Noise in Radio Systems

In the study of radio links in this course, we have seen how radio signals can be reduced in amplitude in the environment. Free-space transmission between two antennas naturally introduces a reduction in power density as you move away from the transmitter in a $1/r^2$ manner. Reflections and diffraction introduce additional losses so that at the receiver the received signal is diminished even further. When is the power too low to be received?

There has always been an assumption that a radio link "stops working" once the received power is too low. This partly true, but it is actually the noise level of a system that sets the minimum acceptable received power that is required to reliably detect the signal. In fact, it is the *signal to noise* power ratio that sets this threshold. If systems had no noise, we could receive signals no matter how low the received power is!

All radio systems receive and create noise, and the goal is to ensure that the received power is differentiable from this noise power at the receiver. Such systems are called *noise-limited* links. Other systems are *interference limited*, especially when other users share the same bandwidth (wireless LANs and cellular systems are two obvious examples). Interference can be essentially modelled as noise, so the study of how noise affects radio systems is important to both types of systems.

The Nature of Noise

Noise can be classified as internal and external to the receiver.

External sources of noise are received by the antenna. Examples include atmospheric noise, galactic/cosmic noise, man-made noise, interference "noise" created from other users in an adjacent channel (adjacent channel interference, ACI) or in the same channel (co-channel interference, CCI), and so on.

Internal noise is generated by components inside the receiver. This noise is the result of random processes such as the flow of charges in a device, or at a more fundamental level, the thermal vibrations in any component at a temperature above absolute zero. Radio receivers are made of components that generate noise. All components, passive (such as resistors), or active (transistor-based circuits) generate noise. The noise in active components actually limits the useful operating range of the device. We will consider internal noise first, and deal with external noise received from the antenna later.

Consider a microwave amplifier with 10 dB of gain. The input/output power characteristics is shown in Figure 1. The output power is 10 dB above the input power over a specific range known as the *dynamic range* of the amplifier. At high input powers, the output power is limited by the maximum power the amplifier can deliver, so the characteristic becomes saturated. At low input powers, the amplifier may also not amplify the signal as expected. For zero input to the amplifier, the amplifier outputs pure noise at a specific level, known as the noise floor of the amplifier. So, if our input power was say, -100 dBm, the output power should be -90 dBm, but the amplifier is incapable of producing a replica of the input signal at this power level because it is essentially drowned out by the noise. The amplifier only operates normally over a certain range of input powers.

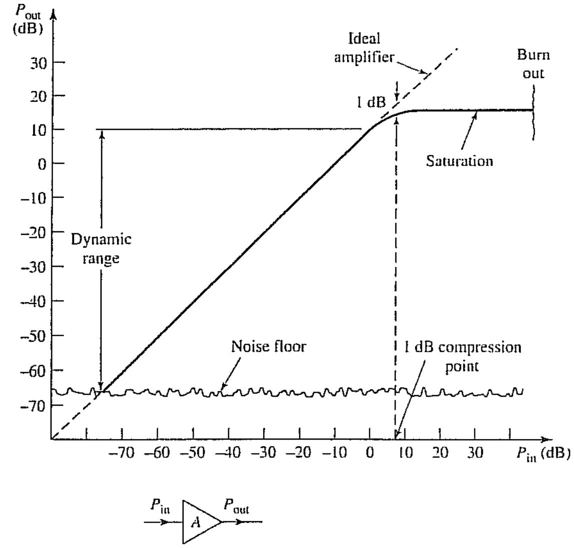

Figure 1: Input / output characteristic of a microwave amplifier

This noise can be generated by:

- Thermal noise, caused by the thermal vibration of bound charges. It is also called Johnson or Nyquist noise and is the most important source of noise we will study.
- Shot noise, caused by random fluctuations of charge carriers in an active device such as a solid-state device or electron tube.
- Flicker noise or 1/f noise, which also exists in active components, is caused by recombination of charge carriers.
- Quantum noise, caused by the discrete or quantized nature of charge carriers and photons, though it is often insignificant compared to the above noise sources.

We will focus on thermal noise, since it is a fundamental source of noise in all circuits. Furthermore, other noise processes (internal and external) can be modelled as thermal noise.

Thermal Noise

Any conductive material at a temperature above absolute zero generates noise. Resistors (or resistive materials) are sources of thermal noise. If you have a resistor of value R at temperature T and measure the open-circuit voltage across the resistor with a voltmeter, the RMS value of the noise is given by

$$v_n = \sqrt{\frac{4hfBR}{e^{hf/kT} - 1}}$$

where the temperature T of the resistor is in degrees Kelvin, the resistance R is in ohms, $h = 6.546 \times 10^{-34}$ J · s is Planck's constant, $k = 1.380 \times 10^{-23}$ J/K is Boltzmann's constant, B is the bandwidth of the system in Hz, and f is the centre frequency of the bandwidth under consideration (also in Hz). The voltage is zero-mean.

Usually, $hf << kT$, an assumption known as the *Rayleigh-Jeans approximation*, from which it follows that

$$e^{hf/kT} - 1 \approx \frac{hf}{kT}$$

which results from a Taylor series approximation. This results in a useful form of the voltage expression for microwave work:

$$v_n = \sqrt{4kTBR}$$

Hence, a resistor can be replaced with a Thevenin equivalent circuit, shown in Figure 2, consisting of a noiseless (ideal) resistor and a generator with a voltage given by the above expression. From what we know of conjugate matching, that maximum available noise power from such a resistor is

$$N = \frac{v_n^2}{4R} = kTB$$

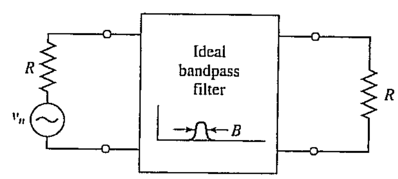

Figure 2: Equivalent circuit of a thermal noise source

We observe the following trends:

- As $B \to 0$, the noise power decreases. Hence, systems with lower bandwidths collect less noise (which is why filters are so important in receivers).
- As $T \to 0$, the noise power decreases. Hence, cooling devices reduces the noise power, which explains why receivers are often cooled in radio astronomy applications when received signals are very low (and quite noise-like, making it very important to reduce receiver noise).
- The power spectral density of thermal noise $N/B = kT$ is constant. At room temperature (290 K) this constant is equal to:

$$kT = 4 \times 10^{-21} \text{ W/Hz} = -204 \text{ dBW-Hz}$$

It is important to note that no net power is created from this process. If we connected two noisy resistors R_1 and R_2 together, the power flowing from R_1 to R_2 is the same as that flowing from R_2 to R_1. However, if R_2 was at a lower temperature, then there would be a power flow from R_1 to R_2, which would continue to heat R_2 until it reaches thermal equilibrium and both resistors are at the same temperature.

Equivalent Noise Temperature

The PSD only depends on the temperature of the resistor. The PSD is constant with frequency (at least, over the limits of the approximation) and called "white" noise because of this quality. In fact, any noise process, even non-thermal, can be modelled using an equivalent noise source by adjusting the temperature of the source appropriately. For instance, we showed earlier that an amplifier generates some noise because it contain components that generate thermal noise, as well as active devices which also generate their own noise (shot noise, flicker noise, etc.). In total, the noise power generated by an arbitrary white noise source can be modelled using an equivalent situation if the noise power is known. We can then say that the noise was generated by a resistor at an equivalent temperature T_e such that

$$T_e = \frac{N}{kB}$$

where N is the actual noise power generated by the noise process. It is important to recognize in this case that the equivalent temperature is in no way representative of a physical temperature.

Example: A Noisy Amplifier Consider a noisy amplifier with bandwidth B and gain G, matched to the source and load resistance. The input resistor is a noiseless resistor which is made for example by using a noisy resistor at a temperature of 0 K. The input power to the amplifier is therefore zero. From what we have discussed about amplifiers so far, the amplifier will output some noise power N_{ao} at the output of the amplifier. If this noise referred to the input side of the amplifier, will be $N_{ai} = N_{ao}/G$ if G is the gain of the amplifier. Hence, we can replace the noisy amplifier (Figure 3(a)) with a noiseless amplifier (Figure 3(b)) connected to a noisy resistor on the input side at a temperature

$$T_e = \frac{N_{ao}}{GkB},$$

where we have imposed an equivalent temperature on the resistor necessary to represent the noisy power actually generated by the amplifier. Physically, the resistor is not at that temperature; it is simply a convenient choice for the temperature since it yields a thermal noise power at the output of the amplifier that matches expectation.

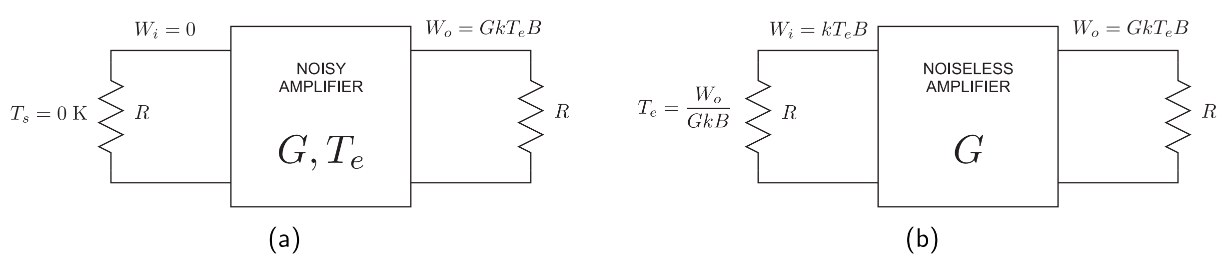

Figure 3: A noisy amplifier

It is important to note that the equivalent temperature of the amplifier T_e was defined with the noise power *referred to the input to the network* (an amplifier in this case). In general, a noisy network with gain G can be represented using a noiseless network with a suitable noise source at the input, as shown below in (a) and (b). Alternatively, the noise temperature could be defined at the output of the network (T'_e), as shown in (c). Obviously, since the noise power there is G times higher, the noise temperature of the resistor is also G times higher, i.e.,

$$T'_e = \frac{N_{out}}{kB}. \tag{1}$$

In any case, the total noise power N_{out} is the same in both cases.

Equivalent noise temperature is almost always defined as the *input* to the noisy network. The exception is the noise temperature of an antenna, which is defined at the output of the receiving antenna, which we will see later. Figure 4 show the various cases of referring the noise sources to the input and output of the network.

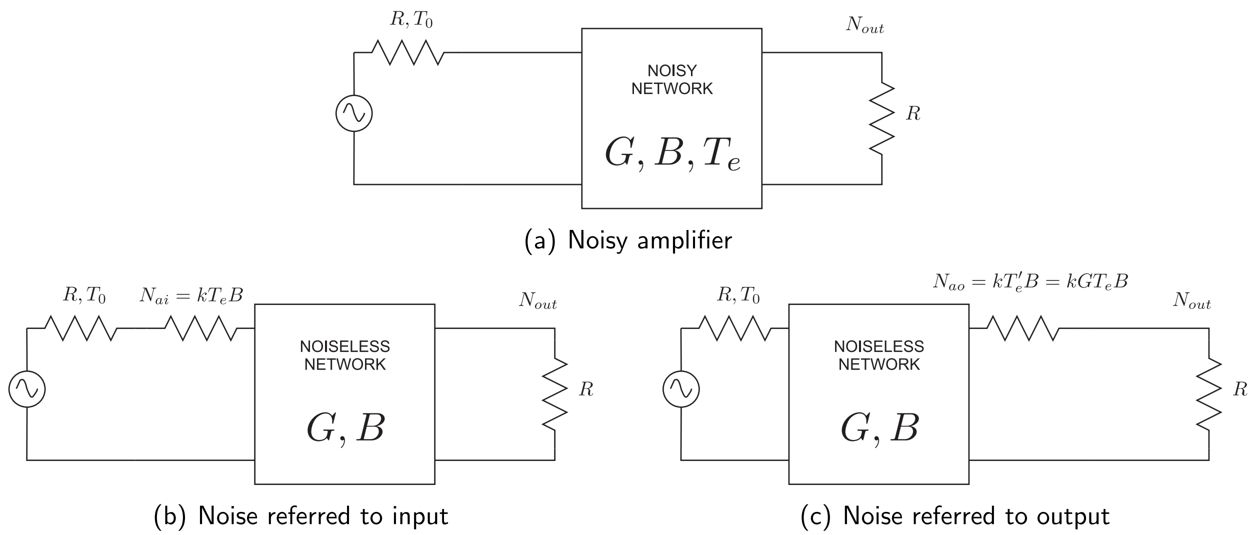

Figure 4: Noise referral cases

Effect of Noise on Radio Systems

As discussed earlier, it is the ratio of the signal power to the noise power that determines the quality of a radio link. Signal-to-noise ratio, or SNR, is defined as

$$SNR = \frac{\text{average signal power}}{\text{average noise power}}.$$

The SNR often expressed in dB since it is a ratio ($SNR_{dB} = 10 \log SNR$).

Noise Figure

It is important to track the signal to noise ratio as we pass through a radio receiver, because components change this signal ratio significantly. For example, an amplifier is theoretically designed to boost signal power, but in practise adds some noise in the process as well. Hence, the signal-to-noise ratio at the output of the amplifier is actually lower than at the input: the signal gets amplified, but so does the noise that was initially present on that signal. Hence, the signal and noise power have been boosted by the amplifier; then, additional noise is added to the output signal as a result of the noisiness of the amplifier. The ratio of the input SNR to the output SNR is called the *noise figure* of a component:

$$F = \frac{S_i/N_i}{S_o/N_o}$$

where S_i and N_i are the input signal and noise powers, and S_o and N_o are the output signal and noise powers. This ratio is always greater than one because the output SNR is never higher than the input SNR:

$$F \geq 1.$$

Noise figure can also be expressed in dB. For a noiseless network, $F = 1$ or $F = 0$ dB.

A noisy two-port network is characterized as having a gain G (or equivalently, a loss $1/G$), bandwidth B, and a noise equivalent temperature T_e, defined earlier. This 2-port network is connected to a system as shown in Figure 5. The input resistor is assumed to be at room temperature ($T_0 = 290$ K), and hence produces a input noise of $N_i = kT_0B$. This is an important, as noise figure is always defined assuming that the input of the network is terminated in a resistor at this *reference temperature* of 290K. Meanwhile the input signal power is S_i. Noise is added to the signal by the noisy 2-port network. As we discussed previously, the noise added by the network is modelled by an additional thermal noise source at an equivalent temperature T_e that adds a power $kGBT_e$ at the output of the network.

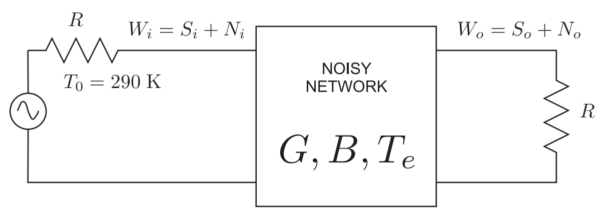

Figure 5: Noisy network with input at reference temperature T_0

At the output of the network, the signal power is $S_o = GS_i$. The total noise power, after being acted upon by the gain of the network, is $kGB(T_0 + T_e)$, since the input noise power kBT_0 is acted upon by the gain of the network. The noise figure of the network is found as

$$F = \frac{S_i}{kT_0B} \cdot \frac{kGB(T_0 + T_e)}{GS_i} = 1 + \frac{T_e}{T_0} \geq 1.$$

This allows us to express the noise temperature of the network as

$$T_e = (F - 1)T_0.$$

Example: Noisy Amplifier Revisited To see the effect of such an amplifier on the spectrum of a narrowband signal, consider the plots in Figure 6. If the gain of the amplifier is 20 dB, and the noise figure is 10 dB (quite bad for an amplifier), we observe the following. The "noise floor" is the noise power on the periphery of the signal of interest. The signal power is initially -60 dBW. Since the amplifier has a gain of 20 dB, after amplification we see that the signal power has increased to -40 dBW, as expected. But the noise floor has also increased. Prior to amplification, the SNR was 40 dB. It has reduced to 30 dB, the difference corresponding to the noise figure of the amplifier (10 dB).

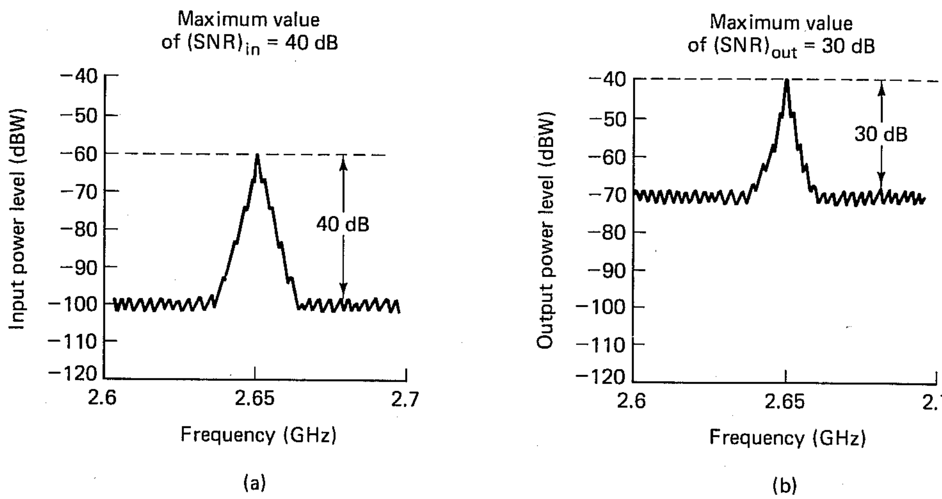

Figure 6: Spectrum of a noisy, narrowband signal applied to a noisy amplifier at (a) the input to the amplifier and (b) the output of the amplifier

Noise Figure of a Lossy Line / Lossy Passive Two-Port Network

If we are considering a passive network, $G < 1$ and it is customary to refer to the loss of the network $L = 1/G$ instead of its gain. An important example is a matched lossy transmission line, or lossy two-port network in general. If the whole system is in thermal equilibrium, then at the output of the line/network the noise power delivered to the load resistance is kTB. The resistor at the input also generates a noise power kTB, which is attenuated by the network so that at the output this power is reduced to $GkTB$ since $G < 1$. The deficit between the output power kTB and the contribution from the source resistor $GkTB$ is the power added by the network:

$$N_o = kTB = GkTB + GN_{added}.$$

Then,

$$N_{added} = \frac{1-G}{G}kTB = \frac{1-\frac{1}{L}}{\frac{1}{L}}kTB = (L-1)kTB.$$

If $N_{added} = kT_eB$ (following in our tradition of defining noise temperature at the input side of the network), then the equivalent noise temperature of the lossy line/network is

$$T_e = \frac{N_{added}}{kB} = (L-1)T,$$

since the gain G has already been accounted for above. Hence, the noise figure of a lossy transmission line/network is

$$F = 1 + (L - 1)\frac{T}{T_0}$$

Often, the lossy transmission line is at room temperature, so that noise figure is simply

$$F = L$$

which is an important and easy-to-remember result: the noise figure of a lossy transmission line is simply the loss of the network itself. Hence if transmission line had 1 dB of loss, the cable would decrease the SNR by 1 dB between the output and the input.

This same result is true of *any* passive network so long as it is impedance-matched to the next stage (which is usually the case of passive components, such as filters, attenuators, etc. in a radio system).

Noise Figure of Cascaded System

What is the expression for the noise figure of multiple systems that are cascaded together? We know that in a real system the signal will be passed on through various stages in a receiver, each of which degrade the SNR further. Consider the cascade of two components first, as shown in Figure 7(a), where network 1 has gain G_1 and equivalent noise temperature T_{e1}, and network 2 has gain G_2 and equivalent noise temperature T_{e2}. The bandwidth B of each component is identical.

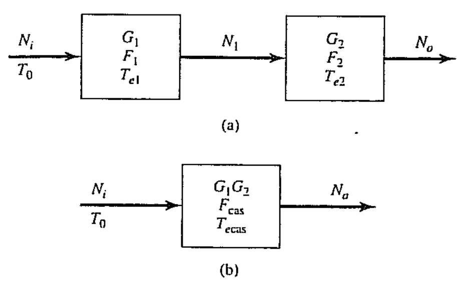

Figure 7: Cascade connection of noisy two-port networks

The noise power at the output of the first stage, which we know already from before, is

$$N_1 = G_1 k T_0 B + G_1 k T_{e1} B$$

The noise from the second network is calculated as follows. The noise from the first system is simply amplified by the second network; the second network then adds its noise $kT_{e2}B$:

$$N_2 = N_o = G_2 N_1 + G_2 k T_{e2} B = G_2(G_1 k T_0 B + G_1 k T_{e1} B) + G_2 k T_{e2} B$$

We are interested in the total noise power at the output of the cascade of the two systems, relative to the noise power at the input (the composite noise figure):

$$N_o = G_1 G_2 k B \left(T_0 + T_{e1} + \frac{1}{G_1} T_{e2} \right)$$

For the equivalent system shown in Figure 7(b) we could represent

$$N_o = G_1 G_2 k B (T_0 + T_{cas})$$

from which it follows that

$$T_{cas} = T_{e1} + \frac{1}{G_1} T_{e2}$$

Since $T_{e1} = (F_1 - 1)T_0$, $T_{e2} = (F_2 - 1)T_0$, and $T_{cas} = (F_{cas} - 1)T_0$,

$$F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1).$$

This is a very important result, because it can be seen that the composite noise figure of the cascade of networks is dominated by the noise figure of the first network. The second network, even if it is quite noisy (high F_2), does not degrade the SNR as much as the first network because its noise figure has been reduced the gain of the first network.

This has a profound impact on the way receivers are designed. The first component in the receiver chain is usually chosen to have:

1. As low a noise figure as possible, to minimize F_1 which has the most effect on the composite noise figure of the system;

2. As high a gain as possible, to reduce the noise contribution from components further down the receiver chain.

Hence, most receiver used a *low-noise amplifier* at the input to the receiver, often before filters, cables, and other components.

The composite noise figure equation can be generalized for n networks instead of just two. The resulting equations are:

$$T_{cas} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots + \frac{T_{en}}{G_1 G_2 \cdots G_{n-1}}$$

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_n - 1}{G_1 G_2 \cdots G_{n-1}}$$

The single most important aspect of coalescing multiple networks into a single network is that now we can represent that single noisy network as an equivalent noiseless network with a single noise source applied at the input to that network. That greatly simplifies the calculation of SNR in a system, since now it no longer matters whether we measure the SNR at the input or output of the noiseless network, since the answer is the same. Hence, link budgets often boil down to knowing the characteristics of the receiver, which are described by this single network, and the external noise added from the antenna, which is discussed next.

Antenna Noise

So far, we have considered the noise added by the receiver – internal sources of noise after the signal has entered the receiver. External noise also affects the link budget of a system, and depends solely on the signal the antenna is receiving. All types of noise exists in the environment, which can subsequently be picked up by an antenna. The amount of noise picked up by the antenna depends on what the antenna is "viewing" through its beams and sidelobes. For example, the antenna can view thermal bodies such as the sun, the moon, parts of the earth, and the galaxy, which each are at a specific temperature. Furthermore, non-thermal sources, such as interference and atmospheric noise, are representable as thermal sources at an equivalent noise temperature. The *antenna temperature* is the effective temperature integrated over the entire antenna pattern. We represent the effective temperature of the antenna as T_A,

$$T_A = \frac{\int_0^{2\pi} \int_0^{\pi} T_B(\theta, \phi) G(\theta, \phi) \sin\theta d\theta d\phi}{\int_0^{2\pi} \int_0^{\pi} G(\theta, \phi) \sin\theta d\theta d\phi},$$

where $T_B(\theta, \phi)$ is a function describing the background sky temperature as a function of angular coordinates.

We now examine some specific phenomenon and how they affect the signal-to-noise ratio at the antenna terminals, prior to processing by the receiver.

Sky Noise

When an antenna is pointed at a thermal body, that body acts as a noise source with an associated noise temperature. Such thermal bodies could be the earth, the sun, the moon, and even the atmosphere. The atmosphere is at a nonzero temperature. We have learnt previously that radio signal attenuation is introduced by two sources in the atmosphere: absorption by atmospheric gases (such as oxygen and water vapour), and absorption by rain. Since absorption is a thermal process (the absorbed radio wave is transformed into heat), these media also introduce noise to the signal as it passes through.

In the absence of the above effects, there is still a small but significant amount of noise introduced by pointing the antenna skyward, from the rest of the galaxy. Such noise if called *galaxy noise* and is a combination of effects that decrease with frequency.

Hence, when considering noise sources at the input to an antenna, the two majors sources are galaxy noise and atmospheric noise. Atmospheric noise becomes significant beyond 10 GHz, due to increased absorption as frequency increases beyond this point. These noise processes can be modelled as having an equivalent noise temperature associated with the noise power that is available at the antenna terminals. This equivalent temperature is plotted in Figure , as a function of frequency and for a variety of elevation angles.

Notice that there is a region where the noise is lowest, between about 1-10 GHz. This region is called the *microwave window* or *space window* and is particularly useful for satellite and deep-space communication. Also notice how rain acts to narrow the microwave window, and how increasing the elevation angle increases the noise temperature, since the effective path length through the

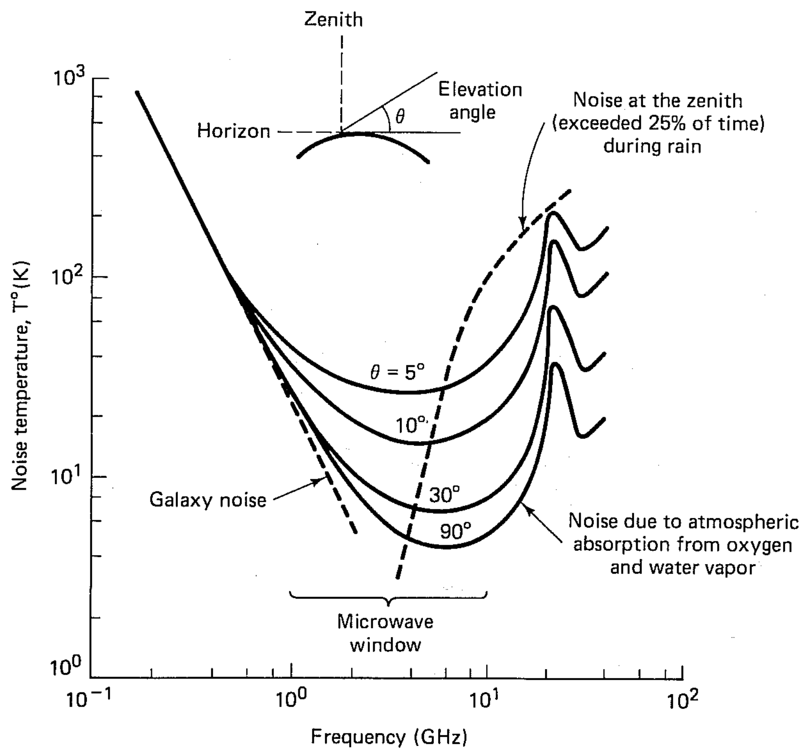

Figure 8: Equivalent noise temperature from sky noise

atmosphere becomes larger at smaller elevation angles.

Antenna Temperature

Consider a signal emanating from space (e.g. from a satellite) that is being received by an earth station. This signal is received by pointing an antenna at the sky; hence, even in the absence of the atmosphere the antenna is going to pick up noise from the background radiation of the galaxy and other celestial bodies. Meanwhile, the radio signal enters the atmosphere and is attenuated while additional noise is added to the signal (as in the case with any lossy network).

What is the total noise power of a signal once it reaches the antenna? This is the noise introduced even before thermal noise in the receive further degrades the signal. We can determine it using the cascaded network approach we have learnt about already. Consider the diagram in Figure 9, which shows the signal path from a signal originating in space (e.g. from a satellite) to a station on the ground.

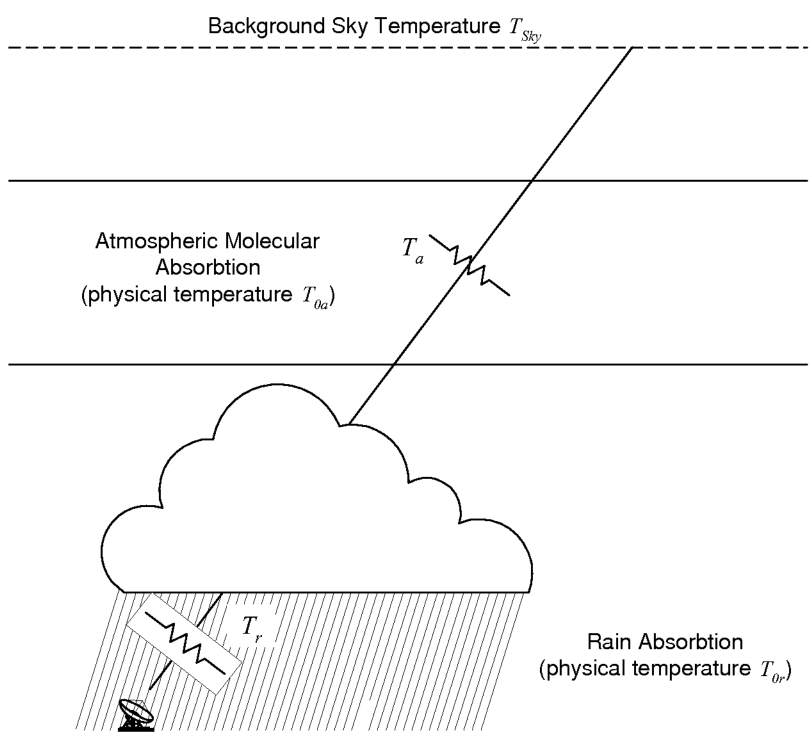

Figure 9: Radio link with attenuation

The signal originates from space. The signal has some noise superimposed on top of it, corresponding to the thermal noise from the sky. The signal is then attenuated by the atmosphere, where some additional noise is added. That signal is then attenuated by the rain cell, which adds additional noise. Using a cascade of lossy networks, we can represent the signal flow as shown in Figure 10. Note that T_{0a} and T_{0r} refer to physical thermal temperatures of the atmosphere and rain cell, respectively.

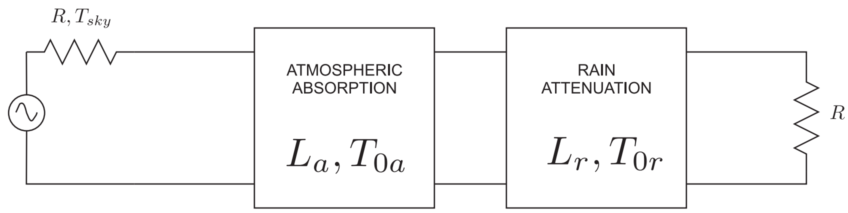

Figure 10: Equivalent network representation of the radio link in Figure 9

Since both attenuation/absorption processes are lossy (they have a gain less than one), we can use the result derived previous for lossy networks. As shown previously, such networks add a noise power $(L-1)kTB$ to the signal at the input side of the network.

Our goal is to find the equivalent temperature of the antenna. In this case, we wish to find a noise source $kT'_{sky}B$ referred to the *output* of all the networks that models all of the noise received by the antenna (sky noise, atmospheric noise, rain noise). The noise powers produced at different stages of the network are:

- Sky noise $kT_{sky}B$ at the input to the system
- Atmospheric noise $G_a(L_a-1)kT_{0a}B$ produced at the output of the atmospheric absorption block
- Rain noise $G_r(L_r-1)kT_{0r}B$ produced at the output of the rain attenuation block

The total noise power dissipated by R is

$$G_aG_r[kT_{sky}B] + G_r[G_a(L_a-1)kT_{0a}B] + [G_r(L_r-1)kT_{0r}B] = kT'_{sky}B$$

Therefore,

$$\begin{aligned} T'_{sky} &= G_aG_rT_{sky} + G_rG_a(L_a-1)T_{0a} + G_r(L_r-1)T_{0r} \\ &= \frac{T_{sky}}{L_aL_r} + \frac{(L_a-1)T_{0a}}{L_aL_r} + \frac{(L_r-1)T_{0r}}{L_r} \end{aligned}$$

defines the equivalent noise temperature at the receiver antenna terminals that models external noise power received by the antenna. The noise from this source is the input noise to the rest of the receiver, prior to accounting for internal (thermal) noise sources.

Antenna noise can also originate from losses in the antenna itself. If an antenna is not 100% efficient ($\eta_A = 1$), then it is effectively a lossy network: the input power (from the incident field) is larger than the output power delivered to the load by the antenna. This network representation is shown in Figure 11. It acts like a network with a gain $G_A = \eta_A$ (not to be confused with the antenna's radiation gain) and hence loss $L_A = 1/\eta_A$. So, if the antenna is at temperature T_0, it is as if the antenna adds a noise power

$$N_{added} = (1/\eta_A - 1)kT_0B$$

at the input side of the network. So, to model the total noise added by the sky, atmospheric attenuation, rain attenuation, and antenna losses, we have the equivalent network shown below: The noise at the "input" to the lossless antenna was already found as $kT'_{sky}B$. Hence, the total

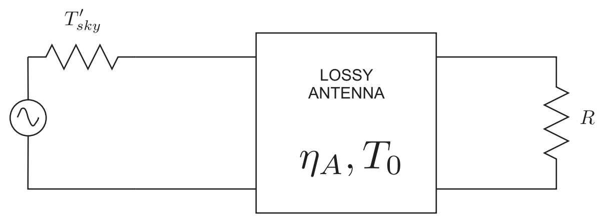

Figure 11: A noisy, lossy antenna

power at the input to the antenna is $kT'_{sky}B + (1/\eta_A - 1)kT_0B$, so that at the noise power at the output of the antenna (input to the receiver) is:

$$\eta_A kT'_{sky}B + \eta_A(1/\eta_A - 1)kT_0B = kT_AB$$

The temperature of all these effects is equivalently modelled as

$$T_A = \eta_A T'_{sky} + (1 - \eta_A)T_0$$

Remember that this is the only time we refer the noise temperatures to the output side of the network instead of the input – it makes more sense to do this for receiving antennas since the "output" is the antenna terminals where the noise is readily measured. The "input" is the fields incident on the antenna, where it is much more difficult to measure noise power.

System Effective Temperature

We have now discussed the bulk of the noise sources considered in a practical communication system. Notice that all of the noise sources have been *referred* to the receiver input port. Noise introduced by amplifiers and other components *after* being input to the receiver was referred *back* to the input of the receiver, modelled by a resistor at an effective temperature T_R, where the subscript R indicates noise introduced by the receiver. Similarly, noise from the sky, atmosphere, and antenna losses was referred *forward* to the receiver input port, modelled as a resistor at an equivalent antenna temperature T_A at the input of receiver. The total equivalent system temperature, as seen at the input port of the receiver / output port of the antenna, is

$$T_S = T_A + T_R$$

This nicely illustrates that part of the unwanted noise in the system is *injected via the antenna* (kT_AB), and *internally generated* in the receiver front end (kT_RB). There is very little that can be done to reduce the noise contribution from the former. If, for example, a hot body such as the sun is in the antenna's field of view, then the antenna temperature is going to be very high and even the best LNA is not going to help improve the system SNR. On the other hand, if the antenna temperature is low, any improvements to the noise must be addressed by reducing the noise internally generated by the receiver.