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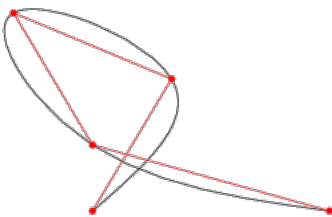
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Created, developed, and
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Cubic Spline

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A cubic spline is a [spline](#) constructed of piecewise third-order [polynomials](#) which pass through a set of m control points. The second [derivative](#) of each [polynomial](#) is commonly set to zero at the endpoints, since this provides a boundary condition that completes the system of $m - 2$ equations. This produces a so-called "natural" cubic spline and leads to a simple [tridiagonal](#) system which can be solved easily to give the coefficients of the polynomials. However, this choice is not the only one possible, and other boundary conditions can be used instead.

Cubic splines are implemented in the [Wolfram Language](#) as `BSplineCurve[pts, SplineDegree -> 3]`.

Consider 1-dimensional spline for a set of $n + 1$ points (y_0, y_1, \dots, y_n) . Following Bartels *et al.* (1998, pp. 10-13), let the i th piece of the spline be represented by

$$Y_i(t) = a_i + b_i t + c_i t^2 + d_i t^3, \tag{1}$$

where t is a parameter $t \in [0, 1]$ and $i = 0, \dots, n - 1$. Then

$$Y_i(0) = y_i = a_i \tag{2}$$

$$Y_i(1) = y_{i+1} = a_i + b_i + c_i + d_i. \tag{3}$$

Taking the derivative of $y_i(t)$ in each interval then gives

$$Y_i'(0) = D_i = b_i \tag{4}$$

$$Y_i'(1) = D_{i+1} = b_i + 2c_i + 3d_i. \tag{5}$$

Solving (2)-(5) for a_i , b_i , c_i , and d_i then gives

$$a_i = y_i \tag{6}$$

$$b_i = D_i \tag{7}$$

$$c_i = 3(y_{i+1} - y_i) - 2D_i - D_{i+1} \tag{8}$$

$$d_i = 2(y_i - y_{i+1}) + D_i + D_{i+1}. \tag{9}$$

Now require that the second derivatives also match at the points, so

$$Y_{i-1}'(1) = y_i \tag{10}$$

$$Y_{i-1}''(1) = Y_i''(0) \tag{11}$$

$$Y_i'(0) = y_i \tag{12}$$

$$Y_{i-1}''(1) = Y_i''(0), \tag{13}$$

for interior points, as well as that the endpoints satisfy

$$Y_0'(0) = y_0 \tag{14}$$

$$Y_{n-1}'(1) = y_n \tag{15}$$

This gives a total of $4(n - 1) + 2 = 4n - 2$ equations for the $4n$ unknowns. To obtain two more conditions, require that the second derivatives at the endpoints be zero, so


$$Y_0''(0) = 0 \tag{16}$$

$$Y_{n-1}''(1) = 0. \tag{17}$$

Rearranging all these equations (Bartels *et al.* 1998, pp. 12-13) leads to the following beautifully symmetric [tridiagonal](#) system

$$\begin{bmatrix} 2 & 1 & & & \\ 1 & 4 & 1 & & \\ & 1 & 4 & 1 & \\ & & 1 & 4 & 1 \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ & & & 1 & 4 & 1 \\ & & & & 1 & 2 \end{bmatrix} \begin{bmatrix} D_0 \\ D_1 \\ D_2 \\ D_3 \\ \vdots \\ D_{n-1} \\ D_n \end{bmatrix} = \begin{bmatrix} 3(y_1 - y_0) \\ 3(y_2 - y_0) \\ 3(y_3 - y_1) \\ \vdots \\ 3(y_{n-1} - y_{n-3}) \\ 3(y_n - y_{n-2}) \\ 3(y_n - y_{n-1}) \end{bmatrix}. \tag{18}$$

If the curve is instead closed, the system becomes

area between the curves $y=1-x^2$ 

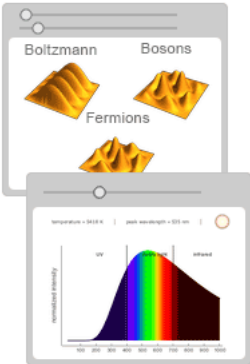
THINGS TO TRY:
= area between the curves $y=1-x^2$ and $y=x$
= express 4.8675 through pi and e
= Leech lattice

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$$\begin{bmatrix} 4 & 1 & & & & & 1 \\ 1 & 4 & 1 & & & & \\ & 1 & 4 & 1 & & & \\ & & 1 & 4 & 1 & & \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ & & & 1 & 4 & 1 \\ 1 & & & & 1 & 4 \end{bmatrix} \begin{bmatrix} D_0 \\ D_1 \\ D_2 \\ D_3 \\ \vdots \\ D_{n-1} \\ D_n \end{bmatrix} = \begin{bmatrix} 3(y_1 - y_0) \\ 3(y_2 - y_1) \\ 3(y_3 - y_2) \\ \vdots \\ 3(y_{n-1} - y_{n-2}) \\ 3(y_n - y_{n-1}) \end{bmatrix}, \quad (19)$$

SEE ALSO:

[Bézier Curve](#), [Spline](#), [Thin Plate Spline](#)

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