Homework #1: Cosmology & Conversions
SOLUTIONS

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1. No solution necessary! Congratulations.

2. Converting between flux, flux density, luminosity and magnitudes are critical skills to professional astronomers, and often prove confusing. Here are a set of exercises to make you more adept at these conversions. You will need to visit this page to download a series of files to complete this set of problems:

www.as.utexas.edu/~cmcasey/ast386/hw1tools/

Template Stellar A0V Spectrum	spectrum_A0V.txt
Subaru g-band filter profile	subaru_g.txt
Subaru r-band filter profile	subaru_r.txt
Subaru i-band filter profile	subaru_i.txt
Subaru z-band filter profile	subaru_z.txt
Subaru Y-band filter profile	subaru_y.txt

The first column of every file is the wavelength (in Å). The second column of the template stellar spectrum is given in S_ν units ($\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}}$), and the second column of the filter profile is representative of the total system response T_λ, or the product of the filter response, detector quantum efficiency, instrument and telescope throughput and atomsphereic transmission. In other words, T_λ is the fraction of light that gets through at a given wavelength.

(a) Plot the spectrum of the A0V star, along with a flat-spectrum source of flux density 3631 Jy, overplotting and labeling each of these filters. Make sure the plot is legible, and add labels/legends sufficient for me to fully understand what is plotted. You will need to rescale the filter curves to be clearly visible on the plot.

I first converted all of the wavelengths into μm from $\AA$, and plotted just the filter system response functions from g through y-bands. The flux density scale for the Vega template is on a completely different scale than the system response (which ranges from 0 to 1), so I multiplied the Vega template by 2×10^{19} to scale nicely against the filter profiles. I overplot a flat spectrum source of 3631 Jy (also multiplied by 2×10^{19} as a dashed line.

(b) the AB magnitude system is defined such that a flat-spectrum source with flux density of 3631 Jy has a measured flux density of (...drumroll...) 3631 Jy in all filters, regardless of filter bandwidth/shape. In the *AB* magnitude system, such a source would have a magnitude of 0 in all bands. If the template spectrum for the A0V star were to represent Vega, what is the magnitude of Vega in the AB system across these five filters? (This is the offset between Vega magnitudes and AB magnitudes for these filters.)

We first compute the flux density of Vega in each of the five bands using the following equation from Tokunaga & Vacca (2005):

$$F_{filter} = \frac{\int \lambda F_\lambda(\lambda) S(\lambda) d\lambda}{\int \lambda S(\lambda) d\lambda} \tag{1}$$

This requires that we convert from F_ν units to F_λ units by multiplying by c/λ^2. Once we have the flux density for Vega in erg s^{-1} cm^{-2} Å^{-1}, we can convert back to F_ν units (or you could have stayed in F_ν units the entire time, and calculate the magnitude of Vega using:

$$M = -2.5 \log \left(\frac{F_{\nu,vega}}{3631 \, \mathrm{Jy}} \right) \tag{2}$$

The results are that Vega should have the following magnitudes in these filters: -0.099 (g), 0.157 (r), 0.399 (i), 0.542 (z), and 0.622 (y).

(c) What do you notice about the values of these offsets and the shape of the spectrum of Vega?

Vega is brighter than the flat spectrum source in g band and fainter in all other bands. So it follows that the magnitude for Vega is <0 in g and positive in all other bands (and is fainter and fainter towards redder filters).

(d) Plot the A0V spectrum in units of νL_ν, assuming it is Vega and Vega is 7.68 pc from us (note that $1\,pc = 3.086 \times 10^{16}\,m$). Overplot λL_λ on the same plot for comparison. From this plot, what can you surmise about how intrinsically bright Vega is compared to the Sun? (this should give you some feeling of why people sometimes plot in units of νL_ν or λL_λ.)

If we convert the spectrum to νL_ν or λL_λ we get the same result: the two quantities are equivalent! From this plot I would surmise that Vega is brightest at 4000Å, at about $\sim 60 L_\odot$. Indeed, the apparent luminosity of Vega is $\sim 57 L_\odot$. Note that it's actual luminosity is $40 L_\odot$, and the difference is due to the star's high rate of rotation and differential emergent flux as a function of line-of-sight with respect to the axis of rotation. In any case, νL_ν is useful for inferring how luminous whatever object actually is bolometrically, and what wavelengths of light dominate that luminosity!

3. This problem will introduce the concept of stellar population synthesis (SPS) models by building up a basic understanding of stellar populations. SPS models are critical to how we understand the stellar emission of high-redshift galaxies as integrated light sources, and so it is very important to understand how they are built. This problem deals with bolometric quantities, and the next problem will introduce some of the mechanics of building SPS models with real templates.

(a) The Salpeter IMF (Salpeter 1955) is parameterized:

$$\xi(\log m) = \frac{d(N/V)}{d \log m} = \frac{dn}{d \log m} \propto m^{-x} \tag{3}$$

where $x = 1.35$. Using this distribution of stellar masses, plot the cumulative stellar mass fraction from high masses to low, in other words $f(>m)$ vs. m. You can stop at the brown dwarf/hydrogen burning limit, $\sim 80\,\mathrm{M_{jup}}$. What is the average mass of a star drawn at random from this Salpeter distribution (i.e. the expectation value)?

To compute the cumulative fraction of stellar mass function, I first convert $\xi(\log m)$ into dn/dm. The stellar mass function that exists at a mass m or above is then equal to:

$$f(>m) = \int_m^\infty \frac{dn}{dm} m \, dm = \int_m^\infty m^{-2.35} m \, dm = \frac{m^{-0.35}}{0.35} \tag{4}$$

This is plotted here: The expectation value for a mass distribution $p(m) = dn/dm$ is defined:

$$E(m) = \frac{\int m\, p(m)\, dm}{\int p(m)\, dm} \tag{5}$$

For this distribution integrated between a stellar mass of $80\,\mathrm{M_{Jup}}$ and $100\,\mathrm{M_\odot}$, the expectation value is $0.27\,\mathrm{M_\odot}$.

(b) The relationship between a star's luminosity and mass can be parameterized roughly as:

$$\frac{L}{L_\odot} \approx 0.23\left(\frac{M}{M_\odot}\right)^{2.3} \quad (M < 0.43M_\odot) \qquad \frac{L}{L_\odot} \approx \left(\frac{M}{M_\odot}\right)^{4} \quad (0.43M_\odot < M < 2M_\odot) \qquad \frac{L}{L_\odot} \approx 1.5\left(\frac{M}{M_\odot}\right)^{3.5} \quad (2M_\odot < M < 20M_\odot) \qquad \frac{L}{L_\odot} \approx 3200\left(\frac{M}{M_\odot}\right) \quad (M > 20M_\odot)$$

What is the highest mass main sequence star you expect to live past 100 Myr? 500 Myr? 1 Gyr?

Using the mass scale I defined for part (a) I generated a luminosity function according to the above scalings. Here's a plot of stellar lifetimes on the main sequence by mass:

Then we can read off this plot the masses of the stars equal to the lifetimes given: $5.3\,\mathrm{M_\odot}$ (100 Myr), $2.8\,\mathrm{M_\odot}$ (500 Myr), and $2.1\,\mathrm{M_\odot}$ (1 Gyr).

(c) If you assume that a stellar population is aged 500 Myr, what would be the average mass of a star drawn at random? How does this differ from your answer to part (a), and how would it differ for a stellar population that is aged 1 Gyr? (You can ignore evolved stars for the purposes of this problem even though we know they're... important.)

Here we just use the expectation value for a mass given a distribution, as in part (a), but we truncate the stellar mass distribution at its maximum mass given an age, as determined in part (b). The result is the following average masses: $0.23\,\mathrm{M_\odot}$, $0.21\,\mathrm{M_\odot}$, and $0.21\,\mathrm{M_\odot}$. In other words, small, and not changing a whole lot since the IMF is very bottom heavy!

(d) What is the fractional contribution for stars of a given mass m to the total light emitted by a given stellar population? Plot this as a cumulative distribution, $f_{\mathrm{L}}(> m)$ as a function of m. Hint: it would be wise to convert $\xi(\log m)$ to dn/dl for this step. What can you say about what types of stars dominate the light of any given stellar population?

For this problem, we need to convert $\xi(\log m)$ to dn/dl, using the luminosity-mass relations given in part (b). This requires some piecewise integration! Since we want to know what the contribution is *above* a certain mass limit, it would be good to start at the high end. The cumulative contribution to luminosity will be:

$$f_L(> m) = \int_{l(m)}^{\infty} \frac{dn}{dl}\, dl = \int_{l(m)}^{l_{\max}} \frac{3200}{ln(10)} \left(\frac{L}{3200}\right)^{-x-1} dl = \frac{3200^{x+2}}{ln(10)} \left(\frac{l_{\max}^{1-x}}{1-x} - \frac{l(m)^{1-x}}{1-x}\right) \tag{6}$$

Here $l(m)$ is the luminosity of a star of mass m and $l_{\max}$ is the luminosity of a $100\,\mathrm{M_\odot}$ star. We proceed to piece together this function $f_L(> m)$ between each mass interval in a similar manner, adding the total light contribution from the higher mass intervals to given piecewise $f_L(> m)$. Here is a plot of the final function:

(e) Generate the same plot in part (d) but adjust it to represent a 500 Myr-old and 1 Gyr-old stellar population. Mark the three curves (including the 0-age curve) clearly.

To plot this, we have to remove the light contribution from all main sequence stars that would have died by the given epoch. Amazingly, >99% of the light is from stars that have lifetimes shorter than 100 Myr, and so this makes a big difference! Computationally, an aged version of the function f_L is calculated by taking $f'_L(> m, \tau) = \frac{f_L(>m)-f_L(>m_{\rm max})}{1-f_L(>m_{\rm max})}$. We get:

Obviously the elephant in the room is the lack of accounting for evolved massive stars, which would add another layer of complexity to this since they are bright, but this exercise still hammers home the point that only the brightest of the brightest stars contribute to the bolometric luminosity of a stellar population. Sad!

4. This problem builds on the previous problem, but now you are asked to build up a stellar population model spectrally. To do this you will need to download and unpack the contents of this folder: `http://www.as.utexas.edu/~cmcasey/ast386/hw2tools/` In that folder there is a file named kurucz93.tar.gz and a file named EEM_dwarf_UBVIJHK_colors_Teff.txt. The first is a directory of stellar atmosphere model spectra from Kurucz (1993) for a range of metallicities, effective temperatures and surface gravities. The txt file should be used to map effective temperature and luminosity back to mass[1]. You can read the readme files in the Kurucz directories for more information, in addition to the annotations of the txt file.

(a) Using the same mass range as in problem 1, plot $T_{\rm eff}$ against stellar mass and stellar luminosity L vs stellar mass (by interpolating the values given in the reference txt file).

To do this, I read in the data from the EEM_dwarf_UBVIJHK_colors_Teff.txt file, and plotted those that had Msun and Teff values. I took the log of both, and I added one data point at 100 $M_\odot$ and 45,000 K, then I performed an interpolation in IDL between the data points. Overplotted on both of these plots are the original points from the EEM file (as red circles) and my interpolated array between (black line). I have also overplotted the analytic approximation to luminosity (as given in problem 1b) on the mass-luminosity plot for fun (in blue), so you can see how they compare.

(b) Now that you know the effective temperature and luminosity across our entire mass range, you can make a composite spectrum for the stellar population as a whole. For each value of your stellar mass grid, you should read in the appropriate Kurucz model, choosing the closest one in effective temperature. For the gravity you can adopt: $T_{\rm eff} \geq 41000\,K$ (column g50), $36000 \geq T_{\rm eff} < 41000\,K$ (column g45), $9000 \geq T_{\rm eff} < 36000\,K$ (column g40), and $T_{\rm eff} < 9000\,K$ (column g45). You'll want to add all of the spectra of the stars together, proportional to how many stars of each type are in the stellar population. Plot the resulting stellar population spectrum in νL_ν units against wavelength. Be sure to make sensible choices for your axes and think about whether or not it would be best to use a linear or a log scale to present your results.

This was a lot of work! First I read in all of the templates in the kp00 directory and made an array of arrays of the flux density (in f_λ, as indicated in the readme file) for

[1] The table quotes $T_{\rm eff}$ for stars between $0.1 < M/M_\odot < 19.6$ with a rough gridding, so you'll have to interpolate between these points to come up with good $T_{\rm eff}$ estimates for all mass points on the scale you used in problem 1. At high masses you should interpolate towards a 100 $M_\odot$ star having a 45,000 K temperature.

each temperature given. I also made an array of those representative temperatures. Then I generated an interpolated spectrum for each mass point of my mass array by combining the spectra in the two nearest temperature bins that span the $T_{\rm eff}$ value for the given mass point shown in part (a). I scaled the flux for the template linearly with temperature between points.

Then to get the total spectrum for the stellar population, I multiplied $\xi(\log m) \times \delta \log m$ by the flux array at that $\log m$ point to get the total contribution from stars of that spectral type. Then I summed all of those contributions up over all masses to create a template spectrum for the whole stellar population. The units of my spectrum are still proportional to f_λ, so to convert to something proportional to νL_ν, I multiplied through by the wavelength in $\mathring{A}$ to get λf_λ, which is $\propto \nu L_\nu$. This is the resulting spectrum I obtain (left panel). I choose a log scale to show the dynamic range and I hone in on the optical through mid-infrared portion of the spectrum:

(c) Now go back and split up this spectrum into the contribution from stars in different mass ranges. You choose 3-4 mass ranges that you think convey the most interesting results. State what mass ranges you assume clearly and label them on your plot.

(Plotted above right) Looking at the luminosities of stars in different mass ranges in part (a) I decided to make five bins of the following mass cuts: $M/M_\odot < 0.5$, $0.5 < M/M_\odot < 2$, $2 < M/M_\odot < 5$, $5 < M/M_\odot < 20$, and $M/M_\odot > 20$. We can see that the contribution of low mass stars is really quite a bit lower than the contribution of high-mass stars.

(d) Using your results from problem 1 now generate a spectrum of this stellar population after it has aged 500 Myr, and dilineate the contributions from each of the mass ranges chosen in part (c). Then, do the same for a 1 Gyr age.

To do this I follow a similar procedure as I used in problem 1, by noting the maximum main sequence stellar mass I expect to still be around at 500 Myr or 1 Gyr. As noted there, these masses are $2.8\,\mathrm{M}_\odot$ and $2.1\,\mathrm{M}_\odot$ respectively. This means the two highest mass ranges from part (c) no longer exist! (errr, well as evolved stars, which we're ignoring). Here are the results of what's left, plotted as the total spectrum in black and the remaining stellar mass ranges colored appropriately.

(e) What differences and similarities do you notice across these stellar population models? What types of stars dominate the spectrum over what wavelength range?

The spectra are overall quite different! The 0-age stellar population model is very blue and basically equal to the aggregate spectra of all of the massive OB stars. As the population ages, the UV/blue light diminishes substantially. I do notice that the SED shape at long wavelengths appears largely similar at all ages though, with the same intrinsic slope. It seems like most of the light is dominated by the highest mass stars that exist at any given time.