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FIR Filter Design - Part II

4.1 Differentiator Filters

4.1.1 Introduction: Idealised Differentiators

While digital filters are most frequenly used to apply frequency-selective filtering of signals, some applications also require information about the first- or higher order derivatives of the input signal. In this exercise session, we'll look at a special class of digital filters, called differentiator filters, that can calculate these derivatives. More information about these filters can also be found in the course notes 'Digital Signal Processing'.

First, let's consider the time domain and frequency domain representations of a typical differentiating operation in the continuous (non-sampled) time/frequency domains:

$$y(t) = \frac{dx(t)}{dt} \tag{4.1}$$

$$Y(j\Omega) = j\Omega X(j\Omega) \tag{4.2}$$

Similar to the frequency domain representation in equation 4.2, an ideal theoretical differentiator in the sampled (or digital) domain would have an amplitude response that is proportional to the frequency, and a phase shift $\pi/2$. If we consider T_s the sampling period of the sampled signal, this results in the following design:

$$Y(e^{j\omega}) = j\frac{\omega}{T_s}X(e^{j\omega}) \tag{4.3}$$

$$Y(e^{j\omega}) = \frac{\omega}{T_s} e^{j\frac{\pi}{2}} X(e^{j\omega}) \tag{4.4}$$

$$H_{Diff}(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\omega}{T_s} e^{j\frac{\pi}{2}} \tag{4.5}$$

Which corresponds to an idealised linear phase filter with zero delay ($\alpha = 0$) and an antisymmetric impulse response.

4.1.2 Exercise

Use the function 'firls' to design a normalised differentiator with filter order 1 (the impulse response of this filter has a length of 2 samples). Analyse the filter's impulse response and its frequency characteristics (amplitude and phase).
The transfer function of this fillter is of the form $1 - z^{-1}$. As we've already seen in the theory sessions of the DSP course, the amplitude transfer function of this differentiator filter has a sinusoidal shape instead of the ideal linear one that was considered in the previous section. What implications does this have on the application of this filter?

Now design differentiator filters with filter orders 2 and 3 (impulse response lengths of 3 and 4 samples), and compare their amplitude characteristics. Denormalise the amplitude characteristic of these differentiators for a sampling frequency of 10 kHz and apply them to sinusoidal signals of 1 kHz and 4 kHz. Compare the results and discuss the implications on their application.

Finally, design a differentiator of order 100 (even order) and order 101 (odd order), inspect their characteristics and apply them on the sinusoidal signals.