3D Taylor-Green Vortex Comparison

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Abstract

As part of the verification of Xcompact3D we simulate the Taylor-Green vortex and compare with results from a reference 6^{th} order compact finite difference code provided by Eric Lamballais.

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1 Introduction

As a canoncical test case, the Taylor-Green vortex provides a check that the time integration of the Navier-Stokes is working correctly. The Taylor-Green vortex is initialised as follows

$$\mathbf{u} = \begin{cases} U \sin(x/\pi) \cos(y/\pi) \cos(z/\pi) \\ -U \cos(x/\pi) \sin(y/\pi) \cos(z/\pi) \\ 0 \end{cases}$$
(1)

in the $-\pi \leq x \leq \pi$ periodic box. To save computational effort, the symmetries inherent in the flow field are exploited to simulate only the impermeable sub-domain $0 \leq x \leq \pi$.

2 Computational setup

2.1 Discretisation

To ensure comparability of the results we must first ensure the same schemes are being used the pertinent variables are fpi2 and ailcaix6 (and y and z) which should be set to $48/7/\pi^2$ and 0.461658 in both codes.

2.2 Runtime parameters

The runtime parameters pertinent to the simulation are given in table 1.

Table 1: Runtime parameters for Taylor-Green Vortex simulations.

Parameter	Value	Notes
xlx	3.14159265358979	yly and zlz the same
nx	65	ny and nz the same
		equivalent to a 129^3 domain
nclx	1	ncly and nclz the same
		corresponds to free-slip
dt	0.001	
Time scheme	RK3	
ilast	20,000	
Output frequency	1,000	Stores snapshots every 1,000 steps

3 Comparison of results

The main statistics of interest for comparison purposes are the kinetic energy and enstrophy, defined as

$$k = \frac{1}{2} \int_{\Omega} \mathbf{u}^2 dV , \qquad (2)$$

and

$$\varepsilon = \int_{\Omega} |\boldsymbol{\omega}|^2 dV , \qquad (3)$$

where

$$\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{u} , \qquad (4)$$

is the vorticity.

The codes compute these statistics online, here a python script has been developed to plot them for comparison. It expects that the data are located in ./x3d/time_evol.dat and ./e3d/time_evol.dat with the following format (comment lines beginning with a # and and additional columns are ignored)

#	\mathbf{TIME}	\mathbf{KE}	DISS	ENST	DISS2
	t1	k1	d1	e1	D1
	t2	k2	d2	e2	D2
	tn	kn	dn	en	Dn

which can be read by following python snippet given in listing 1.

```
def read_stats(filename):
    t = []
    enst = []
    ke = []

with open(filename, "r") as data:
    for row in data:
    if not (row[0]=="#"):
        words = row.split()
        t.append(float(words[0]))
        enst.append(float(words[1]))
        ke.append(float(words[3]))
```

Listing 1: Python code to read statistics for TGV case.

The data are plotted using matplotlib in listing 2.

Listing 2: Python code to plot comparison of Xcompact3D and Eric's reference code.

And finally, the following script (plot_tgv.py) plots the data in fig. 1 and fig. 2.

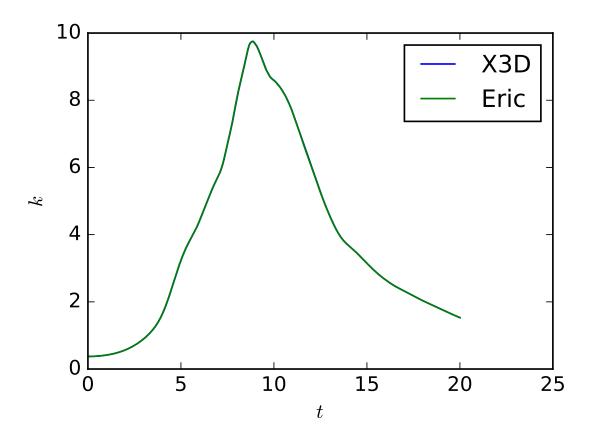


Figure 1: Comparison of kinetic energy

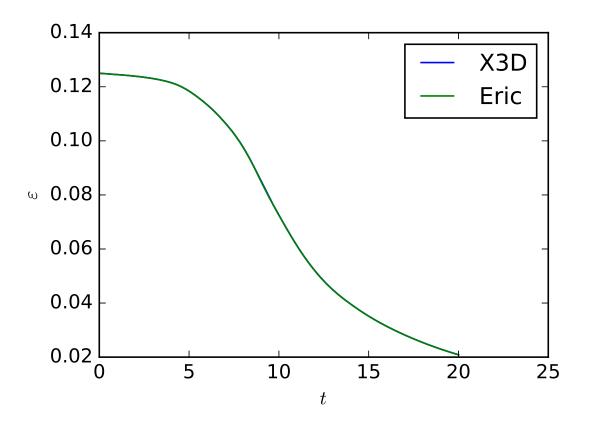


Figure 2: Comparison of enstrophy