## Matlab for Finance Course: Session 3

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## **REVIEW OF SESSION 2**

- MATLAB Basics
  - Scripts and Editor
  - Executing Scripts
  - Debugging Tools
- Data Manipulation
  - Array Operations
  - Matrix Operations
  - Array Indexing
- Programming Concepts
  - Boolean Logic
  - Control Flow (if-else, while, for, switch)
  - Error Handling
- Best Practices
  - Common Mistakes
  - Performance Tips

## **SESSION OBJECTIVES**

- MATLAB Fundamentals
  - Function creation and usage
  - File I/O operations
  - Data formatting and display
- Data Handling
  - Stock data import (getMarketDataViaTiingo)
  - Time series manipulation
  - Matrix operations
- Financial Analysis
  - Returns calculation
  - Statistical analysis
  - Volatility estimation
- Risk Measures
  - VaR and CVaR implementation
  - Backtesting framework

The presentation includes MATLAB implementation for all topics.

## **FUNCTIONS** - Basic Structure

- Functions in MATLAB are defined in separate files with the .m extension
- Basic function structure:

```
function [output1, output2] = myFunction(input1, input2)
    % Function description/help comment
    % input1: description of first input
    % output2: description of second input
    % output1: description of first output
    % output2: description of second output

    % Function body
    output1 = someCalculation(input1);
    output2 = anotherCalculation(input2);
end
```

- Function name must match the filename (e.g., myFunction.m)
- Help comments are displayed when using help myFunction

## **FUNCTIONS** - Example

• Example of a plotting function:

Call the function:

```
[y, h] = mysin_func(0:pi/50:2*pi);
```

#### **FILE TYPES**

Most financial data that will be imported into MATLAB will come in three main forms:

.csv: Comma Separated Values

```
Date, Open, High, Low, Close 2024-01-01, 100.5, 101.2, 99.8, 100.9
```

.tsv: Tab Separated Values

```
Date Open High Low Close 2024-01-01 100.5 101.2 99.8 100.9
```

• .txt: Text data in some format

```
# Financial Data
100.5 101.2 99.8 100.9
```

Other types of data that will be imported include:

- x1s: Excel files
- .xml: Extensible Markup Language
- .mat: MATLAB binary files (loaded using load data.mat)

It is important to understand the organisation of different data types in order to understand the memory requirements for data.

# FILE FUNCTIONS

Command	Meaning	
fopen(filename)	Open a file	
fclose(fid)	Close a file	
fread(fid)	Read binary data	
<pre>fwrite(fid,A,precision)</pre>	Write binary data	
<pre>fprintf(fid, A, precision)</pre>	Write formatted data	
fscanf(fid,format)	Read formatted data	
sprintf(format,A)	Write to a string	
sscanf(s,format)	Read string	
ferror(fid)	Query about errors	
feof(fid)	Test for end of file	
<pre>fseek(fid,offset,origin)</pre>	Set the file position indicator	

## I/O EXAMPLES

• Writing and reading numeric data:

```
A = [1 2 3 4 5];
fid = fopen('some_data.txt', 'w');
fwrite(fid, A);
fclose(fid);

fid = fopen('some_data.txt', 'r');
fread(fid)
fclose(fid);
```

Writing and reading text:

```
str = 'this is a test';
fid = fopen('test.txt', 'w');
fwrite(fid, str, 'char');
fclose(fid);
```

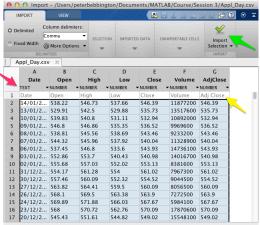
## TIMES TABLE EXAMPLE

```
display('Times Table:')
2 fprintf(1,' X '); % Write to command window
_{3} for i = 0:9
      fprintf(1,'%2d ',i);
  end
6 fprintf(1,'\n');
7 | for i = 0:9
      fprintf(1,'%2d ',i);
      for j = 0:9
9
          fprintf(1,'%2d ',i*j);
10
      end
11
      fprintf(1,'\n');
12
13
  end
```

Output shows formatted multiplication table 0-9

#### IMPORT TOOL

 Simply drag and drop a ".csv" file to the command window of Matlab to import data



 You can edit; data type (pink arrow), data field name (yellow arrow) and import data (green arrow)

## WORKSPACE

 Now that we have the data in Matlab we can create a workspace

```
1 >> whos
   Name
             Size
                     Bytes Class
                                   Attributes
   Date
           252x1
                    2016
                          double
   High
           252x1
                    2016
                          double
   Low
           252x1
                    2016 double
   Open
           252x1
                    2016 double
   Volume
           252x1
                    2016
                          double
   Close
           252x1
                    2016 double
```

>> clear

## TIINGO DATA API

- We'll use Tiingo API to download historical stock data
- Steps to get started:
  - Sign up for free account at Tiingo
  - Generate API key from your account dashboard
  - Set up API key in MATLAB:

```
setenv('TIINGO_API_KEY', 'your_api_key_here');
```

- Advantages of Tiingo:
  - Reliable and maintained API
  - High-quality financial data
  - Free tier available for academic use

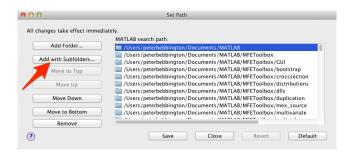
## TIINGO API EXAMPLE

```
% Download one year of daily data for AAPL and MSFT
>> startdt = '2023-01-01';
>> enddt = '2023-12-31';
% Get AAPL data
>> aapl = tiingo_prices('AAPL', startdt, enddt, 'daily');
% Get MSFT data
>> msft = tiingo_prices('MSFT', startdt, enddt, 'daily');
% Data structure example:
>> aapl
      date: [252x1 datetime]
      open: [252x1 double]
      high: [252x1 double]
       low: [252x1 double]
     close: [252x1 double]
    volume: [252x1 double]
```

## **ECONOMETRICS TOOLBOXES**

- Matlab has its own Econometrics toolbox with rich functionality
- There are also third-party toolboxes that can be installed which can help with time series analysis for summer projects
- Two recommended toolboxes:
  - MFEToolbox: www.kevinsheppard.com
  - JPLV7: www.spatial-econometrics.com

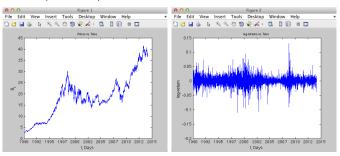
## **INSTALLING TOOLBOXES**



• Click "Add with Subfolders..." (red arrow) and Locate the two toolboxes and save.

## FINANCIAL SERIES

 A good starting point when analyzing financial time series is to plot basic quantities against time, such as price, log-returns, volume, etc...



## SAMPLE STATISTICS: Basic Moments

 Basic statistics measure the shape and central tendencies of returns

```
% Basic Statistics
mean_lr = mean(lreturns); % First moment
std_lr = std(lreturns); % Second moment (volatility)
ske_lr = skewness(lreturns); % Third moment (asymmetry)
kurt_lr = kurtosis(lreturns); % Fourth moment (tail thickness)
```

- For financial returns, we typically expect:
  - Mean close to zero
  - Significant volatility
  - Negative skewness (more extreme losses than gains)
  - High kurtosis (fat tails)

## SAMPLE STATISTICS: Tests

Statistical tests help verify stylized facts of returns

```
% Serial Correlation Tests
sacf_lr = sacf(lreturns, 1, 1, 0); % Return predictability
sacf_lr2 = sacf(lreturns.^2, 1, 1, 0); % Volatility
clustering

% Normality Tests
[jb_lr, pval] = jarquebera(lreturns); % Jarque-Bera test
kst_lr = kstest(lreturns); % Kolmogorov-Smirnov
```

- Test Interpretations:
  - Serial correlation tests check for time dependencies
  - Normality tests verify distribution assumptions

## **NORMALIZING**

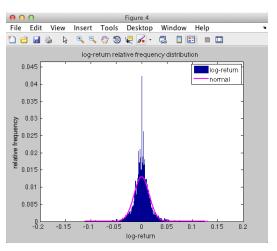
• Any Gaussian distributed random variable can be normalized:

$$X \sim \mathcal{N} \big( \mu, \sigma^2 \big)$$
 
$$Z = \frac{X - \mu}{\sigma} \quad \text{(standardization)}$$
 
$$X = \sigma Z + \mu \quad \text{(reconstruction)}$$

 Analysis of return time series is better in this form for comparison between different time series such as a portfolio

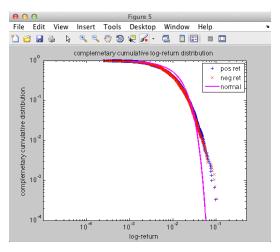
#### COMPARISON WITH A GAUSSIAN

 Here we make a comparison of the empirical histogram against a parametrized normal distribution



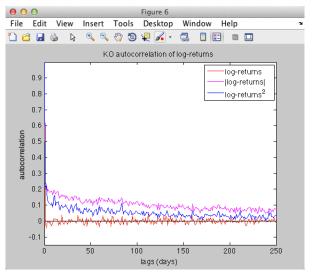
## COMPLEMENTARY CUMULATIVE DISTRIBUTION

 We see in this log-log plot the empirical time series differs from the tails of a normal distribution, indicating heavier tails in the data



## **AUTOCORRELOGRAM**

- Shows correlation between returns at different time lags
- Helps identify patterns and dependencies in the time series



## **VOLATILITY CALCULATION:** The Cumsum Trick

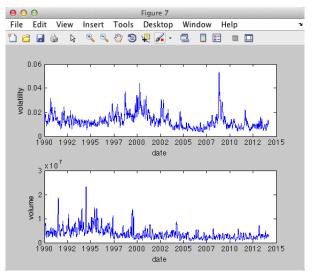
Efficient method to calculate rolling window volatility using cumulative sums

```
% Step 1: Calculate cumulative sums
window = 252; % Trading days in a year
y1 = cumsum(lreturns(:,1), 1); % Cumsum of returns
y2 = cumsum(lreturns(:,1).^2, 1); % Cumsum of squared returns
% Step 2: Calculate rolling window volatility
volatility = sqrt(
    (Y2((window+1):end) - Y2(1:(end-window))) / window %
    Variance
- ((Y1((window+1):end) - Y1(1:(end-window))) / window).^2
    % Mean²);
```

- Key advantages:
  - Vectorized operations (faster than loops)
  - Memory efficient (only stores cumsum)
  - Based on variance formula:  $\sigma^2 = E[X^2] (E[X])^2$
- Window calculations:
  - Y2(t+w) Y2(t): Sum of squared returns in window
  - Y1(t+w) Y1(t): Sum of returns in window
  - Division by window gives rolling means

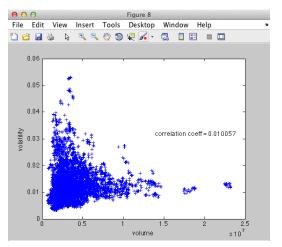
## **VOLATILITY**

- Volatility measures the dispersion of returns over time
- Calculated using a rolling window of 252 trading days



#### VOLATILITY Vs. VOLUME

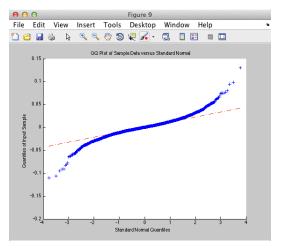
- Higher trading volume often associated with higher volatility
- Correlation coefficient indicates strength of relationship



• Important for trading strategy and risk management

## **QQPLOT (QUANTILE-QUANTILE PLOT)**

- Compares empirical distribution against theoretical normal
- Straight line indicates normality; deviations show fat tails



Financial returns typically show deviations at the tails

## VALUE AT RISK (VaR) - Mathematical Definition

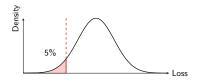
VaR is formally defined as:

$$VaR_{\alpha} \triangleq \inf\{l \in \mathbb{R} : F_L(l) \geq \alpha\}$$

- Breaking down the equation:
  - $VaR_{\alpha}$ : Value at Risk at confidence level  $\alpha$
  - inf: Infimum (minimum value)
  - $l \in \mathbb{R}$ : Loss value in real numbers
  - $F_L(l)$ : Cumulative distribution function of losses
  - $\geq \alpha$ : Probability threshold (e.g., 0.95)
- In simpler terms:
  - VaR is the smallest loss value
  - Where the probability of exceeding this loss
  - Is less than or equal to  $1-\alpha$  (e.g., 5%)

## VALUE AT RISK (VaR) - Visualization

 $\bullet$  VaR represents a threshold where probability of larger losses is  $1-\alpha$ 



- Example interpretation:
  - $\bullet~VaR_{95\%}=\$100$  means there's a 5% chance of losing more than \$100
  - Red area shows probability of extreme losses

## VAR ESTIMATION IN MATLAB

• Parametric estimation (assuming normal distribution):

- Function parameters:
  - PortReturn: Expected portfolio return
  - PortRisk: Portfolio standard deviation
  - RiskThreshold: Confidence level (e.g., 0.95)
  - PortValue: Current portfolio value
- Limitations:
  - Assumes normal distribution
  - May underestimate tail risk
  - Compare with empirical estimation

# CONDITIONAL VALUE AT RISK (CVaR) - Mathematical Definition

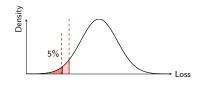
CVaR is formally defined as:

$$\mathsf{CVaR}_\alpha = \mathbb{E}[L|L \geq \mathsf{VaR}_\alpha] = \frac{1}{1-\alpha} \int_\alpha^1 \mathsf{VaR}_\gamma(L) \, d\gamma$$

- Breaking down the equation:
  - $CVaR_{\alpha}$ : Expected loss exceeding VaR
  - $\mathbb{E}[L|L \geq \mathsf{VaR}_{\alpha}]$ : Conditional expectation
  - $\frac{1}{1-\alpha}$ : Normalization factor
  - $VaR_{\gamma}(L)$ : VaR at confidence level  $\gamma$
- In simpler terms:
  - CVaR is the average loss in the worst  $(1 \alpha)\%$  of cases
  - More conservative than VaR
  - Accounts for the shape of the tail distribution

# CONDITIONAL VALUE AT RISK (CVaR) - Visualization

CVaR measures the average loss beyond VaR



- Example interpretation:
  - If  $VaR_{95\%} = \$100$ ,  $CVaR_{95\%} = \$130$  for normally distributed returns
  - CVaR represents average loss in worst 5% of cases
  - Darker red area shows the region CVaR measures

## PARAMETRIC CVaR

Conditional Value at Risk (CVaR) calculations:

```
% Parametric CVaR
m = mean(lreturns(:,1));
s = std(lreturns(:,1));
CVaR_95 = -m + s*(normpdf(norminv(0.05,0,1),0,1))/(1-0.95);
CVaR_99 = -m + s*(normpdf(norminv(0.01,0,1),0,1))/(1-0.99);

% Empirical CVaR
CVaR_95_emp = -mean(slr(1:ceil(N*0.05))); % 5% loss
CVaR_99_emp = -mean(slr(1:ceil(N*0.01))); % 1% loss
```

- CVaR represents the expected loss exceeding VaR
- Also known as Expected Shortfall (ES)
- More coherent risk measure than VaR

## INTERPRETING VAR RESULTS

Sample VaR Results:

Method	95% VaR	99% VaR
Parametric	1.75%	2.49%
Empirical	1.56%	3.10%

- Key Insights:
  - Parametric assumes normal distribution
  - Empirical uses actual historical data
  - Difference suggests non-normal distribution
  - Higher empirical 99% VaR indicates fat tails
- Practical Example (for \$1,000,000 portfolio):
  - 95% Parametric: Max loss of \$17,520
  - 95% Empirical: Max loss of \$15,612
  - 99% Parametric: Max loss of \$24,894
  - 99% Empirical: Max loss of \$30,968
- Recommendation:
  - Rely more on empirical values
  - Better captures actual risk characteristics
  - More conservative at higher confidence levels

## COMPARISON OF RISK MEASURES

- Key differences between VaR and CVaR:
  - VaR: Maximum loss at confidence level
  - CVaR: Average loss beyond VaR

Property	VaR	CVaR
Coherence	No	Yes
Tail Sensitivity	Limited	High
Ease of Calculation	Higher	Lower
Regulatory Use	Basel II	Basel III

## **BACKTESTING - OVERVIEW**

- Purpose of Backtesting:
  - Validate risk model accuracy
  - Meet regulatory requirements
  - Improve risk estimation
- Key Concepts:
  - VaR violation: When actual loss exceeds VaR estimate
  - Expected violation frequency:  $(1 \alpha)$  for  $VaR_{\alpha}$
  - Example: For 95% VaR, expect violations in 5% of cases
- Backtesting Period:
  - Typically 250-500 trading days
  - Basel requirement: Minimum 250 days
  - Need sufficient data for statistical significance

## **BACKTESTING RISK MEASURES**

Verify accuracy of risk measures:

```
% Count VaR violations
violations = sum(returns < -VaR_95);
violation_rate = violations/length(returns);
% Kupiec test
[h,p] = kupiectest(violations, length(returns), 0.05);</pre>
```

#### Testing approaches:

- Violation ratio analysis
  - Compares actual violations to expected frequency
  - For 95% VaR, expect violations 5% of the time
  - ullet Ratio >1 indicates model underestimates risk
- Independence tests
  - Examines clustering of violations
  - Uses Christoffersen's test for independence
  - Clustering suggests model weakness in stress periods
- Dynamic backtesting
  - Uses rolling windows (typically 250 days)
  - Updates risk estimates with new data
  - Better captures changing market conditions

## BACKTESTING - IMPLEMENTATION

#### Basic VaR Violation Test:

```
% Count VaR violations
violations = sum(returns < -VaR_95);
violation_rate = violations/length(returns);

% Expected rate for 95% VaR is 0.05
excess = (violation_rate - 0.05)/0.05;
fprintf('Violation excess: %.2f%%\n', excess*100);</pre>
```

#### Rolling Window Analysis:

```
window = 252; % One trading year
for t = window:length(returns)
    % Estimate VaR using rolling window
    rolling_var = std(returns(t-window+1:t));
    rolling_VaR = norminv(0.05)*rolling_var;

% Check for violation
    violations(t) = returns(t) < -rolling_VaR;
end</pre>
```

## STATISTICAL TESTS FOR BACKTESTING

- Kupiec Test (Unconditional Coverage):
  - Tests if violation frequency matches expected rate
  - Null hypothesis: Observed rate = Expected rate
  - Uses likelihood ratio test
- Christoffersen Test (Conditional Coverage):
  - Tests independence of violations
  - Checks for violation clustering
  - Combines tests for frequency and independence
- Dynamic Quantile Test:
  - Tests if violations are predictable
  - Uses regression-based approach
  - More powerful than basic tests

## ADVANCED BACKTESTING TECHNIQUES

- Purpose of Multipliers:
  - Determines capital requirements for market risk
  - Penalizes banks for inaccurate VaR models
  - Formula: Capital = VaR x Multiplier
- Traffic Light Approach (Basel):

Zone	Violations	Multiplier
Green	0-4	3.00
Yellow	5-9	3.40-3.85
Red	10+	4.00

Duration-Based Tests:

```
% Time between violations
durations = diff(find(violations));
[h,p] = duration_test(durations, alpha);
```

- Multiple VaR Levels:
  - Test at different confidence levels
  - Compare 95%, 99%, 99.9% VaR
  - Check consistency across levels

## KUPIEC TEST (UNCONDITIONAL COVERAGE)

- Purpose:
  - Tests if the observed violation frequency equals expected rate
  - Known as Proportion of Failures (POF) test
  - Fundamental VaR validation tool
- Test Statistics:
  - Let N = number of violations
  - Let T = total number of observations
  - Let p =expected violation rate (e.g., 0.05 for 95% VaR)
  - Let  $\hat{p} = N/T = \text{observed violation rate}$
- Likelihood Ratio Test:

$$LR_{POF} = -2 \ln \left[ \frac{(1-p)^{T-N} p^{N}}{(1-\hat{p})^{T-N} \hat{p}^{N}} \right]$$
$$\sim \chi^{2}(1)$$

## INTERPRETING KUPIEC TEST RESULTS

- Null Hypothesis:
  - $H_0$ : The model's violation rate equals the expected rate
  - H<sub>1</sub>: The model's violation rate differs from expected
- Decision Rules:
  - Reject  $H_0$  if  $LR_{POF} > \chi^2_{1,\alpha}$
  - Typical significance level  $\alpha=0.05$
  - Critical value  $\chi^2_{1.0.05} = 3.841$
- Limitations:
  - Only tests violation frequency
  - Ignores clustering of violations
  - Low power for small samples
  - Should be combined with other tests

## **BACKTESTING RESULTS**

Detailed analysis of VaR model performance:

Metric	95% VaR	99% VaR
Actual Violations	142	69
Expected Violations	187	37
Violation Rate	3.79%	1.84%
Kupiec Test p-value	0.0004	0.0000
Model Rejected	Yes	Yes

#### Distribution characteristics:

• Mean Return: 0.033%

Standard Deviation: 1.078%

Skewness: -0.783Kurtosis: 12.630

#### Key findings:

- 95% VaR overestimates risk (conservative)
- 99% VaR underestimates risk (inadequate)
- Heavy-tailed return distribution
- Both models rejected by Kupiec test