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The WILLIAM STATES LEE COLLEGE of ENGINEERING

Introduction to ML
Lecture 13: Support Vector Machine

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Recap

Perceptron:

- Set t=1, start with the all zero vector w_1.
- Given example x, predict positive iff $w_t \cdot x \geq 0$
- On a mistake, update as follows:
  - Mistake on positive, then update $w_{t+1} \leftarrow w_t + x$
  - Mistake on negative, then update $w_{t+1} \leftarrow w_t - x$

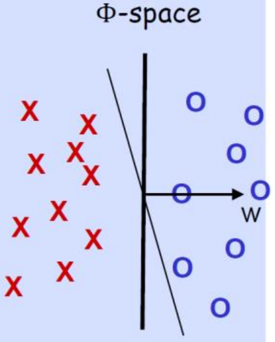
Kernelization:

- Given x, predict + iff

$$\phi(x_{i_{t-1}}) \cdot \phi(x)$$

$$a_{i_1} K(x_{i_1}, x) + \cdots + a_{i_{t-1}} K(x_{i_{t-1}}, x) \geq 0$$

- On the t th mistake, update as follows:
  - Mistake on positive, set $a_{i_t} \leftarrow 1$; store x_{i_t}
  - Mistake on negative, $a_{i_t} \leftarrow -1$; store x_{i_t}



Intuitive Definition

- A linear discriminative classifier would attempt to draw a straight line separating the two sets of data, and thereby create a model for classification.
- The intuition is this: rather than simply drawing a zero-width line between the classes, we can draw around each line a *margin* of some width, up to the nearest point.
- In support vector machines, the line that maximizes this margin is the one we will choose as the optimal model.
- Support vector machines are an example of such a *maximum margin* estimator.

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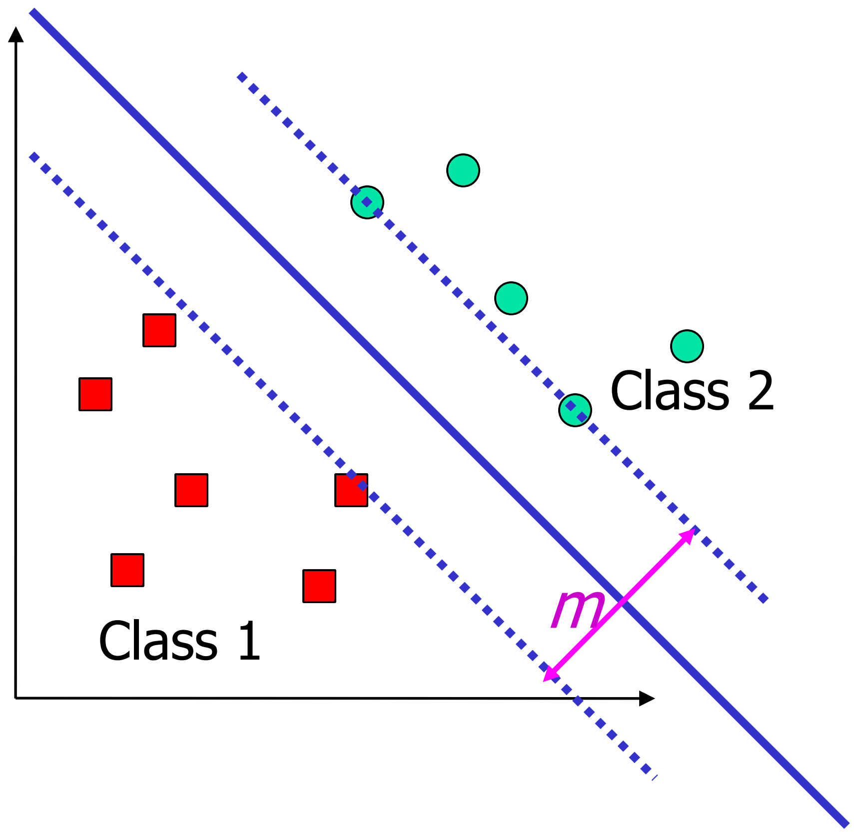
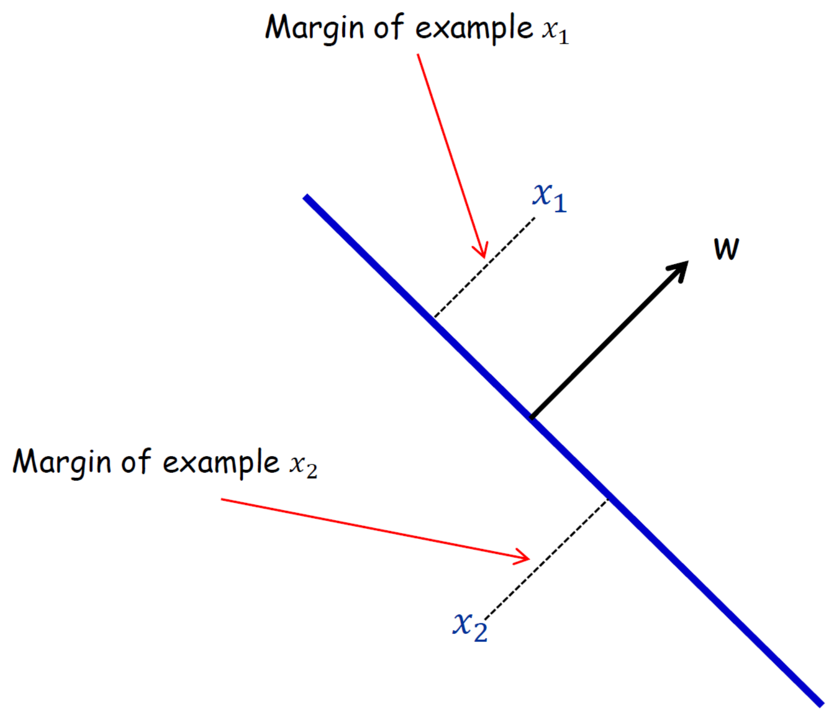
Good Decision Boundary: Margin Should Be Large

- The decision boundary should be as far away from the data of both classes as possible

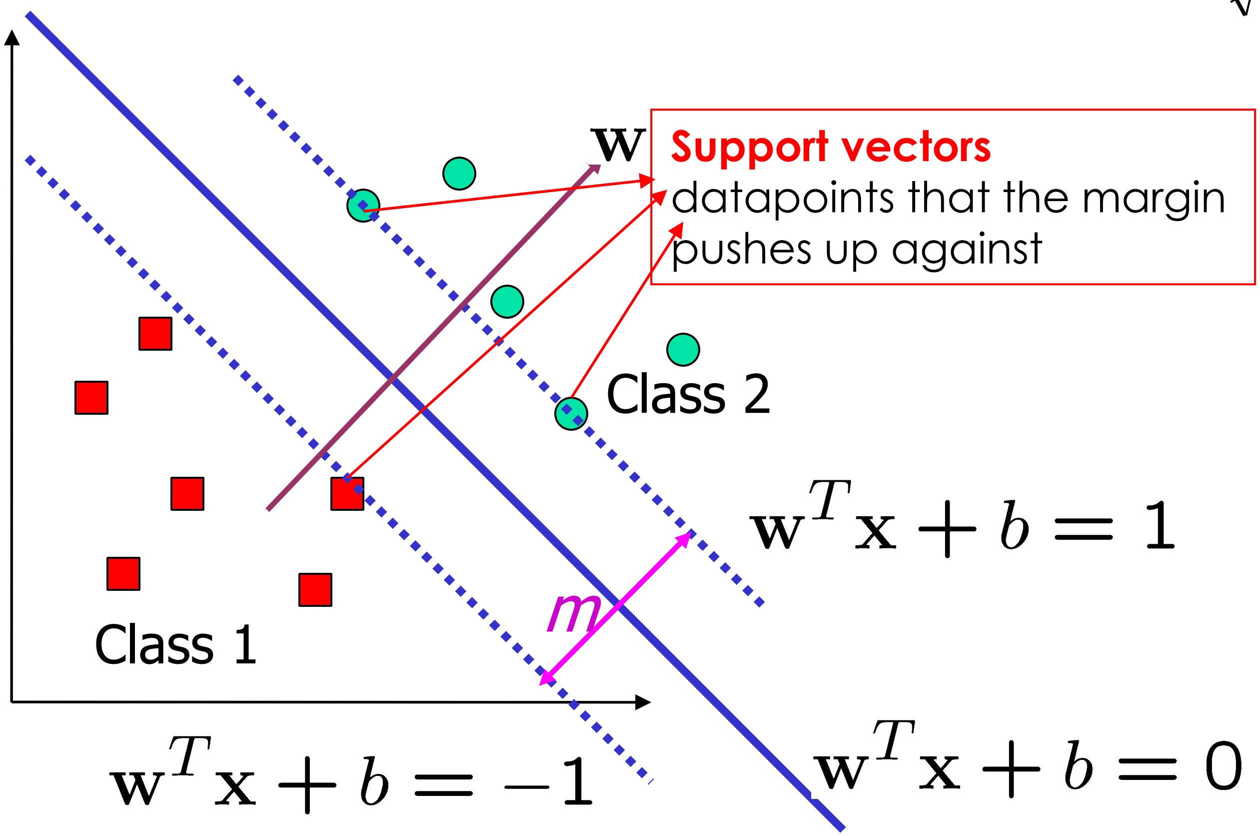




Good Decision Boundary: Margin Should Be Large

- The decision boundary should be as far away from the data of both classes as possible
  - We should maximize the margin, m

$$m = \frac{2}{\sqrt{w.w}} \qquad m = \frac{2}{||\mathbf{w}||}$$



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The Optimization Problem

- Let $\{x_1, ..., x_n\}$ be our data set and let $y_i \in \{1,-1\}$ be the class label of x_i
- The decision boundary should classify all points correctly $\Rightarrow$

$$y_i(\mathbf{w}^T\mathbf{x}_i + b) \geq 1, \qquad \forall i$$

- A constrained optimization problem

$$\text{Minimize } \frac{1}{2}||\mathbf{w}||^2$$

- $||\mathbf{w}||^2 = \mathbf{w}^T\mathbf{w}$

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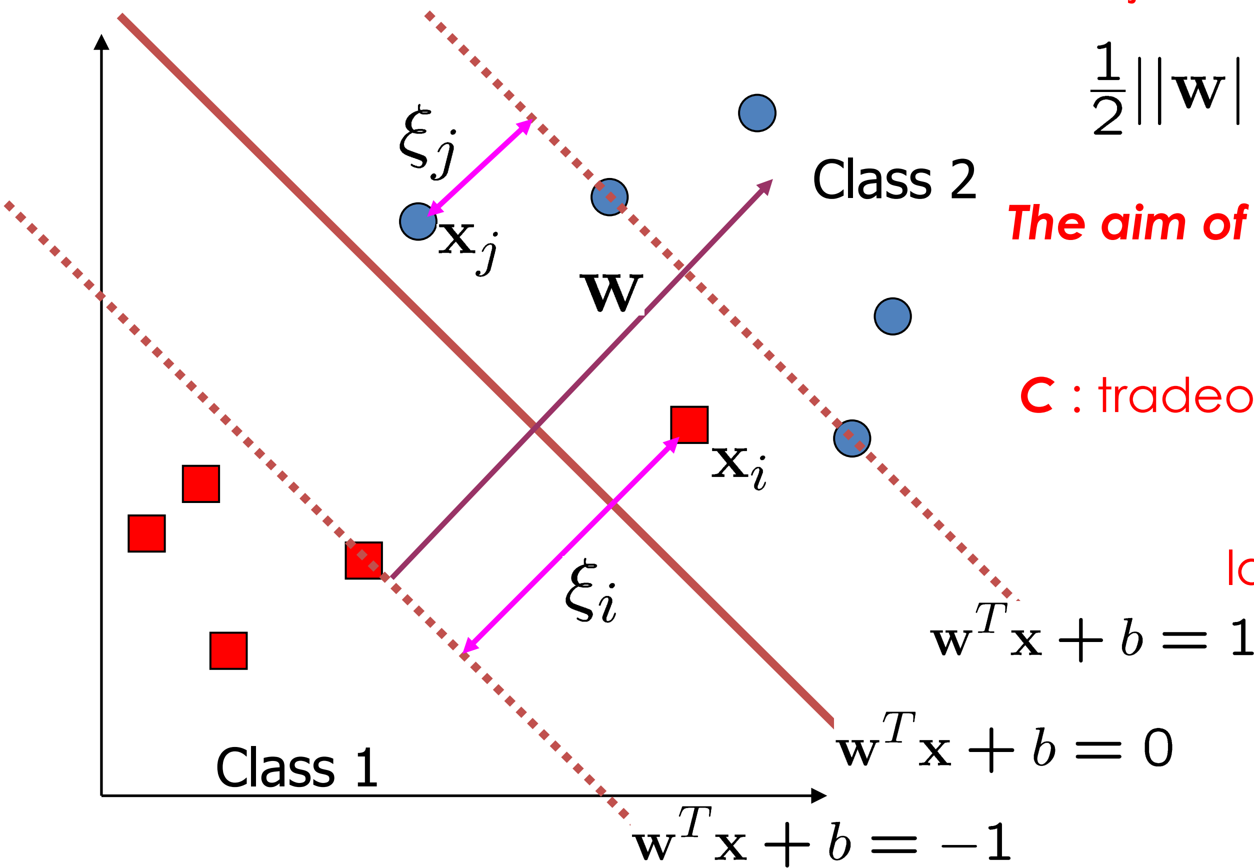
Non-linearly Separable Problems

- We allow "error" ξ_i in classification; it is based on the output of the discriminant function $\mathbf{w}^T\mathbf{x}+b$
- ξ_i approximates the number of misclassified samples



New objective function:

$$\frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{n}\xi_i$$

The aim of SVM is to minimize this objective function.

C : tradeoff parameter between error and margin; chosen by the user; large C means a higher penalty to errors

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The effect of C

- A small C will give a wider margin, at the cost of some misclassifications.

- A huge C will give the hard margin classifier and tolerates zero constraint violation -> Less Generalization Opportunities

- The key is to find the value of such that noisy data does not impact the solution too much.

How to find C？→ search with cross-validation

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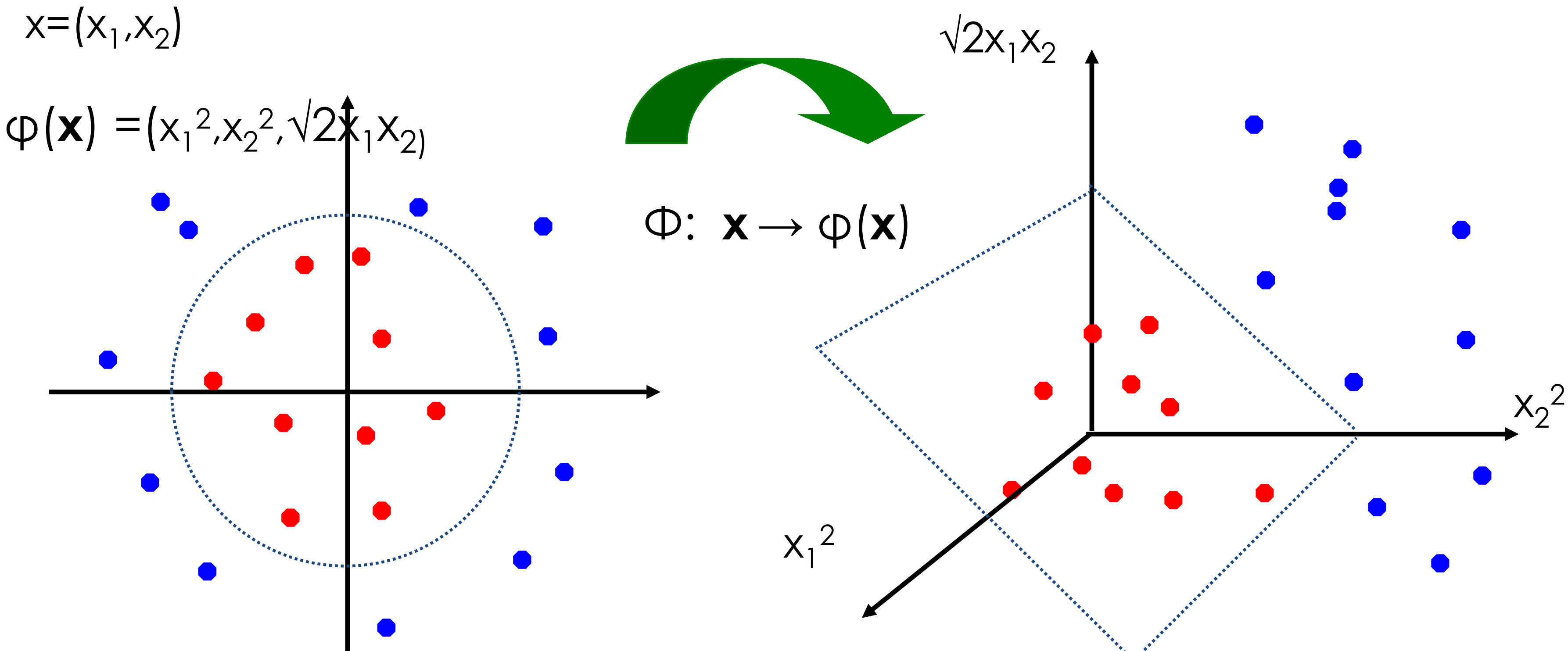
Extension to Non-linear SVMs (Kernel Machines)

Non-linear SVMs: Feature Space

General idea: the original input space (x) can be mapped to some higher-dimensional feature space ($\varphi(\mathbf{x})$)where the training set is separable:



If data are mapped into higher a space of sufficiently high dimension, then they will in general be linearly separable;
N data points are in general separable in a space of N-1 dimensions or more!!!

This slide is courtesy of *www.iro.umontreal.ca/~pift6080/documents/papers/**svm**_tutorial.ppt*

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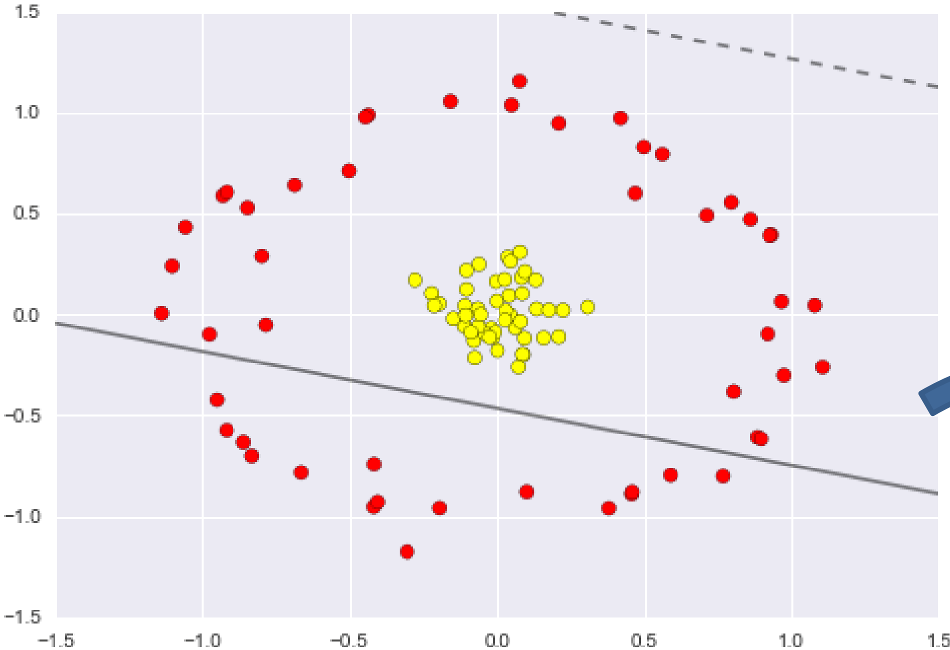
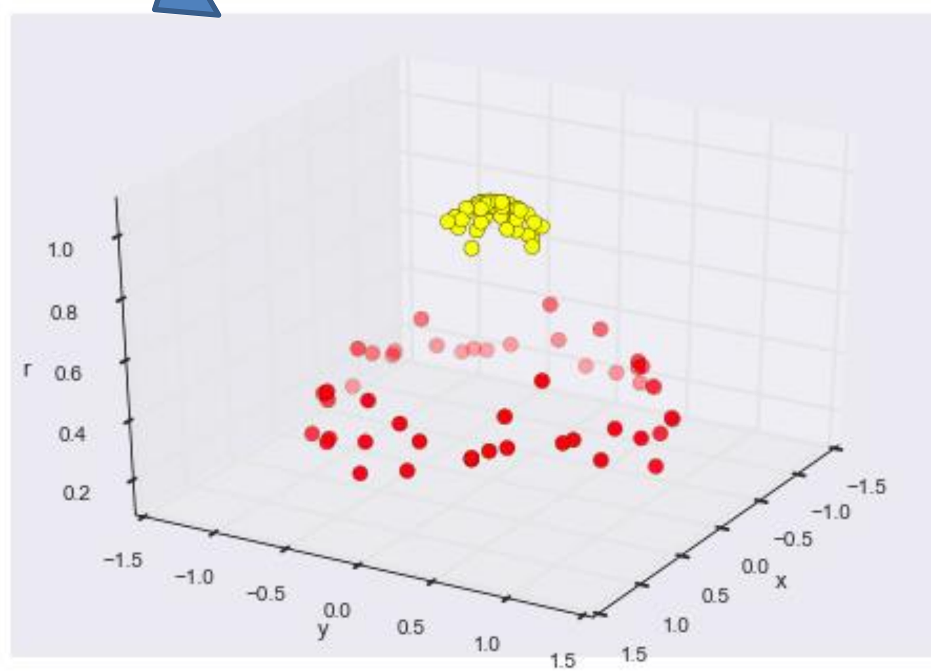

1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


r
1.0
0.8
0.6
0.4
0.2
y
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
x
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

Example Transformation

- Consider the following transformation

$$\phi(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}) = (1, \sqrt{2}x_1, \sqrt{2}x_2, x_1^2, x_2^2, \sqrt{2}x_1x_2)$$

$$\phi(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}) = (1, \sqrt{2}y_1, \sqrt{2}y_2, y_1^2, y_2^2, \sqrt{2}y_1y_2)$$

- Define the kernel function *K* (**x**,**y**) as

$$\begin{aligned} \langle \phi(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}), \phi(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}) \rangle &= (1 + x_1y_1 + x_2y_2)^2 \\ &= K(\mathbf{x}, \mathbf{y}) \end{aligned}$$

$$K(\mathbf{x}, \mathbf{y}) = (1 + x_1y_1 + x_2y_2)^2$$

- The inner product $\phi(.)\phi(.)$ can be computed by *K* without going through the map $\phi(.)$ explicitly!!!

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Examples of Kernel Functions

- Polynomial kernel with degree *d*

$$K(\mathbf{x}, \mathbf{y}) = (\mathbf{x}^T \mathbf{y} + 1)^d$$

- Radial basis function kernel with width σ

$$K(\mathbf{x}, \mathbf{y}) = \exp(-||\mathbf{x} - \mathbf{y}||^2 / (2\sigma^2)) = \exp\left(-\gamma ||x - y||^2\right)$$

- Hyperbolic Tangent (Sigmoid) kernel with parameter κ and θ

$$K(\mathbf{x}, \mathbf{y}) = \tanh(\kappa \mathbf{x}^T \mathbf{y} + \theta)$$

- Research on different kernel functions in different applications is very active

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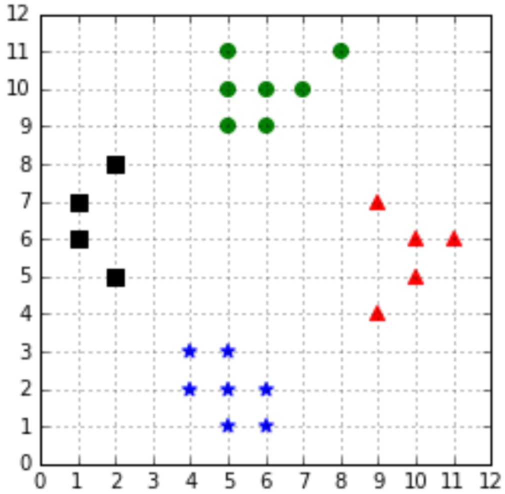
Recap of Steps in SVM

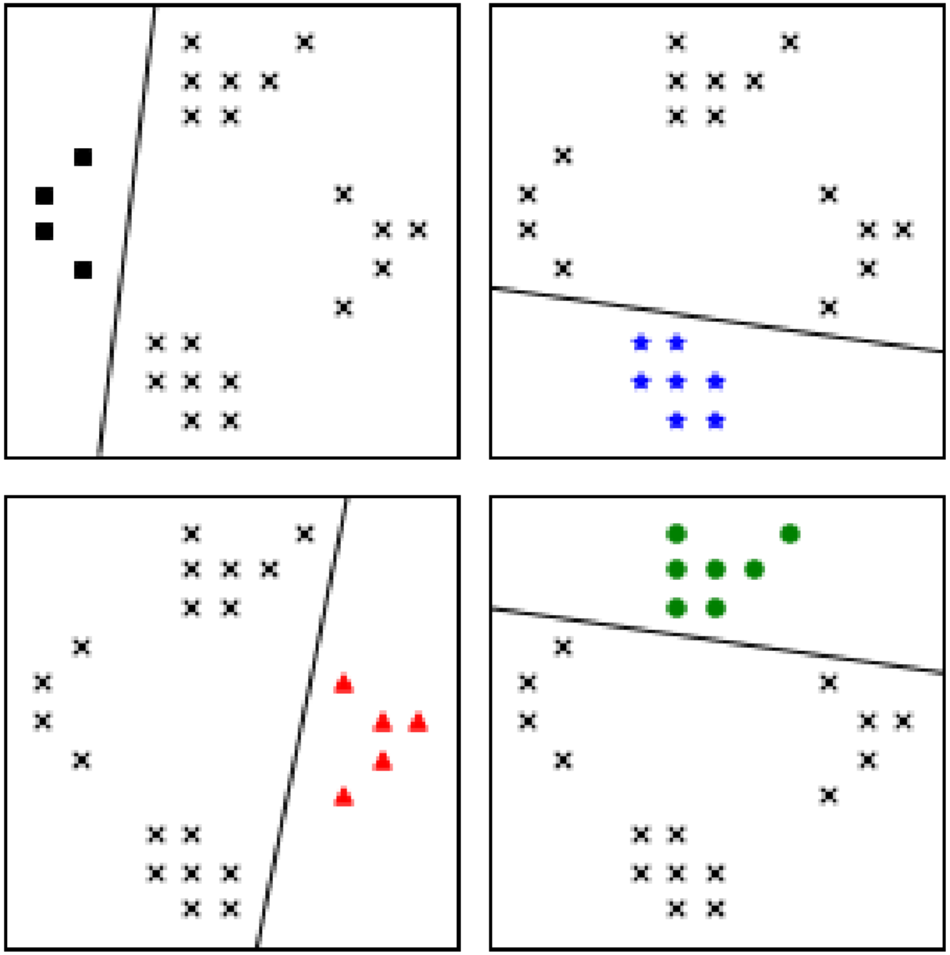
- Prepare data matrix $\{(x_i,y_i)\}$
- Select a Kernel function
- Select the error parameter *C*
- "Train" the system (to find all α_i)
- New data can be classified using α_i and Support Vectors

Extension to Non-linear SVMs (Kernel Machines)

Multi-class SVM

One-against-all (one-versus-the-rest)





Pros and Cons of SVM

- **Pros**
  - Good at dealing with high dimensional data
  - Works well on small data sets
- **Cons**
  - Picking the right kernel and parameters can be computationally intensive

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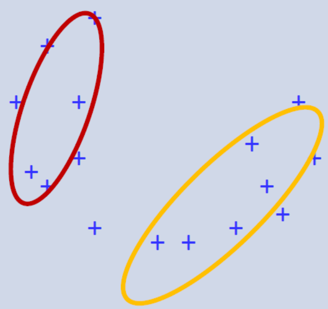
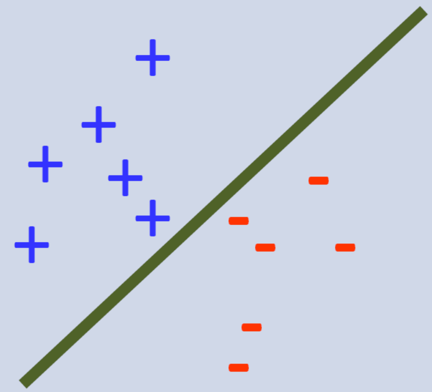
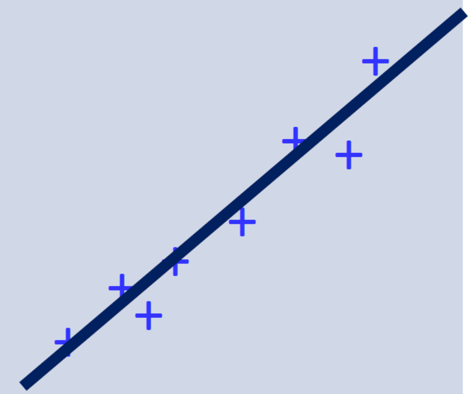
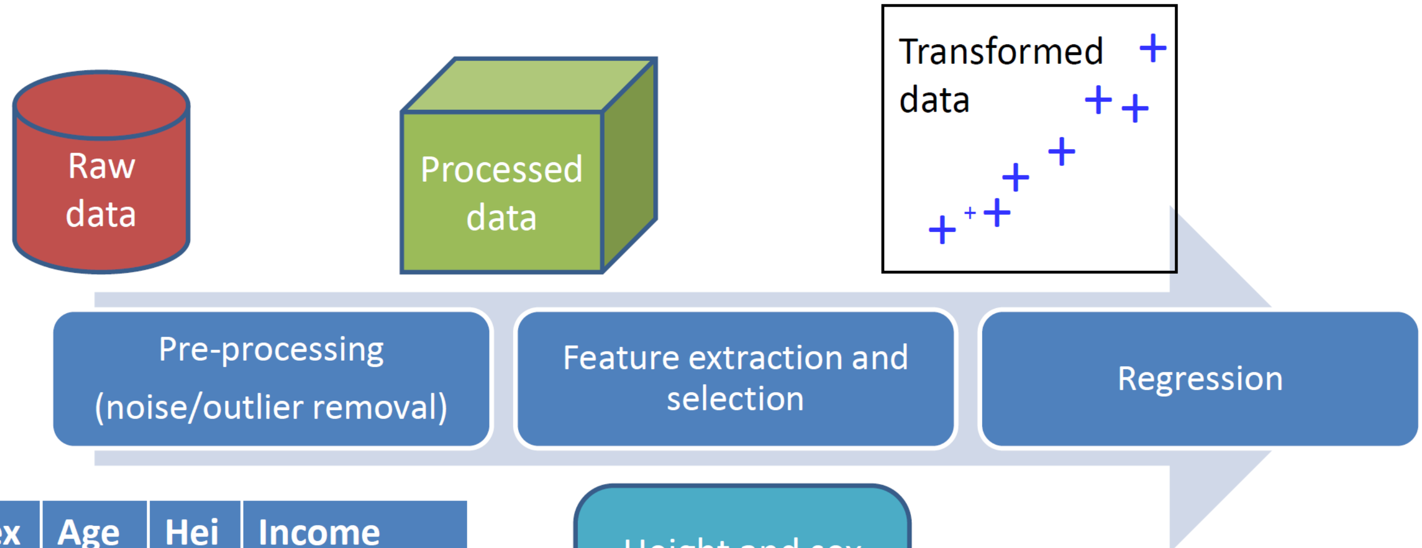
Support Vector Regression

Classification Overview

CLUSTERING	CLASSIFICATION	REGRESSION (THIS TALK)
		
K-means	• Decision tree • Linear Discriminant Analysis • Neural Networks • Support Vector Machines • Boosting	• Linear Regression • Support Vector Regression
Group data based on their characteristics	Separate data based on their labels	Find a model that can explain the output given the input

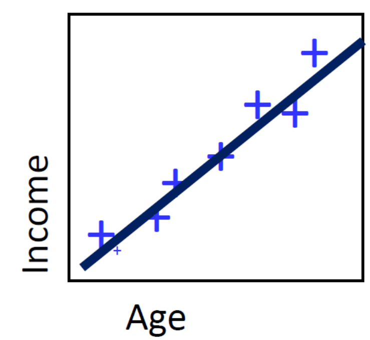
Regression Processing Flow



Sex	Age	Height	Income
M	20	1.7	25,000
F	30	1.6	55,000
M	**125**	1.8	30,000
…			

Mis-entry (should have been 25!!!)

Height and sex seem to be irrelevant.



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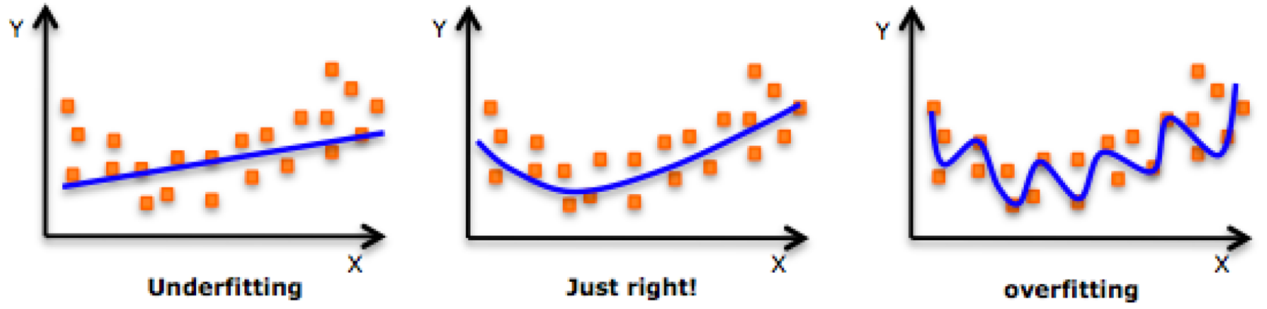
Linear Regression

$$\min \sum_{i=1}^{m} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{m} (y_i - (\hat{\mathbf{w}} \cdot \mathbf{x}_i + \hat{b}))^2$$



- To ovoid over-fitting, a regularization term can be introduced (minimize a magnitude of w)
  - LASSO: $\min \sum_{i=1}^{m} (y_i - \mathbf{w} \cdot \mathbf{x}_i - b)^2 + C \sum_{j=1}^{n} | w_j |$
  - Ridge regression: $\min \sum_{i=1}^{m} (y_i - \mathbf{w} \cdot \mathbf{x}_i - b)^2 + C \sum_{j=1}^{n} | \mathbf{w}_j^2 |$

Support Vector Regression

- Find a function, f(x), with at most ε-deviation from the target y

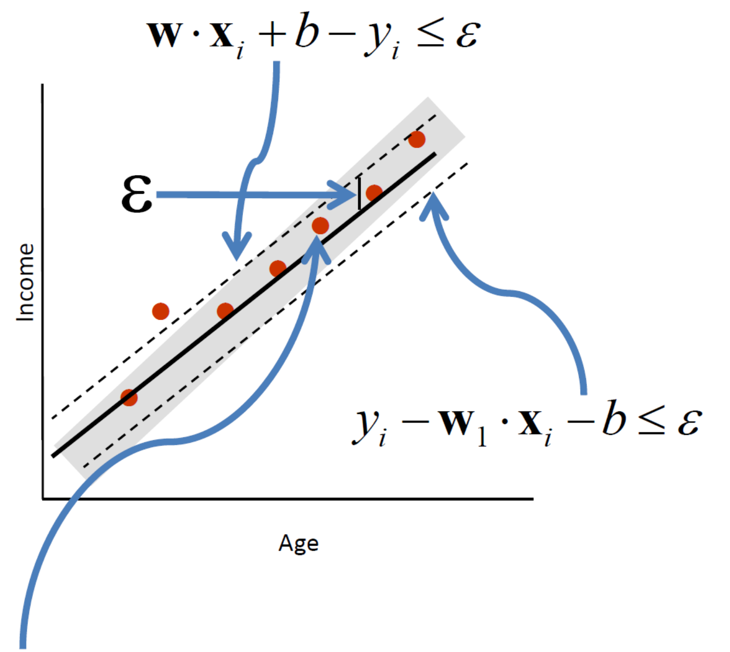
The problem can be written as a convex optimization problem

$$\min \frac{1}{2} \| \mathbf{w} \|^2$$
$$s.t.\ y_i - \mathbf{w}_1 \cdot \mathbf{x}_i - b \le \varepsilon;$$
$$\mathbf{w}_1 \cdot \mathbf{x}_i + b - y_i \le \varepsilon;$$

C: trade off the complexity

What if the problem is not feasible?

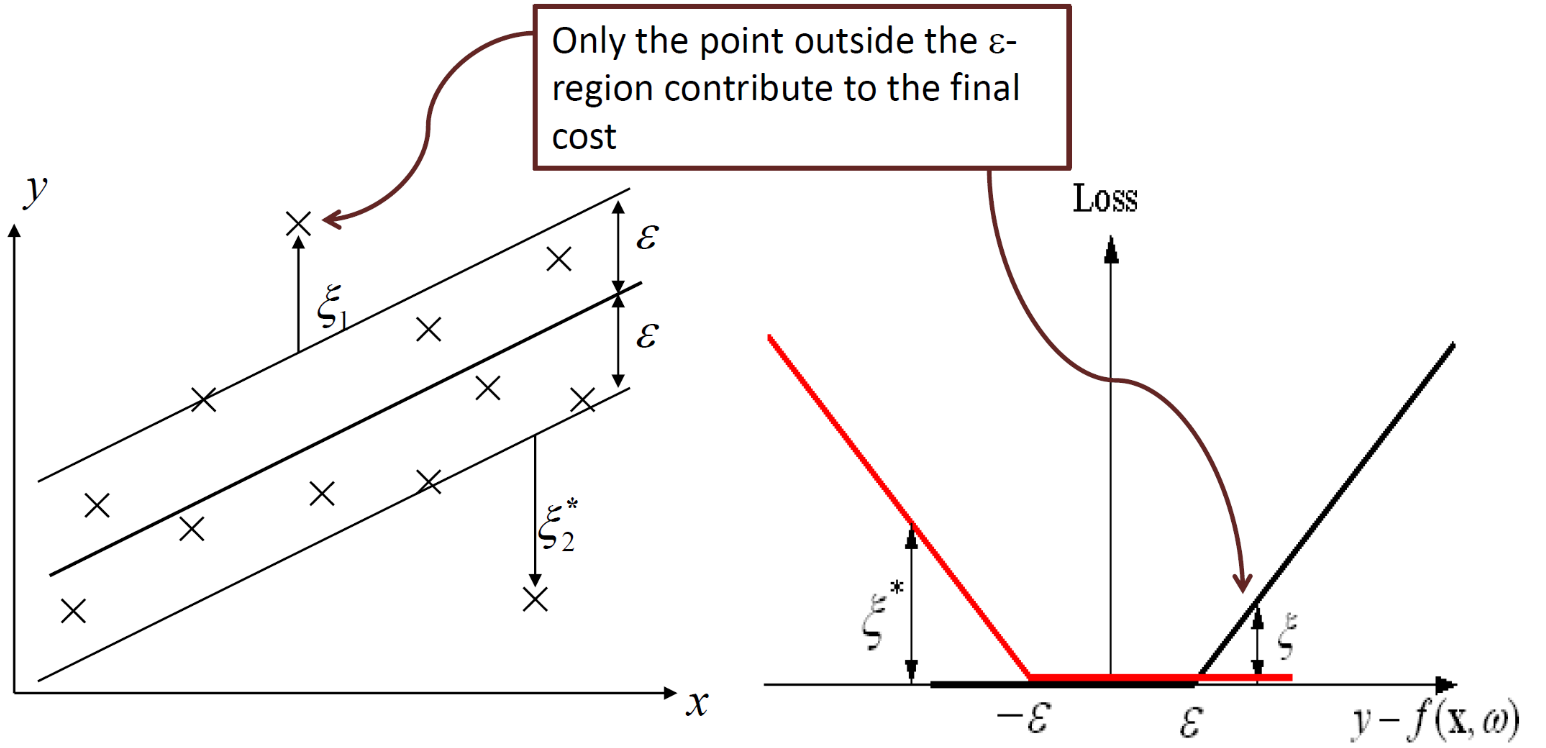
We can introduce slack variables (similar to soft margin loss function).



We do not care about errors as long as they are less than ε

SVR Loss

Assume linear parameterization $f(\mathbf{x}, \omega) = \mathbf{w} \cdot \mathbf{x} + b$



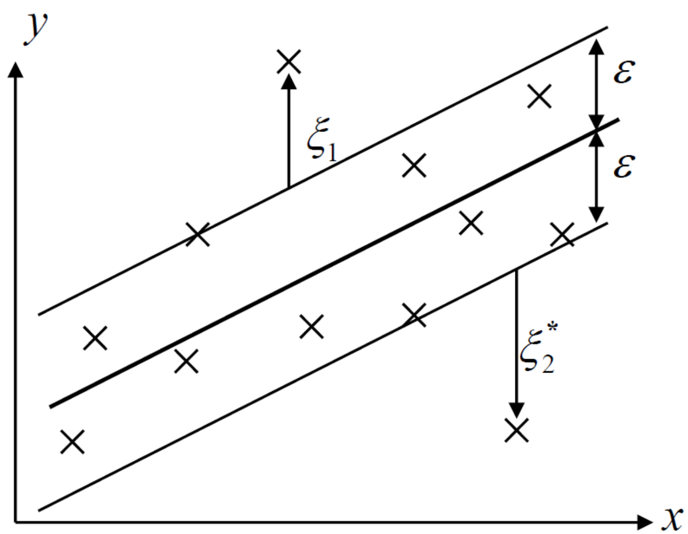
Soft Margin SVR

Given training data

$$(\mathbf{x}_i, y_i) \quad i = 1, \ldots, m$$

Minimize

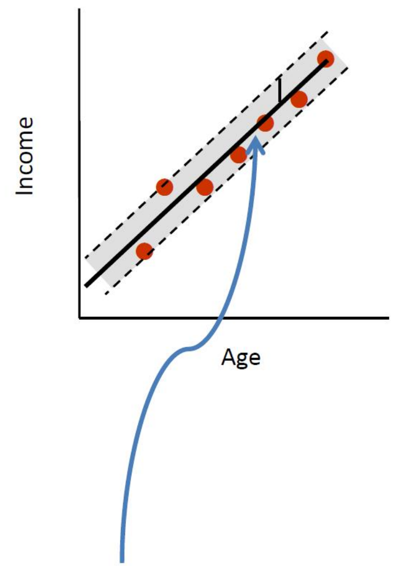
$$\frac{1}{2} \| \mathbf{w} \|^2 + C \sum_{i=1}^{m} (\xi_i + \xi_i^*)$$



Linear vs Non-linear SVR

- **Linear case**

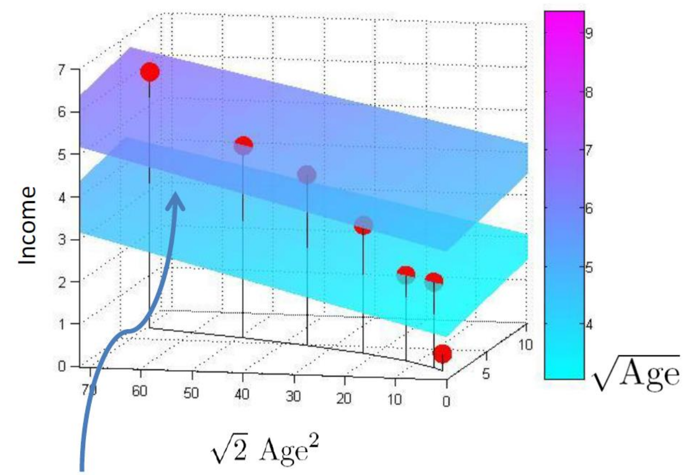
$$f: age \rightarrow income$$



$$y_i = \mathbf{w}_1 \cdot \mathbf{x}_i + b$$

- **Non-linear case**
  - Map data into a higher dimensional space, e.g.,

$$f: (\sqrt{age}, \sqrt{2}age^2) \rightarrow income$$



$$y_i = \mathbf{w}_1 \sqrt{\mathbf{x}_i} + \mathbf{w}_2 \sqrt{2}\mathbf{x}_i^2 + b$$

Kernel Trick

- Linear: $\langle x, y \rangle$
- Non-linear: $\langle \varphi(x), \varphi(y) \rangle = K(x, y)$

Note: No need to compute the mapping function, φ(.), explicitly. Instead, we use the kernel function.

Commonly used kernels:

- Polynomial kernels: $K(x, y) = (x^T y + 1)^d$

- Radial basis function (RBF) kernels:

$$K(x, y) = \exp\left(-\frac{1}{2\sigma^2} \| x - y \|^2\right)$$

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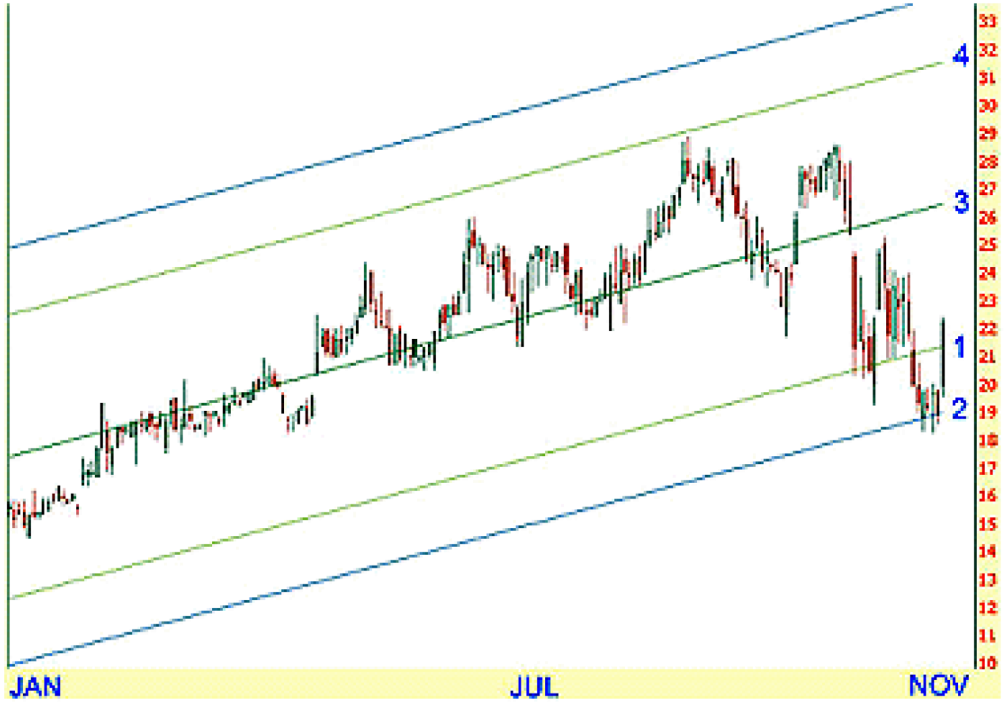
Application

SVR Applications

Stock price prediction



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