Ministry of Education

Saskatchewan Curriculum

Mathematics

5

2008

Mathematics 5
ISBN 978-1-9266310-01-1
1. Study and teaching (Elementary) - Saskatchewan - Curricula. 2. Competency-based education - Saskatchewan.
Saskatchewan. Ministry of Education. Curriculum and E-Learning. Science and Technology Unit.

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A special thank you is extended to the Elders who reviewed and provided advice to strengthen the curriculum outcomes:

- Jonas Bird
- Albert Scott
- Darlene Spiedel
- Allan Adams.

In addition, the Ministry of Education acknowledges the guidance of:

- program team members
- focus groups of teachers
- other educators and reviewers.

Introduction

As a required area of study, Mathematics is to be allocated 210 minutes per week for the entire school year at Grade 5. It is important that students receive the full amount of time allocated to their mathematical learning and that the learning be focused upon students attaining the understanding and skills as defined by the outcomes and indicators stated in this curriculum.

The outcomes in Grade 5 Mathematics build upon students' prior learnings and continue to develop their number sense, spatial sense, logical thinking, and understanding of mathematics as a human endeavour. These continuing learnings prepare students to be confident, flexible, and capable with their mathematical knowledge in new contexts.

Outcomes describe the knowledge, skills, and understandings that students' are expected to attain by the end of a particular grade level.

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. These indicators are a representative list of the kinds of things a student needs to know and/or be able to do in order to achieve the learnings intended by the outcome. New and combined indicators, which remain within the breadth and depth of the outcome, can and should be created by teachers to meet the needs and circumstances of their students and communities.

Indicators are a representative list of the types of things a student should know or be able to do if they have attained the outcome.

This curriculum's outcomes and indicators have been designed to address current research in mathematics education as well as the needs of Saskatchewan students. The Grade 5 Mathematics outcomes have been influenced by the renewal of the Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K-9 Mathematics* outcomes (2006). Changes throughout all of the grades have been made for a number of reasons including:

- decreasing content in each grade to allow for more depth of understanding
- rearranging concepts to allow for greater depth of learning in one year and to align related mathematical concepts
- increasing the focus on numeracy (i.e., understanding numbers and their relationship to each other) beginning in Kindergarten
- introducing algebraic thinking earlier.

Also included in this curriculum is information regarding how Grade 5 Mathematics connects to the K-12 goals for mathematics. These goals define the purpose of mathematics education for Saskatchewan students.

All students need an education in mathematics that will prepare them for a future of great and continual change.
(NCTM, 2000, p, 8)

In addition, teachers will find discussions of the critical characteristics of mathematics education, assessment and evaluation of student learning in mathematics, inquiry in mathematics, questioning in mathematics, and connections between Grade 5 Mathematics and other Grade 5 areas of study within this curriculum.

Finally, the Glossary provides explanations of some of the mathematical terminology you will find in this curriculum.

Core Curriculum

Core Curriculum is intended to provide all Saskatchewan students with an education that will serve them well regardless of their choices after leaving school. Through its various components and initiatives, Core Curriculum supports the achievement of the Goals of Education for Saskatchewan. For current information regarding Core Curriculum, please refer to *Core Curriculum: Principles, Time Allocations, and Credit Policy* (August 2007) on the Ministry of Education website.

Broad Areas of Learning

There are three Broad Areas of Learning that reflect Saskatchewan's Goals of Education. K-12 Mathematics contributes to the Goals of Education through helping students achieve knowledge, skills, and attitudes related to these Broad Areas of Learning.

Building Lifelong Learners

Related to the following Goals of Education:
- *Basic Skills*
- *Lifelong Learning*
- *Self Concept Development*
- *Positive Lifestyle*

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
(NCTM, 2000, p. 20)

Students who are engaged in constructing and applying mathematical knowledge naturally build a positive disposition towards learning. Throughout their study of mathematics, students should be learning the skills (including reasoning strategies) and developing the attitudes that will enable the successful use of mathematics in daily life. Moreover, students should be developing understandings of mathematics that will support their learning of new mathematical concepts and applications that may be encountered within both career and personal interest choices. Students who successfully complete their study of K-12 Mathematics should feel confident about their mathematical abilities and have developed the knowledge, understandings, and abilities necessary to make future use and/or studies of mathematics meaningful and attainable.

In order for mathematics to contribute to this Broad Area of Learning, students must actively learn the mathematical content in the outcomes through using and developing logical thinking, number sense, spatial sense, and understanding of mathematics as a human endeavour (the four goals of K-12 Mathematics). It is crucial that the students discover the mathematics outlined in the curriculum rather than the teacher covering it.

Building a Sense of Self and Community

To learn mathematics with deep understanding, students not only need to interact with the mathematical content, but with each other as well. Mathematics needs to be taught in a dynamic environment where students work together to share and evaluate strategies and understandings. Students who are involved in a supportive mathematics learning environment that is rich in dialogue are exposed to a wide variety of perspectives and strategies from which to construct a sense of the mathematical content. In such an environment, students also learn and come to value how they, as individuals and as members of a group or community, can contribute to understanding and social well-being through a sense of accomplishment, confidence, and relevance. When encouraged to present ideas representing different perspectives and ways of knowing, students in mathematics classrooms develop a deeper understanding of the mathematics. At the same time, students also learn to respect and value the contributions of others.

Mathematics also provides many opportunities for students to enter into communities beyond the classroom by engaging with people in the neighbourhood or around the world. By working towards developing a deeper understanding of mathematics and its role in the world, students will develop their personal and social identity, and learn healthy and positive ways of interacting and working together with others.

Related to the following Goals of Education:

- *Understanding & Relating to Others*
- *Self Concept Development*
- *Positive Lifestyle*
- *Spiritual Development*

Many of the topics and problems in a mathematics classroom can be initiated by the children themselves. In a classroom focused on working mathematically, teachers and children work together as a community of learners; they explore ideas together and share what they find. It is very different to the traditional method of mathematics teaching, which begins with a demonstration by a teacher and continues with children practicing what has been demonstrated.

(Skinner, 1999, p. 7)

Building Engaged Citizens

Mathematics brings a unique perspective and way of knowing to the analysis of social impact and interdependence. Doing mathematics requires students to "leave their emotions at the door" and to engage in different situations for the purpose of understanding what is really happening and what can be done. Mathematical analysis of topics that interest students such as trends in global warming, homelessness, technological health issues (oil spills, hearing loss, carpal tunnel syndrome, diabetes), and discrimination can be used to engage the students in interacting and contributing positively to their classroom, school, community, and world. With the understandings that students can derive through mathematical analysis, they become better informed and have a greater respect for, and understanding of, differing opinions and possible options.

Related to the following Goals of Education:

- *Understanding & Relating to Others*
- *Positive Lifestyle*
- *Career and Consumer Decisions*
- *Membership in Society*
- *Growing with Change*

With these understandings, students can make better informed and more personalized decisions regarding roles within, and contributions to, the various communities in which students are members.

The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater.

(NCTM, 2000, p. 4)

Cross-curricular Competencies

The Cross-curricular Competencies are four interrelated areas containing understandings, values, skills, and processes which are considered important for learning in all areas of study. These competencies reflect the Common Essential Learnings and are intended to be addressed in each area of study at each grade level.

Developing Thinking

K-12 Goals

- *thinking and learning contextually*
- *thinking and learning creatively*
- *thinking and learning critically.*

It is important that, within their study of mathematics, students are engaged in personal construction and understanding of mathematical knowledge. This most effectively occurs through student engagement in inquiry and problem solving when students are challenged to think critically and creatively. Moreover, students need to experience mathematics in a variety of contexts – both real world applications and mathematical contexts – in which students are asked to consider questions such as "what would happen if …", "could we find …", and "what does this tell us?" Students need to be engaged in the social construction of mathematics to develop an understanding and appreciation of mathematics as a tool which can be used to consider different perspectives, connections, and relationships. Mathematics is a subject that depends upon the effective incorporation of independent work and reflection with interactive contemplation, discussion, and resolution.

Developing Identity and Interdependence

K-12 Goals

- *understanding, valuing, and caring for oneself*
- *understanding, valuing, and respecting human diversity and human rights and responsibilities*
- *understanding and valuing social and environmental interdependence and sustainability.*

Given an appropriate learning environment in mathematics, students can develop both their self-confidence and self-worth. An interactive mathematics classroom in which the ideas, strategies, and abilities of individual students are valued supports the development of personal and mathematical confidence. It can also help students take an active role in defining and maintaining the classroom environment and accept responsibility for the consequences of their choices, decisions, and actions. A positive learning environment combined with strong pedagogical choices that engage students in learning serves to support students in behaving respectfully towards themselves and others.

Developing Literacies

Through their mathematics learning experiences, students should be engaged in developing their understandings of the language of mathematics and their ability to use mathematics as a language and representation system. Students should be regularly engaged in exploring a variety of representations for mathematical concepts and should be expected to communicate in a variety of ways about the mathematics being learned. An important part of learning mathematical language is to make sense of mathematics, communicate one's own understandings, and develop strategies to explore what and how others know about mathematics. The study of mathematics should encourage the appropriate use of technology. Moreover, students should be aware of and able to make appropriate use of technology in mathematics and mathematics learning. It is important to encourage students to use a variety of forms of representation (concrete manipulatives, physical movement, oral, written, visual, and symbolic) when exploring mathematical ideas, solving problems, and communicating understandings. All too often, it is assumed that symbolic representation is the only way to communicate mathematically. The more flexible students are in using a variety of representations to explain and work with the mathematics being learned, the deeper students' understanding becomes.

K-12 Goals
- *constructing knowledge related to various literacies*
- *exploring and interpreting the world through various literacies*
- *expressing understanding and communicating meaning using various literacies.*

Ideas are the currency of the classroom. Ideas, expressed by any participant, warrant respect and response. Ideas deserve to be appreciated and examined. Examining an idea thoughtfully is the surest sign of respect, both for the idea and its author.
(Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, Human, 1997, p. 9)

Students gain insights into their thinking when they present their methods for solving problems, when they justify their reasoning to a classmate or teacher, or when they formulate a question about something that is puzzling to them. Communication can support students' learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols. Misconceptions can be identified and addressed. A side benefit is that it reminds students that they share responsibility with the teacher for the learning that occurs in the lesson.

(NCTM, 2000, pp. 60 – 61)

Developing Social Responsibility

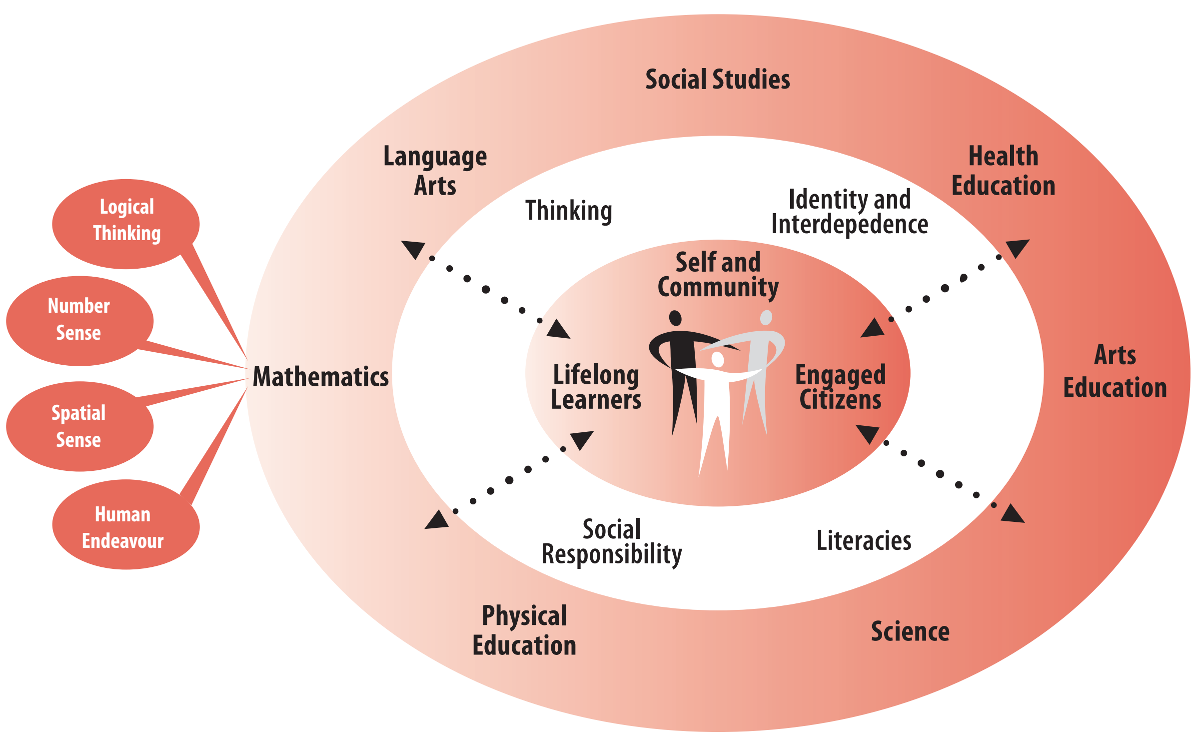
As students progress in their mathematical learning, they need to experience opportunities to share and consider ideas, and resolve conflicts between themselves and others. This requires that the learning environment be co-constructed by the teacher and students to support respectful, independent,

K-12 Goals
- *using moral reasoning*
- *engaging in communitarian thinking and dialogue*
- *contributing to the well-being of self, others, and the natural world.*

and interdependent behaviours. Every student should feel empowered to help others in developing their understanding, while finding respectful ways to seek help from others. By encouraging students to explore mathematics in social contexts, students can be engaged in understanding the situation, concern, or issue and then in planning for responsible reactions or responses. Mathematics is a subject dependent upon social interaction and, as a result, social construction of ideas. Through the study of mathematics, students learn to become reflective and positively contributing members of their communities. Mathematics also allows for different perspectives and approaches to be considered, assessed for situational validity, and strengthened.

Aim and Goals of K-12 Mathematics

The aim of the K-12 mathematics program is to prepare individuals who value mathematics and appreciate its role in society. The K-12 Mathematics curricula are designed to prepare students to cope confidently and competently with everyday situations that demand the use of mathematical concepts including interpreting quantitative information, estimating, performing calculations mentally, measuring, understanding spatial relationships, and problem solving. The Mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.



Defined below are four goals for K-12 Mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are directly related to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes must therefore also promote student achievement with respect to the goals.

Logical Thinking

Through their learning of K-12 Mathematics, students should **develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems.**

This goal encompasses processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- inductive and deductive thinking
- proportional reasoning
- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proving
- exploring, identifying, and describing relationships
- modeling and representing (including concrete, oral, physical, pictorial, and symbolical representations)
- conjecturing and asking "what if" (mathematical play).

A … feature of the social culture of [mathematics] classrooms is the recognition that the authority of reasonability and correctness lies in the logic and structure of the subject, rather than in the social status of the participants. The persuasiveness of an explanation, or the correctness of a solution depends on the mathematical sense it makes, not on the popularity of the presenter.
(Hiebert et al., 1997, p. 10)

In order to develop logical thinking, students need to be actively involved in constructing their mathematical knowledge using the above strategies and processes. Inherent in each of these strategies and processes is student communication and the use of and connections between multiple representations.

Number Sense

Through their learning of K-12 Mathematics, students should **develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers and apply this understanding to new situations and problems.**

Students also develop understanding of place value through the strategies they invent to compute.
(NCTM, 2000, p. 82)

Foundational to students developing number sense is having ongoing experiences with:

- decomposing and composing of numbers
- relating different operations to each other
- modeling and representing numbers and operations

(including concrete, oral, physical, pictorial, and symbolical representations)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- developing and understanding algebraic representations and manipulations as an extension of numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Number sense goes well beyond being able to carry out calculations. In fact, in order for students to become flexible and confident in their calculation abilities, and to transfer those abilities to more abstract contexts, students must first develop a strong understanding of numbers in general. A deep understanding of the meaning, roles, comparison, and relationship between numbers is critical to the development of students' number sense and their computational fluency.

Spatial Sense

Explorations of and relationships between different forms of measurement, both 2-D and 3-D, are emphasized in Grade 5. How can you be sure that students are able to transfer these understandings to new situations?

Through their learning of K-12 Mathematics, students should **develop an understanding of 2-D shapes and 3-D objects, and the relationships between geometrical shapes and objects and numbers, and apply this understanding to new situations and problems.**

Development of a strong spatial sense requires students to have ongoing experiences with:
- construction and deconstruction of 2-D shapes and 3-D objects
- investigations and generalizations about relationships between 2-D shapes and 3-D objects
- explorations and abstractions related to how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- explorations and generalizations about the movement of 2-D shapes and 3-D objects
- explorations and generalizations regarding the dimensions of 2-D shapes and 3-D objects
- explorations, generalizations, and abstractions about different forms of measurement and their meaning.

As students sort, build, draw, model, trace, measure, and construct, their capacity to visualize geometric relationships will develop.
(NCTM, 2000, p. 165)

Being able to communicate about 2-D shapes and 3-D objects is foundational to students' geometrical and measurement understandings and abilities. Hands-on exploration of 3-D objects and the creation of conjectures based upon patterns that are discovered and tested should drive the students' development of spatial sense, with formulas and definitions resulting from the students' mathematical learnings.

Mathematics as a Human Endeavour

Through their learning of K-12 Mathematics, students should **develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs.**

What types of instructional strategies support student attainment of the K-12 Mathematics goals?

How can student attainment of these goals be assessed and the results be reported?

Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:

- encourage and value varying perspectives and approaches to mathematics
- recognize and value one's evolving strengths and knowledge in learning and doing mathematics
- recognize and value the strengths and knowledge of others in doing mathematics
- value and honour reflection and sharing in the construction of mathematical understanding
- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematically and personally)
- build self-confidence related to mathematical insights and abilities
- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics

Students should be encouraged to challenge the boundaries of their experiences, and to view mathematics as a set of tools and ways of thinking that every society develops to meet their particular needs. This means that mathematics is a dynamic discipline in which logical thinking, number sense, and spatial sense form the backbone of all developments and those developments are determined by the contexts and needs of the time, place, and people.

Meaning does not reside in tools; it is constructed by students as they use tools.
(Hiebert et al., 1997, p. 10)

The content found within the grade level outcomes for the K-12 Mathematics programs, and its applications, is first and foremost the vehicle through which students can achieve the four goals of K-12 Mathematics. Attainment of these four goals will result in students with the mathematical confidence and tools necessary to succeed in future mathematical endeavours.

Teaching Mathematics

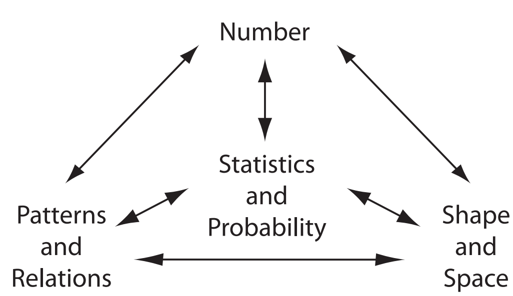
At the National Council of Teachers of Mathematics (NCTM) Canadian Regional Conference in Halifax (2000), Marilyn Burns said in her keynote address, "When it comes to mathematics curricula there is very little to cover, but an awful lot to uncover [discover]." This statement captures the essence of the ongoing call for change in the teaching of mathematics. Mathematics is a dynamic and logic-based language that students need to explore and make sense of for themselves. For many teachers, parents, and former students this is a marked change from the way mathematics was taught to them. Research and experience indicate there is a complex, interrelated set of characteristics that teachers need to be aware of in order to provide an effective mathematics program.

Critical Characteristics of Mathematics Education

The following sections in this curriculum highlight some of the different facets for teachers to consider in the process of changing from covering to supporting students in discovering mathematical concepts. These facets include the organization of the outcomes into strands, seven mathematical processes, the difference between covering and discovering mathematics, the development of mathematical terminology, the continuum of understanding from the concrete to the abstract, modelling and making connections, the role of homework in mathematics, and the importance of ongoing feedback and reflection.

Strands

The content of K-12 Mathematics can be organized in a variety of ways. In this curriculum, the outcomes and indicators are grouped according to four strands: **Number, Patterns and Relations, Shape and Space, and Statistics and Probability.**



Although this organization implies a relatedness among the outcomes identified in each of the strands, it should be noted the mathematical concepts are interrelated across the strands as well as within strands. Teachers are encouraged to design learning activities that integrate outcomes both within a strand and between strands so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate strands.

Mathematical Processes

This Grade 5 Mathematics curriculum recognizes seven processes inherent in the teaching, learning, and doing of mathematics. These processes focus on: communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing along with using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding.

The outcomes in K-12 Mathematics should be addressed through the appropriate mathematical processes as indicated by the bracketed letters following each outcome. Teachers should consider carefully in their planning those processes indicated as being important to supporting student achievement of the various outcomes.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas using both personal and mathematical language and symbols. These opportunities allow students to create links between their own language, ideas, and prior knowledge, the formal language and symbols of mathematics, and new learnings.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology, but only when they have had sufficient experience to develop an understanding for that terminology.

Concrete, pictorial, symbolic, physical, verbal, written, and mental representations of mathematical ideas should be encouraged and used to help students make connections and strengthen their understandings.

Communication works together with reflection to produce new relationships and connections. Students who reflect on what they do and communicate with others about it are in the best position to build useful connections in mathematics.
(Hiebert et al., 1997, p. 6)

> *Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding …. Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching.*
> *(Caine & Caine, 1991, p.5)*

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to other real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

The brain is constantly looking for and making connections. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and prior knowledge, and increase student willingness to participate and be actively engaged.

Mental Mathematics and Estimation [ME]

> *Mental mathematics and estimation strategies need to be developed, shared, and critiqued by students in Grade 5 mathematics to help them attain efficiency in computation.*

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally and reasoning about the relative size of quantities without the use of external memory aids. Mental mathematics enables students to determine answers and propose strategies without paper and pencil. It improves computational fluency and problem solving by developing efficiency, accuracy, and flexibility.

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

> *Mathematical problem-solving often involves moving backwards and forwards between numerical/algebraic representations and pictorial representations of the problem.*
> *(Haylock & Cockburn, 2003, p. 203)*

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you …?”, “Can you …?”, or “What if …?”, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not problem solving but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students actively look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confidence, reasoning, and mathematical creativity.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. High-order inquiry challenges students to think and develop a sense of wonder about mathematics.

What types of conjectures and reasoning for those conjectures might Grade 5 students be able to make, and on what prior knowledge or experiences might these conjectures and reasonings be based?

Mathematical experiences in and out of the classroom should provide opportunities for students to engage in inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Posing conjectures and trying to justify them is an expected part of students' mathematical activity.
(NCTM, 2000, p. 191)

Visualization [V]

The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number sense, spatial sense, and logical thinking. Number visualization occurs when students create mental representations of numbers and visual ways to compare those numbers.

[Visualization] involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.
(Armstrong, 1993, p.10)

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes including aspects such as dimensions and measurements.

Visualization is also important in the students' development of abstraction and abstract thinking and reasoning. Visualization provides a connection between the concrete, physical, and pictorial to the abstract symbolic. Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations as well as the use of communication to develop connections between different contexts, content, and representations.

Technology [T]

Technology tools contribute to the learning of a wide range of mathematical outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators, computers and other forms of technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense
- develop spatial sense
- develop and test conjectures.

Technology should not be used as a replacement for basic understandings and intuition.
(NCTM, 2000, p. 25)

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. It is important for students to understand and appreciate the appropriate use of technology in a mathematics classroom. It is also important that students learn to distinguish between when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding, but should be used to enhance it.

Discovering versus Covering

Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content. Knowing what needs to be covered and what can be discovered is crucial in planning for mathematical instruction and learning. The content that needs to be covered (what the teacher needs to explicitly tell the students) is the social conventions or customs of mathematics. This content includes things such as what the symbol for an operation looks like, mathematical terminology, and conventions regarding recording of symbols.

The content that can and should be discovered by students is the content that can be constructed by students based on their prior mathematical knowledge. This content includes things such as procedures and strategies, rules, and problem solving. Any learning in mathematics that is a result of the logical structure of mathematics can and should be constructed by students.

For example, in Grade 5 the students encounter increasing patterns in outcome P5.2:

> Write, solve, and verify solutions of single-variable, one-step equations with whole number coefficients and whole number solutions. [C, CN, PS, R]

In this outcome, there are many terms that are social conventions of mathematics such as "solve", "verify", and "single-variable, one-step equations". These are terms that the teacher must tell the students about as the students are exploring what the equality shown in this type of situation conveys and how to work within it. Strategies for solving such equations, however, should emerge from the students' analysis of the equations and be based on their prior understandings of quantity, operations, and equality. For students to carry out such an analysis, they need to be engaged in conjecturing, communicating, verifying, and reasoning about their ideas and emerging strategies. This type of learning requires students to work concretely, physically, orally, pictorially, and possibly in writing. It also requires that students share their ideas with their classmates and reflect upon how the ideas and understandings of others relate to, inform, and clarify what students individually understand. In this type of learning, the teacher does not tell the students how to do the mathematics but, rather, invites the students to explore and develop an understanding of the logical structures inherent in the mathematics in increasing patterns. Thus, the teacher's role is to create inviting and rich inquiring tasks and to use questioning to effectively probe and further students' learning.

"The plus sign, for example, and the symbols for subtraction, multiplication, and division are all arbitrary convention. … Learning most of mathematics, however, relies on understanding its logical structures. The source of logical understanding is internal, requiring a child to process information, make sense of it, and figure out how to apply it."
(Burns & Sibley, 2000, p. 19)

Development of Mathematical Terminology

Part of learning mathematics is learning how to speak mathematically. Teaching students mathematical terminology when they are learning for deep understanding requires that the students connect the new terminology with their developing mathematical understanding. As a result, it is important that students first linguistically engage with new mathematical concepts using words that students already know or that make sense to them.

Teachers should model appropriate conventional vocabulary.
(NCTM, 2000, p. 131)

Consider outcome SS5.5:

> Describe and provide examples of edges and faces of 3-D objects and sides of 2-D shapes that are:
> - parallel
> - intersecting
> - perpendicular
> - vertical
> - horizontal.
>
> [C, CN, R, T,V]

Students should already be familiar with the terms "edges", "faces", "sides", "3-D objects", and "2-D shapes" from previous grades; whereas the terminology, at least in a mathematical sense, of "parallel", "intersecting", "perpendicular", "vertical" and "horizontal" will likely be new to most of the students. Initially, students should be encouraged to use their own vocabulary (e.g., "running side by side", "never cross", "cross", "up and down", or "side-to-side") to describe the relationships between the edges, faces, and sides. Through dialogue, the students should be engaged in clarifying these statements by considering questions such as "If you were to connect the back, top, left corner of this cube to the front, bottom right corner with a line, would that line still be up and down?" or "How are the intersecting edges of the prism different from those of a pyramid?". As students become fluent in describing the relationships within the 3-D objects and 2-D shapes, the mathematical terminology can then be easily introduced as "in mathematics, two edges that meet in the way you are describing are said to be … intersecting, parallel, intersecting but not perpendicular…" From there, students can then formulate their own working definitions and examples of where and how to use this terminology. By having the students first construct their understanding, and then introducing the mathematical terminology, the new understandings become more easily transferred to new situations.

The Concrete to Abstract Continuum

How can your classroom environment be arranged so that students have access to tools and materials that are at differing levels of abstraction or concreteness?

It is important that, in learning mathematics, students be allowed to explore and develop understandings by moving along a concrete to abstract continuum. As understanding develops, this movement along the continuum is not necessarily linear. Students may at one point be working abstractly but when a new idea or context arises they need to return to a concrete starting point. Therefore, the teacher must be prepared to engage students at different points along the continuum.

In addition, what is concrete and what is abstract is not always obvious and can vary according to the thinking processes of the individual. For example, when considering a problem about the total number of pencils, some students might find it more concrete to use pictures of pencils as a means of representing the situation. Other students might find coins more concrete because they directly associate money with the purchasing or having of a pencil.

It is important for students to use representations that are meaningful to them.
(NCTM, 2000, p. 140)

As well, teachers need to be aware that different aspects of a task might involve different levels of concreteness or abstractness. Consider the following problem involving multiplication:

> If each student in the class brought 82 sheets of paper at the start of the year, how many sheets of paper would have been brought by the entire class?

Depending upon how the problem is expected to be solved (or if there is any specific expectation), this problem can be approached abstractly (using symbolic number statements), concretely (e.g., using manipulatives, pictures, role play), or both.

Models and Connections

New mathematics is continuously developed by creating new models as well as combining and expanding existing models. Although the final products of mathematics are most frequently represented by symbolic models, their meaning and purpose is often found in the concrete, physical, pictorial, and oral models and the connections between them.

To develop a deep and meaningful understanding of mathematical concepts, students need to represent their ideas and strategies using a variety of models (concrete, physical, pictorial, oral, and symbolic). In addition, students need to make connections between the different representations. These connections are made by having the students try to move from one type of representation to another (how could you write what you've done here using mathematical symbols?) or by having students compare their representations with others around the class.

A major responsibility of teachers is to create a learning environment in which students' use of multiple representations is encouraged.
(NCTM, 2000, pp. 139)

In making these connections, students should also be asked to reflect upon the mathematical ideas and concepts that students already know are being used in their new models (e.g., I know that addition means to put things together into a group, so I'm going to move the two sets of blocks together to determine the sum).

Making connections also involves looking for patterns. For example, in outcome N5.1:

> Represent, compare, and describe whole numbers to 1 000 000 within the contexts of place value and the base ten system, and quantity. [C, CN, R, T, V]

It is important that students explore how to represent, or know what representations might look like, when they extend understanding from whole numbers to 10 000 (Grade 4) to whole numbers to 1 000 000 (Grade 5). Because of the size of these quantities, students may not be able to physically create representations of such large numbers, but they should be able to describe and visualize what possible representations might be. For example, when considering modeling whole numbers using base-ten blocks, a student knows the pattern that ones are represented by cubes, tens by rods, hundreds by flats, and thousands by larger cubes. Students might then generalize that ten thousand could be represented by a rod of ten "thousand cubes", one hundred thousand by a flat of ten "ten thousand rods", and one million by a cube of ten "hundred thousand flats".

Characteristics of Good Math Homework

- *It is accessible to children at many levels.*
- *It is interesting both to children and to any adults who may be helping.*
- *It is designed to provoke deep thinking.*
- *It is able to use concepts and mechanics as means to an end rather than as ends in themselves.*
- *It has problem solving, communication, number sense, and data collection at its core.*
- *It can be recorded in many ways.*
- *It is open to a variety of ways of thinking about the problem although there may be one right answer.*
- *It touches upon multiple strands of mathematics, not just number.*
- *It is part of a variety of approaches to and types of math homework offered to children throughout the year.*

(Raphel, 2000, p. 75)

Role of Homework

The role of homework in teaching for deep understanding is very important and also quite different from homework that is traditionally given to students. Students should be given unique problems and tasks that help students to consolidate new learnings with prior knowledge, explore possible solutions, and apply learnings to new situations. Although drill and practice does serve a purpose in learning for deep understanding, the amount and timing of the drill will vary among different learners. In addition, when used as homework, drill and practice frequently serves to cause frustration, misconceptions, and boredom to arise in students.

As an example of the type or style of homework that can be used to help students develop deep understanding of Grade 5 Mathematics, consider outcome SS5.6:

> Identify and sort quadrilaterals, including:
> - rectangles
> - squares
> - trapezoids
> - parallelograms
> - rhombuses
>
> according to their attributes. [C, R, V]

As a homework task, students might be asked to draw as many different types of four-sided 2-D shapes as possible and to describe what makes the shapes different. The students should be encouraged to consider attributes other than size, which is the most common form of comparison that students have done in prior learning about shape and space, and to share their drawings and descriptions. Together, the students can begin to sort their drawings based upon the descriptions given. The teacher can then introduce new drawings of 2-D shapes with other considerations, such as orientation.

This type of homework task is one that a student can easily engage in, and that parents would feel confident in providing help with, while also preparing the students for new content to be learned in class by having them access related prior knowledge and past experiences.

Ongoing Feedback and Reflection

Ongoing feedback and reflection, both for students and teachers, are crucial in classrooms when learning for deep understanding. Deep understanding requires that both the teacher and students need to be aware of their own thinking as well as the thinking of others.

Feedback from peers and the teacher helps students rethink and solidify their understandings. Feedback from students to the teacher gives much needed information in the teacher's planning for further and future learnings.

Self-reflection, both shared and private, is foundational to students developing a deep understanding of mathematics. Through reflection tasks, students and teachers come to know what it is that students do and do not know. It is through such reflections that not only can a teacher make better informed instructional decisions, but also that a student can set personal goals and make plans to reach those goals

.

Feedback can take many different forms. Instead of saying, "This is what you did wrong," or "This is what you need to do," we can ask questions: "What do you think you need to do? What other strategy choices could you make? Have you thought about…?"

(Stiff, 2001, p. 70)

Not all feedback has to come from outside – it can come from within. When adults assume that they must be the ones who tell students whether their work is good enough, they leave them handicapped, not only in testing situations (such as standardized tests) in which they must perform without guidance, but in life itself.

(NCTM, 2000, p. 72)

Teaching for Deep Understanding

A simple model for talking about understanding is that to understand something is to connect it with previous learning or other experiences… A mathematical concept can be thought of as a network of connections between symbols, language, concrete experiences, and pictures.
(Haylock & Cockburn, 2003, p. 18)

For deep understanding, it is vital that students learn by constructing knowledge, with very few ideas being relayed directly by the teacher. As an example, the symbol for the unit of measure millimetre (mm) is something which the teacher must introduce and ensure that students know. The process of measuring using mm, and relating mm to m and cm should be discovered through the students' investigation of patterns, relationships, abstractions, and generalizations. It is important for teachers to reflect upon outcomes to identify what students need to know, understand, and be able to do. Opportunities must be provided for students to explain, apply, and transfer understanding to new situations. This reflection supports professional decision making and planning effective strategies to promote students' deeper understanding of mathematical ideas.

What types of things might you hear or see in a Grade 5 classroom that would indicate to you that students were learning for deep understanding?

It is important that a mathematics learning environment include effective interplay of:

- reflection
- exploration of patterns and relationships
- sharing of ideas and problems
- consideration of different perspectives
- decision making
- generalizing
- verifying and proving
- modeling and representing.

Mathematics is learned when students are engaged in strategic play with mathematical concepts and differing perspectives. When students learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong connections necessary for learning to be meaningful, easily accessible, and transferable. The learning environment must be respectful of individuals and groups, fostering discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the construction of meaning.

Inquiry

Inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience. The inquiry process focuses on the development of compelling questions, formulated by teachers and students, to motivate and guide inquiries into topics, problems, and issues related to curriculum content and outcomes.

Inquiry is more than a simple instructional method. It is a philosophical approach to teaching and learning, grounded in constructivist research and methods, which engages students in investigations that lead to disciplinary and transdisciplinary understanding.

Inquiry builds on students' inherent sense of curiosity and wonder, drawing on their diverse backgrounds, interests, and experiences. The process provides opportunities for students to become active participants in a collaborative search for meaning and understanding. Students who are engaged in inquiry:

- construct deep knowledge and deep understanding rather than passively receiving it
- are directly involved and engaged in the discovery of new knowledge
- encounter alternative perspectives and conflicting ideas that transform prior knowledge and experience into deep understandings
- transfer new knowledge and skills to new circumstances
- take ownership and responsibility for their ongoing learning and mastery of curriculum content and skills.

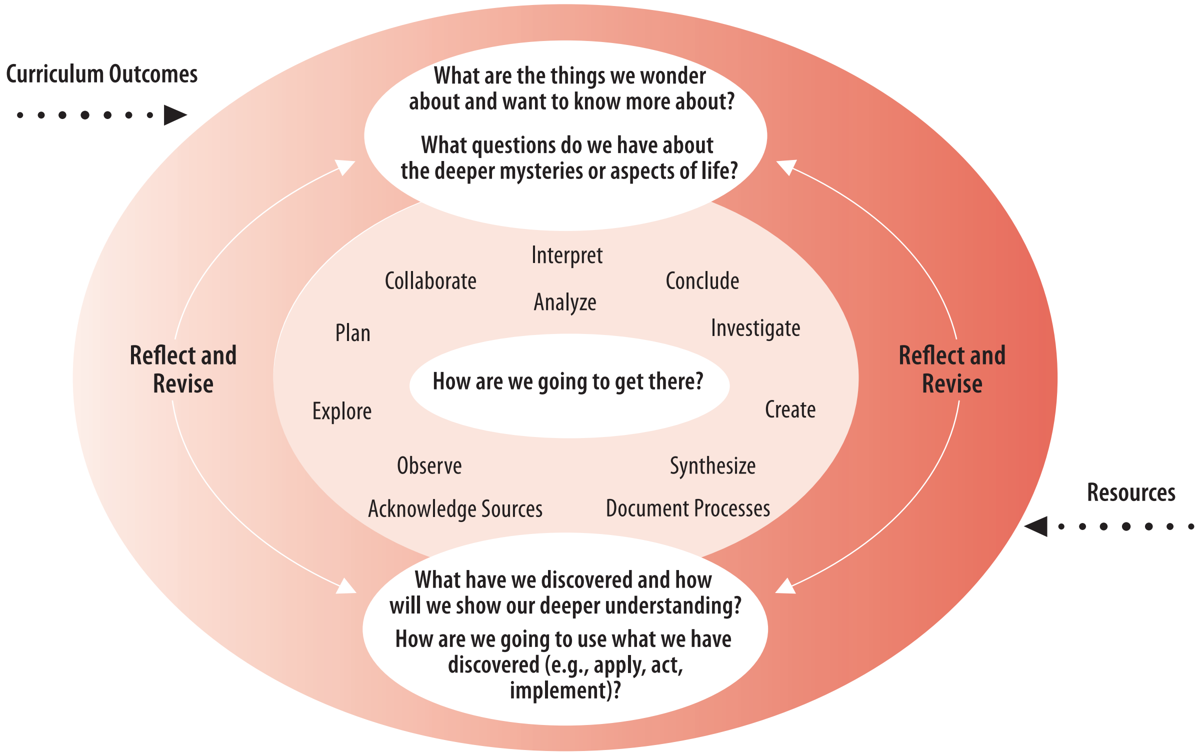
(Adapted from Kuhlthau & Todd, 2008, p. 1)

Inquiry learning is not a step-by-step process, but rather a cyclical process, with various phases of the process being revisited and rethought as a result of students' discoveries, insights, and co-construction of new knowledge. The following graphic shows various phases of this cyclical inquiry process.

Inquiry is a philosophical stance rather than a set of strategies, activities, or a particular teaching method. As such, inquiry promotes intentional and thoughtful learning for teachers and children.
(Mills & Donnelly, 2001, p. xviii)

Something is only a problem if you don't know how to get from where you are to where you want to be. Be sure that Grade 5 students are solving such problems.

Constructing Understanding Through Inquiry



Inquiry prompts and motivates students to investigate topics within meaningful contexts. The inquiry process is not linear or lock-step, but is flexible and recursive. Experienced inquirers will move back and forth through the cyclical process as new questions arise and as students become more comfortable with the process.

Well formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, conclude, document, reflect on learning, and develop new questions for further inquiry.

In Mathematics, inquiry encompasses problem solving. Problem solving includes processes to get from what is known to discover what is unknown. When teachers show students how to solve a problem and then assign additional problems that are similar, the students are not problem solving but practising. Both are necessary in mathematics, but one should not be confused with the other. If the path for getting to the end situation has already been determined, it is no longer problem solving. Students too must understand this difference.

Creating Questions for Inquiry in Mathematics

Teachers and students can begin their inquiry at one or more curriculum entry points; however, the process may evolve into transdisciplinary integrated learning opportunities, as reflective of the holistic nature of our lives and interdependent global environment. It is essential to develop questions that are evoked by students' interests and have potential for rich and deep learning. Compelling questions are used to initiate and guide the inquiry and give students direction for discovering deep understandings about a topic or issue under study.

The process of constructing inquiry questions can help students to grasp the important disciplinary or transdisciplinary ideas that are situated at the core of a particular curricular focus or context. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school.

Effective questions in Mathematics are the key to initiating and guiding students' investigations and critical thinking, problem solving, and reflection on their own learning. Questions such as:

- "Why would you want to know a probability for a situation?"
- "How can you use probability to solve a problem or make a decision?"
- "Why do we use probabilities when they don't give us 'the' answer?"
- "Why don't the results of a probability experiment always turn out the way you expected them to?"

are examples of questions that will move students' inquiry towards deeper understanding. Effective questioning is essential for teaching and student learning and should be an integral part of planning in mathematics. Questioning should also be used to encourage students to reflect on the inquiry process and the documentation and assessment of their own learning.

Questions should invite students to explore mathematical concepts within a variety of contexts and for a variety of purposes. When questioning students, teachers should choose questions that:

Questions may be one of the most powerful technologies invented by humans. Even though they require no batteries and need not be plugged into the wall, they are tools which help us make up our minds, solve problems, and make decisions. – Jamie McKenzie (Schuster & Canavan Anderson, 2005, p. 1)

Effective questions:

- *cause genuine and relevant inquiry into the important ideas and core content.*
- *provide for thoughtful, lively discussion, sustained inquiry, and new understanding as well as more questions.*
- *require students to consider alternatives, weigh evidence, support their ideas, and justify their answers.*
- *stimulate vital, ongoing rethinking of key ideas, assumptions, and prior lessons.*
- *spark meaningful connections with prior learning and personal experiences.*
- *naturally recur, creating opportunities for transfer to other situations and subjects.*

(Wiggins & McTighe, 2005, p. 110)

As teachers of mathematics, we want our students not only to understand what they think but also to be able to articulate how they arrived at those understandings.
(Schuster & Canavan Anderson, 2005, p. 1)

- *help students make sense of the mathematics.*
- *are open-ended, whether in answer or approach. There may be multiple answers or multiple approaches.*
- *empower students to unravel their misconceptions.*
- *not only require the application of facts and procedures but encourage students to make connections and generalizations.*
- *are accessible to all students in their language and offer an entry point for all students.*
- *lead students to wonder more about a topic and to perhaps construct new questions themselves as they investigate this newly found interest.*

(Schuster & Canavan Anderson, 2005, p. 3)

Reflection and Documentation of Inquiry

An important part of any inquiry process is student reflection on their learning and the documentation needed to assess the learning and make it visible. Student documentation of the inquiry process in mathematics may take the form of reflective journals, notes, drafts, models, and works of art, photographs, or video footage. This documentation should illustrate the students' strategies and thinking processes that led to new insights and conclusions. Inquiry-based documentation can be a source of rich assessment materials through which teachers can gain a more in-depth look into their students' mathematical understandings.

When we ask good questions in math class, we invite our students to think, to understand, and to share a mathematical journey with their classmates and teachers alike. Students are no longer passive receivers of information when asked questions that challenge their understandings and convictions about mathematics. They become active and engaged in the construction of their own mathematical understanding and knowledge.
(Schuster & Canavan Anderson, 2005, p. 1)

It is important that students are required and allowed to engage in the communication and representation of their progress within a mathematical inquiry. A wide variety of forms of communication and representation should be encouraged and, most importantly, have links made between them. In this way, student inquiry into mathematical concepts and contexts can develop and strengthen student understanding.

Outcomes and Indicators

Number Strand	
Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour	
Outcomes (What students are expected to know and be able to do.)	**Indicators** (Students who have achieved this outcome should be able to:)
N5.1 Represent, compare, and describe whole numbers to 1 000 000 within the contexts of place value and the base ten system, and quantity. [C, CN, R, T, V]	a. Write and say the numeral for a quantity using proper spacing without commas and without the word "and" (e.g., 934 567, nine hundred thirty-four thousand five hundred sixty-seven). b. Critique the way numbers have been said or numerals written in examples of whole numbers found in various types of media and personal conversations, and provide reasons for why certain errors in speech or writing might occur. c. Describe the patterns related to quantity and place value of adjacent digit positions moving from right to left within a whole number. d. Visualize and explain concrete or pictorial models for the place value positions of 100 000 and 1 000 000. e. Describe the meaning of quantities to 1 000 000 by relating them to self, family, or community and explain the contribution each successive numeral position makes to the actual quantity. f. Pose and solve problems that explore the quantity of whole numbers to 1 000 000 (e.g., a student might wonder: "How does the population of my community compare to those of surrounding communities?"). g. Provide examples of large numbers used in print or electronic media and explain the meaning of the numbers in the context used. h. Visualize a representation of a given numeral and explain how the representation is related to the numeral's expanded form. i. Express a given numeral in expanded notation (e.g., 45 321 = (4 x 10 000) + (5 x 1000) + (3 x 100) + (2 x 10) + (1 x 1) or 40 000 + 5000 + 300 + 20 + 1) and explain how the expanded notation shows the total quantity represented by the given numeral. j. Compare and order examples of whole numbers found in various types of media and print.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
***N5.2** Analyze models of, develop strategies for, and carry out multiplication of whole numbers. [C, CN,ME, PS, R, V]*	a. Describe mental mathematics strategies used to determine multiplication facts to 81 (e.g., skip counting from a known fact, doubling, halving, 9s patterns, repeated doubling, or repeated halving). b. Explain concretely, pictorially, or orally why multiplying by zero produces a product of zero. c. Recall multiplication facts to 81 including within problem solving and calculations of larger products. d. Generalize and apply strategies for multiplying two whole numbers when one factor is a multiple of 10, 100, or 1000. e. Generalize and apply halving and doubling strategies to determine a product involving at least one two-digit factor. f. Apply and explain the use of the distributive property to determine a product involving multiplying factors that are close to multiples of 10. g. Model multiplying two 2-digit factors using an array, base ten blocks, or an area model, record the process symbolically, and describe the connections between the models and the symbolic recording. h. Pose a problem which requires the multiplication of 2-digit numbers and explain the strategies used to multiply the numbers. i. Illustrate, concretely, pictorially, and symbolically, the distributive property using expanded notation and partial products (e.g., 36 x 42 = (30 +6) x (40+2) = 30 x 40 + 6 x 40 +30 x 2 + 6 x 2). j. Explain and justify strategies used when multiplying 2-digit numbers symbolically.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes

***N5.3** Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.*
[C, CN, PS, R]

Indicators

a. Identify situations in one's life, family, or community in which division might be used and explain the reasoning.
b. Model the division process as equal sharing or equal grouping using various models and record the resulting process symbolically.
c. Explain concretely, pictorially, or orally why division by zero is not possible or undefined (e.g., $8 \div 0$ is undefined or not possible to determine).
d. Generalize, relate, and apply concrete, pictorial, and symbolic strategies for dividing 3-digit whole numbers by 1-digit whole numbers.
e. Justify the choice of what to do with a remainder for a quotient depending upon the situation:
   - disregard the remainder (e.g., dividing 22 books among 4 students)
   - round up the quotient (e.g., the number of five passenger cars required to transport 13 people)
   - express remainders as fractions (e.g., five apples shared by two people)
   - express remainders as decimals (e.g., measurement and money).
f. Solve a division problem that is relevant to self, family, or community using personal strategies and record the process symbolically.
g. Recall the division facts to a dividend of 81 including in problem-solving situations.

Goals: Spatial Sense, Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes

***N5.4** Develop and apply personal strategies for estimation and computation including:*
- *front-end rounding*
- *compensation*
- *compatible numbers.*

[C, CN, ME, PS, R, V]

Indicators

a. Describe a situation relevant to self, family, or community for when estimation is used to:
   - make predictions
   - check reasonableness of an answer
   - determine approximate answers.
b. Develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results.
c. Critique the statement "an estimate is never good enough".

Outcomes	Indicators
N5.4 (continued)	d. Identify and describe situations relevant to self, family, or community when it is best to overestimate or when it is best to underestimate and explain the reasoning. e. Determine an approximate solution to a problem not requiring an exact answer and explain the strategies and reasoning used (e.g., number of fish, deer, or elk required to feed a family over a winter; amount of money a family spends on groceries). f. Explain estimation and computation strategies, including compatible numbers, compensation, and front-end rounding, and how each strategy relates to different operations. g. Identify if a strategy used in solving a problem involved estimation or computation. h. Apply and explain the choice of estimation or computation strategy such as compatible numbers, compensation, and front-end rounding.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
***N5.5** Demonstrate an understanding of fractions by using concrete and pictorial representations to:* • *create sets of equivalent fractions* • *compare fractions with like and unlike denominators.* *[C, CN, PS, R, V]*	a. Create concrete, pictorial, or physical models of equivalent fractions and explain why the fractions are equivalent. b. Model and explain how equivalent fractions represent the same quantity. c. Verify whether or not two given fractions are equivalent using concrete materials, pictorial representations, or symbolic manipulation. d. Generalize and verify a symbolic strategy for developing a set of equivalent fractions. e. Determine equivalent fractions for a fraction found in a situation relevant to self, family, or community. f. Explain how to use equivalent fractions to compare two given fractions with unlike denominators. g. Position a set of fractions, with like and unlike denominators, on a number line and explain strategies used to determine the order. h. Justify the statement, "If two fractions have a numerator of 1, the larger of the two fractions is the one with the smaller denominator".

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
***N5.6** Demonstrate understanding of decimals to thousandths by:* • *describing and representing* • *relating to fractions* • *comparing and ordering.* *[C, CN, R, V]*	a. Tell a story (orally, in writing, or through movement) that explains what a concrete or pictorial representation of a part of a set, part of a region, or part of a unit of measure illustrates and record the quantity as a decimal. b. Represent concretely or pictorially a decimal identified in a situation relevant to self, family, or community. c. Recognize and generate equivalent forms (decimal or fraction) of fractions and decimals found in situations relevant to one's life, family, or community. d. Demonstrate, using concrete or pictorial models to explain, how a quantity in tenths or hundredths can also be recorded as hundredths or thousandths (e.g., 0.2 can be written as 0.200). e. Describe the quantity represented by each digit in a given decimal. f. Make and test conjectures about the relationship of equality of quantities written in decimal and fractional form (e.g., 0.7 and $\frac{7}{10}$) and verify concretely, pictorially, or logically. g. Use and explain personal strategies for writing decimals as fractions. h. Use and explain personal strategies for writing fractions with a denominator of 10, 100, or 1000 as a decimal. i. Explain, by providing examples, how to write decimals as a fraction with a denominator of 10, 100, or 1000. j. Identify benchmarks on a number line that could be used to order a given set of decimals and explain the choices made. k. Use benchmarks to order a set of decimals from a situation related to one's life, family, or community.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
***N5.7** Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).* *[C, CN, PS, R, V]*	a. Identify and describe situations relevant to one's life, family, or community experiences in which sums and differences of decimals might be determined. b. Use personal strategies to predict sums and differences of decimals and evaluate the effectiveness of the strategies. c. Create concrete or pictorial models to represent the determination of the sum or difference of two decimal numbers, explain the model, and record the process symbolically. d. Explain how estimation can be used to determine the position of the decimal point in a sum or difference.

Outcomes	**Indicators**
N5.7 (continued)	e. Identify and correct errors in the calculation of sums and differences of decimals and explain the reasoning. f. Explain how understanding place value is necessary in calculating sums and differences of decimals. g. Solve a given problem that involves addition and subtraction of decimals and explain the strategies used.

Patterns and Relations Strand

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	**Indicators**
***P5.1** Represent, analyse, and apply patterns using mathematical language and notation.* *[C, CN, PS, R, V]*	a. Describe situations from one's life, family, or community in which patterns emerge, identify assumptions made in extending the patterns, and analyze the usefulness of the pattern for making predictions. b. Describe, using mathematics language (e.g., one more, seven less) and symbolically (e.g., r + 1, p – 7), a pattern represented concretely or pictorially that is found in a chart. c. Create alternate representations, including concrete or pictorial models, charts, and mathematical expressions, for a given pattern (numeric or geometric). d. Predict subsequent elements (terms or values) in a pattern (with and without concrete materials or pictorial representations) and explain the reasoning including the assumptions being made. e. Verify whether or not a particular number belongs to a given pattern. f. Solve problems and make decisions based upon the mathematical analysis of a pattern and other contributing factors.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	**Indicators**
***P5.2** Write, solve, and verify solutions of single-variable, one-step equations with whole number coefficients and whole number solutions.* *[C, CN, PS, R]*	a. Identify aspects of experiences from one's life, family, and community that could be represented by a variable (e.g., temperature, cost of a DVD, size of a plant, colour of shirts, or performance of a team goalie). b. Describe a situation for which a given equation could apply and identify what the variable represents in the situation. c. Solve single-variable equations with the variable on either side of the equation, explain the strategies used, and verify the solution.

Shape and Space Strand

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS5.1 Design and construct different rectangles given either perimeter or area, or both (whole numbers), and draw conclusions. *[C, CN, PS, V]*	a. Construct (concretely or pictorially) and record the dimensions of two or more rectangles with a specified perimeter and select, with justification, the dimensions that would be most appropriate in a particular situation (e.g., a rectangle is to have a perimeter of 18 units, what are the dimensions of the possible rectangles, which rectangle would be most appropriate if the rectangle is to be the base of a shoe box or a dog pen). b. Critique the statement "A rectangle with dimensions of 1 cm by 8 cm is different from a rectangle with dimensions of 8 cm by 1 cm". (Note: Any dimensions could be used to demonstrate the idea of orientation and point of view.) c. Construct (concretely or pictorially) and record the dimensions of as many rectangles as possible with a specified area and select, with justification, the rectangle that would be most appropriate in a particular situation (e.g., a rectangle is to have an area of 24 units^2, what are the dimensions of the possible rectangles, which rectangle would be most appropriate if the rectangle is to fence off the largest garden possible or be the base of a box on a shelf that is 10 units by 8 units). d. Critique the statement: "A rectangle with dimensions of 3 cm by 4 cm is different from a rectangle with dimensions of 2 cm by 5 cm". (Note: Any dimensions with the same perimeter could be used to demonstrate the idea of same perimeter not necessarily resulting in the same area or shape of the rectangle). e. Generalize patterns discovered through the exploration of the areas of rectangles with the same perimeter and through the exploration of the perimeters of rectangles with the same area (e.g., greater areas do not imply greater perimeters and vice versa, the rectangle for a situation closest to a square will have the greatest area, or the rectangle with the smallest width for a given perimeter will have the smallest area). f. Identify situations relevant to self, family, or community where the solution to problems would require the consideration of both area and perimeter, and solve the problems.

Goals: Spatial Sense, Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
***SS5.2** Demonstrate understanding of measuring length (mm) by:* • *selecting and justifying referents for the unit mm* • *modelling and describing the relationship between mm, cm, and m units.* *[C, CN, ME, PS, R, V]*	a. Choose and use referents for 1 mm to determine approximate linear measurements in situations relevant to self, family, or community and explain the choice. b. Generalize measurement relationships between mm, cm, and m from explorations using concrete materials (e.g., 10 mm = 1 cm, 0.01m = 1 cm). c. Provide examples of situations relevant to one's life, family, or community in which linear measurements would be made and identify the standard unit (mm, cm, or m) that would be used for that measurement and justify the choice. d. Draw, construct, or physically act out a representation of a given linear measurement (e.g., the students might be asked to show 4 m; this could be done by drawing a straight line on the board that is 4 m in length, constructing a box (or different boxes) that has a base with a perimeter of 4 m, or carrying out a physical movement that results in moving 4 m). e. Pose and solve problems that involve hands-on linear measurements using either referents or standard units

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
***SS5.3** Demonstrate an understanding of volume by:* • *selecting and justifying referents for cm^3 or m^3 units* • *estimating volume by using referents for cm^3 or m^3* • *measuring and recording volume (cm^3 or m^3)* • *constructing rectangular prisms for a given volume.* *[C, CN, ME, PS, R, V]*	a. Provide referents for cm^3 and m^3 and explain the choice. b. Describe strategies developed for selecting and using referents to determine approximate volume measurements in situations relevant to self, family, or community. c. Estimate the volume of 3-D objects using personal referents. d. Decide what standard cubic unit is represented by a specific referent, and verify. e. Determine the volume of a 3-D object using manipulatives, describe the strategy used, and explain whether the volume is exact or an estimate. f. Construct possible rectangular prisms for a given volume, identify the dimensions of each prism, and explain which prism would be most appropriate for a particular situation.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
***SS5.4** Demonstrate understanding of capacity by:* • *describing the relationship between mL and L* • *selecting and justifying referents for mL or L units* • *estimating capacity by using referents for mL or L* • *measuring and recording capacity (mL or L).* *[C, CN, ME, PS, R, V]*	a. Show, using concrete materials, that 1000 mL has the same capacity as 1 L. b. Provide referents for 1 millilitre and 1 litre and explain the choice. c. Describe strategies for selecting and using referents to determine approximate capacity measurements in situations relevant to self, family, or community. d. Decide what standard capacity unit is represented by a specific referent, and verify. e. Estimate the capacity of a container using personal referents. f. Determine the capacity of a container using concrete materials that closely take on the shape of the container, describe the strategy used, and explain whether the volume is exact or an estimate (e.g., if beads are used, discuss the impact on accuracy because of the space between the beads compared to the accuracy if water is used). g. Sort a set of containers from least to greatest capacity, explain the strategies used, and verify by determining or estimating the capacity.

Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
***SS5.5** Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are:* • *parallel* • *intersecting* • *perpendicular* • *vertical* • *horizontal.* *[C, CN, R, T, V]*	a. Identify and describe examples of parallel, intersection, perpendicular, vertical, and horizontal lines, edges, and faces of 2-D shapes and 3-D objects found within one's home, school, and community (including 2-D shapes and 3-D objects in the natural environment, print and multimedia texts). b. Sketch a 2-D shape or 3-D object that is relevant to self, family, or others and identify any lines, edges, or faces that are parallel, intersecting, perpendicular, vertical, or horizontal. c. Describe, orally, in writing, or through physical movement, what it means for a line, edge, or face of a 2-D shape or 3-D object to be parallel, intersecting, perpendicular, vertical, or horizontal.

Mathematics 5

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS5.6 Identify and sort quadrilaterals, including: • ***rectangles*** • ***squares*** • ***trapezoids*** • ***parallelograms*** • ***rhombuses*** ***according to their attributes.*** ***[C, R, V]***	a. Identify and provide examples for the types of quadrilaterals that are found in one's home, school, and community. b. Compare different quadrilaterals using concrete materials and pictures, identify common and differing attributes, and sort the quadrilaterals according to one of the attributes (e.g., relationships between side lengths, or number of pairs of parallel sides). c. Analyze a set of sorted quadrilaterals and determine where a new quadrilateral would belong in the sorted set. d. Describe, orally or in writing, the attributes of different quadrilaterals including rectangles, squares, trapezoids, parallelograms, and rhombuses. e. Create a model to illustrate the relationships between different quadrilaterals (e.g., demonstrating that a square is a rectangle and a parallelogram is a trapezoid) including rectangles, squares, trapezoids, parallelograms, and rhombuses.

Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS5.7 Identify, create, and analyze single transformations of 2-D shapes (with and without the use of technology). ***[C, CN, R, T, V]***	a. Carry out different transformations (translations, rotations, and reflections) concretely, pictorially (with or without the use of technology), or physically and generalize statements regarding the position and orientation of the transformed image based upon the type of transformation. b. Determine if a given 2-D shape and its transformed image match a set of transformation instructions and explain the conclusion reached. c. Draw a 2-D shape, translate the shape, and record the translation by describing the direction and magnitude of the movement. d. Draw a 2-D shape, rotate the shape, and describe the direction of the turn (clockwise or counter clockwise), the fraction of the turn, and the point of rotation. e. Draw a 2-D shape, reflect the shape, and identify the line of reflection and the distance of the image from the line of reflection. f. Predict the result of a single transformation of a 2-D shape and verify the prediction.

Outcomes	Indicators
SS5.7 (continued)	g. Describe a single transformation that could be used to replicate the given image of a 2-D shape. h. Identify transformations found within one's home, classroom, or community, describe the type and amount of transformations evident (e.g., translation to the left and up, ¼ of a rotation in a clockwise direction, and reflection about the right side of the shape), and create a concrete or pictorial model of the same set of transformations.

Statistics and Probability Strand

Goals: Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SP5.1 Differentiate between first-hand and second-hand data. ***[C, R, T, V]***	a. Provide examples of data relevant to self, family, or community and categorize the data, with explanation, as first-hand or second-hand data. b. Formulate a question related to self, family, or community which can best be answered using first-hand data, describe how that data could be collected, and answer the question (e.g., "What game will we play at home tonight?" "I can survey everyone at home to find out what games everyone wants to play."). c. Formulate a question related to self, family, or community, which can best be answered using second-hand data (e.g., "Which has the larger population – my community or my friend's community?"), describe how those data could be collected (I could find the data on the StatsCan website), and answer the question. d. Find examples of second-hand data in print and electronic media, such as newspapers, magazines, and the Internet, and compare different ways in which the data might be interpreted and used (e.g., statistics about health-related issues, sports data, or votes for favourite websites).

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SP5.2 Construct and interpret double bar graphs to draw conclusions. ***[C, PS, R, T, V]***	a. Compare the attributes and purposes of double bar graphs and bar graphs based upon situations and data that are meaningful to self, family, or community. b. Create double bar graphs, without the use of technology, based upon data relevant to one's self, family, or community. Pose questions, and support answers to those questions using the graph and other identified significant factors. c. Pose and solve problems related to the construction and interpretation of double bar graphs.

Goals: Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SP5.3 Describe, compare, predict, and test the likelihood of outcomes in probability situations. ***[C, CN, PS, R]***	a. Describe situations relevant to self, family, or community which involve probabilities and categorize different outcomes for the situations as being impossible, possible, or certain (e.g., it is possible that my little sister will be put to bed by 8:00 tonight or it is impossible that I will have time to watch a movie tonight because I have two hockey games). b. Design and conduct probability experiments to determine the likelihood of a specific outcome and explain what the results tell about the outcome including whether the outcome is impossible, possible, or certain. c. Identify all possible outcomes in a probability experiment and classify the outcomes as less likely, equally likely, or more likely to occur and explain the reasoning (e.g., for an upcoming Pow Wow, list the dances that could be done and then classify the likelihood of each of the dances occurring, or of the dances occurring while you are in attendance). d. Predict how the likelihood of two outcomes in a probability experiment, carry out the experiment, compare the results to the prediction, and identify possible reasons for discrepancies.

Assessment and Evaluation of Student Learning

Assessment and evaluation require thoughtful planning and implementation to support the learning process and to inform teaching. All assessment and evaluation of student achievement must be based on the outcomes in the provincial curriculum.

Assessment involves the systematic collection of information about student learning with respect to:

- ☑ Achievement of provincial curriculum outcomes
- ☑ Effectiveness of teaching strategies employed
- ☑ Student self-reflection on learning.

Evaluation compares assessment information against criteria based on curriculum outcomes for the purpose of communicating to students, teachers, parents/caregivers, and others about student progress and to make informed decisions about the teaching and learning process.

> *Assembling evidence from a variety of sources is more likely to yield an accurate picture.*
> *(NCTM, 2000, p. 24)*

Reporting of student achievement must be based on the achievement of curriculum outcomes. Assessment information which is not related to outcomes can be gathered and reported (e.g., attendance, behaviour, general attitude, completion of homework, effort) to complement the reported achievement related to the outcomes of Grade 5 Mathematics. There are three interrelated purposes of assessment. Each type of assessment, systematically implemented, contributes to an overall picture of an individual student's achievement.

Assessment for learning involves the use of information about student progress to support and improve student learning and inform instructional practices and:

- is teacher-driven for student, teacher, and parent use
- occurs throughout the teaching and learning process, using a variety of tools
- engages teachers in providing differentiated instruction, feedback to students to enhance their learning, and information to parents in support of learning.

> *Assessment should not merely be done to students; rather it should be done for students.*
> *(NCTM, 2000, p. 22)*

Assessment as learning involves student reflection on and monitoring of her/his own progress and:

- students self-reflect and critically analyze learning related to curricular outcomes without anxiety or censure
- is student-driven with teacher guidance for personal use
- occurs throughout the learning process
- engages students in reflecting on learning, future learning, and thought processes (metacognition).

> *What are examples of assessments as learning that could be used in Grade 5 mathematics and what would be the purpose of those assessments?*

> *Assessment should become a routine part of the ongoing classroom activity rather than an interruption.*
> *(NCTM, 2000, p. 23)*

Assessment of learning involves teachers' use of evidence of student learning to make judgements about student achievement and:

- provides opportunity to report evidence of achievement related to curricular outcomes
- occurs at the end of a learning cycle, using a variety of tools
- provides the foundation for discussion on placement or promotion.

In mathematics, students need to be regularly engaged in assessment as learning. The assessments used should flow from the learning tasks and provide direct feedback to the students regarding their progress in attaining the desired learnings as well as opportunities for the students to set and assess personal learning goals related to the mathematical content for Grade 5.

Connections with Other Areas of Study

There are many possibilities for connecting Grade 5 mathematical learning with the learning occurring in other subject areas. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learnings. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. Following are just a few of the ways in which mathematics can be connected to other subject areas (and other subject areas connected to mathematics) at Grade 5.

Arts Education – Grade 5 Arts Education and Mathematics complement and extend many of the ideas that students explore and learn in each of the subjects. As an example, both subjects involve the students in the study of different forms of measurement. In Grade 5 Mathematics, students study many forms of measurement, including linear measurement as found in outcome SS5.2:

> Demonstrate understanding of measuring length (mm) by:
> - selecting and justifying referents for the unit of mm
> - modeling and describing the relationship between mm, cm, and m units.
>
> [C, CN, ME, PS, R, V]

In learning about this aspect of linear measurement, many of the understandings that students are developing, such as the impact and importance of the different units of measurement can be linked in purpose to the organization of music by the use of tempo, also a form of measurement, in Grade 5 Arts Education. In both instances, students are learning how to assess the particular measurement in appropriate contexts, and what that measurement tells about the particular happening or object being measured. The tempo of a piece of music can reveal emotion, while linear measurements and units can reveal relative size (and help determine the appropriateness or usefulness of a particular object). Students should be engaged in making connections in this way between the two subjects areas to strengthen their understanding of measurement and why it is done.

Connections can also be made when students explore different types of sources of ideas for the arts and the mathematical notions of first-hand and second-hand data found in outcome SP5.1:

> Differentiate between first-hand and second-hand data. [C, R, T, V]

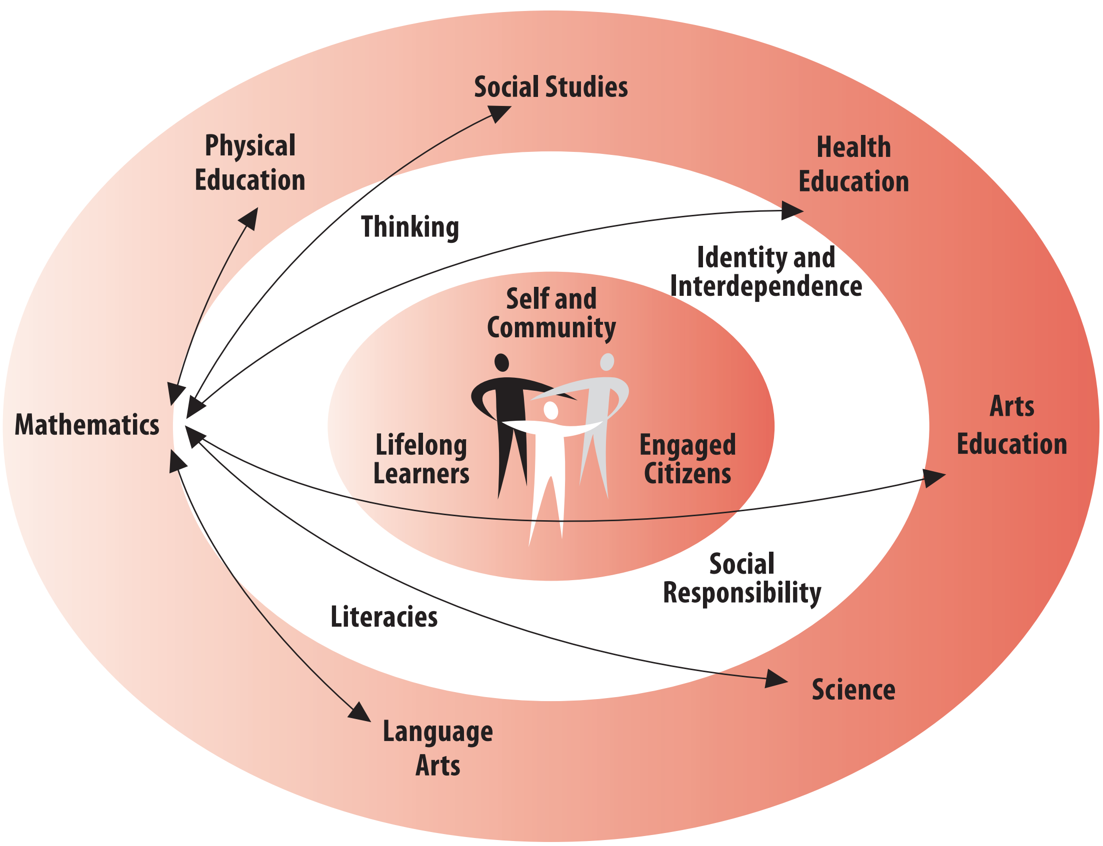
The sources of ideas that the students consider in Grade 5 Arts Education are excellent examples of first-hand and second-hand data. First-hand data the students explore come from contexts such as personal experiences, observations, and feelings while second-hand data comes from contexts such as research or stories. In mathematics, focus is often placed on surveying and reading about surveys or experiments. Although these are excellent sources of first- and second-hand data, making connections between the students' study of Arts Education and Mathematics in terms of the type of data being collected and interpreted will give Grade 5 students a richer understanding and sense of purpose and use for a wider variety of data.

English Language Arts (ELA) – ELA and Mathematics share a common interest in students developing their abilities to reflect upon and communicate about their learnings through viewing, listening, reading, representing, speaking, and writing. The strategies for constructing meaning that students study and implement as part of Grade 5 ELA (e.g., activating and building upon prior knowledge, constructing, monitoring, and confirming meaning, and reflecting on cause and effect relationships) are the same strategies that the students should be engaging in as they construct their understandings of the different outcomes in Grade 5 Mathematics.

In addition, at Grade 5, the two subjects also have outcomes that focus on the collection, display, and interpretation of data. Specifically, there are strong connections between an ELA outcome related to viewing, demonstrating comprehension of, and critically evaluating visual and multimedia texts and the Mathematics outcome SP5.2:

> Construct and interpret double bar graphs to draw conclusions. [C, PS, R, T, V]

Within this learning outcome, students construct, view, interpret, analyze, and draw conclusions based on double bar graphs. In ELA, students are also viewing, interpreting, and analyzing double bar graphs as one example of the visual and multimedia texts. This connection between the subjects provides an excellent opportunity for students to develop two key understandings. First, there are many ways in which data can be represented, and second, that all representations should be evaluated for accuracy and purpose.



Health Education – As a part of their Grade 5 Health Education, students are to analyze their personal eating practices with respect to information that is available to them such as food labels and food guides. A strong connection can be made between this learning and the students' exploration of outcomes found in the Statistics and Probability Strand. For example, when students are learning to construct and interpret double bar graphs (in outcome SP5.2), the students can use first- and second-data collected in Health Education (also connected to mathematics outcome SP5.1) to construct double bar graphs. The resulting graphs can be used by the students to analyze their eating practices. This analysis could be used by some or all students as the basis for one of the five day action plans that they are to create and carry out.

Students could also use personal and researched data regarding the eating practices of themselves and family members or friends to reflect on the likelihood of engaging in different eating practices. This information could then be used as the students learn about outcome SP5.3:

> Describe, compare, predict, and test the likelihood of outcomes in probability situations. [C, CN, PS, R]

by using the collected data to make a prediction regarding a certain eating practice to occur within a specified period of time. Again, data could be collected against which the estimated probability could be compared. Students could then discuss why the estimated probability did or did not reflect the actual outcome.

Physical Education – There are many opportunities for teachers to create learning experiences that connect Physical Education and Mathematics. These learning experiences can provide students the opportunity to expand both their mathematical and physical skills and understandings. As students are studying lines in 2-D shapes and 3-D objects in outcome SS5.5:

> Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are:
> - parallel
> - intersecting
> - perpendicular
> - vertical
> - horizontal
>
> [C, CN, R, T, V]

they can be using physical movements, including those being targeted in their study of body management activities in

Physical Education, to demonstrate the students' understandings of the different types of lines. Moreover, students can use their understanding of the mathematical terminology to accurately describe and communicate about the positioning of the students' bodies, and the adjustments and movements they are making, within their Physical Education classes.

Students can also use their understanding of first-hand and second-hand data (mathematics outcome SP5.1), as well as the creation and interpretation of double bar graphs (mathematics outcome SP5.2), as students collect data for and design a health-related fitness plan in physical education class.

Science – Mathematics and Science have many common topics and features, such as the recognition and description of patterns, sorting and categorizing, measurement, and the use of multiple representations. Opportunities for connections occur in the Grade 5 Science topics of Properties and Changes of Materials, Forces and Simple Machines, and Weather.

In Grade 5 Science, students investigate the properties of materials and explore whether or not those properties change with the change of state of those materials. The concept of change is also important in Grade 5 Mathematics, particularly in outcome SS5.7:

> Identify, create, and analyze single transformations of 2-D shapes (with and without the use of technology). [C, CN, R, T, V]

An important understanding that students should gain from their investigation of transformations is that although the orientation of the 2-D shape might change, other properties or characteristics of the shape do not. The results of transformations on the properties of shapes can be compared to the results of changes in the properties of materials in Science.

The students' study of Weather in Grade 5 Science also has strong ties to SP5.3:

> Describe, compare, predict, and test the likelihood of outcomes in probability situations. [C, CN, PS, R]

In Mathematics class, students can use observations and measurements from Science to make predictions about the probabilities of different weather occurrences. Second-hand data can also be researched and compared to the collected first-hand data and analyzed through the use of double-bar graphs (mathematics outcomes SP5.1 and SP5.2).

Social Studies – Social Studies and Mathematics often connect through the investigation of patterns and trends and in the representation of data. In Grade 5, students in Social Studies are focusing their study on Canada. In doing so, students are developing understandings of the cultural diversity of Canada, how Canada's population distributions have changed over time, and the importance of the natural resources in early Canada. These topics can be connected to the students' study of Grade 5 Mathematics through the outcomes in the Statistics and Probability Strand:

> SP5.1 Differentiate between first-hand and second-hand data.[C, R, T, V]
>
> SP5.2 Construct and interpret double bar graphs to draw conclusions. [C, PS, R, T, V]
>
> SP5.3 Describe, compare, predict, and test the likelihood of outcomes in probability situations. [C, CN, PS, R]

Using both data the students have collected themselves and statistical data that they have researched, students can compare their communities or Canada's cultural composition in its earlier years to that of today using a double bar graph. By analyzing these double bar graphs, students can make and justify predictions for future population distributions.

Such investigations and inquiries can provide opportunities for students to develop their understandings related to outcome N5.1:

> Represent, compare, and describe whole numbers to 1 000 000 within the contexts of place value and the base ten system, and quantity. [C, CN, R, T, V]

By engaging in the comparison of population sizes over time and within a community, students will strengthen their understanding of the quantity (or relative size) of whole numbers while also refining their understanding of the base ten system.

The students' mathematical learnings might also be extended if some researched data express different cultural population sizes in terms of fractions of the entire population. In such cases, students can use their learnings related to outcome N5.5:

Demonstrate an understanding of fractions by using concrete and pictorial representations to:
- create sets of equivalent fractions
- compare fractions with like and unlike denominators.

[C, CN, PS, R, V]]

to compare and order the different population data. These comparisons can then be used to help students understand the relationships that developed between the different cultural groups in Canada and explain the actions and behaviours of different groups.

Glossary

Attributes: Characteristics of 2-D shapes and 3-D objects that can be used to compare and sort sets of 2-D shapes and 3-D objects (e.g., colour, relative size, number of corners, number of lines of symmetry).

Benchmarks: Numeric quantities used to compare and order other numeric quantities. For example, 0, 5, 10, and 20 are often used as benchmarks when placing whole numbers on a number line.

Compatible Numbers: Numbers that can be grouped to make computation or estimation easier. For example, when estimating 807 ÷ 4 a student might estimate that 807 is approximately 800 and then determine 800 ÷ 4 as the estimate. As a computation strategy, students would first look for groupings of numbers and operations that are easily determined. For example, in determining 25 × 17 × 4 the student might consider 25 × 4 first and then multiply this product by 17. Another example would be when adding a series of quantities. Students might look for groupings of quantities that are easily added first. For example, in 37 + 28 + 13 + 15, some students might first recognize 37 + 13 as being 50. Other students might look at the ones digits and quickly recognize 7 + 8 + 5 as 20. In these types of situations, students will often combine using compatible numbers with compensation.

Compensation: A computation strategy for addition and subtraction of numbers that involves adding or subtracting a quantity from one number and then compensating for the change by doing the opposite to the other number. For example, when determining 789 + 832, a student might add 11 to 789 to get 800, but then would subtract 11 from 832 to compensate for the change in 789. So 789 + 832 would become 800 + 821.

Dividend: In a division statement, the dividend is the quantity that is being divided into equal groups. For example, in the expression 38 ÷ 4, 38 is the dividend (which is being divided into groups of 4 items or being divided into 4 equal groups).

Divisor: In a division statement, the divisor is the number of groups to be created, or the number of items to be within a group. For example, in the expression 38 ÷ 4, 4 is the divisor.

Double Bar Graph: A bar graph which compares two different sets of data according to the same criteria. For example, in a double bar graph, internet usage of boys and girls at a particular school could be compared over a number of years (the criteria).

Equality as a Balance and Inequality as Imbalance: The equal sign represents the idea of equivalence. For many students, it means do the question. For some students, the equal sign in an expression such as 2 + 5 = means to add. When exploring equality and inequality, by using objects on a balance scale, students discover the relationships between and among the mass of the objects. The equal sign in an equation is like a scale: both sides, left and right, must be the same in order for the scale to stay in balance and the equation to be true. When the scale is imbalanced, the equation is not true. Using 2 + 5 = □, rather than simply 2 + 5 = helps students understand that the equal sign (=) represents equality rather than "do the work" or "do the question".

Equation: A statement of equality involving one or more variables. For example, $x - 5 = 7$ is an equation.

Expression: A statement of a mathematical relationship or pattern involving one or more variables that is not a statement of equality or inequality. For example $x + 2$ is an expression that describes two more than an unknown value.

First-hand Data: Any data (information that is either quantitative or qualitative) that have been directly collected by the individual using the information.

Front-end Rounding: A strategy used to estimate computations by rounding the quantities involved to an appropriate power of 10. For example, for the sum 8437 + 2056 students might estimate using 8000 + 2000 or by using 8400 + 2100. In some cases, truncation rather than rounding is used in this type of estimation. For the same example, a truncated front-end estimation would be 8400 + 2000.

Interdisciplinary: Disciplines connected by common concepts and skills embedded in disciplinary outcomes.

Multidisciplinary: Discipline outcomes organized around a theme and learned through the structure of the disciplines.

Number, Numeral, Digit: A number is the name that we give to quantities. For example, there are 7 days in a week, or I have three brothers – both seven and three are numbers in these situations because they are defining a quantity. The symbolic representation of a number, such as 287, is called the numeral. If 287 is not being used to define a quantity, we call it a numeral.

Numerals, as the symbolic representation of numbers, are made up of a series of digits. The Hindu-Arabic number system that we use has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. (Note: sometimes students are confused between these digits and their finger digits – this is because they count their fingers starting at one and get to ten rather than zero to nine.) These digits are also numerals and can be numbers (representing a quantity), but all numbers and all numerals are combinations of digits. The placement of a digit in a number or numeral affects the place value of the digit and, hence, how much of the quantity that it represents. For example, in 326, the 2 is contributing 20 to the total, while in 236 the 2 contributes 200 to the total quantity.

Object: Object is used to refer to a three-dimensional geometrical figure. To distinguish this meaning from that of shape, the word "object" is preceded by the descriptor "3-D".

Personal Strategies: Personal strategies are strategies that the students have constructed and understand. Outcomes and indicators that specify the use of personal strategies convey the message that there is not a single procedure that is correct. Students should be encouraged to explore, share, and make decisions about what strategies to use in different contexts. Development of personal strategies is an indicator of the attainment of deeper understanding.

Polygon: 2-D shapes that have straight line edges that only intersect at the endpoints of the line segments and that form a closed shape.

Quadrilaterals: Four-sided polygons.

Referents: A concrete approximation of a quantity or unit of measurement. For example, the width of a thumb nail is often used as a referent for 1 cm. This referent can then be used to approximate a linear measurement.

Representations: Mathematical ideas can be represented and manipulated in a variety of forms including concrete manipulatives, visual designs, sounds and speech, physical movements, and symbolic notations (such as numerals and operation signs). Students need to have experiences in working with many different types of representations, and in transferring and translating knowledge between the different forms of representations.

Second-hand data: Any data (either quantitative or qualitative) that has been obtained from an outside source such as information from a friend or a report in a newspaper.

Shape: In this curriculum, shape is used to refer to two-dimensional geometric figures and is thus preceded by "2-D". The term shape is sometimes also used in mathematics resources and conversations to refer to three-dimensional geometric figures. It is important that students learn to be clear in identifying whether their use of the term shape is in reference to a 2-D or 3-D geometrical figure.

Solution: The quantity or quantities that, when assigned to the variable in an equation, result in a true statement. For example, 3 is a solution to $5 = 2 + x$ because $5 = 2 + 3$ is a true statement.

Transdisciplinary: All knowledge interconnected and interdependent; real-life contexts emphasized and investigated through student questions.

Transformations: The changing of 2-D shapes (and 3-D objects) according to position and orientation. At Grade 5, students study translations, rotations, and reflections as transformations of 2-D shapes. Other transformations, such as transformations in size (dilations), are also possible but are not part of the Grade 5 curriculum.

Variable: A quantity that is unknown but can vary according to the context it is in. In mathematics, variables are indicated within expressions and equations by using a small letter.

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Document Title: **Mathematics Grade 5 Curriculum**

1. Please indicate your role in the learning community

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   ☐ teacher librarian ☐ school community council member

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   What was your purpose for looking at or using this curriculum?

2. a) Please indicate which format(s) of the curriculum you used:

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The curriculum content is:	Strongly Agree	Agree	Disagree	Strongly Disagree
a. appropriate for its intended purpose	1	2	3	4
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c. clear and well organized	1	2	3	4
d. visually appealing	1	2	3	4
e. informative	1	2	3	4

5. Explain which aspects you found to be:

   Most useful:

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6. Additional comments:

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Thank you for taking the time to provide this valuable feedback.

Please return the completed feedback form to:

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