Radiometry
- Measurement of light

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Concepts - informal

- Light ~ energy
- Light pr. time ~ Flux, Power
- Light pr. area ~ Irradians, Radiant Exitance
- Light in special direction ~ Radient Intensity
- Light along Ray ~ Radiance
- Light pr. wavelength

- Special cases
  - Diffuse, Isotropic

- Area: planar surface, sphere, half sphere
- Solid angle - steradian

Figure 8: The plane angle

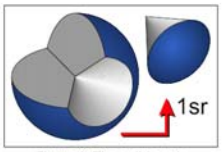


Figure 9: The solid angle

Table 2: Solid angles of some objects.
(Monitor at 60 cm, watch at 30 cm, beam-angle of spotlight)

Physics	**Radiometry**			**Photometry**	
Energy Energi	Radiant Energy Strålingsenergi	**Q**	Joules J kgm^2/s^2	Luminous Energy Lysmængde	Talbot T Lumen.s
Flux (Power) Effekt	Radiant Flux(Power) Strålingsstrøm	**Φ=dQ/dt**	Watt W W = J/s	Luminous Flux Lysstrøm	Lumen L L = T/s
Flux Density Modtaget effekttæthed	Irradiance Irradians	**E=dΦ/dA**	W/m^2	Illuminance Illuminans	Lux = L/m^2
Flux Density Udsendt effekttæthed	Radiant Exitance Udstråling	**M=dΦ/dA**	W/m^2	Luminous Exitance Lysudstråling	Lux = L/m^2
	Radiosity Udstråling – konstant	**B**	W/m^2	Luminosity	Lux = L/m^2
Ang.Flux dens Effekttæthed/ rumvinkel	Radiance Radians	**$L=d\Phi/(dA^{\perp}d\omega)$**	$W/(m^2sr)$	Luminance Luminans	Nit=L/m^2sr = cd/m^2
Intensity Effekt pr. rumvinkel	Radiant Intensity Strålingsstyrke	**$I=d\Phi/d\omega$**	W/sr	Lumious Intensity Lysstyrke	Candela cd = L/sr

Differential Flux - Throughput

- Differential Flux
- Radiance along ray
  - Constant
    - No absorption,
    - No scattering
  - Conservation of energy
    - Along pencil
- Throughput T

$$d\Phi = L(x,\omega)\cos\theta \, d\omega \, dA$$

$$L_1 d\omega_1 \, dA_1 = L_2 d\omega_2 \, dA_2$$

T Throughput

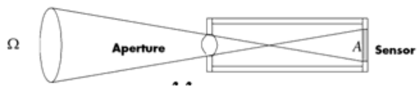
$$T = d\omega_1 \, dA_1 = d\omega_2 \, dA_2 = \frac{dA_1 \, dA_2}{r^2}$$

$$L_1 = L_2$$

Sensor Response R

- R =LT
- ~ Radiance of visible surface
- Walls equally bright over a wide range of viewing distances

$$R = \int_A \int_\Omega L \cos\theta \, d\omega \, dA$$
$$= LT$$



Light - wave

$$y = A_0 \sin[\frac{2\pi}{\lambda}(x + vt)]$$
$$v = \lambda f$$
$$y = A_0 \sin[\frac{2\pi}{\lambda}(x + \lambda f t)]$$

$$k = \frac{2\pi}{\lambda}$$
$$\omega = 2\pi f$$

$$y = A_0 \sin(kx + \omega t)$$
$$q = hf = \frac{hc}{\lambda}$$

$$h = 6.63.10^{-34}[Js]$$

$$q[500nm] = 6.63 \cdot 10^{-34} \cdot f(500)$$

Flux Φ - photon

$$q[\lambda] = hf = \frac{hc}{\lambda}$$

Flux Φ = Power P

- 100 W pære
-

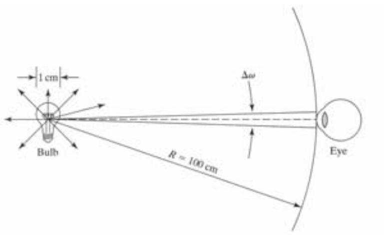
$$f = \frac{c}{\lambda} = \frac{3 \cdot 10^8}{500 \cdot 10^{-9}} \frac{m/s}{m} = 6 \cdot 10^{14} s^{-1}$$

$$q[500] = hf = \frac{hc}{\lambda} = 6.63 \cdot 10^{-34}[Js] \;\; 6 \cdot 10^{14}[s^{-1}] \approx 4 \cdot 10^{-19}[J]$$

$$N_{photon}[100W] = \frac{100\,[W]}{4 \cdot 10^{-19}[J\,/\,photon]} \approx 10^{20}[photon/s]$$

Exercise - Light Bulb

- Krypton flashlight bulb ~ sphere
- 2.4 V, 0.7 A,
- d=1cm



Radiant Flux/Power Φ

$$\Phi = \frac{dQ}{dt} \qquad [W] = [J/s]$$

$$Q = \int dQ = \int \Phi dt$$

$$\Phi = P = Volt \; x \; Ampere \; [W]$$

Radiant Flux/Power Φ

$\Phi = P = Volt\ x\ Ampere$

Radient Intensity - I

$$I = \frac{d\Phi}{d\omega} \quad \left[\frac{W}{sr}\right]$$

$$\int_\Omega d\Phi = \int_\Omega I \cdot d\omega = I \int_\Omega d\omega \quad Isotropic$$

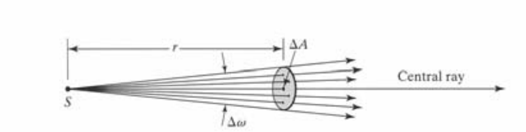
$$\Phi = I\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta\, d\theta = -I \cdot 2\pi \cdot \cos\theta \Big|_0^{\pi} = I \cdot 4\pi$$

See next slides

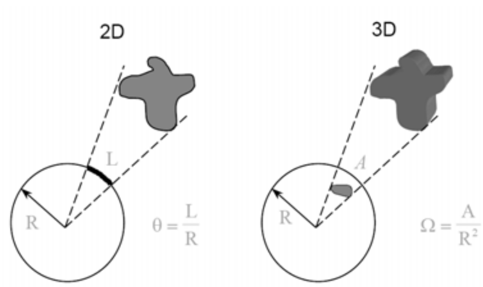
$$I = \frac{\Phi}{4\pi} \quad \text{Sphere, Isotropic}$$

Solid Angle

$$d\omega = \frac{dA}{r^2}$$



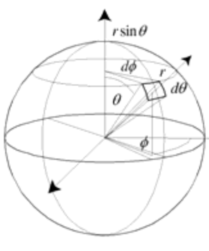
Steradian - Radian



Full circle = 2π radians

Full sphere = 4π steradians

Sphere - Solid Angles – sr



$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

$$S = \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi = 4\pi$$

Angles – Solid Angles

$ds = r\,d\theta$ section on circle

$dA = (rd\theta)(r\sin\theta\,d\phi) = r^2 \sin\theta\,d\theta\,d\phi$ Area Element on sphere

$d\theta = \frac{ds}{r}$ Circle: $\frac{2\pi r}{r} = 2\pi$ [radians]

$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$ Sphere: $\frac{4\pi r^2}{r^2} = 4\pi$ [sr]

Solid Angle - Sphere

$$\int_\Omega d\omega = \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi$$

$$= -\int_0^{2\pi} d\phi \int_0^{\pi} \mathrm{d}(\cos\theta)$$

$$= -2\pi \cdot \cos\theta\Big|_0^{\pi}$$

$$= 4\pi \quad [sr]$$

Radient Intensity - I

$$I = \frac{d\Phi}{d\omega}$$

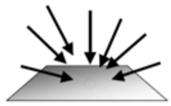
$$\int_\Omega d\Phi = \int_\Omega I \cdot d\omega \; = I\int_\Omega d\omega$$

$$\Phi = I\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta \, d\theta = -I \cdot 2\pi \cdot \cos\theta\Big|_0^{\pi} = I \cdot 4\pi$$

$$I = \frac{\Phi}{4\pi} \quad \text{Isotropic}$$

Irradiance E

$$E = \frac{d\Phi}{dA} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$



$$\Phi = \int_\Omega d\Phi = \int_\Omega E \cdot dA = E\int_\Omega dA = E \cdot 4\pi r^2$$

$$E = \frac{\Phi}{4\pi r^2} \quad \text{Sphere, Isotropic}$$

Irradiance at point on sphere - Inverse square law

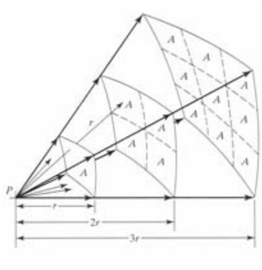
$$E = \frac{d\Phi}{dA}$$

$$\Phi = \int_\Omega d\Phi = \int_\Omega E \cdot dA = E\int_\Omega dA = E \cdot 4\pi r^2$$

$$E = \frac{\Phi}{A} = \frac{\Phi}{4\pi r^2} \quad \text{Sphere, Isotropic}$$

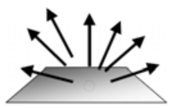
$$I = \frac{\Phi}{\omega} = \frac{\Phi}{4\pi} \quad \text{Sphere, Isotropic}$$

$$E = \frac{\Phi}{A} = \frac{\Phi}{4\pi r^2} = \frac{I}{r^2} \quad \textit{Sphere, Isotropic}$$



Radient Exitance - M

$$M = \frac{d\Phi}{dA} \quad \left[\frac{W}{m^2}\right]$$



$$\Phi = \int_\Omega d\Phi = \int_\Omega M \cdot dA = M\int_\Omega dA = M \cdot 4\pi r^2$$

$$M = \frac{\Phi}{4\pi r^2} \quad \text{Sphere, Isotropic}$$

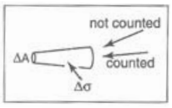
Radiance - L

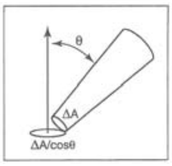
$$L = \frac{d\Phi}{dA^{\perp} d\omega} = \frac{d\Phi}{dA \cos\theta \, d\omega},$$

$$\Phi = \int_{\Omega} d\Phi = \int_A \int_{\Omega} L d\omega \cos\theta \, dA$$

$$\Phi = L \int_A dA \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \, d\theta$$

$$= -LA \, 2\pi \frac{\cos^2\theta}{2} \bigg|_0^{\frac{\pi}{2}} = LA \, \pi \quad \text{diffuse emitter}$$





In figure
$d\omega = \Delta\sigma$
$dA^{\perp} = \Delta A$
$dA = \Delta A / \cos\theta$

Radiance – Diffuse emitter

$$L = \frac{d\Phi}{dA^{\perp} d\omega} = \frac{d\Phi}{dA \cos\theta \, d\omega},$$

$$\Phi = \int_{\Omega} d\Phi = \int_A \int_{\Omega} L d\omega \cos\theta \, dA$$

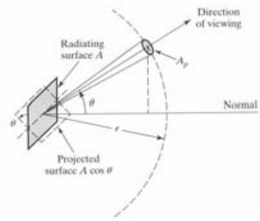
$$\Phi = L \int_A dA \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \, d\theta$$

$$= -LA \, 2\pi \frac{\cos^2\theta}{2} \bigg|_0^{\frac{\pi}{2}} = LA \, \pi \quad \text{diffuse emitter}$$

$$L = \frac{d\Phi}{dA^{\perp} d\omega} = \frac{d\Phi}{dA \cos\theta \, d\omega} = \frac{dI}{dA \cos\theta}$$

$$\frac{dI(\theta)}{dA \cos\theta} = \frac{dI(0)\cos\theta}{dA \cos\theta} = \frac{dI(0)}{dA} = \text{constant}$$

Used Lambert's cosine law



Radiance – L : Sphere

$$L = \frac{d\Phi}{dA^{\perp} d\omega}$$

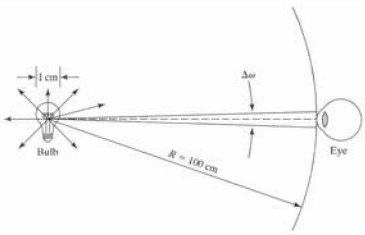
$$\Phi = \int_{\Omega} d\Phi = \int_{\Omega} dA^{\perp} \int_{\Omega} L \, d\omega$$

$$\Phi = L \int_{\Omega} dA \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta \, d\theta = L \, 4\pi r^2 \, 4\pi$$

$$= \frac{1.68}{4\pi r^2 \, 4\pi} \left[\frac{W}{m^2 sr}\right] = \frac{1.68}{4\pi \, (0.5 \cdot 10^{-2})^2 \, 4\pi} \left[\frac{W}{m^2 sr}\right] = 430 \left[\frac{W}{m^2 sr}\right]$$

Exercise - Light Bulb

- Krypton flashlight bulb ~ sphere
- 2.4 V, 0.7 A,
- d=1cm



Photometry

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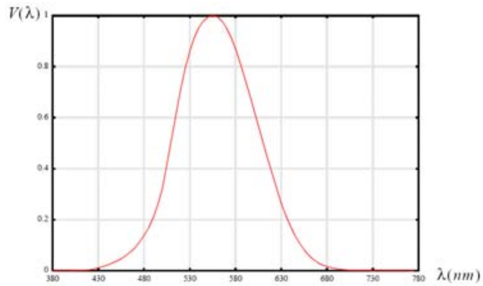
Physics	Radiometry			Photometry	
Energy Energi	Radiant Energy Strålingsenergi	**Q**	Joules J kgm^2/s^2	Luminous Energy Lysmængde	Talbot T Lumen.s
Flux (Power) Effekt	Radiant Flux(Power) Strålingsstrøm	**Φ=dQ/dt**	Watt W W = J/s	Luminous Flux Lysstrøm	Lumen L L = T/s
Flux Density Modtaget effekttæthed	Irradiance Irradians	**E=dΦ/dA**	W/m^2	Illuminance Illuminans	Lux L/m^2
Flux Density Udsendt effekttæthed	Radiant Exitance Udstråling	**M=dΦ/dA**	W/m^2	Luminous Exitance Lysudstråling	Lux L/m^2
	Radiosity Udstråling – konstant	**B**	W/m^2	Luminosity	Lux L/m^2
Ang.Flux dens Effekttæthed/ rumvinkel	Radiance Radians	**L=dΦ/(dA$^\perp$dω)**	W/(m^{2}sr)	Luminance Luminans	Nit L/m^{2}sr cd/m^2
Intensity Effekt pr. rumvinkel	Radiant Intensity Strålingsstyrke	**I=dΦ/dω**	W/sr	Lumious Intensity Lysstyrke	Candela cd L/sr

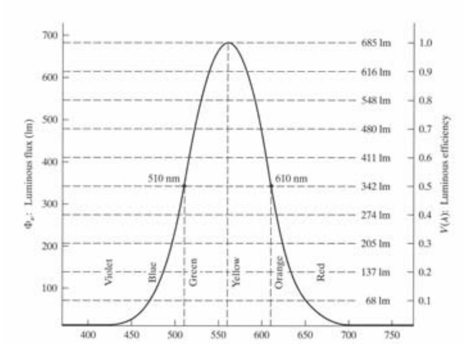
Example - Illuminance

- Light bulb
- 100 W
- 2 m from surface
- Surface perpendicular
- λ=650 nm
- K(λ) =685 V(λ)
- K(λ) Luminous efficacy
- V(λ) Luminous efficiency
- V(650) = 0.1

Luminous efficiency curve



Luminous efficiency curve



Radiometri - photometri

$$Photomerisk = K(\lambda)\ radiomerisk$$

$$K(\lambda) = 685\, V(\lambda)$$

$$Illuminance = E_\upsilon = K(\lambda)\, Irradiance = 685\, V(\lambda)\, E$$

Irradiance E - Illuminance E_v

$$E = \frac{d\Phi}{dA}$$

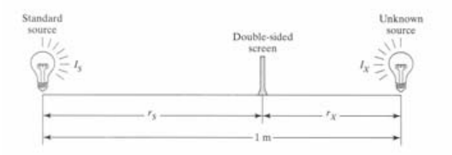
$$\Phi = \int_\Omega d\Phi = \int_\Omega E \cdot dA = E \int_\Omega dA = E \cdot 4\pi r^2$$

$$E = \frac{\Phi}{4\pi r^2} \quad \text{Sphere, Isotropic}$$

$$E_\upsilon = K(\lambda)\, E = 685\, V(\lambda)\, E$$

Photometer - simple

- Light sources
- I_s = 40 lm/sr = 40 cd
- I_x = ?
- r_s = 35 cm
- r_x = 65 cm



Lumineous Intensity - I_v

$$E = \frac{d\Phi}{dA}$$

$$\Phi = \int_\Omega d\Phi = \int_\Omega E \cdot dA = E \int_\Omega dA = E \cdot 4\pi r^2$$

$$E = \frac{\Phi}{4\pi r^2} = \frac{4\pi\, I}{4\pi\, r^2} = \frac{I}{r^2}$$

$$\frac{I_s / K(\lambda)}{r_s^{\,2}} = \frac{I_x / K(\lambda)}{r_x^{\,2}} \quad \Rightarrow \frac{I_s}{r_s^{\,2}} = \frac{I_x}{r_x^{\,2}}$$

Example

- Diffuse emitter
- Radiance equal in all directions from all surface points
- L=6000 W/(m^2 sr)
- Power?
- Radiosity?

Radiance – Diffuse emitter

$$L = \frac{d\Phi}{dA^{\perp} d\omega} = \frac{d\Phi}{dA \cos\theta\, d\omega},$$

$$\Phi = \int_\Omega d\Phi = \int_A \int_\Omega L d\omega \cos\theta\, dA$$

$$\Phi = L \int_A dA \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta\, d\theta$$

$$= -LA\, 2\pi \frac{\cos^2\theta}{2}\Bigg|_0^{\frac{\pi}{2}} = LA\,\pi \quad \text{diffuse emitter}$$

Radiosity B – Diffuse emitter

$$L = \frac{d\Phi}{dA^{\perp} d\omega} \quad \text{, Here diffuse emitter}$$

$$\Phi = \int_\Omega d\Phi = \int_A \int_\Omega L d\omega \cos\theta\, dA$$

$$\Phi = L \int_A dA \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta\, d\theta$$

$$= -LA\, 2\pi \frac{\cos^2\theta}{2}\Bigg|_0^{\frac{\pi}{2}} = LA\,\pi = BA$$

Non-diffuse emitter - Example

- L =6000 cos θ [W/m² sr]

- Radiosity?
- Power?

Radiosity

$$L = \frac{d\Phi}{dA^{\perp} d\omega} = \frac{d\Phi}{dA \cos\theta \, d\omega} \ , B = \frac{d\Phi}{dA}$$

$$B = \int_{\Omega} L \cos\theta \, d\omega$$

Example – Sun, Mars

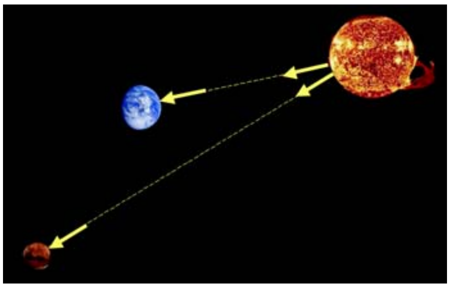


Fig 2.5

Example