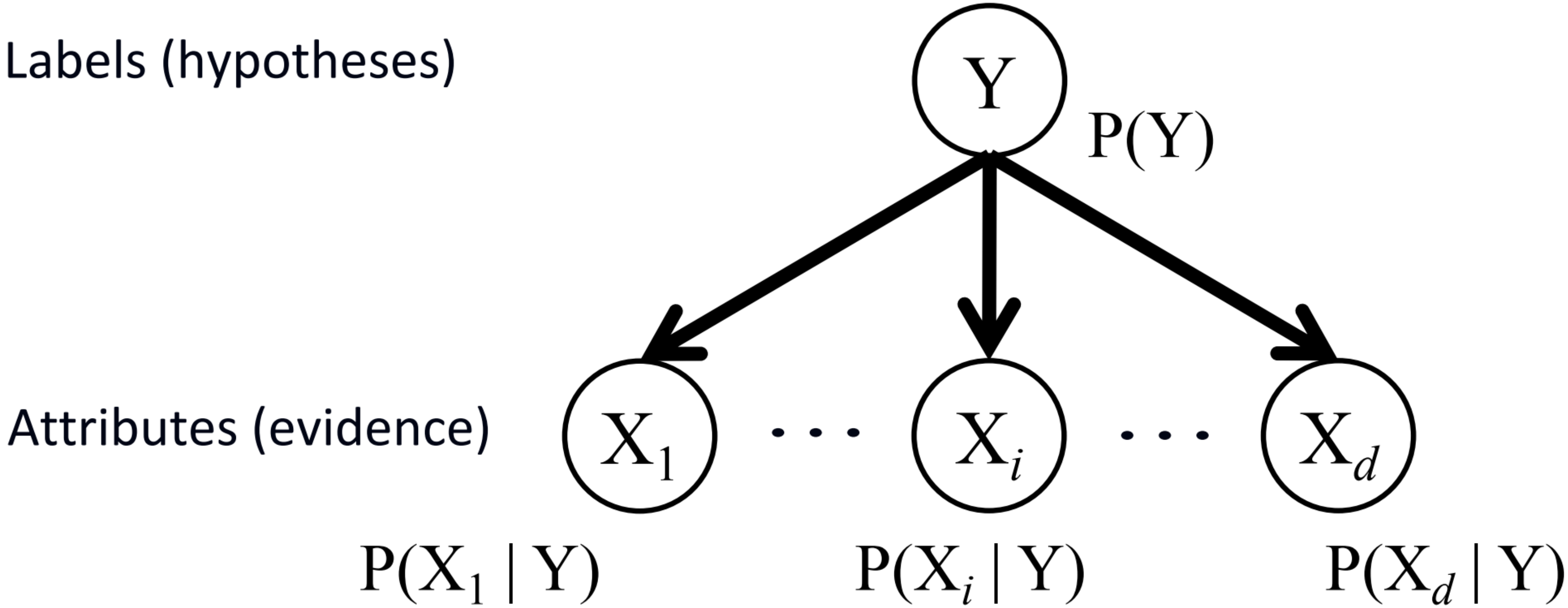
$$\frac{0 + (1)}{1 + (2)}$$

P(Humid = high | ¬play) = ?

...

$$P(X_i = x|Y = y) = \frac{Count(X_i = x, Y = y) \boxed{+1}}{\sum_{x'} Count(X_i = x', Y = y) \boxed{+ \ |\text{values}(X_i)|}}$$

The Naïve Bayes Graphical Model

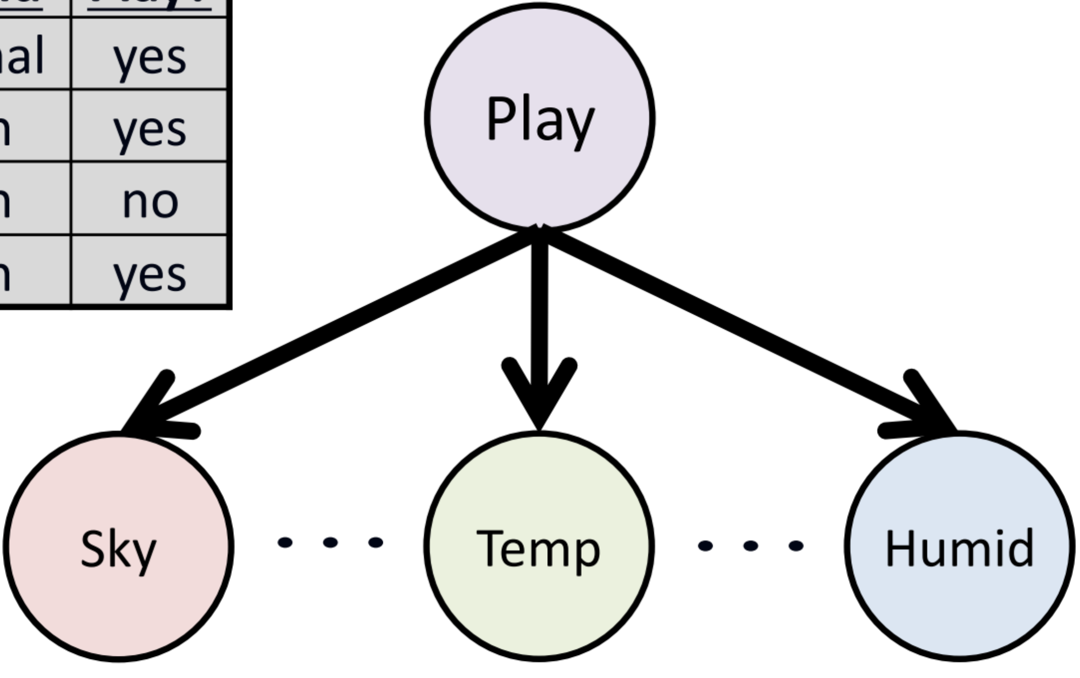


- Nodes denote random variables
- Edges denote dependency
- Each node has an associated conditional probability table (CPT), conditioned upon its parents

Example NB Graphical Model

Data:

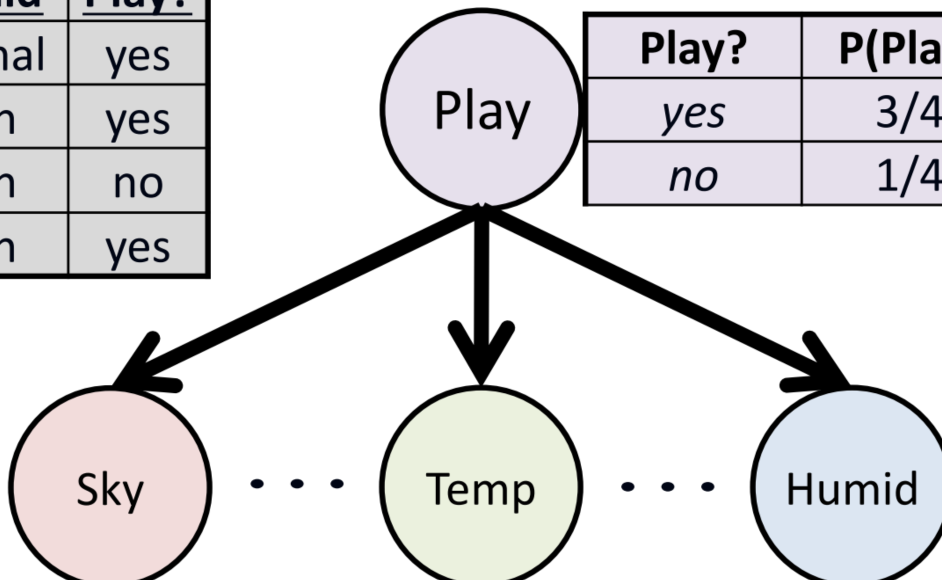
Sky	Temp	Humid	Play?
sunny	warm	normal	yes
sunny	warm	high	yes
rainy	cold	high	no
sunny	warm	high	yes



Example NB Graphical Model

Data:

Sky	Temp	Humid	Play?
sunny	warm	normal	yes
sunny	warm	high	yes
rainy	cold	high	no
sunny	warm	high	yes



Play?	P(Play)
yes	3/4
no	1/4

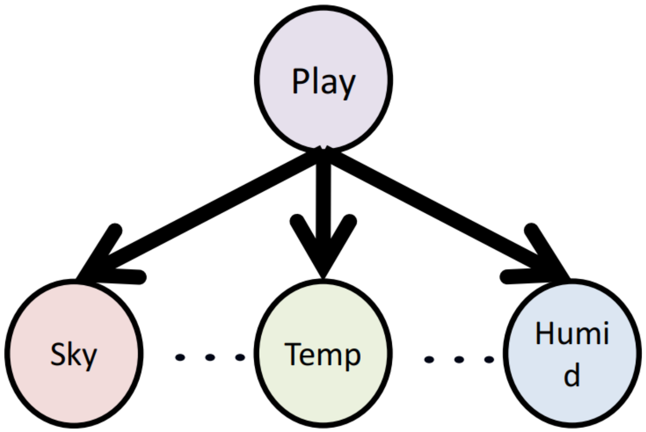
Laplace smoothing is applied

Sky	Play?	P(Sky | Play)
sunny	*yes*	4/5
rainy	*yes*	1/5
sunny	*no*	1/3
rainy	*no*	2/3

Temp	Play?	P(Temp | Play)
warm	*yes*	4/5
cold	*yes*	1/5
warm	*no*	1/3
cold	*no*	2/3

Humid	Play?	P(Humid | Play)
high	*yes*	3/5
norm	*yes*	2/5
high	*no*	2/3
norm	*no*	1/3

Example Using NB for Classification



Play?	P(Play)
yes	3/4
no	1/4

Temp	Play?	P(Temp | Play)
warm	*yes*	4/5
cold	*yes*	1/5
warm	*no*	1/3
cold	*no*	2/3

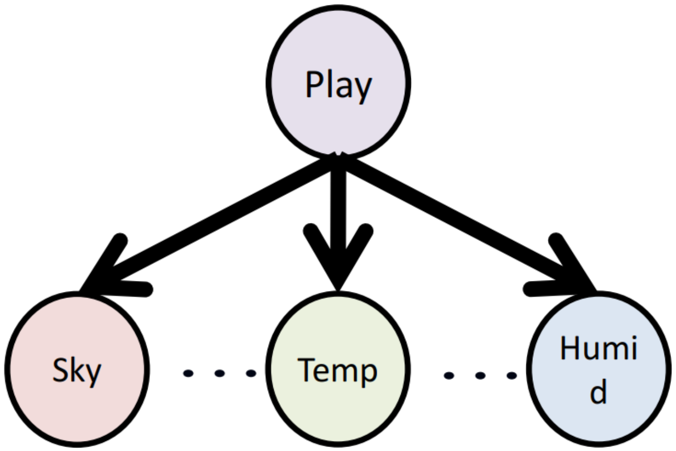
Sky	Play?	P(Sky | Play)
sunny	*yes*	4/5
rainy	*yes*	1/5
sunny	*no*	1/3
rainy	*no*	2/3

Humid	Play?	P(Humid | Play)
high	*yes*	3/5
norm	*yes*	2/5
high	*no*	2/3
norm	*no*	1/3

$$h(\mathbf{x}) = \arg\max_{y_k} \; \log P(Y = y_k) + \sum_{j=1}^{d} \log P(X_j = x_j \mid Y = y_k)$$

Goal: Predict label for $\mathbf{x}$ = (rainy, warm, normal)

Example Using NB for Classification



Play?	P(Play)
yes	3/4
no	1/4

Temp	Play?	P(Temp | Play)
warm	*yes*	4/5
cold	*yes*	1/5
warm	*no*	1/3
cold	*no*	2/3

Sky	Play?	P(Sky | Play)
sunny	*yes*	4/5
rainy	*yes*	1/5
sunny	*no*	1/3
rainy	*no*	2/3

Humid	Play?	P(Humid | Play)
high	*yes*	3/5
norm	*yes*	2/5
high	*no*	2/3
norm	*no*	1/3

Predict label for:

$\mathbf{x}$ = (rainy, warm, normal)

$$P(\text{play} \mid \mathbf{x}) \propto \log P(\text{play}) + \log P(\text{rainy} \mid \text{play}) + \log P(\text{warm} \mid \text{play}) + \log P(\text{normal} \mid \text{play})$$

$$\propto \log 3/4 + \log 1/5 + \log 4/5 + \log 2/5 = \boxed{-1.319 \quad \textbf{predict PLAY}}$$

$$P(\neg\text{play} \mid \mathbf{x}) \propto \log P(\neg\text{play}) + \log P(\text{rainy} \mid \neg\text{play}) + \log P(\text{warm} \mid \neg\text{play}) + \log P(\text{normal} \mid \neg\text{play})$$

$$\propto \log 1/4 + \log 2/3 + \log 1/3 + \log 1/3 = -1.732$$

Continuous features (Gaussian Naive Bayes)

- Suppose the training data contains a continuous attribute x
- We first segment the data by the class, and then compute the mean and variance of x in each class
- For each class, compute

μ_k : the mean of x in class C_k

σ_k^2 : the variance of x in class C_k

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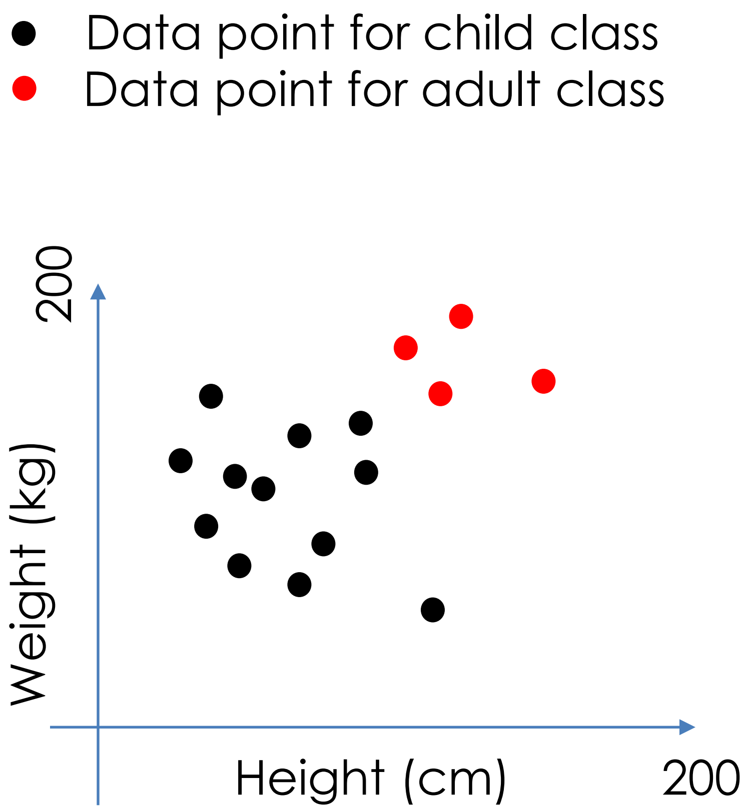
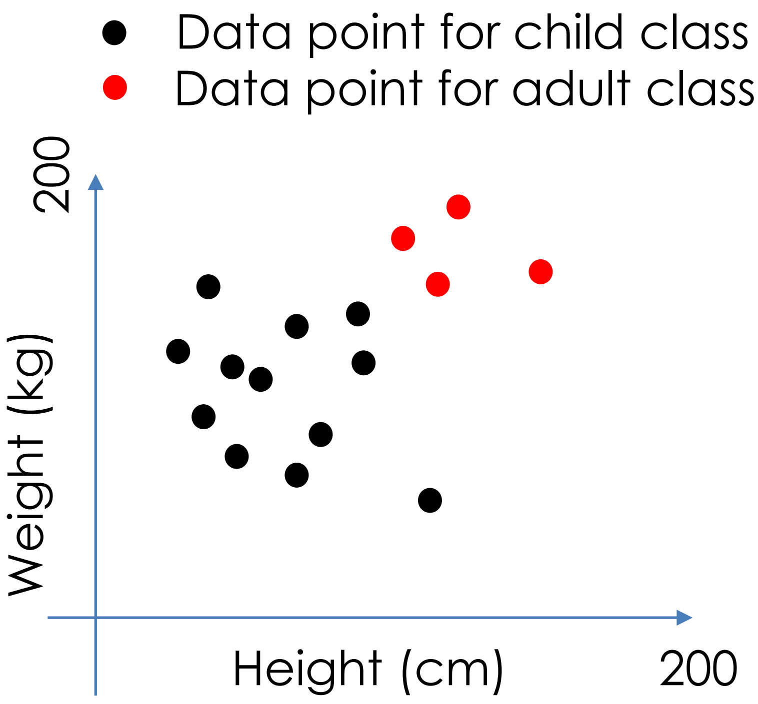
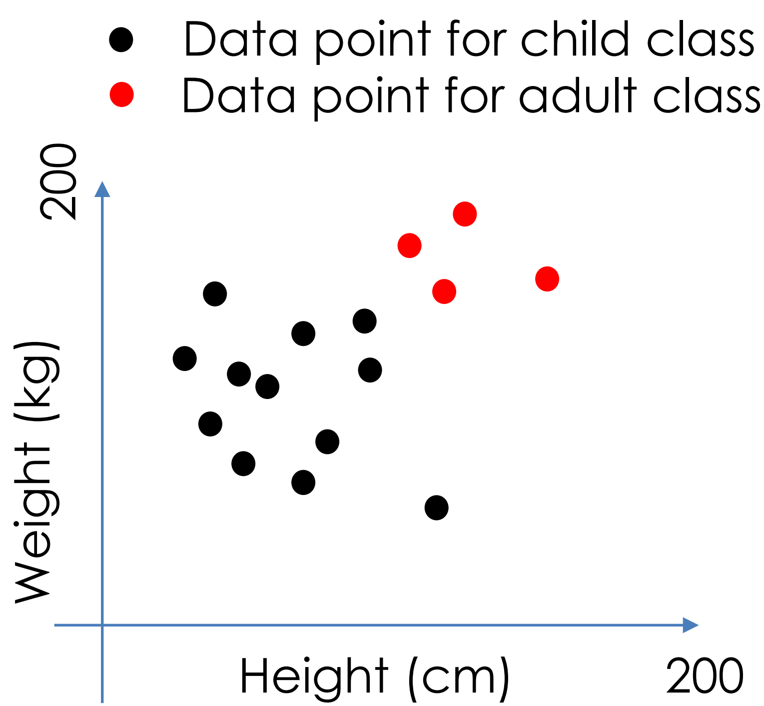
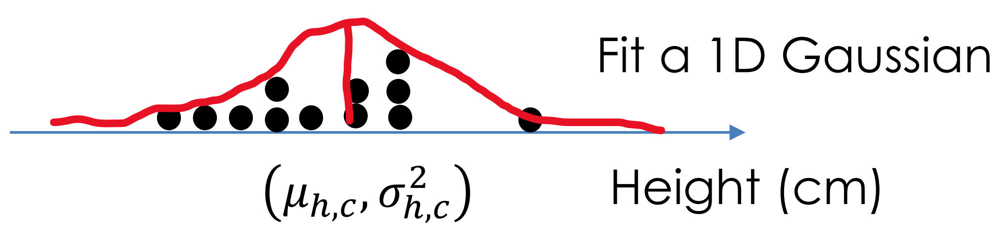
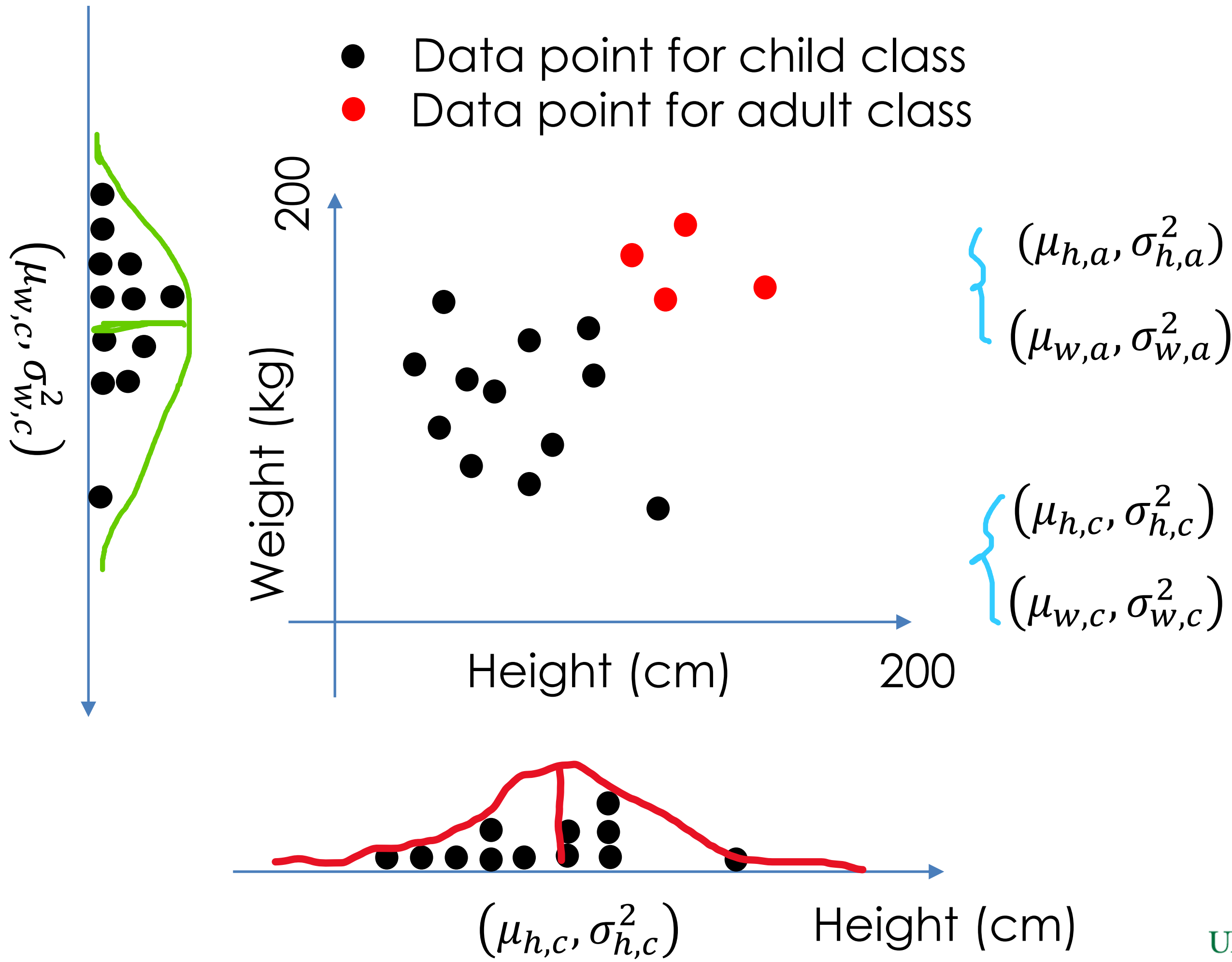

Data point for child class
Data point for adult class
200
Weight (kg)
Height (cm)
200



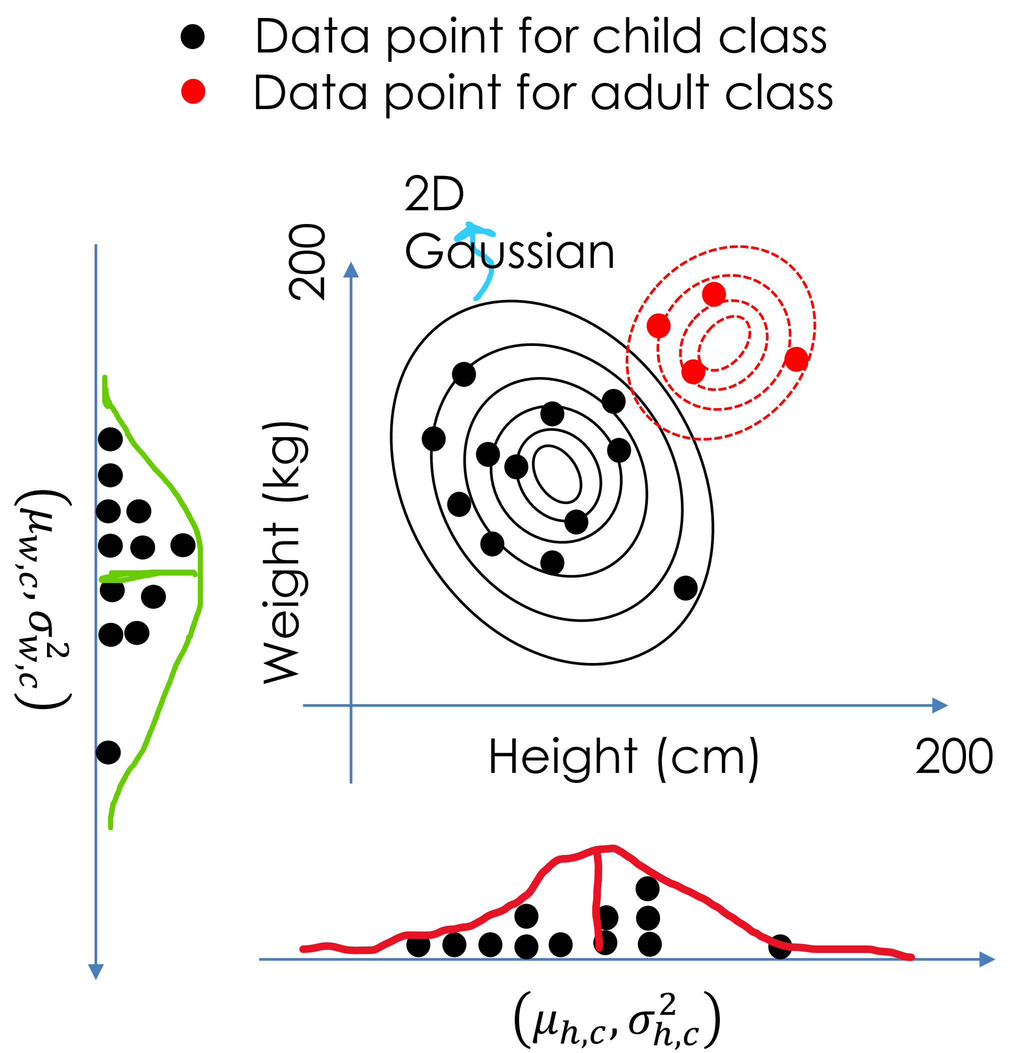
UNC CHARLOTTE


Data point for child class
Data point for adult class
200
Weight (kg)
Height (cm)
200


Fit a 1D Gaussian
$(\mu_{h,c}, \sigma^2_{h,c})$
Height (cm)



UNC CHARLOTTE



Data point for child class
Data point for adult class
2D
Gaussian
200
Weight (kg)
Height (cm)
200
$(\mu_{w,c}, \sigma^2_{w,c})$
$(\mu_{h,c}, \sigma^2_{h,c})$

Example

- Example

Problem: classify whether a given person is **a male or a female** based on the measured features. The features include height, weight, and foot size.

Person	height (feet)	weight (lbs)	foot size(inches)
male	6	180	12
male	5.92 (5'11")	190	11
male	5.58 (5'7")	170	12
male	5.92 (5'11")	165	10
female	5	100	6
female	5.5 (5'6")	150	8
female	5.42 (5'5")	130	7
female	5.75 (5'9")	150	9

	x1	x2	x3
Person	height (feet)	weight (lbs)	foot size(inches)
male	6	180	12
male	5.92 (5'11")	190	11
male	5.58 (5'7")	170	12
male	5.92 (5'11")	165	10
female	5	100	6
female	5.5 (5'6")	150	8
female	5.42 (5'5")	130	7
female	5.75 (5'9")	150	9

Class – male
Training samples

Class – female
Training samples

Use Gaussian distribution assumption

Person	height (feet)	weight (lbs)	foot size(inches)
male	6	180	12
male	5.92 (5'11")	190	11
male	5.58 (5'7")	170	12
male	5.92 (5'11")	165	10
female	5	100	6
female	5.5 (5'6")	150	8
female	5.42 (5'5")	130	7
female	5.75 (5'9")	150	9

The sample mean of a set $\{x_1, \ldots, x_n\}$ of n observations from a given distribution is defined by

$$\bar{x} \equiv \frac{1}{n} \sum_{k=1}^{n} x_k .$$

unbiased estimator for $\sigma^2 \equiv \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2$

UNC CHARLOTTE

Person	height (feet)	weight (lbs)	foot size(inches)
male	6	180	12
male	5.92 (5'11")	190	11
male	5.58 (5'7")	170	12
male	5.92 (5'11")	165	10
female	5	100	6
female	5.5 (5'6")	150	8
female	5.42 (5'5")	130	7
female	5.75 (5'9")	150	9

% mean for height (male)
>> (6+5.92+5.58+5.92)/4
a = 5.855

% variance for height (male)
>> ((6-a)^2+(5.92-a)^2+(5.58-a)^2+(5.92-a)^2)/3
ans = 0.035033

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Person	height (feet)	weight (lbs)	foot size(inches)
male	6	180	12
male	5.92 (5'11")	190	11
male	5.58 (5'7")	170	12
male	5.92 (5'11")	165	10
female	5	100	6
female	5.5 (5'6")	150	8
female	5.42 (5'5")	130	7
female	5.75 (5'9")	150	9

Person	mean (height)	variance (height)	mean (weight)	variance (weight)	mean (foot size)	variance (foot size)
male	5.855	$3.5033*10^{-2}$	176.25	$1.2292*10^{2}$	11.25	$9.1667*10^{-1}$
female	5.4175	$9.7225*10^{-2}$	132.5	$5.5833*10^{2}$	7.5	1.6667

We have equiprobable classes so P(male)= P(female) = 0.5

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Below is a sample to be classified as male or female.

Person	height (feet)	weight (lbs)	foot size(inches)
sample	6	130	8

Goal: We wish to determine which posterior is greater, male or female.

For the classification as **male** the posterior is given by

$$\text{posterior (male)} = \frac{P(\text{male})\, p(\text{height} \mid \text{male})\, p(\text{weight} \mid \text{male})\, p(\text{foot size} \mid \text{male})}{evidence}$$

Note: In Bayesian statistics, the posterior probability of a random event or an uncertain proposition is the conditional probability that is assigned after the relevant evidence or background is taken into account.

Below is a sample to be classified as male or female.

Person	height (feet)	weight (lbs)	foot size(inches)
sample	6	130	8

Goal: We wish to determine which posterior is greater, male or female.

For the classification as **female** the posterior is given by

$$\text{posterior (female)} = \frac{P(\text{female})\, p(\text{height} \mid \text{female})\, p(\text{weight} \mid \text{female})\, p(\text{foot size} \mid \text{female})}{evidence}$$

Below is a sample to be classified as male or female.

Person	height (feet)	weight (lbs)	foot size(inches)
sample	6	130	8

Goal: We wish to determine which posterior is greater, male or female.

The evidence (also termed normalizing constant) can be calculated:

The evidence is a constant and thus scales both posteriors equally. It therefore does not affect classification and can be ignored.

A new sample to be classified:

Person	height (feet)	weight (lbs)	foot size(inches)
sample	6	130	8

We now determine the probability distribution for the sex of the sample.

$$P(\text{male}) = 0.5$$

$$p(\text{height} \mid \text{male}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(6-\mu)^2}{2\sigma^2}\right) \approx \underline{1.5789}$$

It is probability density rather than probability, because height is a continuous variable – therefore it is OK to greater than 1.

where $\mu = 5.855$ and $\sigma^2 = 3.5033 \cdot 10^{-2}$

Person	mean (height)	variance (height)	mean (weight)	variance (weight)	mean (foot size)	variance (foot size)
male	5.855	$3.5033*10^{-2}$	176.25	$1.2292*10^{2}$	11.25	$9.1667*10^{-1}$
female	5.4175	$9.7225*10^{-2}$	132.5	$5.5833*10^{2}$	7.5	1.6667

They are the parameters of normal distribution which have been previously determined from the training set.

$$P(\text{male}) = 0.5$$

$$p(\text{height} \mid \text{male}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(6-\mu)^2}{2\sigma^2}\right) \approx 1.5789$$

$$p(\text{weight} \mid \text{male}) = 5.9881 \cdot 10^{-6}$$

$$p(\text{foot size} \mid \text{male}) = 1.3112 \cdot 10^{-3}$$

$$\text{posterior numerator (male)} = \text{their product} = 6.1984 \cdot 10^{-9}$$

$$P(\text{female}) = 0.5$$

$$p(\text{height} \mid \text{female}) = 2.2346 \cdot 10^{-1}$$

$$p(\text{weight} \mid \text{female}) = 1.6789 \cdot 10^{-2}$$

$$p(\text{foot size} \mid \text{female}) = 2.8669 \cdot 10^{-1}$$

$$\text{posterior numerator (female)} = \text{their product} = 5.3778 \cdot 10^{-4}$$

Person	height (feet)	weight (lbs)	foot size(inches)
sample	6	130	8

→ Female

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