Two degrees of freedom control of a ball and beam system

2013 IOP Conf. Ser.: Mater. Sci. Eng. 53 012070

(http://iopscience.iop.org/1757-899X/53/1/012070)

Two degrees of freedom control of a ball and beam system

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Abstract. In this paper, a two-degree of freedom (2DOF) controller is designed for a ball and beam system. The controller is developed based on the algebraic method. The ball and beam system is one of the most popular laboratory experiments for control education. The controller is designed such that the ball can track a square wave with a certain design specification. The advantages of 2DOF controller are the feedback controller takes care of the uncertainty and the feedforward filter ensures the tracking of the reference command. Though the control method is not new, the application of the technique on such unstable system is of interest. Controlling the ball on the beam is a challenging task because of the instability of the system and the fact that the output, i.e. the ball position, increases almost without limit for a fixed input beam angle. The controller needs to regulate the position of the ball by changing the angle of the beam at the pivot point. It is a difficult control task as the ball moves with an acceleration which is proportional to the tilt angle of the beam. Two types of 2DOF controllers are considered; based on dominant pole design and based on integral time absolute error (ITAE) design. The performances of the resulting controllers are compared. Simulations were run over various frequencies. The results indicate the effectiveness of the designed 2DOF controllers in achieving the design specification.

1. Introduction

Two degrees of freedom control is a control technique that can provide pole and zero matching. In this type of controller three conditions must be satisfied such that the desired closed-loop transfer function and the plant transfer function are implementable, i.e. pole-zero excess inequality, retainment of non-minimum phase errors and the Hurwitz requirement of the desired closed-loop transfer function [1].

Typically the structure of the closed-loop system is shown in Figure 1.

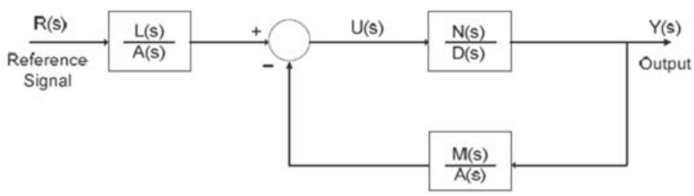

Figure 1 Two degree of freedom controller structure

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The controllers are described by two transfer functions L(s)/A(s) and M(s)/A(s), in the feedforward and feedback connection, respectively.
This type of controller has been used in applications, such as liquid tank controller [2]. The technique offers the advantage of pole and zero matching between the actual and desired closed-loop transfer function. For this reason, it is also called model matching technique.

The ball on beam balancer system is one of the most enduringly popular and important laboratory models for teaching control systems engineering. The system control job is automatically regulating the position of the ball on the beam by changing the angle of the beam. An example of modeling and controller design for a ball and beam system can be found in [1].
A ball is placed on a beam and is allowed to roll with one degree of freedom along the length of the beam. The model of the ball and beam system is shown in Figure 2. A lever arm is attached to the beam at one end and a servo gear at the other. As the servo gear turns by an angle theta, the lever changes the angle of the beam by alpha. When the angle is changed from the horizontal position, gravity causes the ball to roll along the beam. In this project, the objective is to design a controller to control a ball and beam system.

Controlling the ball on the beam is a challenging task because of the instability of the system and the fact that the output, i.e. the ball position, increases almost without limit for a fixed input beam angle. The controller needs to regulate the position of the ball by changing the angle of the beam at the pivot point. It is a difficult control task as the ball moves with an acceleration which is proportional to the tilt angle of the beam.

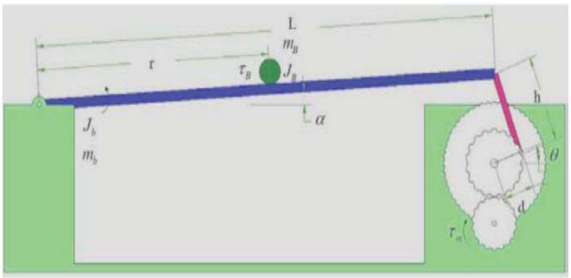

Figure 2. Ball and beam system

The paper is organized as follows. Section 2 provides the mathematical model of the ball and beam system. Section 3 describes the controller design. Section 4 presents the results and discussion. Section 5 concludes the paper

2. Mathematical model
Lagrange method is used to derive the dynamic equations of ball and beam system that is based on energy balance of the system. The derivation in this section is based on [3-4].

2.1. Equations of motion
For deriving the Euler-Lagrange equation, we need to find the kinetic energy and potential energy for the ball and beam system first. The kinetic energy of the system is:

$$T = T_b + T_B, \tag{1}$$

$$T_b = \frac{1}{2} J_b \dot{\alpha}^2 \tag{2}$$

$$T_B = \frac{1}{2}(m_B r^2)\dot{\alpha}^2 + \frac{1}{2}m_B \dot{r}^2 + \frac{1}{2}J_B {\omega_B}^2, \tag{3}$$

$$\dot{r} = R\omega_B, \tag{4}$$

$$\omega_B = \frac{\dot{r}}{R}, \tag{5}$$

$$J_B = \frac{2}{5}m_B R^2. \tag{6}$$

Substituting equation (5) and (6) into equation (3),

$$T_B = \frac{1}{2}(m_B r^2)\dot{\alpha}^2 + \frac{1}{2}m_B \dot{r}^2 + \frac{1}{5}m_B \dot{r}^2 = \frac{1}{2}\left[m_B r^2 \dot{\alpha}^2 + \frac{7}{5}m_B \dot{r}^2\right] \tag{7}$$

Substituting equation (2) and (7) into equation (1),

$$T = \frac{1}{2}J_b \dot{\alpha}^2 + \frac{1}{2}\left[m_B r^2 \dot{\alpha}^2 + \frac{7}{5}m_B \dot{r}^2\right] = \frac{1}{2}\left[(J_b + m_B r^2)\dot{\alpha}^2 + \frac{7}{5}m_B \dot{r}^2\right] \tag{8}$$

The potential energy of the system is exhibited by the rolling ball alone:

$$U = m_B g r \sin\alpha + \frac{L}{2}m_b g \sin\alpha = \left(m_B r + \frac{L}{2}m_b\right) g \sin\alpha \tag{9}$$

where parameters $m_B, m_b, J_b, J_B, R, \omega_B, \dot{r}, g, L, r, \alpha$ are the ball and beam mass, beam moment of inertia, ball moment of inertia, radius of the ball, rotational and radial velocities of the ball, gravity acceleration, length of the beam, linear motion of the beam along the beam and beam angle, respectively.

The Lagrange equation is the difference between kinetic and potential energy that is defined by equation (10).

$$L = T - U. \tag{10}$$

By substituting equation (8) and (9) into equation (10), we will get equation (11).

$$L = \frac{1}{2}\left[(J_b + m_B r^2)\dot{\alpha}^2 + \frac{7}{5}m_B \dot{r}^2\right] - \left(m_B r + \frac{L}{2}m_b\right) g \sin\alpha \tag{11}$$

Since there is no external force on the ball in radial direction, Lagrange's equations of motion are formed as:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\alpha}}\right] - \frac{\partial L}{\partial \alpha} = \tau, \tag{12}$$

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{r}}\right] - \frac{\partial L}{\partial r} = 0, \tag{13}$$

where τ is the torque produced by the motor applied at the end of the beam. For equation (12):

$$(J_b + m_B r^2)\ddot{\alpha} + 2m_B r\dot{r}\dot{\alpha} + \left(m_B r + \frac{L}{2}m_b\right) g \cos\alpha = \tau \tag{14}$$

From equation (14), we can obtain

$$\frac{7}{5}\ddot{r} - r\dot{\alpha}^2 + g\sin\alpha = 0. \tag{15}$$

2.2. Linearization around operating-point of the system

Jacobian linearization method is used to find the linear approximation of the dynamic equations. The linear dynamics equation of ball and beam system is presented in state space realization based on equation (16) and (17).

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\,, \tag{16}$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\,, \tag{17}$$

where matrix **A** defines the dynamic behavior of the system, matrix **B** defines the position and behavior of system actuator, matrix **C** defines the relation between states and output of system and matrix **D** is feedforward matrix that is equal to zero in this case.

Jacobian linearization gives the linear dynamic equation around the operating point that is the middle of the beam. Besides, the state space formulation must be derived around operating point. The assumption for state space formulation is to define the operating point of the system. Thus, we used (*) notation to refer to operating point of the system that is as follows.

$$\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\boldsymbol{x} + \mathbf{B}\Delta u\,, \tag{18}$$

$$\Delta\mathbf{y} = \mathbf{C}\Delta\mathbf{x} + \mathrm{D}\Delta u\,, \tag{19}$$

$$\Delta\mathbf{x} = \begin{bmatrix}\Delta x_1\\ \vdots \\ \Delta x_n\end{bmatrix} = \begin{bmatrix}x_1 - x_1{}^*\\ \vdots \\ x_n - x_n{}^*\end{bmatrix}\,, \tag{20}$$

$$\Delta u = u - u^*, \text{and}\, y = y - h(x^*, u^*).$$

The operating point: $x_1{}^* = 0$, $x_2{}^* = \delta$, $x_3{}^* = 0$, $x_4{}^* = 0$.

Let: $x_1 = \alpha$; $x_2 = r$; $x_3 = \dot{\alpha}$; $x_4 = \dot{r}$

Thus, equation (14) can be rewritten as

$$(J_b + m_B x_2{}^2)\dot{x}_3 + 2m_B x_2 x_4 x_3 + \left(m_B x_2 + \frac{L}{2}m_b\right) g\cos x_1 = \tau \tag{21}$$

or

$$\dot{x}_3 = \frac{\tau - 2m_B x_2 x_4 x_3 - \left(m_B x_2 + \frac{L}{2}m_b\right) g\cos x_1}{J_b + m_B x_2{}^2}\,. \tag{22}$$

Using small angle approximation,

$$\dot{x}_3 = \frac{\tau - 2m_B x_2 x_3 x_4 - m_B x_2 g - \frac{L}{2}m_b g}{J_b + m_B x_2{}^2}\,. \tag{23}$$

Likewise, equation (15) can be rewritten as

$$\frac{7}{5}\dot{x}_4 - x_2 x_3{}^2 + g\sin x_1 = 0 \tag{24}$$

Using the small angle approximatinon, i.e. $\sin x_1 \approx x_1$

$$\dot{x}_4 = \frac{5x_2 x_3{}^2 - 5g x_1}{7}\,, \tag{25}$$

$$\dot{x}_1 = \dot{\alpha} = x_3, \tag{26}$$

$$\dot{x}_2 = \dot{r} = x_4. \tag{27}$$

The dynamic equation of the system is defined as:

$$\dot{x}_1 = x_3 = f_1\,, \tag{28}$$

$$\dot{x}_2 = x_4 = f_2 \quad , \tag{29}$$

$$\dot{x}_3 = \frac{\tau - 2m_B x_2 x_3 x_4 - m_B x_2 g - \frac{L}{2} m_b g}{J_b + m_B {x_2}^2} = f_3 \ , \tag{30}$$

$$\dot{x}_4 = \frac{5x_2 {x_3}^2 - 5gx_1}{7} = f_4. \tag{31}$$

The static load required in the operating point is defined by τ^*.

$$\tau^* = g\left(\frac{m_b L}{2} + m_B r\right). \tag{32}$$

Modeling of DC servomotor can be divided into electrical and mechanical subsystems. The electrical subsystem is based on Kirchhoff's voltage law that is defined by equation (33).

$$V = L_m \dot{I} + R_m I + K_m \dot{\Theta}_m \ , \tag{33}$$

$$\tau_m = K_i I \quad . \tag{34}$$

where V, I, Θ_m are motor voltage, motor current and motor angle. L_m, R_m, K_m, K_i are motor constants. Compared to $R_m I$and $K_m \dot{\Theta}_m$, the term $L_m \dot{I}$ is very small. In order to simplify the modeling, we neglected the term $L_m \dot{I}$. Thus,

$$V = R_m I + K_m \dot{\Theta}_m \quad . \tag{35}$$

Rearranging equation (35), we will get

$$I = \frac{V - K_m \dot{\Theta}_m}{R_m} \quad . \tag{36}$$

Substituting equation (36) into equation (34), we get

$$\tau_m = K_i \frac{V - K_m \dot{\Theta}_m}{R_m} \quad . \tag{37}$$

The mechanical subsystem is defined in the following equation.

$$\tau_b = \frac{K_g K_i \eta_{total}}{R_m} \frac{L}{d} V - \frac{{K_g}^2 K_i K_m \eta_{total}}{R_m} \frac{L^2}{d^2} \dot{\alpha}. \tag{38}$$

Substituting equation (38) into equation (28), we get

$$f_3 = \dot{x}_3 = \frac{\frac{K_g K_i \eta_{total} L}{R_m \ d} V - \frac{{K_g}^2 K_i K_m \eta_{total} L^2}{R_m \ d^2} x_3 - 2m_B x_2 x_3 x_4 - m_B x_2 g - \frac{L}{2} m_b g}{J_b + m_B {x_2}^2} \tag{39}$$

Based on our assumption, voltage V^* should be defined for the operating point ${x_1}^* = 0, \quad {x_2}^* = \delta$, ${x_3}^* = 0, \quad {x_4}^* = 0$, which is shown as in the following equations.

$$\frac{\frac{K_g K_i \eta_{total}}{R_m}\frac{L}{d}V - \frac{K_g{}^2 K_i K_m \eta_{total}}{R_m}\frac{L^2}{d^2}x_3 - 2m_B x_2 x_3 x_4 - m_B x_2 g - \frac{L}{2}m_b g}{J_b + m_B {x_2}^2} = 0$$

$$\frac{K_g K_i \eta_{total}}{R_m}\frac{L}{d}V - \frac{K_g{}^2 K_i K_m \eta_{total}}{R_m}\frac{L^2}{d^2}x_3 - 2m_B x_2 x_3 x_4 - m_B x_2 g - \frac{L}{2}m_b g = 0$$

$$V^* = \left(m_B \delta g + \frac{L}{2}m_b g\right)\frac{R_m d}{K_g K_i \eta_{total} L} \tag{40}$$

Thus, the linearization of f_1, f_2, f_3 and f_4 around operating point ${x_1}^* = 0,\quad {x_2}^* = \delta,\quad {x_3}^* = 0,$ ${x_4}^* = 0, \delta = \delta_2$, are as follows.

$$\frac{\partial f_3}{\partial x_2} = \frac{-m_B g\left(J_b + m_B {\delta_2}^2\right) - 2m_B \delta_2\left(\frac{K_g K_i \eta_{total}}{R_m}\frac{L}{d}\left(m_B \delta g + \frac{L}{2}m_b g\right)\frac{R_m d}{K_g K_i \eta_{total} L} - m_B \delta_2 g - \frac{L}{2}m_b g\right)}{\left(J_b + m_B {\delta_2}^2\right)^2}$$

$$\frac{\partial f_3}{\partial x_2} = -\frac{m_B J_b g + {m_B}^2 g {\delta_2}^2}{\left(J_b + m_B {\delta_2}^2\right)^2} \tag{41}$$

$$\frac{\partial f_3}{\partial x_3} = \frac{-\frac{K_g{}^2 K_i K_m \eta_{total}}{R_m}\frac{L^2}{d^2}\left(J_b + m_B {\delta_2}^2\right)}{\left(J_b + m_B {\delta_2}^2\right)^2} = -\frac{K_g{}^2 K_i K_m \eta_{total}}{R_m\left(J_b + m_B {\delta_2}^2\right)}\frac{L^2}{d^2} \tag{42}$$

$$\frac{\partial f_4}{\partial x_1} = \frac{-5g(7)}{7^2} = -\frac{5g}{7} \tag{43}$$

$$\frac{\partial f_3}{\partial V} = \frac{\frac{K_g K_i \eta_{total}}{R_m}\frac{L}{d}\left(J_b + m_B {\delta_2}^2\right)}{\left(J_b + m_B {\delta_2}^2\right)^2} = \frac{K_g K_i \eta_{total}}{R_m\left(J_b + m_B {\delta_2}^2\right)}\frac{L}{d} \tag{44}$$

The state space matrices and vectors that describe the system are as the following.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{m_B J_b g + {m_B}^2 g {\delta_2}^2}{\left(J_b + m_B {\delta_2}^2\right)^2} & -\frac{K_g{}^2 K_i K_m \eta_{total}}{R_m\left(J_b + m_B {\delta_2}^2\right)}\frac{L^2}{d^2} & 0 \\ -\frac{5g}{7} & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{K_g K_i \eta_{total}}{R_m\left(J_b + m_B {\delta_2}^2\right)}\frac{L}{d} \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \text{ and } D = [0]$$

The beam moment of inertia is defined as

$$J_b = \frac{1}{2}m_b L^2 \tag{45}$$

The ball and beam system parameters and DC motor specifications in the present study are shown in Table 1.

Table 1. Ball and beam system parameters

Symbol	Quantity	Value
g	Gravity acceleration	9.8 (m/s^2)
m_B	Ball mass	0.064 (kg)
m_b	Beam mass	0.65 (kg)
R	Ball radius	0.0254 (m)
l	Beam length	0.425 (m)
d	Lever length	0.12 (m)
δ	Equilibrium point of ball position	0.2 (m)
K_m	Back EMF constant	0.00767 (V.sec/rad)
K_i	Torque constant	0.00767 (N.m)
K_g	Gear ratio	14
R_m	Motor resistance	2.6 (Ω)
η_{motor}	Motor efficiency	0.69
$\eta_{gearbox}$	Gearbox efficiency	0.85

The state space matrix representing the ball and beam system is:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -10.2378 & -1.3983 & 0 \\ -7 & 0 & 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ 3.6769 \\ 0 \end{bmatrix},\ C = [0 \quad 1 \quad 0 \quad 0],\quad D = [0].$$

The transfer function of the system is denoted as $G(s)$,

$$G(s) = \frac{Y(s)}{V(s)} = \frac{-25.7382}{s^4 + 1.3893s^3 - 71.6646}.$$

3. Two DOF controller design

In this paper, two degree of freedom controller based on algebraic method is designed to control the position of the ball on the beam such that it tracks a square wave with amplitude of 5cm. Though the control method is not new, the application of the technique on such unstable system is of interest. In achieving the objective, the following design specifications need to be satisfied:

- The settling time is less than 2 seconds,
- The percent overshoot is less than 5 %
- Velocity error constant is larger than 150.

The plant transfer function is defined as:

$$G(s) = \frac{N(s)}{D(s)} = \frac{-25.7382}{s^4 + 1.3893s^3 - 71.6646} \quad .$$

Diophantine equation, $D_0(s) = \overline{D_p(s)}D_p(s) = A(s)D(s) + M(s)N(s)$.
In this case, the order of the plant, $n = 4$. Thus, the order of $\overline{D_p(s)}D_p(s) \geq 2n - 1 = 2(4) - 1 = 7$.

The closed-loop transfer function of the system is 7th order system. Since the damping ratio, $\zeta = 0.7$ and settling time, $T_s = 0.2$sec. So, the natural frequency, $\omega_n = 28.57$rads/sec.

Thus, we obtain the desired characteristics:

$$a_d(s) = s^2 + 2\zeta\omega_n s + \omega_n{}^2 = s^2 + 39.998s + 816.245$$

The dominant poles are, $p = -20 \pm j20.403$
Since this system is 7th order system, we have to add 5 more poles at least 2-6 times faster than dominant poles. So, the rest of poles are: $p = [-40 \;\; -60 \;\; -80 \;\; -100 \;\; -120]$.
Then, the desired characteristics equation is:

$$\begin{aligned} a_d(s) &= (s^2 + 39.998s + 816.245)(s+40)(s+60)(s+80)(s+100)(s+120) \\ &= s^7 + 440.398s^6 + 78975.5s^5 + 7471176s^4 + 405094220s^3 + 1.283948192\times 10^{10}s^2 \\ &\quad +2.29423392\times 10^{11}s + 1.88064\times 10^{12} \end{aligned}$$

Thus, the overall transfer function $G_0(s)$is:

$$G_0(s) = \frac{1.88064\times 10^{12}}{s^7 + 440.398s^6 + 78975.5s^5 + 7471176s^4 + 405094220s^3 + 1.283948192\times 10^{10}s^2 + 2.29423392\times 10^{11}s + 1.88064\times 10^{12}}$$

The necessary condition for $S_m$to have full row ranks is $n - 1 \leq m$, for this case, $m \geq 3$.

Thus, order of $A(s)$ is 3 since the highest order of plant denominator $D(s)$ is 4.

$$S_m C_m = \begin{bmatrix} D_0 & N_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ D_1 & N_1 & D_0 & N_0 & 0 & 0 & 0 & 0 \\ D_2 & N_2 & D_1 & N_1 & D_0 & N_0 & 0 & 0 \\ D_3 & N_3 & D_2 & N_2 & D_1 & N_1 & D_0 & N_0 \\ D_4 & N_4 & D_3 & N_3 & D_2 & N_2 & D_1 & N_1 \\ 0 & 0 & D_4 & N_4 & D_3 & N_3 & D_2 & N_2 \\ 0 & 0 & 0 & 0 & D_4 & N_4 & D_3 & N_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & D_4 & N_4 \end{bmatrix} \begin{bmatrix} A_0 \\ M_0 \\ A_1 \\ M_1 \\ A_2 \\ M_2 \\ A_3 \\ M_3 \end{bmatrix} = \begin{bmatrix} F_0 \\ F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \end{bmatrix}$$

$$\begin{bmatrix} -71.6646 & -25.7382 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -71.6646 & -25.7382 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -71.6646 & -25.7382 & 0 & 0 \\ 1.3983 & 0 & 0 & 0 & 0 & 0 & -71.6646 & -25.7382 \\ 1 & 0 & 1.3983 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1.3983 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1.3983 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_0 \\ M_0 \\ A_1 \\ M_1 \\ A_2 \\ M_2 \\ A_3 \\ M_3 \end{bmatrix} = \begin{bmatrix} 1.88064\times 10^{12} \\ 2.29423392\times 10^{11} \\ 1.283948192\times 10^{10} \\ 405094220 \\ 7471176 \\ 78975.5 \\ 440.398 \\ 1 \end{bmatrix}$$

From which the coefficients of the controller transfer function are:
$A_0 = 7361602909$, $A_1 = 78361.6467$, $A_2 = 438.9997$, $A_3 = 1$, $M_0 = -7.3089\times 10^{10}$
$M_1 = -8.9139\times 10^9$, $M_2 = -498.8505\times 10^6$, and $M_3 = -15.3391\times 10^6$.

Thus, the transfer function of the second controller:

$$C_2(s) = \frac{M(s)}{A(s)} = \frac{-15.3391\times10^6 s^3 - 498.8505\times10^6 s^2 - 8.9139\times10^9 s - 7.3089\times10^{10}}{s^3 + 438.9997s^2 + 78361.6467s + 7361602.909}.$$

Since $L(s)N(s) = N_0(s)$, where $N_0(s) = 1.88064\times10^{12}$ and $N(s) = -25.7382$,
thus, $L(s) = \frac{N_0(s)}{N(s)} = \frac{1.88064\times10^{12}}{-25.7382} = -7.3068\times10^{10}$.

The transfer function of the first controller,

$$C_1(s) = \frac{L(s)}{A(s)} = \frac{-7.3068\times10^{10}}{s^3 + 438.9997s^2 + 78361.6467s + 7361602.909}$$

As for satisfying the requirement of the velocity error K_v larger than 150, let $E(s)$ be the error signal, where $E(s) = C_1(s)R(s) - C_2(s)Y(s)$. As $Y(s) = E(s)G(s)$, thus,
$E(s) = C_1(s)R(s) - C_2(s)E(s)G(s)$. Substituting $C_1(s)$ and $C_2(s)$ gives

$$E(s) = \frac{C_1(s)R(s)}{1+C_2(s)G(s)} = \frac{\frac{L(s)}{A(s)}R(s)}{1+\frac{M(s)}{A(s)}G(s)}.$$

Therefore, the steady state error, e_{ss}

$$e_{ss} = \lim_{s\to0} sE(s) = \lim_{s\to0} s\frac{\frac{L(s)}{A(s)}\frac{1}{s}}{1+\frac{M(s)}{A(s)}G(s)},$$

$$= \frac{\frac{-7.3068\times10^{10}}{7\ 361602909}}{1+\frac{-7.308\ 9\times10^{10} - 25.7\ 38\ 2}{7\ 361602909\ -7\ 16646}} = 2.7843.$$

From which the uncompensated velocity error constant is

$$K_{vu\ ncompensated} = \frac{1}{e_{ss}} = \frac{1}{2.7843} = 0.359.$$

Since

$$K_{vcompensated} = K_{vu\ ncompensated}\left(\frac{z}{p}\right),$$

and from the specification, $K_{vcompensated}$ must be greater than 150, we choose $K_{vcompensated} = 160$.

$$160 = 0.359\left(\frac{z}{p}\right),$$

$$\frac{z}{p} = 445.68.$$

So, we let $z = 0.001$ which is closed to zero so that it will not give too much effect to the system transient response.

$$p = \frac{0.001}{445.68} = 2.244\times10^{-6} \approx 0.$$

The additional zero and pole are added to the overall transfer function to satisfy the velocity error requirement. Thus, the third controller is placed just after the summing point and just before the plant block in Figure 1. This extra controller in fact provides an additional integrator to the system, or precisely it is given by the following transfer function.

$$C_3(s) = \frac{s+0.001}{s}.$$

4. Results and discussion

From the simulation result for the 2DOF controller, the values for percent overshoot and settling time satisfy the desired specification which percent overshoot need to be less than 5% and settling time less than 2 sec. From the desired specification, we need to consider the velocity error constant K_v which must be greater than 150. From section 4 which is controller 1 design, we obtained steady state error, e_{ss} of 2.7843 and uncompensated velocity error constant, $K_{v,u\ ncompensated}$ of 0.359. Here, we choose the compensated velocity error constant to be 160. From the calculation, we need to add zero of -0.001 and pole of -2.244×10^{-6}. The value of zero must be nearly to zero so that it will not affect the transient response of the closed loop system. This is proven by performance criteria obtained from Figure 4. The value for settling time remains the same and percent overshoot is increased by 0.03%. Figure 4 and 5 shows that the output of closed loop system tracks the square wave input from +5cm to -5cm. The response of the closed-loop system indicates that the percent of overshoot is 1.93%, settling time is 0.207 second and zero steady state error.

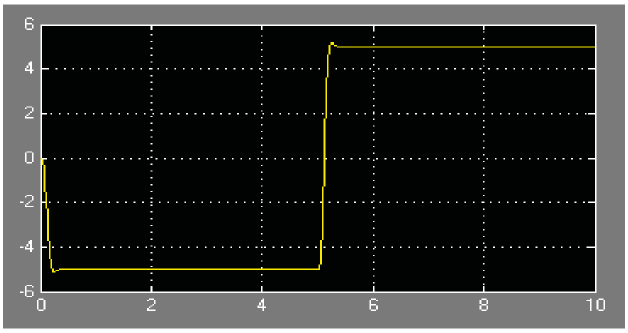

Figure 4. Response for closed loop system with 2 DOF controller without K_v requirement

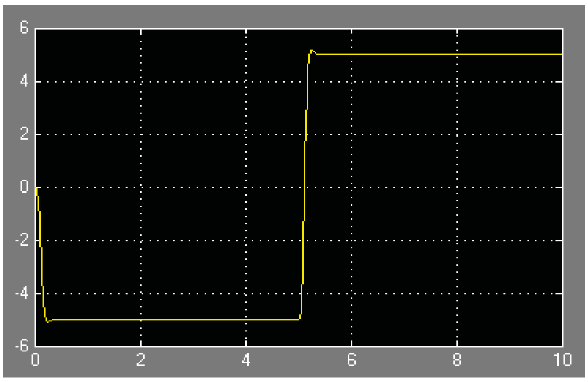

Figure 5. Simulink result for closed loop system with 2 DOF controller with K_v requirement

Comparison between 2DOF and pole placement method

We have designed controller using 2DOF and pole placement methods. For evaluating the performance between pole placement and 2DOF controller, we used different values of natural frequency, ω_n with same value of damping ratio which is 0.707. The optimum coefficient for damping ratio is equal to 0.707. The performance for both controllers is compared based on their percent overshoot, settling time and steady state error. The results obtained from both controllers are tabulated in Table 2.

Table 2. Performance result for different value of frequency response

Controller	Frequency response ω_n (rad/sec)	Percent overshoot (%)	Settling time, T_s (sec)	Steady state error, e_{ss}
Pole placement	5.00	4.20	1.270	0
	10.00	3.54	0.670	0
	15.00	2.83	0.465	0
	20.00	2.12	0.355	0
	25.00			
	28.57			
	30.00			
2 DOF	5.00	4.24	1.270	0
	10.00	4.01	0.670	0
	15.00	3.60	0.471	0
	20.00	3.02	0.368	0
	25.00	2.40	0.303	0
	28.57	1.90	0.207	0
	30.00	1.80	0.190	0

From Table 2, for pole placement controller, the percent overshoot, settling time and steady state error for frequency response, ω_n equals to 25 rad/sec, 28.57 rad/sec and 30 rad/sec are left blank (shaded) as the responses become overdamped. As we can see, as the frequency response below 25 rad/sec increases, the values for percent overshoot and settling time are decreasing. As for the steady state error, the values remain 0.

Further performance evaluation of the closed-loop responses is conducted by comparing the 2 DOF controller with dominant pole design and the ITAE design.

Table 3. Performance of 2DOF controller with different poles selection

Controller	Frequency response ω_n (rad/sec)	Percent overshoot (%)	Settling time, T_s (sec)	Steady state error, e_{ss}
Dominant poles	5.00	4.24	1.270	0
	10.00	4.01	0.670	0
	15.00	3.60	0.471	0
	20.00	3.02	0.368	0
	25.00	2.40	0.303	0
	28.57	1.90	0.207	0
	30.00	1.80	0.190	0
Integral of time multiplied by absolute error (ITAE)	5.00	10.90	2.000	0
	10.00	10.90	0.999	0
	15.00	10.90	0.666	0
	20.00	10.90	0.500	0
	25.00	10.90	0.400	0
	28.57	10.90	0.350	0
	30.00	10.90	0.333	0

IOP Conf. Series: Materials Science and Engineering **53** (2013) 012070 doi:10.1088/1757-899X/53/1/012070

Table 3 indicates that the dominant pole design provides better responses than the ITAE.

5. Conclusion

In conclusion, the objectives of the research have been achieved. The 2 DOF controller and pole placement approaches have been presented. Evaluation of the performance of the closed loop system has been also presented. The performance of the designed 2DOF controller is comparable to that of the pole placement controller. However, the advantage of using 2DOF architecture is that the controller designer has more freedom in achieving pole and zero matching.

References

[1] Chen, C T 1993 Control System Design: Transfer Function, State-Space and Algebraic Methods, Saunders-HBJ

[2] Gunes N S, Uraz A 2010 Two Degree of Freedom Control of a Liquid TankCanadian Journal on Electrical and Electronics Engineering Vol. 1 No. 3

[3] Keshmiri, M, Jahromi, A. F., Mohebbi, A., Amoozgar, M. H. , Xie, W-F. 2012 Modeling and Control of Ball and Beam System using Model Based and Non-Model Based Control Approached *International Journal on Smart Sensing and Intelligent System*, Vol. 5, No. 1

[4] Control Tutorials for MATLAB and Simulink http://www.library.cmu.edu/ctms/ctms/index.htm