Financial and Workplace Mathematics 120

Curriculum

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New Brunswick Department of Education and
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Curriculum Overview for Grades 10-12 Mathematics

BACKGROUND AND RATIONALE

Mathematics curriculum is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society.

It is essential the mathematics curriculum reflects current research in mathematics instruction. To achieve this goal, *The Common Curriculum Framework for Grades 10–12 Mathematics: Western and Northern Canadian Protocol* has been adopted as the basis for a revised mathematics curriculum in New Brunswick. The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others.

The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the NCTM.

There is an emphasis in the New Brunswick curriculum on particular key concepts at each grade which will result in greater depth of understanding and ultimately stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

The intent of this document is to clearly communicate high expectations for students in mathematics education to all education partners. Because of the emphasis placed on key concepts at each grade level, time needs to be taken to ensure mastery of these concepts. *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM Principles and Standards, 2000).*

BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

The New Brunswick Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

- mathematics learning is an active and constructive process;
- learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates;
- learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort; and
- learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and aspirations.

Students construct their understanding of mathematics by developing meaning based on a variety of learning experiences. This meaning is best developed when learners encounter mathematical experiences that proceed from simple to complex and from the concrete to the abstract. The use of manipulatives, visuals and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels of understanding students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions also provide essential links among concrete, pictorial and symbolic representations of mathematics. The learning environment should value, respect and address all students' experiences and ways of thinking, so that students are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. It is important to realize that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood.

Goals for Mathematically Literate Students

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding through:

- taking risks
- thinking and reflecting independently
- sharing and communicating mathematical understanding
- solving problems in individual and group projects
- pursuing greater understanding of mathematics
- appreciating the value of mathematics throughout history.

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Striving toward success, and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Diverse Cultural Perspectives

Students come from a diversity of cultures, have a diversity of experiences and attend schools in a variety of settings including urban, rural and isolated communities. To address the diversity of knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students, a variety of teaching and assessment strategies are required in the classroom. These strategies must go beyond the incidental inclusion of topics and objects unique to a particular culture.

For many First Nations students, studies have shown a more holistic worldview of the environment in which they live (Banks and Banks 1993). This means that students look for connections and learn best when mathematics is contextualized and not taught as discrete components. Traditionally in Indigenous culture, learning takes place through active participation and little emphasis is placed on the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is important that teachers understand and respond to both verbal and non-verbal cues to optimize student learning and mathematical understandings.

Instructional strategies appropriate for a given cultural or other group may not apply to all students from that group, and may apply to students beyond that group. Teaching for diversity will support higher achievement in mathematics for all students.

Adapting to the Needs of All Learners

Teachers must adapt instruction to accommodate differences in student development as they enter school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom. The reality of individual student differences must not be ignored when making instructional decisions.

As well, teachers must understand and design instruction to accommodate differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

Universal Design for Learning

The New Brunswick Department of Education and Early Childhood Development‘s definition of inclusion states that every child has the right to expect that his or her learning outcomes, instruction, assessment, interventions, accommodations, modifications, supports, adaptations, additional resources and learning environment will be designed to respect his or her learning style, needs and strengths.

Universal Design for Learning is a "…framework for guiding educational practice that provides flexibility in the ways information is presented, in the ways students respond or demonstrate knowledge and skills, and in the ways students are engaged." It also "...reduces barriers in instruction, provides appropriate accommodations, supports, and challenges, and maintains high achievement expectations for all students, including students with disabilities and students who are limited English proficient" (CAST, 2011).

In an effort to build on the established practice of differentiation in education, the Department of Education and Early Childhood Development supports *Universal Design for Learning* for all students. New Brunswick curricula are created with universal design for learning principles in mind. Outcomes are written so that students may access and represent their learning in a variety of ways, through a variety of modes. Three tenets of universal design inform the design of this curriculum. Teachers are encouraged to follow these principles as they plan and evaluate learning experiences for their students:

- **Multiple means of representation:** provide diverse learners options for acquiring information and knowledge
- **Multiple means of action and expression:** provide learners options for demonstrating what they know
- **Multiple means of engagement:** tap into learners' interests, offer appropriate challenges, and increase motivation

For further information on *Universal Design for Learning*, view online information at http://www.cast.org/.

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, and physical education.

NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: **change**, **constancy**, **number sense**, **patterns**, **relationships, spatial sense** and **uncertainty**.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Students need to learn that new concepts of mathematics as well as changes to already learned concepts arise from a need to describe and understand something new. Integers, decimals, fractions, irrational numbers and complex numbers emerge as students engage in exploring new situations that cannot be effectively described or analyzed using whole numbers.

Students best experience change to their understanding of mathematical concepts as a result of mathematical play.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Many important properties in mathematics do not change when conditions change. Examples of constancy include:

- the conservation of equality in solving equations
- the sum of the interior angles of any triangle
- the theoretical probability of an event.

Number Sense

Number sense, which can be thought of as deep understanding and flexibility with numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p. 146). Continuing to foster number sense is fundamental to growth of mathematical understanding.

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Students with strong number sense are able to judge the reasonableness of a solution, describe relationships between different types of numbers, compare quantities and work with different representations of the same number to develop a deeper conceptual understanding of mathematics.

Number sense develops when students connect numbers to real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. Evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing mathematically rich tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all of the mathematical topics, and it is through the study of patterns that students can make strong connections between concepts in the same and different topics.

Working with patterns also enables students to make connections beyond mathematics. The ability to analyze patterns contributes to how students understand their environment. Patterns may be represented in concrete, visual, auditory or symbolic form. Students should develop fluency in moving from one representation to another.

Students need to learn to recognize, extend, create and apply mathematical patterns. This understanding of patterns allows students to make predictions and justify their reasoning when solving problems. Learning to work with patterns helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics.

Relationships

Mathematics is used to describe and explain relationships. Within the study of mathematics, students look for relationships among numbers, sets, shapes, objects, variables and concepts. The search for possible relationships involves collecting and analyzing data, analyzing patterns and describing possible relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense involves the representation and manipulation of 3-D objects and 2-D shapes. It enables students to reason and interpret among 3-D and 2-D representations.

Spatial sense is developed through a variety of experiences with visual and concrete models. It offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions.

Spatial sense is also critical in students' understanding of the relationship between the equations and graphs of functions and, ultimately, in understanding how both equations and graphs can be used to represent physical situations.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately. This language must be used effectively and correctly to convey valuable messages.

ASSESSMENT

Ongoing, interactive assessment (*formative assessment*) is essential to effective teaching and learning. Research has shown that formative assessment practices produce significant and often substantial learning gains, close achievement gaps and build students' ability to learn new skills (Black & William, 1998, OECD, 2006). Student involvement in assessment promotes learning. Interactive assessment, and encouraging self-assessment, allows students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes:

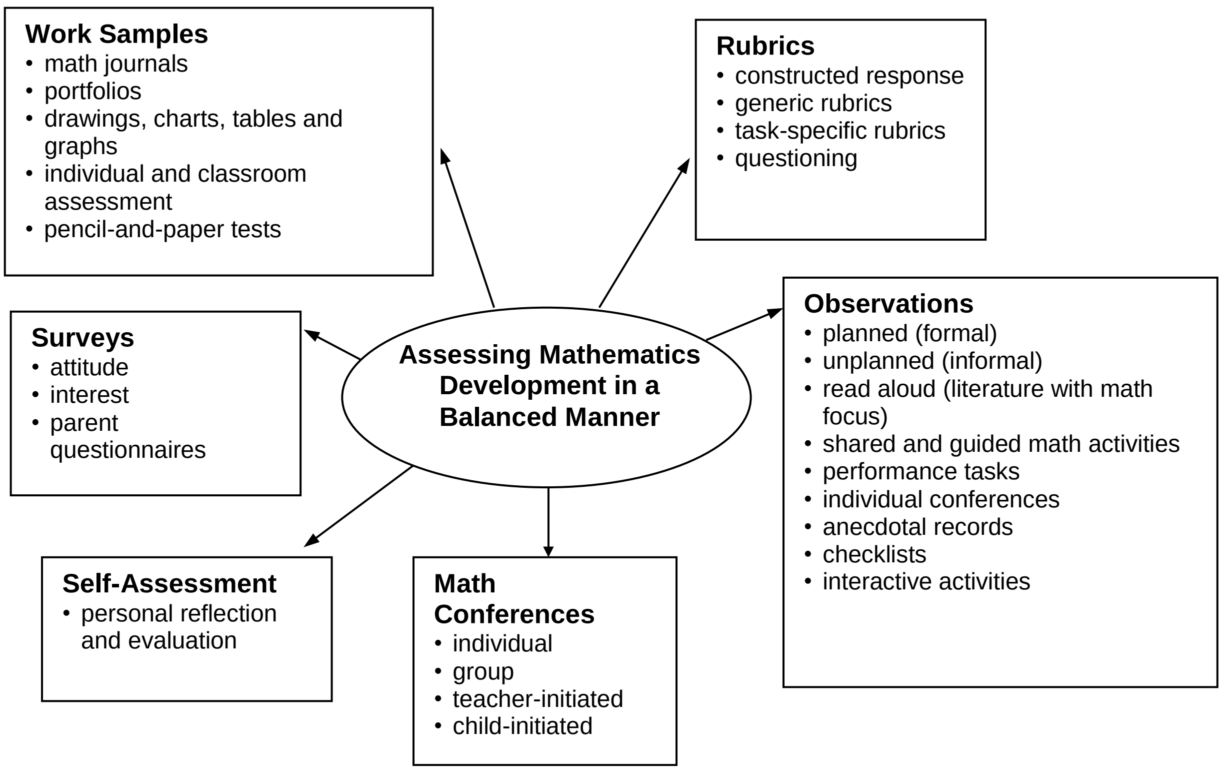
- providing clear goals, targets and learning outcomes
- using exemplars, rubrics and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning (Davies, 2000)

Formative assessment practices act as the scaffolding for learning which, only then, can be measured through summative assessment. *Summative assessment,* or assessment *of* learning, tracks student progress, informs instructional programming and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning and produce achievement gains.

Student assessment should:

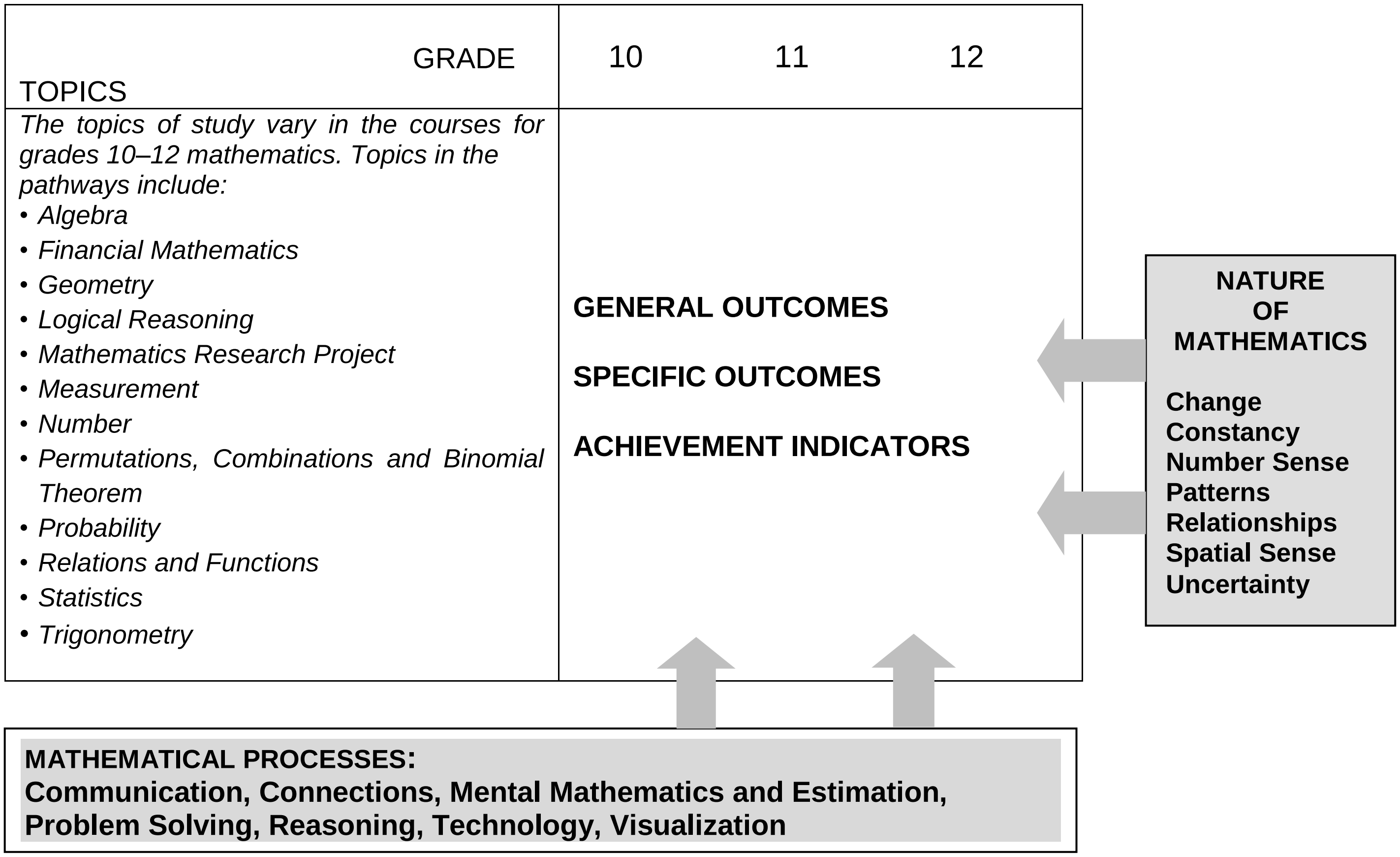
- align with curriculum outcomes
- use clear and helpful criteria
- promote student involvement in learning mathematics during and after the assessment experience
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction

  (adapted from: NCTM, Mathematics Assessment: A practical handbook, 2001, p.22)



CONCEPTUAL FRAMEWORK FOR 10-12 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.
Students are expected to:

- communicate in order to learn and express their understanding of mathematics (Communications: C)
- develop and apply new mathematical knowledge through problem solving (Problem Solving: PS)
- connect mathematical ideas to other concepts in mathematics, to everyday
- experiences and to other disciplines (Connections: CN)
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation: ME)
- select and use technologies as tools for learning and solving problems (Technology: T)
- develop visualization skills to assist in processing information, making connections and solving problems (Visualization: V).
- develop mathematical reasoning (Reasoning: R)

The New Brunswick Curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Emerging technologies enable students to engage in communication beyond the traditional classroom to gather data and share mathematical ideas.

Problem Solving [PS]

Problem solving is one of the key processes and foundations within the field of mathematics. Learning through problem solving should be the focus of mathematics at all grade levels. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts. Problem solving is to be employed throughout all of mathematics and should be embedded throughout all the topics.

When students encounter new situations and respond to questions of the type, *How would you*...*?* or *How could you* ...*?,* the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. Students should not know the answer immediately. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement. Students will be engaged if the problems relate to their lives, cultures, interests, families or current events.

Both conceptual understanding and student engagement are fundamental in moulding students' willingness to persevere in future problem-solving tasks. Problems are not just simple computations embedded in a story, nor are they contrived. They are tasks that are rich and open-ended, so there may be more than one way of arriving at a solution or there may be multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill or understanding. Problem solving can vary from being an individual activity to a class (or beyond) undertaking.

In a mathematics class, there are two distinct types of problem solving: solving contextual problems outside of mathematics and solving mathematical problems. Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem.

Problem solving can also be considered in terms of engaging students in both inductive and deductive reasoning strategies. As students make sense of the problem, they will be creating conjectures and looking for patterns that they may be able to generalize. This part of the problem-solving process often involves inductive reasoning. As students use approaches to solving the problem they often move into mathematical reasoning that is deductive in nature. It is crucial that students be encouraged to engage in both types of reasoning and be given the opportunity to consider the approaches and strategies used by others in solving similar problems.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk-takers.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"*Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*" (Caine and Caine, 1991, p. 5).

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental mathematics" (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "*become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving*" (Rubenstein, 2001).

Mental mathematics "*provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers*" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to learn which strategy to use and how to use it.

Technology [T]

Technology can be used effectively to contribute to and support the learning of a wide range of mathematical outcomes. Technology enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- generate and test inductive conjectures
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- model situations
- develop number and spatial sense.

Technology contributes to a learning environment in which the curiosity of students can lead to rich mathematical discoveries at all grade levels. The use of technology should not replace mathematical understanding. Instead, technology should be used as one of a variety of approaches and tools for creating mathematical understanding.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers. Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and spatial reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills.
Measurement sense includes the ability to determine when to measure and when to estimate and involves knowledge of several estimation strategies (Shaw and Cliatt, 1989, p. 150).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations. It is through visualization that abstract concepts can be understood concretely by the student. Visualization is a foundation to the development of abstract understanding, confidence and fluency.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking.

Questions that challenge students to think, analyze and synthesize help them develop an understanding of mathematics. All students need to be challenged to answer questions such as, *Why do you believe that's true/correct?* or *What would happen if ….*

Mathematical experiences provide opportunities for students to engage in inductive and deductive reasoning. Students use inductive reasoning when they explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Students use deductive reasoning when they reach new conclusions based upon the application of what is already known or assumed to be true. The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines.

ESSENTIAL GRADUATION LEARNINGS

Graduates from the public schools of Atlantic Canada will be able to demonstrate knowledge, skills, and attitudes in the following essential graduation learnings. These learnings are supported through the outcomes described in this curriculum document.

Aesthetic Expression
Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.

Citizenship
Graduates will be able to assess social, cultural, economic, and environmental interdependence in a local and global context.

Communication
Graduates will be able to use the listening, viewing, speaking, reading and writing modes of language(s) as well as mathematical and scientific concepts and symbols to think, learn, and communicate effectively.

Personal Development
Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.

Problem Solving
Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language, mathematical, and scientific concepts.

Technological Competence
Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems

PATHWAYS AND TOPICS

The Common Curriculum Framework for Grades 10–12 Mathematics on which the New Brunswick Grades 10-12 Mathematics curriculum is based, includes pathways and topics rather than strands as in *The Common Curriculum Framework for K–9 Mathematics*. In New Brunswick all Grade 10 students share a common curriculum covered in two courses: *Geometry, Measurement and Finance 10* and *Number, Relations and Functions 10*. Starting in Grade 11, three pathways are available: *Finance and Workplace*, *Foundations of Mathematics*, and *Pre-Calculus*.

Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. Students are encouraged to cross pathways to follow their interests and to keep their options open. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings.

Goals of Pathways

The goals of all three pathways are to provide prerequisite attitudes, knowledge, skills and understandings for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Design of Pathways

Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study and for direct entry into the work force.

The content of each pathway has been based on the *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings* and on consultations with mathematics teachers.

Financial and Workplace Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into some college programs and for direct entry into the work force. Topics include financial mathematics, algebra, geometry, measurement, number, statistics and probability.

Foundations of Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, number, logical reasoning, relations and functions, statistics and probability.

Pre-calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Students develop of a function tool kit including quadratic, polynomial, absolute value, radical, rational, exponential, logarithmic and trigonometric functions. They also explore systems of equations and inequalities, degrees and radians, the unit circle, identities, limits, derivatives of functions and their applications, and integrals.

Outcomes and Achievement Indicators

The New Brunswick Curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes and achievement indicators.

General Curriculum Outcomes (GCO) are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the pathway.

Specific Curriculum Outcomes (SCO) are statements that identify specific concepts and related skills underpinned by the understanding and knowledge attained by students as required for a given grade.

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome. In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for clarification and are not requirements that must be addressed to fully meet the learning outcome. The word *and* used in an outcome indicates that both ideas must be addressed to fully meet the learning outcome, although not necessarily at the same time or in the same question.

Instructional Focus

Each pathway in *The Common Curriculum Framework for Grades 10–12 Mathematics* is arranged by topics. Students should be engaged in making connections among concepts both within and across topics to make mathematical learning experiences meaningful. Teachers should consider the following points when planning for instruction and assessment.

- The mathematical processes that are identified with the outcome are intended to help teachers select effective pedagogical approaches for the teaching and learning of the outcome.
- All seven mathematical processes must be integrated throughout teaching and learning approaches, and should support the intent of the outcomes.
- Wherever possible, meaningful contexts should be used in examples, problems and projects.
- Instruction should flow from simple to complex and from concrete to abstract.
- The assessment plan for the course should be a balance of assessment for learning, assessment as learning and assessment of learning.

The focus of student learning should be on developing a conceptual and procedural understanding of mathematics. Students' conceptual understanding and procedural understanding must be directly related.

Pathways and Courses

The graphic below summarizes the pathways and courses offered.

Mathematics K-9

Grade 10
- *2 x 90 hr courses; required to pass both*
- *May be taken in any order or in the same semester*

Geometry, Measurement and Finance 10

Number, Relations and Functions 10

Grade 11
- *3 x 90 hr courses offered in 3 pathways*
- *Students are required to pass at least one of "Financial and Workplace Mathematics 11" **or** "Foundations of Mathematics 11".*
- *Pre-requisite Grade 10 course(s) must be passed before taking Grade 11 courses.*

Financial and Workplace Mathematics 110

Pre-requisite: Geometry, Measurement and Finance 10

Foundations of Mathematics 110

*Pre-requisites: Geometry, Measurement and Finance 1 **and** Number, Relations and Functions 10*

Pre-Calculus 110

Pre-requisite or Co-requisite: Foundations of Mathematics 110

Grade 12
- *5 x 90 hr courses offered in 3 pathways*
- *Pre-requisite Grade 11 or Grade 12 course must be passed before taking Grade 12 courses.*

Financial and Workplace Mathematics 120

*Pre-requisite: Financial and Workplace Mathematics 110 **or** Foundations of Mathematics 110*

Foundations of Mathematics 120

Pre-requisite: Foundations of Mathematics 110

Pre-Calculus A 120

Pre-requisite: Pre-Calculus 110

Pre-Calculus B 120

Pre-requisite or Co-requisite: Pre-Calculus A 120

Calculus 120

*Pre-requisites: Pre-Calculus A 120 **and** Pre-Calculus B 120*

SUMMARY

The Conceptual Framework for Grades 10–12 Mathematics describes the nature of mathematics, the mathematical processes, the pathways and topics, and the role of outcomes and achievement indicators in grades 10–12 mathematics. Activities that take place in the mathematics classroom should be based on a problem-solving approach that incorporates the mathematical processes and leads students to an understanding of the nature of mathematics.

CURRICULUM DOCUMENT FORMAT

This guide presents the mathematics curriculum by grade level so that a teacher may readily view the scope of the outcomes which students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings at a particular grade level are part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The heading of each page gives the General Curriculum Outcome (GCO), and Specific Curriculum Outcome (SCO). The key for the mathematical processes follows. A Scope and Sequence is then provided which relates the SCO to previous and next grade SCO's. For each SCO, Elaboration, Achievement Indicators, Suggested Instructional Strategies, and Suggested Activities for Instruction and Assessment are provided. For each section, the *Guiding Questions* should be considered.

GCO: General Curriculum Outcome
SCO: Specific Curriculum Outcome

Mathematical Processes

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math **[T]** Technology
	[V] Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Previous Grade or Course SCO's	Current Grade SCO	Following Grade or Course SCO's

Elaboration

Describes the "big ideas" to be learned and how they relate to work in previous Grades

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Achievement Indicators

Describes observable indicators of whether students have met the specific outcome

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

GCO: General Curriculum Outcome
SCO: Specific Curriculum Outcome

Suggested Instructional Strategies

General approach and strategies suggested for teaching this outcome

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Suggested Activities for Instruction and Assessment

Some suggestions of specific activities and questions that can be used for both instruction and assessment.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

Financial and Workplace Mathematics 120

Specific Curriculum Outcomes

SCO: M1: Demonstrate an understanding of the limitations of measuring instruments, Including precision, accuracy, uncertainty, and tolerance; and solving problems.
[C, PS, R, T, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Measurement

M1: Demonstrate an understanding of the limitations of measuring instruments, including precision, accuracy, uncertainty, and tolerance; and solve problems.

Scope and Sequence of Outcomes:

Grade Ten	Financial and Workplace Mathematics 120
M3: Solve problems, using SI and Imperial units, that involve linear measurement using estimation and measurement strategies. (GMF10)	**M1:** Demonstrate an understanding of the limitations of measuring instruments, including precision, accuracy, uncertainty, and tolerance; and solve problems.

ELABORATION

This is the students' first experience with the terms **precision, accuracy** and **tolerance** in the context of a mathematics class. They will explore these concepts as they apply to a variety of situations.

Precision is the indication of how close you are to the true measurement. It is determined by the smallest division of the scale on the measuring device. Both the type of tool chosen and the precision of the tool used will impact on the precision of a measurement.

For example, to measure the width of a barn, a measuring tape of appropriate length would be more appropriate than a standard ruler. Also depending on how fine the scale divisions are on the measuring tape, the precision of the measurement will vary. If the tape measure is marked off in 10 cm intervals, measurements will be far less **precise** than with a tape measure marked off in $1mm$ intervals.

Accuracy means careful exactness, and is related to the amount of error made in a measurement. The closer to the correct division mark on the scale you are, the less error you have in your measurement and the more accurate it is.

Any measuring device must be used correctly. If the tape measure is not placed correctly on the item to be measured, then no matter how small the scale divisions, the answer will be **inaccurate**. Similarly, if the scale is dropped on the floor before using it, damage caused by the fall could give inaccurate measurements. **Accuracy** of a measurement depends on both how well the measuring tool has been calibrated and on the skill of the person using measuring tool and does not necessarily imply precision.

For example the length of a board may be measured with **precision** to the nearest mm at $15.3\ cm$, but if the actual measure needed was $16.3\ cm$ the measurement is not **accurate**.

When a measurement is written to a given decimal place there is an **implied precision** that the true measurement falls within a halfway measure of that point and the next

SCO: M1: Demonstrate an understanding of the limitations of measuring instruments, Including precision, accuracy, uncertainty, and tolerance; and solving problems.
[C, PS, R, T, V]

highest and lowest decimal measure. So, for example when a measurement is written as 0.01, the implication is that the true measurement is between 0.005 and 0.015. When a measurement is written as 12 the implication is that the true value is between 11.5 and 12.5. However, if the measurement is written as 12.0, then the implication is that the measurement is more precise and the true measurement falls between 11.95 and 12.05.

Sometimes when you are making a measurement, you are given an amount of tolerance that is acceptable. **Tolerance** is an allowance for error. Depending on the situation different levels of variation can be tolerated. The **tolerance range** is the greatest range of variation that can be allowed.

For example, when machining pistons, actual measurements cannot range more than $1\ mm$ from the stated measure. However, when knitting mittens, actual measurements up to 1-$2\ cm$ more or less of the intended measurement are of no consequence.

As another example, if a walking stick is labelled as $85\ cm\ \mathrm{long}$, the **implied precision** is that the stick is between $84.5\ cm$ and $85.5\ cm$. If this range of up to $0.5\ cm$ is acceptable, then this is the **tolerance range**.

Baking provides a useful example of the relevance of **levels of precision** and **range of tolerance.** Most recipes **tolerate** considerable variation in measurements.

For example when baking a cake there is a lot of variation in the weight of an egg.

The amount of butter can also vary considerably. If a cake recipe calls for $150\ g$ butter, an amount from $145\ g$ to $155\ g$ of butter is acceptable, and the measurement **tolerance** for butter is $5\ g$. If the piece of butter placed on a scale measures $152\ g$, this would fall within the **range of tolerance** and it is not necessary to shave a little butter from the piece. Assuming the scale is accurate, a measure of $152\ g$ indicates that the butter weighs between $151.5\ g$ and $152.5\ g$ which is more **precision** than the cook needs.

On the other hand, the recipe also calls for $10\ g$ cinnamon. This requirement has smaller **error tolerance**. No cook will be happy with, say, a reading of $12\ g$ because this could mean as much as 25% more cinnamon than is called for. The cook will remove a little cinnamon from the scale, until it displays $10\ g$, accepting a range of tolerance of from 9.5-$10.5\ g$.

In this example, the scale is more precise than required to weigh the butter, but the one-gram precision is required to weigh the cinnamon.

Students should be given the opportunity to apply their understanding of the meanings of **accuracy**, **precision** and **tolerance** to a wide variety of contextual problems.

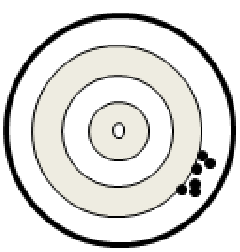
SCO: M1: Demonstrate an understanding of the limitations of measuring instruments, Including precision, accuracy, uncertainty, and tolerance; and solving problems.
[C, PS, R, T, V]

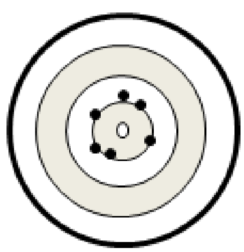
ACHIEVEMENT INDICATORS

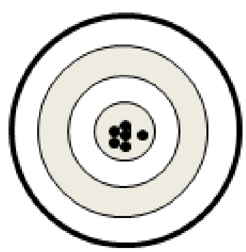
- Explain, in a given context, what measurement error can be tolerated, and what degree of precision is required.
- Explain why, in a given context, a certain degree of precision is required.
- Explain, using examples, the difference between precision and accuracy.
- Compare the degree of accuracy possible using two different instruments to measure the same attribute.
- Relate the degree of accuracy to the uncertainty of a given measure.
- Analyze precision and accuracy in a contextual problem.
- Calculate maximum and minimum values, using a given degree of tolerance in context.
- Describe, using examples, the limitations of measuring instruments used in a specific trade or industry; e.g., tape measure versus Vernier caliper.
- Solve a problem that involves precision, accuracy or tolerance.

Suggested Instructional Strategies

- Arrange with the trades teachers in your school a visit to the shop to examine instruments used for measuring and the specific jobs for which they are used.
- Precision and accuracy can be explained with reference to a dart board/bulls-eye as shown below:


high precision
low accuracy


lower precision
better accuracy


high precision
high accuracy

Provide a wide range of measuring instruments (or ask students to bring them in) – calipers, metre stick, clear 15 cm ruler, measuring tape etc. – and have students determine which instrument is best for measuring various objects. Or conversely have students bring in various objects to be measured and have them select the best measurement tool to measure that object.

- The document found at http://www.georgianc.on.ca/coned09/wp-content/uploads/01-CTS-M18-Accuracy-Tolerance-Precision.pdf has been produced by the Ontario Ministry of Training, Colleges and Universities. It provides examples and solutions of how accuracy, precision and tolerance are used in the trades.

SCO: M1: Demonstrate an understanding of the limitations of measuring instruments, Including precision, accuracy, uncertainty, and tolerance; and solving problems.
[C, PS, R, T, V]

- The following link provides a free download of the PDF “Essential Skills Workbook for the Trades” provided by Human Resources and Skills Development Canada. http://www.esdc.gc.ca/eng/jobs/les/tools/assessment/WP-167-workbook.shtml.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act To demonstrate accuracy, have each student carefully measure the length of some object in the classroom (about $1\ m$ long) and record their measurements. Students should do this individually, without input from their peers, and record their lengths on small pieces of paper, submitted anonymously. In the next class, draw a number line and mark each measurement with a solid dot above the corresponding position on the scale (i.e. a dot plot). Students will see that some of them measured more **accurately** than others.

Q Given a piece of lumber labelled as being $2.0\ m$, what is the:

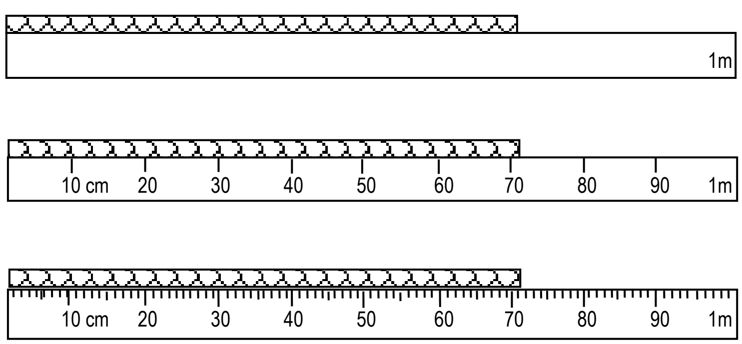
a) Implied Tolerance
b) Longest the piece of lumber could be, and still satisfy the implied error tolerance?
c) Shortest the piece of lumber could be, and still satisfy the implied error tolerance?
d) Tolerance range.

(Note to teacher: This might work better with a true piece of lumber, and with three different measurements specified. Have students measure the lumber themselves. There will be a few surprises!)

Answers: a) $0.05\ m$ or $5\ cm$ b) $2.05\ m$ c) $1.95\ m$ d) $1.95m$ to $2.05\ m$

Q Using the three different meter sticks shown below:

a) What is the measure of the ribbon in each case?
b) Which meter stick would give the highest degree of precision?
c) Does it matter which metre stick is used to measure the length of a nail? Why or why not?
d) Does it matter which metre stick is used to measure the approximate width of a room? Why or why not?



Answers:
a) *First stick 1m Second stick 70 cm Third stick 71 cm*
b) *The metre stick with the smallest units (cm)would give the highest degree of precision.*
c) *Yes, the metre stick with the highest precision is the best choice because a nail is less than 1 m (quite small)and requires great precision.*
d) *No, it does not matter because a rom is larger than 1 m. The metre stick sith the highest precision is still best, but the question asked for an approximate width of the room.*

SCO: M1: Demonstrate an understanding of the limitations of measuring instruments, Including precision, accuracy, uncertainty, and tolerance; and solving problems.
[C, PS, R, T, V]

Q Give an example of a measurement that is:

a) accurate but not precise.
b) precise but not accurate.

Answers:
a) "Mr. Hachey is six feet tall." *He may be six feet, but the unit* foot *is not very precise.*
b) *"The width of pencil graphite is* $0.015m$*." The measurement is precise, but not the true measurement.*

Q Two students use a metre stick marked to cm to measure the width of a lab table. One records $84.0\ cm$ being as precise as possible. The other student records $80\ cm$ being as precise as possible. How could they get different answers and who is right?

Answer: The first student has indicated that they have measured to the nearest cm. The second student has only measured to the nearest $10\ cm$*. The first student is correct.*

SCO: **G1:Solve problems that involve triangles, quadrilaterals, regular polygons.**
[C, CN, PS, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Geometry

G1: Solve problems that involve triangles, quadrilaterals, regular polygons.

Scope and Sequence of Outcomes:

Financial and Workplace Mathematics 110	Financial and Workplace Mathematics 120
. **G1:** Solve problems that involve two and three right triangles	**G1:** Solve problems that involve triangles, quadrilaterals, regular polygons.

ELABORATION

For this outcome students will solve problems through exploration of the properties of triangles, and of quadrilaterals which will include **squares**, **rhombus**, **rectangles**, **parallelograms**, **trapezoids**, and **kites**. They will extend this to include **regular polygons**. This outcome provides students with a good opportunity to explore patterns of geometric shapes, and to practice logical thinking.

Students should be able to describe and illustrate the following triangle properties:

- The largest angle is always opposite the largest side
- The smallest angle is always opposite the smallest side.
- The sum of the angles of a triangle is always $180°$.
- All angles in an equilateral triangle are equal to $60°$.
- In an isosceles triangle, the angles opposite to the equal sides are equal.
- The exterior angle of a triangle is equal to the sum of interior angles across from it.
- The sum of the lengths of any 2 sides of a triangle must be greater than the 3rd side.

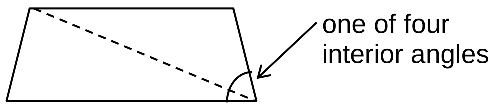
Once triangle properties are established, students can explore angle and side measures of other polygons. This will include quadrilaterals **(squares, rhombus, rectangles, parallelograms, trapezoids,** and **kites**), and **regular polygons**.

Quadrilateral	***Properties***				
	Angles equal	Sides equal	Sides parallel	Diagonal bisect angle is $90°$	Diagonal lengths are equal
square	$all = 90°$	all	$opposites$	yes	yes
rhombus	$opposites\ equal$	all	$opposites$	yes	no
rectangle	$all = 90°$	$opposites$	$opposites$	no	no
parallelogram	$opposites\ equal$	$opposites$	$opposites$	no	no
trapezoid	no	no	$one\ pair\ of\ opposites$	no	no
kite	$one\ pair\ of\ opposites$	$two\ pairs\ a\ djacent$	no	yes	no

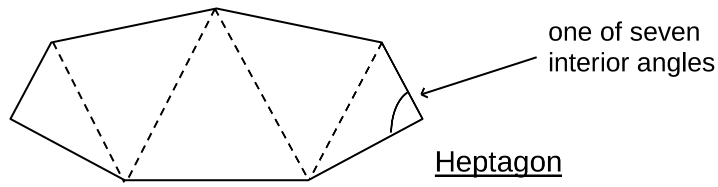
As students explore **polygons** with increasing numbers of sides they will discover that the sum of the **interior angles** of any polygon can be determined as $180° \times$ the number of triangles formed by joining the vertices of the polygon.

SCO: **G1:Solve problems that involve triangles, quadrilaterals, regular polygons.** [C, CN, PS, V]

For example, a quadrilateral can be divided into 2 triangles, so the sum of the **interior angles** is $2 \times 180° = 360°$. A seven sided figure (heptagon) can be divided into 5 triangles, so the sum of the **interior angles** will be $5 \times 180° = 900°$.



Quadrilateral



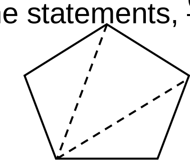
Heptagon

Students should be able to explain why the number of triangles formed is always 2 less than the number of sides of the polygon giving a general formula of: $(n-2) \times 180° = sum\ of\ interior\ angles$.

Regular polygons have sides and angles that are equal. Equilateral triangles and squares are members of this group.

The sum of the **interior angles** of a **regular polygon** can be determined by the method shown above. For regular polygons in which all angles are equal, each **interior angle** can also be determined by dividing the sum of the interior angles, by the number of sides or angles.

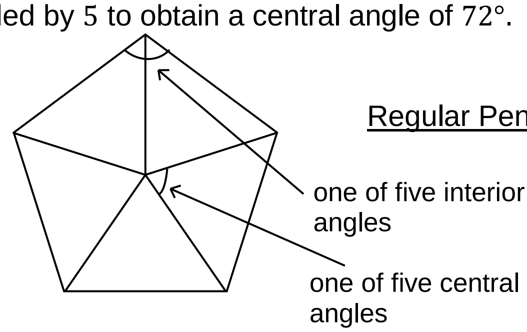
For example, a regular pentagon can be divided into 3 triangles, so the sum of the interior angles will be $(5-2) \times 180° = 540°$, and the measure of each interior angle will be $540° \div 5 = 108°$. Combing the statements, $\frac{(5-2) \times 180°}{5} = 108°$.



Regular Pentagon

Central angles are the angles formed by the equilateral triangle that come together at the center of a regular polygon. To determine the measure of the **central**, the student can consider the polygon as the sum of non-overlapping congruent triangles. The **central angles** are all equal, and add up to $360°$, so their measure can be determined by dividing $360°$ by the number of sides of the polygon.

For example, to find the measure of each of the **central angles** of a regular pentagon, $360°$ is divided by 5 to obtain a central angle of $72°$.



Regular Pentagon

The generalized formulas for any **regular polygon** ($n = number\ of\ sides$) are:

Central angles = $360° \div n$

Interior angles = $\frac{(n \times 180°) - 360°}{n}$ OR $\frac{(n-2) \times 180°}{n}$

SCO: **G1:Solve problems that involve triangles, quadrilaterals, regular polygons.**
[C, CN, PS, V]

ACHIEVEMENT INDICATORS

- Describe and illustrate properties of triangles, including isosceles and equilateral.
- Describe and illustrate properties of quadrilaterals in terms of angle measures, side lengths, diagonal lengths and angles of intersection.
- Describe and illustrate properties of regular polygons.
- Explain, using examples, why a given property does or does not apply to certain polygons.
- Identify and explain an application of the properties of polygons in construction, industrial, commercial, domestic and artistic contexts.
- Solve a contextual problem that involves the application of the properties of polygons.

Suggested Instructional Strategies

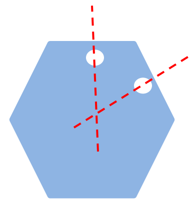
- For examples of triangles, quadrilaterals, and regular polygons in artistic situations, visit the site: http://www.dartmouth.edu/~matc/math5.geometry/unit5/unit5.html
- Students should have the opportunity to construct their own understanding of the various properties through explorations. Once exploration has taken place, the following website summarizes these properties and allows students to confirm their conjectures: http://www.coolmath.com/reference/polygons.html; http://www.mathwarehouse.com/geometry/triangles/
- Interactive site for properties of quadrilaterals: http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html
- Explorations – manually and/or through the use of computer software such as Geometer's SketchPad.
- The following provides some good exercises to explore properties of polygons and also provides some good applications (two examples shown below) http://www.kendallhunt.com/uploadedFiles/Kendall_Hunt/Content/PreK-12/Product_Samples/DG_%20Student_Edition_Sample_Chapter.pdf

SCO: **G1:Solve problems that involve triangles, quadrilaterals, regular polygons.**
[C, CN, PS, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Holes are punched in a regular hexagonal steel plate as indicated. Through what angle must the plate be rotated to locate the second hole?

$Answer$: $\frac{360°}{8} = 45°$

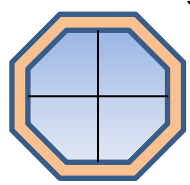


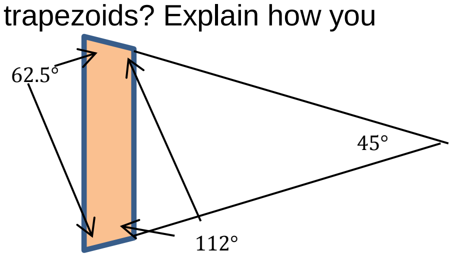
(Source for previous question: "Mathematics for the Trades: A Guided Approach" by Carman, Saunders and Mills, 2005 Pearson/Prentice Hall. Copies purchased for NB High Schools 2007)

Q If the sum of the interior angle of a regular polygon equals $1440°$, how many sides does the polygon have?

$Answer$: $1440° = 180°(n-2) \quad n = 10$

Q To build a window frame for an octagonal window, you will cut 8 identical trapezoidal pieces. What are the measures of the angles of the trapezoids? Explain how you found these measures.



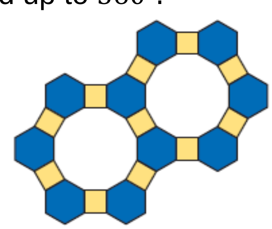


$Answer$:
Each side of the frame is a trapezoid and the end of a triangle. for which the central angle $= \frac{360°}{8} = 45°.$

The other two vertices of the triangle $= \frac{180° - 45°}{2} = 62.5°.$

The two other angles of the trapezoid $= \frac{360° - (2 \times 62.5°)}{2} = 112.5°$

Q Name the regular polygons that appear in the tiling shown below. Find the measures of the angles that surround any vertex point in the tiling. Explain why the angles at every vertex add up to $360°$.



$Answer$:
squares $(90°)$ *and hexagons* $(120°)$*around dodecagons* $(150°)$
$360°$ *is a full rotation on a 2D shape*

(Previous two questions adapted from :
http://www.kendallhunt.com/uploadedFiles/Kendall_Hunt/Content/PreK-12/Product_Samples/DG_%20Student_Edition_Sample_Chapter.pdf)

SCO: **G2: Solve problems by using the sine law and cosine law, excluding the ambiguous case.** [CN, PS, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

G2: Solve problems by using the sine law and cosine law, excluding the ambiguous case.

Scope and Sequence of Outcomes:

Grade 11 Mathematics	Financial and Workplace Mathematics 120
G1: Solve problems that involve two and three right triangles (FWM11) **A1:** Solve problems that require the manipulation and application of formulas related to: slope and rate of change, Rule of 72, finance charges, the Pythagorean theorem and trigonometric ratios. (FWM11) **G3:** Solve problems that involve the cosine law and the sine law, including the ambiguous case (Found11)	**G2:** Solve problems by using the sine law and cosine law, excluding the ambiguous case.

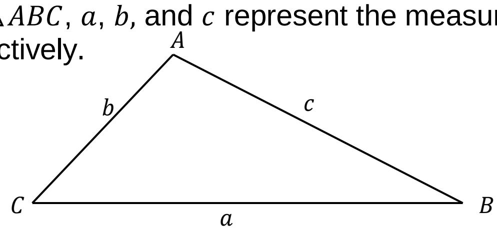
ELABORATION

In previous courses students have encountered problems involving right triangles that could be solved using the Pythagorean Theorem and the primary trigonometric ratios. They have also worked with similar triangles, and angles of elevation and depression and notation such as $S\ 41°W$.

In grade 11, students in the *Financial and Workplace Mathematics 110* course solved problems involving two and three right triangles and right triangle problems in 3-D. This outcome will be a review for students who took *Foundations of Mathematics 110* as a pre-requisite.

For this outcome, students will use the **sine law** or **cosine law** to solve problems involving triangles that are not right triangles. Students are not expected to derive these laws, but should be able to recognize when each law is appropriate. Students should only be given problems that have unique solutions, excluding the ambiguous case of the **sine law**.

For $\triangle ABC$, a, b, and c represent the measures of the sides opposite $\angle A$, $\angle B$, and $\angle C$ respectively.



The **sine law** describes the relationship between the sides and angles in any triangle. It can be used when students have:

- 2 angles and any side (AAS or ASA) **or**
- 2 sides and an angle opposite to one of the sides (SSA - not including any ambiguous case)

The **sine law** can be expressed as:

SCO: **G2: Solve problems by using the sine law and cosine law, excluding the ambiguous case.** [CN, PS, V]

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The **cosine law** describes the relationship between the cosine of an angle and the lengths of the three sides of any triangle. It can be used when students have:
- 3 sides (SSS) **or**
- 2 sides and their contained angle (SAS)

The **Cosine Law** can be expressed as:

$c^2 = a^2 + b^2 - 2ab\ \cos C$ or

$a^2 = b^2 + c^2 - 2bc\ \cos A$ or

$b^2 = a^2 + c^2 - 2ac\ \cos B$

ACHIEVEMENT INDICATORS

- Identify and describe the use of the sine law and cosine law in construction, industrial, commercial and artistic applications.
- Solve problems, using the sine law or cosine law, when a diagram is given.

Suggested Instructional Strategies

- Start with a review of trigonometric ratios (SOH CAH TOA) and Pythagorean Theorem. Give students an oblique triangle to solve and allow them to reach the conclusion that these methods cannot be used, and another method must be used. This allows for the introduction of the sine law, followed by the cosine law.
- When introducing these new laws, provide students with diagrams illustrating different contextual situations and have them determine which law should be applied to solve a problem.
- Students from *Foundations of Mathematics 110* will have seen both laws already. Teachers may want to pair these students up with those from *Financial and Workplace Mathematics 110* to work together when solving problems.
- Geometer's Sketch Pad, Geogebra, or other graphing software can be used to investigate the sine and cosine laws.
- Invite guest speakers who use trigonometry in their work e.g., engineers, military personnel, architects, or professions that use navigation such as forestry, air traffic or aerospace technicians and engineers.

SCO: **G2: Solve problems by using the sine law and cosine law, excluding the ambiguous case.** [CN, PS, V]

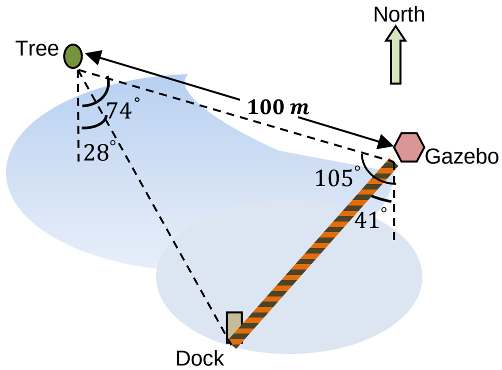
Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act In groups, have students create a sine law application question and a cosine law application question on chart paper to give to another group to solve.

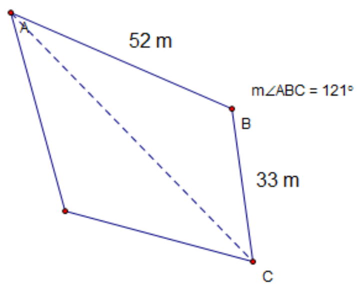
Q A bridge is to be built across a small lake from a gazebo to a dock (see figure). From the gazebo, the bearing to the dock is $S\ 41°W$, and to a tree $100\ m$ away is $S105°W$. From the tree, the bearing to the gazebo is $S\ 74°E$ and to the dock is $S\ 28°E$. Find the distance of the bridge.

Answer: $\frac{100}{\sin[180° - (46° + 64°)]} = \frac{bridge\ length}{\sin 46°}$

$\therefore$ *Bridge is* $77.4m$



Q The diagram below shows the dimensions and shape of a piece of land that is to be split between two siblings along the diagonal. To clearly mark this division between the properties they want to install a fence. What length of fencing would be needed?

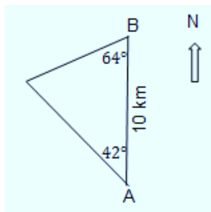


Answer: $b^2 = 52^2 + 33^2 - 2(52)(33)\cos 121$ *Fence would need to be* $74.6m$.

Q Fire towers A and B are $10\ km$ apart, B directly north of A. The rangers at tower A spot a fire at $N42°W$. The rangers at tower B spot the same fire at $S64°W$. How far is each tower from the fire?

Answer: $\frac{a}{\sin 42°} = \frac{b}{\sin 64°} = \frac{10km}{\sin 74°}$ $b \approx 9.4\ km$ $a \approx 7.0km$

Tower B is $7.0km$ *from the fire. Tower A is* $9.4km$ *from the fire.*



SCO: **G3: Demonstrate an understanding of transformations of a 2-D shape or a 3-D object, including translations, rotations, reflections, dilations.** [C, CN, R, T, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

G3: Demonstrate an understanding of transformations of a 2-D shape or a 3-D object, including translations, rotations, reflections, dilations.

Scope and Sequence of Outcomes:

Financial and Workplace Mathematics 110	Financial and Workplace Mathematics 120
G2: Solve problems that involve scale. **G3:** Model and draw 3-D objects and their views. **G4:** Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects. **A3:** Solve problems by applying proportional reasoning and unit analysis.	**G3:** Demonstrate an understanding of transformations of a 2-D shape or a 3-D object, including translations, rotations, reflections, dilations.

ELABORATION

In previous grades, students have performed and identified transformations of 2-D shapes and 3-D objects and have created and identified tessellations. In Grade11, students drew reduction and enlargements of shapes given a scale factor and worked with orthographic and isometric drawings of 3-D objects.

In grade 12, previous work will be reviewed and extended to explore **dilations**, **line symmetry** and **rotational symmetry**.

Dilations are transformations in which a polygon is enlarged or reduced by a scale factor around a given center point, such as a camera zooming in or out.

A figure has line symmetry when there is a line that divides it into two reflected parts. Shapes can have no line of symmetry or multiple lines of symmetry. These lines can exist in any orientation (vertical, horizontal, slanted).

A figure has rotational symmetry if it can be turned about its centre so that it fits in its original outline. The **order of rotation** is the number of times a figure fits onto itself in one complete turn. The **angle of rotation** is the minimum angle required to turn a figure onto itself.

SCO: **G3: Demonstrate an understanding of transformations of a 2-D shape or a 3-D object, including translations, rotations, reflections, dilations.** [C, CN, R, T, V]

ACHIEVEMENT INDICATORS

- Identify a single transformation that was performed, given the original 2-D shape or 3-D object and its image.
- Draw the image of a 2-D shape that results from a given single transformation.
- Draw the image of a 2-D shape that results from a given combination of successive transformations.
- Create, analyze and describe designs, using translations, rotations and reflections in all four quadrants of a coordinate grid.
- Identify and describe applications of transformations in construction, industrial, commercial, domestic and artistic contexts.
- Explain the relationship between reflections and lines or planes of symmetry.
- Determine and explain whether a given image is a dilation of another given shape, using the concept of similarity.
- Draw, with or without technology, a dilation image for a given 2-D shape or 3-D object, and explain how the original 2-D shape or 3-D object and its image are proportional.
- Solve a contextual problem that involves transformations.

Suggested Instructional Strategies

- Provide students with a multitude of 2-D and 3-D shapes and have them perform transformations prior to identifying transformations on a given object.
- Ask students to create a design involving translations, reflections, rotations, and/or dilation. Have them exchange with a classmate who will then describe the transformations.

SCO: **G3: Demonstrate an understanding of transformations of a 2-D shape or a 3-D object, including translations, rotations, reflections, dilations.** [C, CN, R, T, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Give students the dimensions of a 3D object. Have them choose a scale factor and enlarge and then reduce all dimensions by that scale factor.

Q Plot these points on a coordinate grid: $A(-1,3), B(2,4), C(3,-2), D\ (-4,-5)$. Join the points to draw polygon $ABCD$.

a) Translate the polygon 5 units right and 3 units up. Write the coordinates of each vertex of the image polygon $A'B'C'D'$.

b) Reflect the image polygon $A'B'C'D'$ in the x-axis. Write the coordinates of each vertex of the image polygon $A''B''C''D''$.

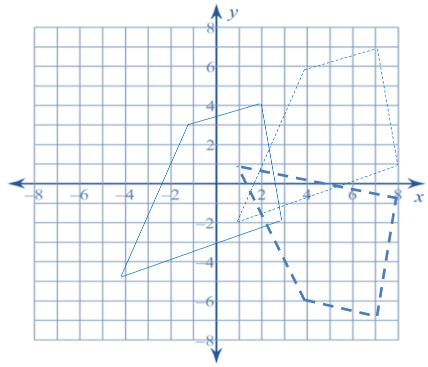
Answer:

$A(-1,3)\ \text{–>}\ A'\ (4,6)\ \text{–>}\ A''\ (4,-6)$

$B(2,4)\ \text{–>}\ B'\ (7,7)\ \text{–>}\ B''\ (7,-7)$

$C\ (3,-2)\ \text{–>}\ C'(8,1)\ \text{–>}\ C''(8,-1)$

$D\ (-4,-5)\ \text{–>}\ D'\ (1,-2)\ \text{–>}\ D''(1,2)$



Q Plot these points on a coordinate grid: $L(-5,8), M(0,8), N(0,5), O(-5,5)$. Join the points to draw the polygon LMNO.

a) Describe the polygon you have sketched.
b) Dilate the polygon LMNO using a scale factor of 2.5 and center point of (0, 0).
c) Write the coordinates of the image polygon L'M'N'O'.
d) Rotate the image polygon L'M'N'O' 90° clockwise around the center point and label the resulting coordinates.

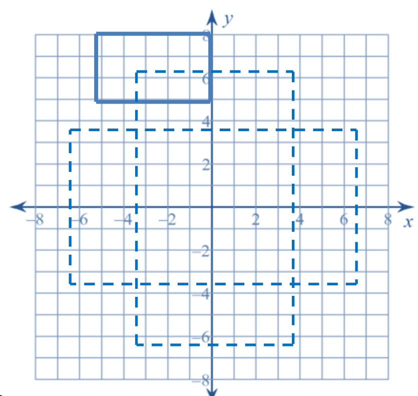
Answer:
a) a rectangle
$b)c) \rightarrow d)$

$L(-5,8) \rightarrow\ L'\ (-6.25, 3.75) \rightarrow\ L''\ (3.75, 6.25)$

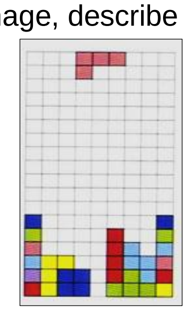
$\mathrm{M}\ (0,8) \rightarrow\ \mathrm{M}'\ (0, 3.75) \rightarrow M''\ (3.75,-6.25)$

$N\ (0,5) \rightarrow\ N'\ (6.25,-3.75) \rightarrow N''\ (-3.75,-6.25)$

$O(-5,8)\ \text{– –>}\ 0'\ (-6.25\ \ -3.75) \rightarrow O''\ (-3.75, 6.25)$



Q The computer game *Tetris* is a tiling game that uses squares. The playing screen is a 9 x 20 grid of squares. During the game, tiles fall from the top of the screen and must be translated or rotated to fill complete rows at the bottom of the screen. Each time a row is filled, it disappears. Given the following image, describe the transformations that must occur to fill *three* complete rows.



Answer: Rotate clockwise 90° and translate down 13 units

SCO: **N1: Analyse puzzles and games that involve logical reasoning, using problem-solving strategies. [**C, CN, PS, R]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Number

N1: Analyze puzzles and games that involve logical reasoning, using problem-solving strategies.

Scope and Sequence of Outcomes:

Grade 11 Mathematics	Financial and Workplace Mathematics 120
N1: Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. (FWM11 and **LR2** Found11)	**N1:** Analyze puzzles and games that involve logical reasoning, using problem-solving strategies.

ELABORATION

Puzzles and games provide opportunities to explore patterns. In grade 10, students focused on playing and analyzing puzzles and games that involved spatial reasoning. They have experience with discussing strategies used to solve a puzzle or win a game. In grade 11, they extended these skills to games and puzzles which involved numerical reasoning.

In grade 12, students will focus on puzzles and games involving **logical reasoning**. **It is intended that this outcome be integrated throughout the course** by using puzzles and games such as Sudoku, Mastermind, Nim and logic puzzles.

Provide time for whole-class or group discussion to identify errors and suggest possible strategies for winning the game or solving the puzzle.

ACHIEVEMENT INDICATORS

- Determine, explain and verify a strategy to solve a puzzle or to win a game: e.g.,
  - guess and check
  - look for a pattern
  - make a systematic list
  - draw or model
  - eliminate possibilities
  - simplify the original problem
  - work backward
  - develop alternative approaches.
- Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.
- Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

SCO: **N1: Analyse puzzles and games that involve logical reasoning, using problem-solving strategies. [**C, CN, PS, R]

Suggested Instructional Strategies

- Use puzzles or games as warm-up activities throughout the term.
- After solving a puzzle or game, have students reflect on their method and reasoning, and share this either verbally or in writing.
- Before assigning games to students, try games together as a group to ensure students have a clear understanding of the instructions and rules of the game.
- Starting with simpler versions of games, gradually increase the difficulty.
- Have students look for patterns and then develop a strategy to create these patterns.
- Have students develop a game for classmates to play.
- Have students change a rule or parameter to a well-known or familiar game and then explain how it affects the outcome of the game.
- Have students critique the quality of game they have found online.
- Do not confine students to paper-and-pencil games and puzzles. Have other options that are more “hands-on” such as chess, Rubik’s cubes, Chinese checkers and backgammon.
- Have students try online flash games that require strategies to solve.
- Have students create their own brainteaser to share with class.

SCO: **N1: Analyse puzzles and games that involve logical reasoning, using problem-solving strategies. [**C, CN, PS, R]

Questions (Q) and Activities (Act) for Instruction and Assessment

Act Have students explain the strategies they used to solve the game or puzzle.

Act Have students describe a variation to improve or enhance an existing game or puzzle.

Act Have students create a strategy journal organized into sections for individual or group, and number or logic games and puzzles. They should use this journal to track strategies and solutions used throughout the course.

Act Have students access online links such as the following to explore various reasoning puzzles and games:

https://www.cariboutests.com/
http://www.puzzles.com/projects/logicproblems.html
http://www.logic-puzzles.org/index.php
http://www.folj.com
http://www.thelogiczone.plus.com

Q Zookeeper George was in charge of feeding all of the animals in the morning. He had a regular schedule that he followed every day. Can you figure out his schedule from the clues?

1. The giraffes were fed before the zebras but after the monkeys.
2. The bears were fed 15 minutes after the monkeys.
3. The lions were fed after the zebras.

	6:30 AM	6:45 AM	7:00 AM	7:15 AM	7:30 AM
Bears					
Giraffes					
Lions					
Monkeys					
Zebras					

$Answers: 6:30\ AM\ \ monkeys\ ,\ \ 6:45\ AM\ \ bears,\ \ 7:00\ AM\ \ giraffes,\ \ 7:15\ AM\ \ zebras\ ,\ \ 7:30\ AM\ lions$

From: http://www.puzzlersparadise.com/puzzles/feedingtime.html

SCO: **N2: Critique the viability of small business options by considering expenses, sales, profit or loss.** [C, CN, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

N2: Critique the viability of small business options by considering expenses, sales, profit or loss.

Scope and Sequence of Outcomes

Financial and Workplace Mathematics 110	Financial and Workplace Mathematics 120
N2: Analyze costs and benefits of renting, leasing and buying. **N3:** Analyze an investment portfolio in terms of interest rate, rate of return, total return. **N4:** Solve problems that involve personal budgets. **S1:** Solve problems that involve creating and interpreting graphs, including: bar graphs, histograms, line graphs and circle graphs.	**N2:** Critique the viability of small business options by considering expenses, sales, profit or loss.

ELABORATION

In grade 11, students explored personal budgets, investment portfolios, and costs and benefits of renting, leasing, and buying. In grade 12, students will extend this knowledge to analyze the viability of small business options, including expenses, sales, profit, and loss.

ACHIEVEMENT INDICATORS

- Identify expenses in operating a small business option, such as a hot dog stand.
- Identify feasible small business options for a given community.
- Generate options that might improve the profitability of a small business.
- Determine the break-even point for a small business.
- Explain factors, such as seasonal variations and hours of operation that might impact the profitability of a small business.

SCO: **N2: Critique the viability of small business options by considering expenses, sales, profit or loss.** [C, CN, R]

Suggested Instructional Strategies

- Bring in guest speakers from the *Canadian Youth Business Foundation* and/or local small businesses to talk about their experiences.
- Teachers could provide case studies for students to analyze and discuss. Teachers should complete an example of this prior to giving students their own study.
- As a class, discuss small businesses that already exist or may be considered for future development.
- Students can work in groups to develop a small business plan and discuss expenses, sales, and profits in a contextual situation. The small business could be run during the school year and any profit raised could be donated to a selected charity/school group.
- Lesson plans such as the one found at the following website can help teachers get started on this unit. In this one students learn about the importance of financially managing a small business. (http://www.thirteen.org/edonline/lessons/fe_start/index.html)
- Discuss possible expenses with students for small businesses. The Canada Revenue Agency has sites for small business owners where possible business expenses are described in detail:

  http://www.cra-arc.gc.ca/tx/bsnss/tpcs/slprtnr/bsnssxpnss/menu-eng.html
- The following website provides good lessons and worksheets on this topic: www.moneyinstructor.com/startbusiness.asp
- Take advantage of opportunities to work with the *Entrepreneurship* or *Accounting* class and use case studies on small businesses available in the resource for *Accounting 120*.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Have students discuss the following questions in large or small groups:

a) What small businesses currently operate locally?
b) For existing businesses, do you think they are operating effectively? Why or why not?
c) For businesses that are struggling, have students suggest options for improvement.

Act Have students access the Tourism website of any town of their choice. After reading about the town, have them find business opportunities for residents who are new to the area.

a) Select two business options and list the resources needed to start each business.

b) Determine which of the two businesses requires the least amount of resources.

c) Explain how the number of resources needed could impact their decision in set up.

SCO: **A1: Demonstrate an understanding of linear relations by recognizing patterns and trends, graphing, creating tables of values, writing equations, interpolating and extrapolating, and solving problems.** [CN, PS, R, T, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Algebra

A1: Demonstrate an understanding of linear relations by recognizing patterns and trends, graphing, creating tables of values, writing equations, interpolating and extrapolating, and solving problems.

Scope and Sequence of Outcomes:

Financial and Workplace Mathematics 110	Financial and Workplace Mathematics 120
A1: Solve problems that require the manipulation and application of formulas related to: slope and rate of change, Rule of 72, finance charges, the Pythagorean theorem, and trigonometric ratios. **A2:** Demonstrate an understanding of slope as rise over run, as rate of change, and by solving problems. **S1:** Solve problems that involve creating and interpreting graphs, including: bar graphs, histograms, line graphs and circle graphs.	**A1:** Demonstrate an understanding of linear relations by recognizing patterns and trends, graphing, creating tables of values, writing equations, interpolating and extrapolating, and solving problems**.**

ELABORATION

In previous grades, student have explored the concepts of discrete and continuous data. Interpolation and extrapolation of data is introduced in *Mathematics 9* and scatter plots are introduced in *Number, Relations and Functions 10*. They would also have been introduced to rate of change in *Mathematics 9*, and would have related slope to rate of change in *Numbers, Relations and Function 10*.

For this outcome students will learn to identify trends by looking at scatter plots. They will also distinguish between linear and nonlinear functions based on various representations including graphs, tables of values, number patterns and equations.

This will be an introduction to the concepts of direct and partial variation.

Direct variation is the relationship between two variables in which one variable is a constant multiple of the other. When graphing, the line always passes through the origin.

Partial variation is the relationship between two variables in which one variable is a constant multiple of the other *plus a constant*, traditionally expressed as $y = mx + b$. When graphing, the line does not pass through the origin.

Students will learn that the slope of a linear function is its **rate of change**. When solving contextual problems, students will learn to substitute into an equation to solve for the unknown.

SCO: **A1: Demonstrate an understanding of linear relations by recognizing patterns and trends, graphing, creating tables of values, writing equations, interpolating and extrapolating, and solving problems.** [CN, PS, R, T, V]

ACHIEVEMENT INDICATORS

- Identify and describe the characteristics of a linear relation represented in a graph, table of values, number pattern or equation.
- Sort a set of graphs, tables of values, number patterns and/or equations into linear and non-linear relations.
- Write an equation for a given context, including direct or partial variation.
- Create a table of values for a given equation of a linear relation.
- Sketch the graph for a given table of values.
- Explain why the points should or should not be connected on the graph for a context.
- Create, with or without technology, a graph to represent a data set, including scatter plots.
- Describe the trends in the graph of a data set, including scatter plots.
- Sort a set of scatter plots according to the trends represented (linear, nonlinear or no trend).
- Solve a contextual problem that requires interpolation or extrapolation of information.
- Relate slope and rate of change to linear relations.
- Match given contexts with their corresponding graphs, and explain the reasoning.
- Solve a contextual problem that involves the application of a formula for a linear relation.

Suggested Instructional Strategies

- Since the topic of linear functions was previously covered in grades 9 and 10, initiate a discussion by asking students to provide examples of a linear relation in real life. This could be a way to assess prior knowledge on this topic. From this, a discussion about the specific characteristics of linear functions in the various forms could follow. This would also be an opportunity to look at non-linear functions and to compare the similarities and differences between linear and non-linear functions.
- Have students research an internet job search site (e.g., www.newbrunswickjobshop.com). Have them find a job where they would be paid:

  a) a salary
  b) an hourly wage
  c) a straight commission
  d) a salary plus commission
  e) an hourly wage plus commission
  f) by piece work.

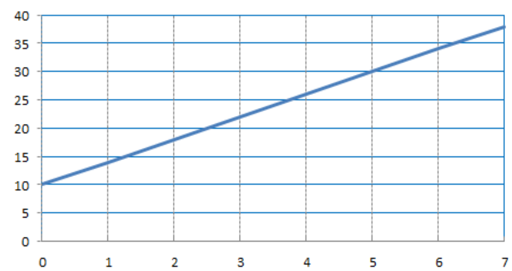
  From this, have students create a graphical representation and an equation to represent the earnings.

SCO: **A1: Demonstrate an understanding of linear relations by recognizing patterns and trends, graphing, creating tables of values, writing equations, interpolating and extrapolating, and solving problems.** [CN, PS, R, T, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Given the equation $y = 4x + 10$, create a table of values and draw the corresponding function. Using these representations, describe the patterns found between the table of values and the graphs.

Answer: The line crosses the y acis at 10, and as the x valure increases by 1, the y values increase by 4. The rise is 4, run is 1, the rate of change or slope is 4.



Q John works part-time at a bike shop assembling bicycles. He is initially paid $\$10$ per bicycle. After working there for one month, the bike shop changes the way he is paid. He is now paid a base salary of $\$50$ per week and an additional $\$6$ for each bicycle.

a) Write an equation that describes how much John would make in each situation if he assembled 20 bicycles per week.

b) Create a table of values to represent how much John makes per week for each pay scale, if he assembles 5, 10, 15, 20 bicycles per week.

c) On the same graph, graph the number of bikes assembled in a week and John's salary for each pay scale

d) If John usually assembles 12 bikes a day, which payment plan would he prefer?

Answers:

a) Initially $y = 10x$, for 20 bicycles he would earn $\$200$
After one month $y = 6x + 50$, for 20 bicycles he would earn $\$144$

b) Initially for 5, 10, 15 20 he would make $\$50, \$100, \$150, \$200$
After one month for 5,10,15,20 he would make $\$80, \$110, \$140, \$170$

c) The points on the graph should be discrete

d) $y = 10(12) = \$120 \quad y = 6(12) + 50 = \$122 \quad \therefore$ the second payment plan

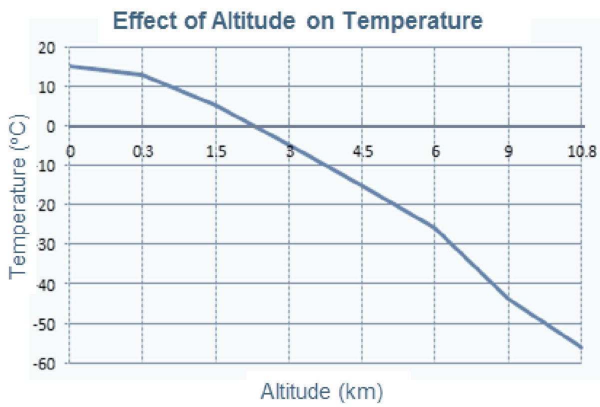
SCO: **A1: Demonstrate an understanding of linear relations by recognizing patterns and trends, graphing, creating tables of values, writing equations, interpolating and extrapolating, and solving problems.** [CN, PS, R, T, V]

Q On a spring day in a balloon is released in Fredericton, recording the following temperatures as it increases in altitude:

$Altitude (km)$	0	0.3	1.5	3.0	4.5	6.0	9.0	10.8
$Temperature$ (°C)	15	13	5	−5	−15	−26	−44	−56

a) Draw a graph that describes how temperature varies with altitude.
b) Explain whether or not you should join the points.

Answer: The points should be joined as it is continuous data.



SCO: **S1: Solve problems that involve the measures of central tendency, including mean, median, mode, weighted mean, trimmed mean.** [C, CN, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Statistics

S1: Solve problems that involve the measures of central tendency, including mean, median, mode, weighted mean, trimmed mean.

Scope and Sequence of Outcomes:

Financial and Workplace Mathematics 110	Financial and Workplace Mathematics 120
S1: Solve problems that involve creating and interpreting graphs, including bar graphs, histograms, line graphs, and circle graphs.	**S1:** Solve problems that involve the measures of central tendency, including mean, median, mode, weighted mean, trimmed mean.

ELABORATION

This outcome is a continuation of the *Financial and Workplace Mathematics 110* outcome **S1**, in which students created appropriate data displays. In this outcome students will extend their understanding of **measures of central tendency** to include the **mean**, **median**, **mode**, **weighted mean**, **trimmed mean** and **outlier**.

Mode is the observation that occurs most often (applies only to discrete data)

Median is the middle data point of a data set, sorted from smallest to largest. If there is an even number of data points, the two middle observations are averaged to find the median.

Mean is the sum of the data values, divided by the number of data points.

Weighted mean is the mean determined by counting each category as a percentage of the total.

Trimmed mean is determined by finding the mean, with the exclusion of a given percentage of the data from both the top and the bottom of the data set (10% or 5%). If this trimmed mean is very different from the mean of the full set of data, this is an indication that there are **outliers** – values that are much smaller or larger than the other data values. These outliers may represent a recording error, or an extreme event. The trimmed mean may give a better picture of what typically happens.

Mean and **median** apply to all numeric data, however **mode** applies only to discrete data.

For example, if students in a class are asked how many pizzas they ate last week, the **mean**, **median** and **mode** all have sensible interpretations.

If asked to record their foot length to the nearest mm, the **mean** and **median** give the average and typical foot lengths. The **mode** is the observation that is repeated most often, but does not indicate a typical size for the class.

If data is collected on student eye colour, which is non-numeric e.g., blue/grey, brown, green or hazel, then **mean** and **median** have no meaning, but **mode** does.

SCO: **S1: Solve problems that involve the measures of central tendency, including mean, median, mode, weighted mean, trimmed mean.** [C, CN, PS, R]

ACHIEVEMENT INDICATORS

- Explain, using examples, the advantages and disadvantages of each measure of central tendency.
- Determine the mean, median and mode for a set of data.
- Identify and correct errors in a calculation of a measure of central tendency.
- Identify the outlier(s) in a set of data.
- Explain the effect of outliers on mean, median and mode.
- Calculate the trimmed mean for a set of data, and compare with the untrimmed mean. Are there any outliers in this data set?
- Explain, using examples such as course marks, why some assigned work or tests should be given greater weighting when calculating a student's average mark for a course.
- Calculate the mean of a set of numbers after allowing the data to have different weightings (weighted mean).
- Explain, using examples from print and other media, how measures of central tendency and outliers are used to provide different interpretations of data. (The classic example here is income data, where means are typically larger than medians.)
- Solve a contextual problem that involves measures of central tendency.

Suggested Instructional Strategies

- Have students calculate the mean, trimmed mean and median for a set of numeric data, with and without an outlier, to see the effect of outliers (change just the lowest or highest value to an outlier). They should see that the median is unaffected but the mean gets either much higher or much lower.
- If you use a categorical marking scheme, show students how their overall mark is currently calculated. Have them bring in the grading schemes from other courses as examples. You may also want to explore how the same grades may yield a different overall percentage when calculated on a non-weighted system.
- Have students find various articles and/or advertisements using measures of central tendency to report findings. Students should be able to determine the meaning of what was reported and if it was the best indicator of what was measured.

SCO: **S1: Solve problems that involve the measures of central tendency, including mean, median, mode, weighted mean, trimmed mean.** [C, CN, PS, R]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q The following data set describes the heart rates of grade 12 students before they wrote a final exam, in beats per minute. Calculate the mean, median, and mode, and explain which measure is most useful and why.

63	59	71	64	68	76	88	72	85	83	93	73
90	57	68	75	67	61	72	88	63	74	72	84

$Answer: \; mean = 73.6 \quad median = 72 \quad mode = 72$

$The\ mean\ is\ the\ most\ relevant, as\ it\ can\ be\ compared\ to\ the\ mean\ when\ students\ are\ in\ a\ different\ situation\ e.g., after\ listening\ to\ relaxing\ music.$

Q Mr. Trig has been doing a lot of marking lately. On one assignment, the scores were: $45, 58, 78, 69, 0, 25, 14, 74, 85, 96, 96, 85, 100, 12, 46, 78, 65, 70, 41, 55$**.**

a) Determine the class mean

b) Determine the trimmed mean with a 10% exclusion

$Answer: \quad a)\ 59.6\% \quad b)\ For\ this\ data\ set\ which\ has\ 20\ values, a\ 10\%\ exclusion\ would\ remove\ the\ top\ 2\ values\ and\ bottom\ 2\ values, 0, 12, 96\ and\ 100.\ The\ trimmed\ mean = 61.5\%$

Q The course syllabus indicates that tests will count for 40%, assignments for 40%, and the project for 20% of the final mark. John has an average of 75% on his tests and 60% on his assignments, and 30% on his project. What is his final mark for the course?

$Answer: \quad (0.4 \times 75\%) + (0.4 \times 60\%) + (0.2 \times 30\%) = 60\%$

SCO: **S2: Analyze and describe percentiles.** [C, CN, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

S2: Analyze and describe percentiles.

Scope and Sequence of Outcomes:

Financial and Workplace Mathematics 110	Financial and Workplace Mathematics 120
	S2: Analyze and describe percentiles.

ELABORATION

Although students have seen and studied percentages, the concept of a percentile will be new to them. A **percentile** is a value below which that percentage of the data falls, and the rest of the data is at or above that value.

For example, at the 20th percentile. 20% of the data falls below 20% and 80% of the data is found at or above that value.

Students should be clear on the difference between **percentages** and **percentiles**. Students should be able to explain why the **median** is the 50th **percentile.**

To determine the **percentile rank** of a specific value, x, the values in the data set should first be ordered from least to greatest, as students practised in **S1**. The percentage of values at or below that value gives the percentile rank. Students should be encouraged to determine this rank with simple percentiles like the 50th or 75th percentile, before applying the following formula to determine percentile rank.

$$\textbf{percentile rank} = \frac{\text{number of values} \leq \text{x}}{\text{total number of values}} \times 100$$

ACHIEVEMENT INDICATORS

- Explain, using examples, percentile ranks in a context.
- Explain decisions based on a given percentile rank.
- Explain, using examples, the difference between percent and percentile rank.
- Explain the relationship between median and percentile.
- Solve a contextual problem that involves percentiles.

SCO: **S2: Analyze and describe percentiles.** [C, CN, PS, R]

Suggested Instructional Strategies

- Students can be asked to line up at the front of the classroom from tallest to shortest (or shortest to tallest). Explore the concepts of percentiles by using the heights by where they lie on the continuum. For example, use the student in the middle (or the two in the middle) to explain the connection between median and the 50^{th} percentile.
- Growth charts for infants, children and adolescents can be found online. They are based on percentiles and separated by gender. Given the appropriate chart, students can to find their own percentile rank.
- The following site gives growth charts for girls: http://www.dietitians.ca/Downloadable-Content/Public/HFA-WFA_2-19_GIRLS_EN_bw.aspx and boys: http://www.dietitians.ca/Downloadable-Content/Public/HFA-WFA_2-19_BOYS_EN_bw.aspx
- Have students research a situation of interest to them involving percentiles and report back to the class. Examples include aptitude test scores and salary ranks.
- For examples and further explanation go to: www.regentsprep.org/regents/math/algebra/ad6/quartiles.htm

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q The final exam scores for a class were:

$62, 66, 71, 75, 75, 78, 81, 83, 84, 85, 85, 87, 89, 89, 91, 92, 93, 94, 95, 99.$

Find the percentile rank for a score of 85 on this test.

$Answer\text{:}\ A\ score\ of\ 85\ is\ the\ middle\ value\ \ \therefore 85\ is\ the\ 50th\ percentile$

Q If Jason graduated 25^{th} out of a class of 150 students, what would his percentile rank be?

$Answer\text{:}\ If\ he\ was\ 25th, there\ would\ be\ 125\ students\ at\ or\ below\ him.\ p$

$\therefore his\ perentile\ rank = \frac{125}{150} \times 100 = \ \ 83.3$

Q Given the following set of data, find the number that lies at the 28^{th} percentile and the 72^{nd} percentile. $(20.1,\ 18.5,\ 21.5,\ 20.3,\ 19.0,\ 18.4,\ 18.2,\ 18.0,\ 17.6,\ 18.5,\ 16.8,\ 20.1,\ 19.0,\ 18.0,\ 18.5,\ 20.0,\ 22.0,\ 18.4)$

$Answer\text{:}\ \ 18\ numbers\ ordered -$

$16.8\ \ 17.6\ \ 18.0\ \ 18.0\ \ 18.2\ \ 18.4\ \ 18.4\ \ 18.5\ \ 18.5\ \ 18.5\ \ 19.0\ \ 19.0\ \ 20.0\ \ 20.1\ \ 20.1\ \ 20.3\ \ 21.5\ \ 22.0$

$18 \times \frac{28}{100} = 5.04 \rightarrow 5^{th} number, 18.2 \qquad 18 \times \frac{72}{100} = 12.96 \rightarrow 13^{th} number, 20.0$

SCO: **P1: Analyze and interpret problems that involve probability.** [C, CN, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Probability

P1: Analyze and interpret problems that involve probability.

Scope and Sequence of Outcomes:

Financial and Workplace Mathematics 110	Financial and Workplace Mathematics 120
	P1: Analyze and interpret problems that involve probability.

ELABORATION

Probability was last addressed in Grade 9. Students focused on the role of probability in society by looking at the probability of events occurring and examining decisions that are based on those predictions. Students were exposed to a variety of examples from daily life in which probability is used. These examples should be revisited in grade 12.

Students entering grade 12 should already understand the difference between theoretical and experimental probability (covered in grade 8). Also, they will have had experience expressing probability in fractions, percent and decimals. This outcome will add writing probability as a written statement.

Students should already know that in situations where all outcomes are equally likely:
probability of an event = P(event)= (#of favourable outcomes)/(total # of outcomes.).

This outcome includes determining the probability of an event given the **odds for** and **odds against**.

Odds for an event are written as a ratio of:
number of favourable outcomes : number of unfavourable outcomes

Odds against an event are written as a ratio of:
number of unfavourable outcomes : number of favourable outcomes

Students will apply this knowledge to determine the probability of an event, given the odds for or against that event.

For example, if an electrical circuit has $50:1$ odds against failure, the probability of a defective electrical circuit is $\frac{1}{51}$.

It should be noted that it is easy to make up games, experiments or probability problems that are too hard and in which some outcomes are more likely than others. To create fair games, all outcomes must be equally likely.

SCO: **P1: Analyze and interpret problems that involve probability.** [C, CN, PS, R]

ACHIEVEMENT INDICATORS

- Describe and explain the applications of probability; e.g., medication, warranties, insurance, lotteries, weather prediction, 100-year flood, failure of a design, failure of a product, vehicle recalls, approximation of area.
- Calculate the probability of an event based on a data set e.g., determine the probability that a randomly chosen light bulb will be defective.
- Express a given probability as a fraction, decimal, percent and in a statement.
- Explain the difference between odds and probability.
- Determine the probability of an event, given the odds for or against.
- Explain, using examples, how decisions may be based on a combination of theoretical probability calculations, experimental results and subjective judgments.
- Solve a contextual problem that involves a given probability.

Suggested Instructional Strategies

- Use newspaper articles and internet resources to find examples of the use of probability and odds in daily life.
- The work of students should be focused on situations which are familiar and contextual to them.
- Use multiple representations (verbal, numerical, visual)
- Do simulations on TI-83, or conducting simple experiments with dice, coins, etc.

SCO: **P1: Analyze and interpret problems that involve probability.** [C, CN, PS, R]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q The following data set lists the number of hours worked by twenty Grade 12 students in a week.

12	25	0	19	2
15	4	8	11	19
30	4	40	3	6
7	1	10	40	21

a) What is the probability of selecting a student who works 30 or more hours per week? Express the answer as a fraction, decimal and percent.

b) What are the odds of selecting a student who works fewer than 30 hours per week?

c) Write a probability statement in sentence form based on the answers from a) and b).

Answer: a) $\frac{1}{10}$ *or* 0.15 *or* 15%

b) 9:1

c) *If a student is chosen randomly from this group, there is a* 15% *probability that they will work at least* 30 *hours per week.*

If a student is chosen at random from this group, the odds are 9 *to* 1 *that they will work less than* 30 *hours per week.*

Q Explain the difference between odds and probability.

Answer: *Probability expresses the fraction of the time you can expect any event to occur. It will never be greater than one or less than* 0, *but the higher it is, the more probable the event is. Odds compare the number of times an event will occur to the number of times it won't occur. If the odds are* 1:1, *the outcomes is as likely to occur as not occur and there is* 50% *probability of it occurring or not.*

Q The odds of winning a prize are 1:24. What is the probability of winning? What is the probability of losing?

Answer: $P(winning) = \frac{1}{25}$ $P(losing) = \frac{24}{25}$

SCO: **RP1: Research and give a presentation on a historical event or an area of interest that involves mathematics.** [C, CN, ME, PS, R, T, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Mathematical Research Project

RP1: Research and give a presentation on a historical event or an area of interest that involves mathematics.

Scope and Sequence of Outcomes:

Financial and Workplace Mathematics 110	Financial and Workplace Mathematics 120
	RP1: Research and give a presentation on a historical event or an area of interest that involves mathematics.

ELABORATION

This purpose of this outcome is to allow students to research an area of interest to them and to demonstrate an understanding of this topic as an oral or written presentation using diagrams or illustrations. A wide range of topics can be explored by the students or a more focused topic can be determined for the class.

Whatever approach is taken it is important that the students research topics that are interesting and engaging to them, and that highlight the usefulness and relevance of mathematics in their lives. This could include budgeting, trades mathematics, mathematics in music and art, or the history of mathematical concepts.Visits to colleges, or from people in the workforce could spark an interest for students.

For the previous curriculum, teachers developed numerous resources to support the independent studies units in *Applications in Mathematics113* and *Geometry and Applications 111/112*. Many of these have been shared and can be found on the High School Mathematics Portal, listed by course, under “Support Resources”.

ACHIEVEMENT INDICATORS

- Collect primary (collected by you) or secondary data (collected by others) or information related to a topic of interest.
- Organize and present the research project, with or without technology.

SCO: **RP1: Research and give a presentation on a historical event or an area of interest that involves mathematics.** [C, CN, ME, PS, R, T, V]

Suggested Instructional Strategies

- Teachers may want to provide a list of suggested topics.
- It may be helpful to offer guiding questions such as the following when students are researching their topic.
  - Why is this topic of interest to you?
  - When researching the history of the topic consider Who? What? When? Where? Why? How?
  - Use diagrams and illustrations where appropriate, to give a detailed explanation of this event or topic of interest.
  - How does this event or topic of interest impact on the world today?
  - What careers involve the use of this knowledge or skill?
- Deadlines should be determined for the work that has to be completed, and for bringing completed work to class. Regular conferences with the teacher regarding progress are crucial.
- Teachers may want to develop a rubric with students so that students take more ownership in the project.
- The assessment for this outcome could be the submission and/or presentation of the final project. The method of presentation could vary – students could present to the whole class or projects could be set up as a math fair.
- Self-assessment could be used to provide students opportunity for reflection.
- Have students provide feedback to their peers after the presentation using exit slips. For example, students could use index cards to write about something new they learned, suggestions for improvement, and overall feedback.

SUMMARY OF CURRICULUM OUTCOMES

Financial and Workplace Mathematics 120

[C] Communication, **[PS]** Problem Solving, **[CN]** Connections, **[R]** Reasoning,
[ME] Mental Mathematics and Estimation, **[T]** Technology **[V]** Visualization

Measurement

General Outcome: Develop spatial sense through direct and indirect measurement.

Specific Outcome
M1: Demonstrate an understanding of the limitations of measuring instruments, including precision, accuracy, uncertainty, tolerance, and solve problems.

Geometry

General Outcome: Develop spatial sense.

Specific Outcomes:
G1: Solve problems that involve triangles, quadrilaterals, regular polygons.
G2: Solve problems by using the sine law and cosine law, excluding the ambiguous case.
G3: Demonstrate an understanding of transformations o a 2-D shape or a 3-D object, including translations, rotations, reflections, dilations.

Number

General Outcome: Develop number sense and critical thinking skills.

Specific Outcomes:
N1: Analyze puzzles and games that involve logical reasoning, using problem-solving strategies.
N2: Critique the viability of small business options by considering expenses, sales, profit or loss.

Algebra

General Outcome: Develop algebraic reasoning.

Specific Outcomes
A1: Demonstrate an understanding of linear relations by recognizing patterns and trends, graphing, creating tables of values, writing equations, interpolating and extrapolating, solving problems.

Statistics

General Outcome: Develop statistical reasoning.
Specific Outcomes
S1: Solve problems that involve measures of central tendency, including mean, median, mode, weighted mean, trimmed mean.
S2: Analyze and describe percentiles.

Probability

General Outcome: Develop critical thinking skills related to uncertainty.

Specific Outcomes
P1: Analyze and interpret problems that involve probability.

Mathematical Research Project

General Outcome: Develop an appreciation of the role of mathematics in society.

Specific Outcomes
RP1: Research and give a presentation on a historical event or an area of interest that involves mathematics .

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