

SEARCH FOR $t\bar{t}Z' \rightarrow t\bar{t}t\bar{t}$ PRODUCTION IN THE MULTILEPTON FINAL STATE IN
 pp COLLISIONS AT $\sqrt{s} = 13$ AND 13.6 TEV WITH THE ATLAS DETECTOR

By

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ABSTRACT

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PREFACE

This is my preface. remarks remarks remarks

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KEY TO ABBREVIATIONS

Chapter 1. Introduction

1.1 Motivation

1.2 Analysis strategy

1.2.1 Profile likelihood fit

1.2.2 Analysis regions

Chapter 2. Theoretical Overview

2.1 The Standard Model

- SM describes fundamental forces & elementary particles
- more descriptions (a bit of history + recent developments - higgs & neutrino masses) -
- limitations: gravity & general relativity, arbitrary free parameters

2.1.1 Elementary particles

- Bosons (Bose-Einstein statistics, integer spin) & fermions (Fermi-Dirac statistics, half-integer spin)
- Fermions - building blocks: quarks & leptons [protons/neutrons constituents?]
- Bosons - force carriers & interaction mediators (elementary bosons == gauge bosons
(chart of elementary particles here))

Fermions

- elementary particles
- half-integer spin

Quarks

- building blocks for hadrons & bosons
- up down — charm strange — bottom top [by order of discovery and mass]
- charge doublets: $+2/3$ and $-1/3$ charge
- color charge & color confinement in hadrons
- interacts with all 4 fundamental forces

Leptons

- electron — muon — tau + neutrino [by order of mass]
- charge -1, neutrinos charge neutral
- interacts with all forces except strong, neutrinos only weak and gravitational

Bosons

- force mediators
- integer spin

Scalar

- spin 0
- Higgs massive, charge neutral, provides rest mass for all elementary particles,

Vector

- spin 1
- W/Z (weak), photon (QED/electrodynamic), gluons (QCD/strong)
- photon/gluon massless, charge neutral, gluon carries color charge out of 8 combinations of quark colors (color octet)
- W/Z massive, charged/neutral

2.1.2 Mathematical formalism

- QFT: treats particles as excitations of corresponding quantum fields: fermion field ψ , electroweak boson fields $W_{1/2/3}$ & B , gluon field G_α , Higgs field ϕ
- Lagrangian: gauge QFT containing local gauge symmetries of $SU(3)_C \times SU(2)_L \times U(1)_Y$ and global Poincar symmetry: translational symmetry, rotational symmetry & special relativity frame invariance

- Noether's theorem: local symmetries - $U(1)$ strong/weak/EM, Poincaré $U(1)$ momentum, angular momentum & energy conservation
- unexpanded Lagrangian with description of each part: kinetic terms, coupling terms, mass/Higgs terms)

Quantum chromodynamics

- strong interaction, $SU(3)_C$ gauge group under Yang-Mills theory
- C = color charge conservation
- QCD Lagrangian, expansion & brief explanation

Electroweak

- unified weak & electromagnetic interactions, $SU(2)_L \times U(1)_Y$ gauge group
- L = left-handed chirality $U(1)$ weak isospin (I) conservation
- Y = weak hyper charge conservation
- Q = charge conservation = $I_3 + 1/2Y$
- QED Lagrangian, expansion & brief explanation

Higgs mechanism

fermions & bosons still massless from previous section, resolved by introduction of the Higgs mechanism

(show Higgs field, potential & Lagrangian)

(show minimum of Higgs potential aka VEV)

—————[continue later]—————

2.1.3 Beyond the Standard Model

2.2 Four-top quark production

- Top: heaviest particle, strong coupling to many BSM particles in BSM models.
- 4top: xsec relevant to and enhanced by many BSM models
- Predicted by SM and observed [observation paper]
- Predicted xsec and observed xsec
- (insert Feynman diagrams)
- Decay products & final state topologies

Top-philic vector resonance

- (briefly introduce composite pseudo-nambu-Goldstone boson and motivation)
- hypothesis: top quark large mass results from high mixing between a "true" top quark and a colored, fermionic composite state
- composite vector resonance can be modeled as a top-philic Z' boson (without QCD color) or top-philic KK-gluon (with QCD color)
- color singlet vector boson (Z') model coupling strongly to top and weakly or not at all to others
- (show Lagrangian for interaction)
- two body decay Z' into $t\bar{t}$ with $m_{Z'}$ in TeV range \rightarrow top mass
- decay channels: $t\bar{t}Z'$ s & t channels, tWZ' , tjZ'
- (show decay width at LO)
- (Feynman diagrams here)

Higgs-top Yukawa coupling

(show Lagrangian of Higgs-top Yukawa coupling)

(show dependence of $t\bar{t}t\bar{t}$ xsec on Yukawa coupling at LO)

Effective field theory

SMEFT expanding on SM Lagrangian using higher order operators

(show EFT Lagrangian)

quick overview on SMEFT dimension-6 four-fermion operators for BSM interpretation

and Higgs oblique parameter

2.3 Collider physics

[pp collision, pdf, cross section, luminosity]

Luminosity

Proton-proton collisions

jets, parton shower, hadronization

Parton distribution function

Cross section

Chapter 3. LHC & ATLAS Experiment

3.1 The Large Hadron Collider

theoretical predictions are tested with experimental data obtained from particle accelerators world's largest accelerator built by CERN situated on the border of Switzerland and France has been operating since xxxx lifetime divided into 3 runs, currently on Run 3 with planned upgrades on the horizon responsible for a number of discoveries aka Higgs, etc.

3.1.1 Overview

[Basic info: location, size, main working mechanism, main detectors, main physics done]

- 27km circumference, reusing LEP tunnels 175m below ground level
- 7-13-13.6 TeV center of mass energies for pp collisions
- other than pp, also collides pPb, PbPb at 4 points with 4 main detectors: ATLAS, CMS (general purpose detectors), ALICE (heavy ion physics, ion collisions), LHCb (*b*-physics)

3.1.2 LHC operations

- focuses mainly on pp collisions for this thesis - beams split into bunches of 1.1×10^{11} protons with instantaneous luminosity of up to $2 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$
- beam energies ramp up in other accelerators before injection, full ramp up to 6.5 GeV about 20 minutes

(insert full diagram of accelerator chain)

Linac 4: hydrogen atoms, accelerated up to 160 MeV

PSB: H atoms stripped of electrons before injection, accelerated to 2 GeV

PS: 26 GeV, SPS: 450 GeV

LHC: injection in opposite directions, 6.5 TeV per beam

Run 1: 2010-2012, Run 2: 2015-2018, Run 3: 2022-2025, HL-LHC: 2029-?

COM energies: 7 & 8 TeV, 13 TeV, 13.6 TeV, 13.6 & 14 TeV

inbetween periods: long shutdowns (LS1, LS2, LS3)

(add HL-LHC timeline graph)

(insert LHC SM processes cross sections chart)

Top quark production at the LHC

history (CDF/D0)

LHC as a top factory: show luminosity and cross section for top processes

couples to Higgs as heaviest elementary particle

Higgs produced mainly from ggH (90%) via top loop and from ttH

(Feynman diagram of related processes)

3.2 The ATLAS detector

44m long, 25m diameter inner detector, solenoid/toroid magnet, EM & hadronic calorimeters, muon spectrometer [insert figure]

right-handed cylindrical system, z-axis follows beamline, azimuthal and polar (0 in the beam direction) angles measured with respect to beam axis.

pseudorapidity $\eta = -\ln \tan(\theta/2)$, approaches $\pm \infty$ along and 0 orthogonal to the beamline

distance $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$

transverse energy $E_T = \sqrt{p_T^2 + m^2}$

transverse momentum p_T component of momentum orthogonal to the beam axis $p_T = \sqrt{p_x^2 + p_y^2}$

3.2.1 Inner detector

- measures tracks of charged particles with high momentum resolution ($\sigma_{p_T}/p_T = 0.05\% \pm 1\%$)
- covers particles with $p_T > 0.5$ GeV, $|\eta| < 2.5$
pixel detector -> semiconductor tracker -> transition radiation tracker, innermost to outermost
- pixel detector:
 - innermost, 250 μm silicon pixel layers
 - detects charged particles from electron-hole pair production in silicon
 - measures impact parameter resolution & vertex identification for reconstruction of short-lived particles
 - spatial resolution of 10 μm in the $R - \phi$ plane and 115 μm in the z-direction
 - 80.4m readout channels
- sct:
 - surrounds pixel detector, silicon microstrip layers with 80 μm strip pitch
 - particle tracks cross 8 strip layers

- measures particle momentum, impact parameters, vertex position
 - spatial resolution of 17 μm in the $R - \phi$ plane and 580 μm in the z-direction
 - 6.3m readout channels.
- trt:
 - outermost, layers of 4 mm diameter gaseous straw tubes with transition radiation material (70% $Xe + 27\% CO_2 + 3\% O_2$) & 30 μm gold-plated wire in the center
 - tubes 144 cm length in barrel region ($|\eta| < 1$), 37 cm in the endcap region ($1 < |\eta| < 2$), arranged in wheels instead of parallel to beamline)
 - gas mixture produces transition radiation when ionized for electron identification
 - resolution/accuracy of 130 μm for each straw tube in the $R - \phi$ plane
 - 351k readout channels

3.2.2 Calorimeter systems

surrounds the inner detector & solenoid magnet, covers $|\eta| < 4.9$ and full ϕ range. Alternates passive and active material layers. Incoming particles passing through calorimeter produce EM cascades or hadronic showers in passive layer. Energies deposited and convert to electric signals in active layers for readout.

EM calorimeter:

- innermost, lead-LAr detector (passive-active)
- measures EM cascades (bremsstrahlung & pair production) produced by electrons/photons

- divided into barrel region ($|\eta| < 1.475$) & endcap regions ($1.375 < |\eta| < 3.2$) with transition region ($1.372 < |\eta| < 1.52$) containing extra cooling materials for inner detector
 - end-cap divided into outer wheel ($1.372 < |\eta| < 2.5$) & inner wheel ($2.5 < |\eta| < 3.2$)
 - higher granularity in ID ($|\eta| < 2.5$) range for electrons/photons & precision physics, coarser elsewhere for jet reconstruction & MET measurements
- hadronic calorimeter:
- outermost
 - measures hadronic showers from inelastic QCD collisions
 - thick enough to prevent most particles showers from reaching muon spectrometer
- split into tile calorimeter in barrel region ($|\eta| < 1.0$) & extended barrel region ($0.8 < |\eta| < 1.7$), LAr hadronic end-cap calorimeter (HEC) in end-cap regions ($1.5 < |\eta| < 3.2$) & LAr forward calorimeters (FCal) in $3.1 < |\eta| < 4.9$ range.
 - tile calorimeters: steel-plastic scintillating tiles, readout via photomultiplier tubes
 - hec: behind tile calorimeters, 2 wheels per end-cap. copper plates-LAr. overlap with other calorimeter systems to cover for gaps between subsystems
 - fcal: 1 copper module & 2 tungsten modules-LAr. copper optimized for EM measurements, tungsten for hadronic.

3.2.3 Muon spectrometer

- ATLAS outermost layer. measures muon momenta & charge in range $|\eta| < 2.7$

- momentum measured by deflection in track from toroid magnets producing magnetic field orthogonal to muon trajectory
 - large barrel toroids in $|\eta| < 1.4$, strength 0.5 T
 - 2 smaller end-cap toroids in $1.6 < |\eta| < 2.7$, strength 1 T
 - transition region $1.4 < |\eta| < 1.6$, deflection provided by a combination of barrel and end-cap magnets
- chambers installed in 3 cylindrical layers, around the beam axis in barrel region & in planes perpendicular to beam axis in the transition and end-cap regions
- split into high-precision tracking chambers (monitored drift tubes & cathode strip chambers) & trigger chambers (resistive plate chambers & thin gap chambers)
- trigger chambers provide fast muon multiplicity & approximate energy range information with L1 trigger logic
 - mdt:
 - * muons pass through tube, ionizing gas and providing signals. Combining signals from tubes forms track
 - * range $|\eta| < 2.7$, innermost layer $|\eta| < 2.0$
 - * precision momentum measurement
 - * maximum drift time from wall to wire 700 ns
 - * layers of 30 mm drift tubes filled with 93% *Ar* & 7% *CO*₂, with a 50 μ m gold-plated tungsten-rhenium wire at the center
 - csc:
 - * resolution: 35 μ m per chamber, 80 μ m per tube

- * forward region $2.0 < |\eta| < 2.7$, – rpc:
 - highest particle flux and density * range $|\eta| < 1.05$
 - region * provide fast meas
- * multiwire proportional chambers – tgc:
 - with higher granularity, filled * range $1.05 < |\eta| < 2.7$
 - with 80% *Ar* & 20% *CO*₂
- * shorter drift time than MDT,
 - plus other features making CSC
 - suitable for high particle den-
 - sities and consequently able to
 - handle background conditions
- * resolution: 40 μm in bending η -
 - plane, 5 mm in nonbending ϕ -
 - plane due to coarser cathode seg-
 - mentation, per CSC plane

3.2.4 Forward detectors

- LUCID (LUminosity measurement using Cherenkov Integrating Detector): ± 17 m from interaction point, measures luminosity using pp scattering in the forward region
- ALFA (Absolute Luminosity for ATLAS): ± 240 m, measures pp scattering at small angles
- ZDC (Zero-Degree Calorimeter): ± 140 m, measures centrality in heavy-ion collisions

3.2.5 Magnetic systems

superconducting solenoid & toroid magnets cooled to 4.5 K with liquid helium

solenoid: 2.56 m diameter, 5.8 m length, 2 T strength, encloses inner detector

toroid = barrel + endcap toroid x2

barrel toroid: 9.2/20.1 m inner/outer diameter, 25.3 m length, 0.5 T strength

endcap toroid: 1.65/10.7 m inner/outer diameter, 5 m length, 1 T strength

(show magnet system diagram)

3.2.6 Trigger & data acquisition (TDAQ)

LHC produces large amount of data (40 MHz with 25 ns bunch crossing), necessitates a way to filter out trash from interesting events

handles online processing, selecting and recording interesting events for further offline processing and more in-depth analyses

- Level-1 (L1) trigger: online, fast hardware-based trigger, reduces to 100 kHz
 - L1 calorimeter triggers (L1Calo): selects high energy objects & MET
 - L1 muon triggers (L1Muon): selects using hit information from RPC & TGC
 - L1 topological trigger (L1Topo): select based on topological selection synthesized using information from L1Calo & L1Muon
 - Central Trigger Processor (CTP): uses L1Calo/Muon/Topo for final L1 trigger decision within $2.5 \mu\text{s}$ latency. Also identify regions of interest in η and ϕ to be processed directly by HLT

- L1 trigger information read out by Front-End (FE) detector electronics then sent to ReadOut Drivers (ROD) for preprocessing and subsequently to ReadOut System (ROS) to buffer
- High-Level Trigger (HLT): offline, software-based trigger, using dedicated algorithms and L1 output as input, reduces to 1 kHz
- Send to storage for analyses after HLT

overall trigger process reduces original collision data rate by a factor of about 10000 after HLT

(show TDAQ diagram)

Chapter 4. Data & Simulated Samples

4.1 Data samples

LHC Run 2 data collected at $\sqrt{s} = 13$ TeV between 2015-2018

luminosity 140 fb^{-1}

(include uncertainty for Run 2 only)

4.2 Monte Carlo samples

4.2.1 $t\bar{t}Z'$ signal samples

Run 2 $t\bar{t}Z'$ sample

samples: 6 samples for each mass poin from $[1, 1.25, 1.5, 2, 2.5, 3]$ TeV

generator: MADGRAPH5_AMC@NLO v.2.8.1p3.atlas9 at LO with NNPDF3.1LO pdf

event: PYTHIA8 [v.244p3.rangefix] using A14 tune & NNPDF2.3LO pdf

parameters:

- chirality θ : does not affect the strong production mode for $t\bar{t}Z'$, therefore picking default value $\pi/4$ to suppress loop-production of the Z' resonance
- top coupling $c_t = 1$

resonance width computed with MADGRAPH5_AMC@NLO to be 4% of model configuration with these parameters

4.2.2 Background samples

Run 2 mc20 samples (2015-2018)

(show MC sample table)

(go in depth into each sample? briefly explain different generator, pdf and ps?)

Chapter 5. Particle Reconstruction & Identification

5.1 Vertex & track reconstruction

Charged particles deposit energy in different layers of the inner detector and muon spectrometer

5.2 Jets

5.3 Electrons

[isolation criteria along with muon]

5.4 Muons

5.5 Missing transverse momentum

5.6 Topological clustering

5.7 Pile-up & overlap removal

5.8 b -tagging

- b-jets important object for many analyses (Higgs decay/top quark id) - special properties of b-jets allowing for identification: decays suppressed by CKM factor, allowing for longer lifetime with a displaced secondary decay vertex, with higher track multiplicity

5.8.1 GN2 algorithm

- GN2 graph neural network based b-tagging algorithm - brief explanation on how GN2 works - working points, trade off between efficiency and purity

Calibration

- correct for b-tagging efficiency disparity between data and MC in the form of a correction scale factor - ttbar calibration [find ttbar calibration paper] - pTrel and high pT calibration
- impact parameter $-i$, signed transverse impact parameter significance - calibration results

Chapter 6. Event Selection

[event selection criteria]

6.1 Object definition

[lepton pt cut study here]

6.2 Background estimation

6.2.1 Fake & non-prompt leptons

6.2.2 Irreducible background

6.3 Analysis regions

6.3.1 Control regions

$t\bar{t}W$ CRs

6.3.2 Signal regions

[include blinding strategy]

6.3.3 Validation region

6.4 Signal extraction

SM MVA

BSM MVA

Chapter 7. Systematic Uncertainties

7.1 Experimental uncertainties

7.2 Modeling uncertainties

7.2.1 Signal modeling uncertainties

7.2.2 Background modeling uncertainties

Chapter 8. Results

8.1 Likelihood fit

8.2 Limits

8.3 Interpretation

Chapter 9. Summary

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APPENDIX A. Statistical analysis

A.1 Statistical inference

A.2 Hypothesis testing

A.3 χ^2 template fitting