Foundations of Mathematics 110

Curriculum

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Curriculum Overview for Grades 10-12 Mathematics

BACKGROUND AND RATIONALE

Mathematics curriculum is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society.

It is essential the mathematics curriculum reflects current research in mathematics instruction. To achieve this goal, *The Common Curriculum Framework for Grades 10–12 Mathematics: Western and Northern Canadian Protocol* has been adopted as the basis for a revised mathematics curriculum in New Brunswick. The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others.

The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the NCTM.

There is an emphasis in the New Brunswick curriculum on particular key concepts at each grade which will result in greater depth of understanding and ultimately stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

The intent of this document is to clearly communicate high expectations for students in mathematics education to all education partners. Because of the emphasis placed on key concepts at each grade level, time needs to be taken to ensure mastery of these concepts. *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM Principles and Standards, 2000).*

BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

The New Brunswick Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

- mathematics learning is an active and constructive process;
- learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates;
- learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort; and
- learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and aspirations.

Students construct their understanding of mathematics by developing meaning based on a variety of learning experiences. This meaning is best developed when learners encounter mathematical experiences that proceed from simple to complex and from the concrete to the abstract. The use of manipulatives, visuals and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels of understanding students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions also provide essential links among concrete, pictorial and symbolic representations of mathematics. The learning environment should value, respect and address all students' experiences and ways of thinking, so that students are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. It is important to realize that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood.

Goals for Mathematically Literate Students

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding through:

- taking risks
- thinking and reflecting independently
- sharing and communicating mathematical understanding
- solving problems in individual and group projects
- pursuing greater understanding of mathematics
- appreciating the value of mathematics throughout history.

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Striving toward success, and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Diverse Cultural Perspectives

Students come from a diversity of cultures, have a diversity of experiences and attend schools in a variety of settings including urban, rural and isolated communities. To address the diversity of knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students, a variety of teaching and assessment strategies are required in the classroom. These strategies must go beyond the incidental inclusion of topics and objects unique to a particular culture.

For many First Nations students, studies have shown a more holistic worldview of the environment in which they live (Banks and Banks 1993). This means that students look for connections and learn best when mathematics is contextualized and not taught as discrete components. Traditionally in Indigenous culture, learning takes place through active participation and little emphasis is placed on the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is important that teachers understand and respond to both verbal and non-verbal cues to optimize student learning and mathematical understandings.

Instructional strategies appropriate for a given cultural or other group may not apply to all students from that group, and may apply to students beyond that group. Teaching for diversity will support higher achievement in mathematics for all students.

Adapting to the Needs of All Learners

Teachers must adapt instruction to accommodate differences in student development as they enter school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom. The reality of individual student differences must not be ignored when making instructional decisions.

As well, teachers must understand and design instruction to accommodate differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

Universal Design for Learning

The New Brunswick Department of Education and Early Childhood Development‘s definition of inclusion states that every child has the right to expect that his or her learning outcomes, instruction, assessment, interventions, accommodations, modifications, supports, adaptations, additional resources and learning environment will be designed to respect his or her learning style, needs and strengths.

Universal Design for Learning is a "…framework for guiding educational practice that provides flexibility in the ways information is presented, in the ways students respond or demonstrate knowledge and skills, and in the ways students are engaged." It also "...reduces barriers in instruction, provides appropriate accommodations, supports, and challenges, and maintains high achievement expectations for all students, including students with disabilities and students who are limited English proficient" (CAST, 2011).

In an effort to build on the established practice of differentiation in education, the Department of Education and Early Childhood Development supports *Universal Design for Learning* for all students. New Brunswick curricula are created with universal design for learning principles in mind. Outcomes are written so that students may access and represent their learning in a variety of ways, through a variety of modes. Three tenets of universal design inform the design of this curriculum. Teachers are encouraged to follow these principles as they plan and evaluate learning experiences for their students:

- **Multiple means of representation:** provide diverse learners options for acquiring information and knowledge
- **Multiple means of action and expression:** provide learners options for demonstrating what they know
- **Multiple means of engagement:** tap into learners' interests, offer appropriate challenges, and increase motivation

For further information on *Universal Design for Learning*, view online information at http://www.cast.org/.

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, and physical education.

NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: **change**, **constancy**, **number sense**, **patterns**, **relationships**, **spatial sense** and **uncertainty**.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain, (Steen, 1990, p. 184).

Students need to learn that new concepts of mathematics as well as changes to already learned concepts arise from a need to describe and understand something new. Integers, decimals, fractions, irrational numbers and complex numbers emerge as students engage in exploring new situations that cannot be effectively described or analyzed using whole numbers.

Students best experience change to their understanding of mathematical concepts as a result of mathematical play.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Many important properties in mathematics do not change when conditions change. Examples of constancy include:

- the conservation of equality in solving equations
- the sum of the interior angles of any triangle
- the theoretical probability of an event.

Number Sense

Number sense, which can be thought of as deep understanding and flexibility with numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p. 146). Continuing to foster number sense is fundamental to growth of mathematical understanding.

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Students with strong number sense are able to judge the reasonableness of a solution, describe relationships between different types of numbers, compare quantities and work with different representations of the same number to develop a deeper conceptual understanding of mathematics.

Number sense develops when students connect numbers to real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. Evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing mathematically rich tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all of the mathematical topics, and it is through the study of patterns that students can make strong connections between concepts in the same and different topics.

Working with patterns also enables students to make connections beyond mathematics. The ability to analyze patterns contributes to how students understand their environment. Patterns may be represented in concrete, visual, auditory or symbolic form. Students should develop fluency in moving from one representation to another.

Students need to learn to recognize, extend, create and apply mathematical patterns. This understanding of patterns allows students to make predictions and justify their reasoning when solving problems. Learning to work with patterns helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics.

Relationships

Mathematics is used to describe and explain relationships. Within the study of mathematics, students look for relationships among numbers, sets, shapes, objects, variables and concepts. The search for possible relationships involves collecting and analyzing data, analyzing patterns and describing possible relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense involves the representation and manipulation of 3-D objects and 2-D shapes. It enables students to reason and interpret among 3-D and 2-D representations.

Spatial sense is developed through a variety of experiences with visual and concrete models. It offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions.

Spatial sense is also critical in students' understanding of the relationship between the equations and graphs of functions and, ultimately, in understanding how both equations and graphs can be used to represent physical situations.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately. This language must be used effectively and correctly to convey valuable messages.

ASSESSMENT

Ongoing, interactive assessment (*formative assessment*) is essential to effective teaching and learning. Research has shown that formative assessment practices produce significant and often substantial learning gains, close achievement gaps and build students' ability to learn new skills (Black & William, 1998, OECD, 2006). Student involvement in assessment promotes learning. Interactive assessment, and encouraging self-assessment, allows students to reflect on and articulate their understanding of mathematical concepts and ideas.

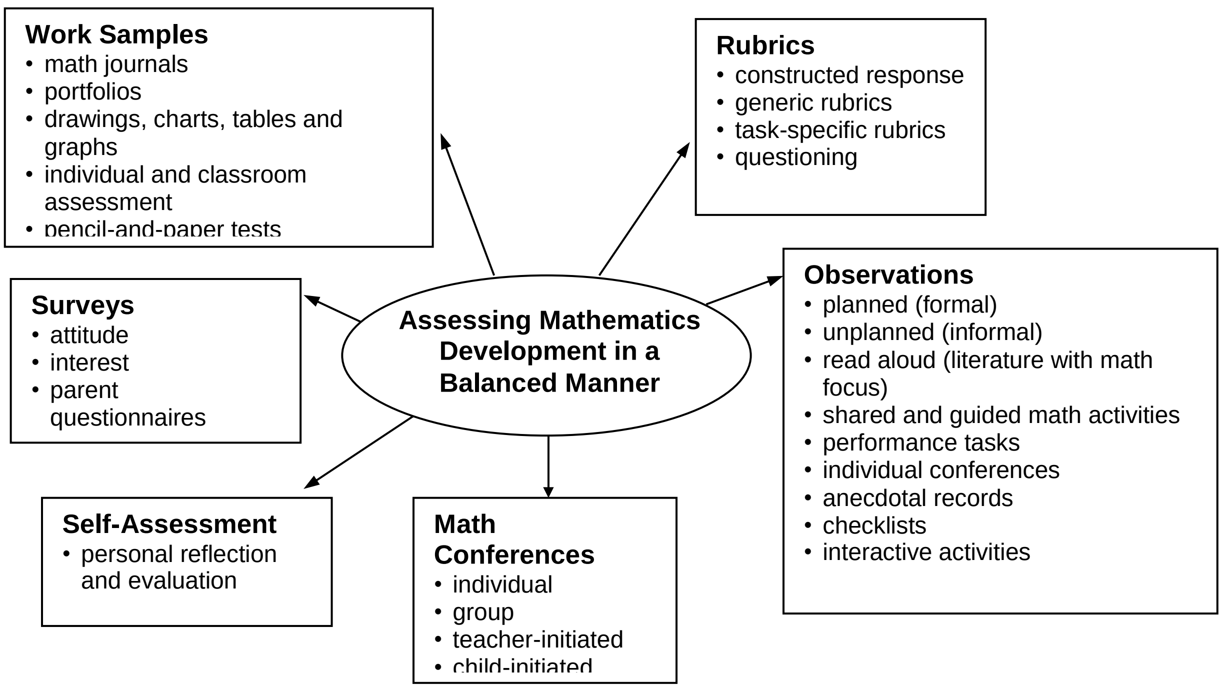
Assessment in the classroom includes:

- providing clear goals, targets and learning outcomes
- using exemplars, rubrics and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning (Davies, 2000)

Formative assessment practices act as the scaffolding for learning which, only then, can be measured through summative assessment. *Summative assessment,* or assessment *of* learning, tracks student progress, informs instructional programming and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning and produce achievement gains.

Student assessment should:

- align with curriculum outcomes
- use clear and helpful criteria
- promote student involvement in learning mathematics during and after the assessment experience
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction (adapted from: NCTM, Mathematics Assessment: A practical handbook, 2001, p.22)



CONCEPTUAL FRAMEWORK FOR 10-12 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.

GRADE TOPICS	10 11 12
The topics of study vary in the courses for grades 10–12 mathematics. Topics in the pathways include: • *Algebra* • *Financial Mathematics* • *Geometry* • *Logical Reasoning* • *Mathematics Research Project* • *Measurement* • *Number* • *Permutations, Combinations and Binomial Theorem* • *Probability* • *Relations and Functions* • *Statistics* • *Trigonometry*	**GENERAL OUTCOMES** **SPECIFIC OUTCOMES** **ACHIEVEMENT INDICATORS**

NATURE OF MATHEMATICS

Change
Constancy
Number Sense
Patterns
Relationships
Spatial Sense
Uncertainty

MATHEMATICAL PROCESSES:
Communication, Connections, Mental Mathematics and Estimation, Problem Solving, Reasoning, Technology, Visualization

MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.
Students are expected to:

- communicate in order to learn and express their understanding of mathematics (Communications: C)
- develop and apply new mathematical knowledge through problem solving (Problem Solving: PS)
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines (Connections: CN)
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation: ME)
- select and use technologies as tools for learning and solving problems (Technology: T)
- develop visualization skills to assist in processing information, making connections and solving problems (Visualization: V).
- develop mathematical reasoning (Reasoning: R)

The New Brunswick Curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Emerging technologies enable students to engage in communication beyond the traditional classroom to gather data and share mathematical ideas.

Problem Solving [PS]

Problem solving is one of the key processes and foundations within the field of mathematics. Learning through problem solving should be the focus of mathematics at all grade levels. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts. Problem solving is to be employed throughout all of mathematics and should be embedded throughout all the topics.

When students encounter new situations and respond to questions of the type, *How would you...?* or *How could you ...?,* the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. Students should not know the answer immediately. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement. Students will be engaged if the problems relate to their lives, cultures, interests, families or current events.

Both conceptual understanding and student engagement are fundamental in moulding students' willingness to persevere in future problem-solving tasks. Problems are not just simple computations embedded in a story, nor are they contrived. They are tasks that are rich and open-ended, so there may be more than one way of arriving at a solution or there may be multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill or understanding. Problem solving can vary from being an individual activity to a class (or beyond) undertaking.

In a mathematics class, there are two distinct types of problem solving: solving contextual problems outside of mathematics and solving mathematical problems. Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem.

Problem solving can also be considered in terms of engaging students in both inductive and deductive reasoning strategies. As students make sense of the problem, they will be creating conjectures and looking for patterns that they may be able to generalize. This part of the problem-solving process often involves inductive reasoning. As students use approaches to solving the problem they often move into mathematical reasoning that is deductive in nature. It is crucial that students be encouraged to engage in both types of reasoning and be given the opportunity to consider the approaches and strategies used by others in solving similar problems.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk-takers.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"*Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*" (Caine and Caine, 1991, p. 5).

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental mathematics" (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "*become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving*" (Rubenstein, 2001).

Mental mathematics "*provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers*" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to learn which strategy to use and how to use it.

Technology [T]

Technology can be used effectively to contribute to and support the learning of a wide range of mathematical outcomes. Technology enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- generate and test inductive conjectures
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- model situations
- develop number and spatial sense.

Technology contributes to a learning environment in which the curiosity of students can lead to rich mathematical discoveries at all grade levels. The use of technology should not replace mathematical understanding. Instead, technology should be used as one of a variety of approaches and tools for creating mathematical understanding.

Visualization [V]

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world” (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers. Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and spatial reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills.
Measurement sense includes the ability to determine when to measure and when to estimate and involves knowledge of several estimation strategies (Shaw and Cliatt, 1989, p. 150).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations. It is through visualization that abstract concepts can be understood concretely by the student. Visualization is a foundation to the development of abstract understanding, confidence and fluency.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking.

Questions that challenge students to think, analyze and synthesize help them develop an understanding of mathematics. All students need to be challenged to answer questions such as, *Why do you believe that's true/correct?* or *What would happen if ….*

Mathematical experiences provide opportunities for students to engage in inductive and deductive reasoning. Students use inductive reasoning when they explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Students use deductive reasoning when they reach new conclusions based upon the application of what is already known or assumed to be true. The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines.

ESSENTIAL GRADUATION LEARNINGS

Graduates from the public schools of Atlantic Canada will be able to demonstrate knowledge, skills, and attitudes in the following essential graduation learnings. These learnings are supported through the outcomes described in this curriculum document.

Aesthetic Expression
Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.

Citizenship
Graduates will be able to assess social, cultural, economic, and environmental interdependence in a local and global context.

Communication
Graduates will be able to use the listening, viewing, speaking, reading and writing modes of language(s) as well as mathematical and scientific concepts and symbols to think, learn, and communicate effectively.

Personal Development
Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.

Problem Solving
Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language, mathematical, and scientific concepts.

Technological Competence
Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems

PATHWAYS AND TOPICS

The Common Curriculum Framework for Grades 10–12 Mathematics on which the New Brunswick Grades 10-12 Mathematics curriculum is based, includes pathways and topics rather than strands as in *The Common Curriculum Framework for K–9 Mathematics*. In New Brunswick all Grade 10 students share a common curriculum covered in two courses: *Geometry, Measurement and Finance 10* and *Number, Relations and Functions 10*. Starting in Grade 11, three pathways are available: *Finance and Workplace*, *Foundations of Mathematics*, and *Pre-Calculus*.

Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. Students are encouraged to cross pathways to follow their interests and to keep their options open. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings.

Goals of Pathways

The goals of all three pathways are to provide prerequisite attitudes, knowledge, skills and understandings for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Design of Pathways

Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study and for direct entry into the work force.

The content of each pathway has been based on the *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings* and on consultations with mathematics teachers.

Financial and Workplace Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into some college programs and for direct entry into the work force. Topics include financial mathematics, algebra, geometry, measurement, number, statistics and probability.

Foundations of Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, number, logical reasoning, relations and functions, statistics and probability.

Pre-calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Students develop a function tool kit including quadratic, polynomial, absolute value, radical, rational, exponential, logarithmic and trigonometric functions. They also explore systems of equations and inequalities, degrees and radians, the unit circle, identities, limits, derivatives of functions and their applications, and integrals.

Outcomes and Achievement Indicators

The New Brunswick Curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes and achievement indicators.

General Curriculum Outcomes (GCO) are overarching statements about what students are expected to learn in each course.

Specific Curriculum Outcomes (SCO) are statements that identify the specific knowledge, skills and understanding that student are required to attain by the end of a given course. The word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for clarification and are not requirements that must be addressed to fully meet the learning outcome. The word *and* used in an outcome indicates that both ideas must be addressed to fully meet the learning outcome, although not necessarily at the same time or in the same question.

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome. The word *and* used in an achievement indicator implies that both ideas should be addressed at the same time or in the same question.

Instructional Focus

Each pathway in *The Common Curriculum Framework for Grades 10–12 Mathematics* is arranged by topics. Students should be engaged in making connections among concepts both within and across topics to make mathematical learning experiences meaningful. Teachers should consider the following points when planning for instruction and assessment.

- The mathematical processes that are identified with the outcome are intended to help teachers select effective pedagogical approaches for the teaching and learning of the outcome.
- All seven mathematical processes must be integrated throughout teaching and learning approaches, and should support the intent of the outcomes.
- Wherever possible, meaningful contexts should be used in examples, problems and projects.
- Instruction should flow from simple to complex and from concrete to abstract.
- The assessment plan for the course should be a balance of assessment for learning, assessment as learning and assessment of learning.

The focus of student learning should be on developing a conceptual and procedural understanding of mathematics. Students' conceptual understanding and procedural understanding must be directly related.

Pathways and Courses

The graphic below summarizes the pathways and courses offered.

Mathematics K-9

Grade 10

- *2 x 90 hr courses; required to pass both*
- *May be taken in any order or in the same semester*

Geometry, Measurement and Finance 10

Number, Relations and Functions 10

Grade 11

- *3 x 90 hr courses offered in 3 pathways*
- *Students are required to pass at least one of "Financial and Workplace Mathematics 11" **or** "Foundations of Mathematics 11".*
- *Pre-requisite Grade 10 course(s) must be passed before taking Grade 11 courses.*

Financial and Workplace Mathematics 110

Pre-requisite:
Geometry, Measurement and Finance 10

Foundations of Mathematics 110

Pre-requisites:
*Geometry, Measurement and Finance 1 **and** Number, Relations and Functions 10*

Pre-Calculus 110

Pre-requisite or Co-requisite:
Foundations of Mathematics 110

Grade 12

- *5 x 90 hr courses offered in 3 pathways*
- *Pre-requisite Grade 11 or Grade 12 course must be passed before taking Grade 12 courses.*

Financial and Workplace Mathematics 120

Pre-requisite:
*Financial and Workplace Mathematics 110 **or** Foundations of Mathematics 110*

Foundations of Mathematics 120

Pre-requisite: Foundations of Mathematics 110

Pre-Calculus A 120

Pre-requisite: Pre-Calculus 110

Pre-Calculus B 120

Pre-requisite or Co-requisite:
Pre-Calculus A 120

Calculus 120

Pre-requisites:
*Pre-Calculus A 120 **and** Pre-Calculus B 120*

SUMMARY

The Conceptual Framework for Grades 10–12 Mathematics describes the nature of mathematics, the mathematical processes, the pathways and topics, and the role of outcomes and achievement indicators in grades 10–12 mathematics. Activities that take place in the mathematics classroom should be based on a problem-solving approach that incorporates the mathematical processes and leads students to an understanding of the nature of mathematics.

CURRICULUM DOCUMENT FORMAT

This guide presents the mathematics curriculum by grade level so that a teacher may readily view the scope of the outcomes which students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings at a particular grade level are part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The heading of each page gives the General Curriculum Outcome (GCO), and Specific Curriculum Outcome (SCO). The key for the mathematical processes follows. A Scope and Sequence is then provided which relates the SCO to previous and next course SCO's. For each SCO, Elaboration, Achievement Indicators, Suggested Instructional Strategies, and Suggested Activities for Instruction and Assessment are provided. For each section, the *Guiding Questions* should be considered.

GCO: General Curriculum Outcome
SCO: Specific Curriculum Outcome

Mathematical Processes

[C] Communication Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math **[T]** and Estimation

Scope and Sequence

Previous Grade or Course SCO's	Current Grade SCO	Following Grade or Course SCO's

Elaboration

Describes the "big ideas" to be learned and how they relate to work in previous Grades

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Achievement Indicators

Describes observable indicators of whether students have met the specific outcome

Guiding Questions:
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

GCO: General Curriculum Outcome
SCO: Specific Curriculum Outcome

Suggested Instructional Strategies

General approach and strategies suggested for teaching this outcome

Guiding Questions
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

Suggested Activities for Instruction and Assessment

Some suggestions of specific activities and questions that can be used for both instruction and assessment.

Guiding Questions
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Guiding Questions
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

Foundations of Mathematics 110

Specific Curriculum Outcomes

SCO	**LR1: Analyze and prove conjectures using logical reasoning, to solve problems.** [C, CN, PS, R]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Logical Reasoning

LR1: Analyze and prove conjectures using logical reasoning, to solve problems.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Eleven/Twelve
G1: Analyze puzzles and games that involve spatial reasoning, using a variety of problem-solving strategies. *(GMF10)*	**LR1:** Analyze and prove conjectures using logical reasoning, to solve problems.	**LR2:** Solve problems that involve the application of set theory. *(FM12)* **LR3:** Solve problems that involve conditional statements. *(FM12)*

ELABORATION

Students have had no formal instruction with this topic in previous mathematics courses. This outcome introduces them to **inductive reasoning**, through the investigation of geometric situations and observation of patterns to make conjectures. They will be required to justify the reasoning used for all **conjectures**.

Students will be given situations to explore and make conjectures, where these situations lead to a contradiction. They will also explore the role that **counterexamples** play in disproving conjectures.

This outcome also introduces students to **deductive reasoning** and the notion of formal **proof**. Deductive and inductive reasoning will be compared, and the **two-column proof** format introduced.

Conjecture – *Based on evidence that you have gathered, the more support you have, the stronger your conjecture, but it does not necessarily prove it.*

Inductive reasoning – *draws a general conclusion by observing patterns and identifying properties from specific examples.*

Deductive reasoning – *draws a specific conclusion through logical reasoning by starting with a general assumption that is known to be true.*

Proof – *is a mathematical argument showing that a statement is valid in all cases.*

Two-column proof – *is a presentation of a logical argument involving deductive reasoning, in which the statements of the argument are written in one column and the justifications for the statements are written in the other column.*

Transitive property – *If two quantities are equal to the same quantity, then they are equal to each other. If $a = b$ and $b = c$, then $a = c$.*

Counterexample – *is an example that invalidates a conjecture*

Students will examine arguments and proofs and judge whether or not the reasoning presented is valid. They will determine if there is an error in the reasoning used and if so, identify the error.

Contextual problems will be solved using inductive or deductive reasoning.

SCO	**LR1: Analyze and prove conjectures using logical reasoning, to solve problems.** [C, CN, PS, R]

ACHIEVEMENT INDICATORS

- Make conjectures by observing patterns and identifying properties, and justify the reasoning.
- Explain why logical reasoning may lead to a false conjecture.
- Compare, using examples, inductive and deductive reasoning
- Provide and explain a counter example to disprove a given conjecture.
- Prove algebraic and number relationships, such as divisibility rules, number properties, mental mathematics strategies or algebraic number tricks.
- Prove a conjecture, using deductive reasoning (not limited to two column proofs).
- Determine if a given argument is valid, and justify the reasoning.
- Identify errors in a given proof; e.g., a proof that ends with $2 = 1$.
- Solve a contextual problem involving inductive or deductive reasoning.

Suggested Instructional Strategies

- Give students a copy of Pascal's triangle and ask them to identify patterns and make conjectures.
- Examine optical illusions and how they "trick" your eyes to provide an opportunity to raise the issue of valid versus invalid conjectures.
- Introduce students to some common fallacious proofs and have them identify the flaw(s) in the reasoning.
- Use a stereotype to introduce the concept of "counterexample".
- Find examples of different types of reasoning on the internet e.g., for basic examples of inductive reasoning go to http://www.basic-mathematics.com/examples-of-inductive-reasoning.html

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Joe says that if a quadrilateral has 4 sides of equal length then it must be a square. Is Joe correct? Explain.

Answer: Joe is incorrect because you can have a figure of 4 *equal sides with interior angles not equal to* $90°$. *For example a rhombus, or a parallelogram.*

Q Give a counterexample to the following conjecture:

The difference between two positive numbers is always a positive number.

Answer: $(+11) - (+6) = 5, \quad (+6) - (+11) = -5$

Act Have students construct different sized polygons (a triangle up to an octagon). Measure the interior angles and find the sum of the interior angles for each polygon. Make a conjecture to determine the sum of the interior angles for a polygon with "n" sides.

SCO	**LR2: Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.** [CN, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

LR2: Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
G1: Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. *(GMF10)*	**LR2:** Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.	**LR1:** Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. *(FWM12, FM12)*

ELABORATION

Puzzles and games provide opportunities to explore patterns and to link spatial and numerical concepts. In grade 10, students focused on playing and analyzing puzzles and games that involved spatial reasoning. They discussed strategies used to solve a puzzle or win a game. In grade 11, they will extend these skills to games and puzzles which involve numerical reasoning.

Students will use familiar problem solving strategies to explain and verify a strategy to solve the puzzle or win the game. All puzzles/games should utilize mathematical concepts that students are familiar with from previous courses so that the emphasis is placed on reasoning.

This outcome provides tremendous opportunity to differentiate as students tackle games or puzzles appropriate to their current level of ability and understanding. Teachers should try games in advance as the difficulty and the instructions to games or puzzles are not always clear.

It is intended that this outcome be integrated throughout the course giving students a related activity every week or two as time permits

ACHIEVEMENT INDICATORS

- Determine, explain and verify a strategy to solve a puzzle or to win a game such as:
  - guess and check
  - look for a pattern
  - make a systematic list
  - draw or model
  - eliminate possibilities
  - simplify the original problem
  - work backward
  - develop alternative approaches.
- Develop alternative approaches to solving puzzles.
- Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.
- Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

SCO	**LR2: Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.** [CN, PS, R, V]

Suggested Instructional Strategies

- Have students look for numerical or other patterns and then develop a strategy to fit these patterns.
- Have students develop a game for classmates to play.
- Using a known game, change a rule or parameter and explain how it affects the outcome of the game.
- Find a game online and critique the quality of the game.
- Plan to have a "games and puzzles day" every other week. Students should switch partners periodically to provide opportunities for new strategies to be shared.
- Have students keep a games and puzzles journal that they are required to write in every "games and puzzles day". Have them reflect on the strategies they used to solve the puzzle or win the game. The following gives an example of how the journal could be set up.

Games Journal			
Date	Game	Win or Lose OR Score	Explain your strategies

SCO	**LR2: Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.** [CN, PS, R, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Solve the following puzzles:

a)

	2	4	
1			3
4			2
	1	3	

b)

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

c)

80×		3	5−		2÷
	11+		1−		
9×	2	3−		30×	
		11+		2÷	
6	8×		13+		8+
10×				1	

Answers:

a)

3	2	4	1
1	4	2	3
4	3	1	2
2	1	3	4

b)

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

c)

80× 5	4	3 3	5− 1	6	2÷ 2
4	11+ 6	5	1− 2	3	1
9× 3	2 2	3− 1	4	30× 5	6
1	3	11+ 6	5	2÷ 2	4
6 6	8× 1	2	13+ 3	4	8+ 5
10× 2	5	4	6	1 1	3

Act The internet is a good source form numerical reasoning problems or games. Here are a few examples of useful sites:
http://nlvm.usu.edu/
http://samgine.com/free/number-puzzles/
http://www.fibonicci.com/numeracy/number-sequences-test/medium/
http://www.mindjolt.com
http://education.jlab.org/nim/index.html
http://www.brocku.ca/caribou/games/game_menu.php
http://dtai.cs.kuleuven.be/projects/ALP/newsletter/archive_93_96/humour/index-num.html
www.combinationlock.com

SCO **G1: Derive proofs that involve the properties of angles and triangles.** [CN, R, V]

[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Math
[T] Technology **[V]** Visualization **[R]** Reasoning and Estimation

Geometry

G1: Derive proofs that involve the properties of angles and triangles.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
G4: Solve problems that involve angle relationships between parallel, perpendicular and transversal lines. *(GMF10)*	**G1:** Derive proofs that involve the properties of angles and triangles.	

ELABORATION

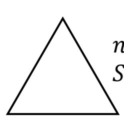
In Grade 6 students were introduced to congruent triangles (equal corresponding sides and angles) but have not developed this topic further in later grades.

In Grade 10, students were introduced to the concepts of complementary and supplementary angles, investigated congruent and supplementary angles, and solved problems involving parallel, perpendicular and transversal lines, and pairs of angles formed between them.

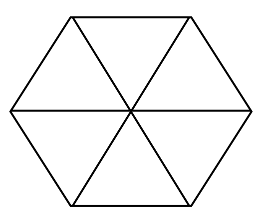
In Grade 11 the focus is on using inductive and deductive reasoning to generalize angle relationships and to derive proofs. For angles formed by transversals and parallel lines students will build on the work done in Grade 10 to develop generalizations for relationships between pairs of angles, and will prove properties of angles including the sum of the angles in a triangle.

Some proofs involving parallel lines will require students to prove triangles congruent so the necessary and sufficient conditions for congruency will need to be taught. These include Side-Angle-Side (SAS), Angle-Side-Angle (ASA), Side-Side-Side (SSS), and Right Angle-Hypotenuse-Side (RHS). Congruent triangles can also be defined as similar triangles with a 1:1 correspondence of sides. Students will use deductive reasoning to prove the relationships between sides and/or angles using congruent triangle properties.

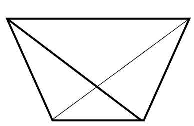
Students will also use inductive reasoning to generalize a rule for the relationship between the number of sides in a polygon and the sum of the interior angles.



$n = 3$
$Sum\ of\ angles\ =\ 180°$



$n = 6$
$Sum\ of\ angles\ in\ each\ triangle = 180°$
$Sum\ of\ angles\ in\ centre = 360°\ (full\ rotation)$
$\therefore Sum\ of\ interior\ angles\ of\ hexagon$
$= 180° \times 6\ - 360°$
$= (180° \times 6) - (180° \times 2)$
$= 180°(6 - 2)$
$= 720°$



$n = 4$
$Sum\ of\ interior\ angles\ of\ quadrilateral$
$=\ 180° \times 4 - 360°$
$= (180° \times 4) - (180° \times 2)$
$= 180°(4 - 2)$
$= 360°$

$Sum\ of\ interior\ angles\ of\ a\ convex\ polygon$
$= 180°(n - 2)$

SCO **G1: Derive proofs that involve the properties of angles and triangles.** [CN, R, V]

ACHIEVEMENT INDICATORS

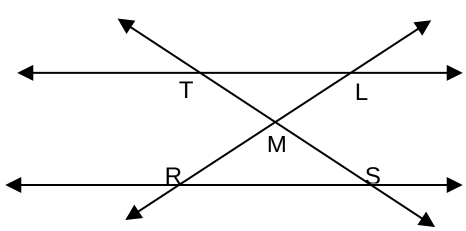
- Generalize, using inductive reasoning, the relationships between pairs of angles formed by transversals and parallel lines, with or without technology.
- Prove, using deductive reasoning, properties of angles formed by transversals and parallel lines, including the sum of the angles in a triangle.
- Prove, using deductive reasoning, relationships between sides and/or angles using congruent triangle properties.
- Generalize, using inductive reasoning, a rule for the relationship between the sum of the interior angles and the number of sides (*n*) in a polygon, with or without technology.
- Identify and correct errors in a given proof of a property involving angles and/or congruent triangles.
- Verify, with examples, that if lines are not parallel, the angle properties do not apply.

Suggested Instructional Strategies

- Examine proofs of other angle relationships in polygons and correct errors (such as Proof of the Isosceles Triangle Theorem, Proof of The Exterior Angles of a Polygon Theorem,...).
- Use Geometer's sketchpad (site license required) or GeoGebra (free download) http://www.geogebra.org/cms/en/download to investigate angle properties.
- The following sites provide interactive activities that can be used to investigate the angles properties.
  http://math4teaching.com/wp-content/uploads/2011/05/transversal2.html
  http://math4teaching.com/wp-content/uploads/2011/05/transversal-1.html

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

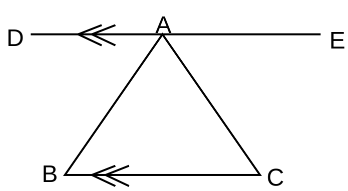
Q If $TL \parallel RS$ and M is the midpoint of LR, prove that M is also the midpoint of TS.



Answer:

$\overline{RM} = \overline{ML}$	Given
$\angle RMS = \angle LMT$	Opposite Angles
$\angle TLR = \angle LRS$	Alternate Angles
$\therefore \triangle TML \cong \triangle SMR$	ASA
$\therefore TM = MS$	$\triangle$ congruency
M is the midpoint	

Q In the diagram below, $DE \parallel BC$. Prove that the sum of the measures of the interior angles of triangle ABC is $180°$.

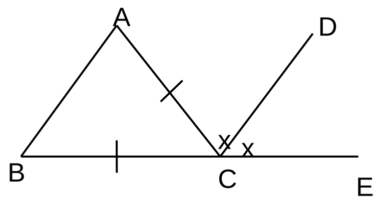


Answer:

$\angle DAB = \angle ABC$	Alternate Angles
$\angle EAC = \angle ACE$	Alternate Angles
$\angle BAC = 180° - \angle DAB - \angle CAE$	Supplementary Angles
$\therefore \angle ABC + \angle ACB + \angle BAC = 180°$	

SCO	**G1: Derive proofs that involve the properties of angles and triangles.** [CN, R, V]

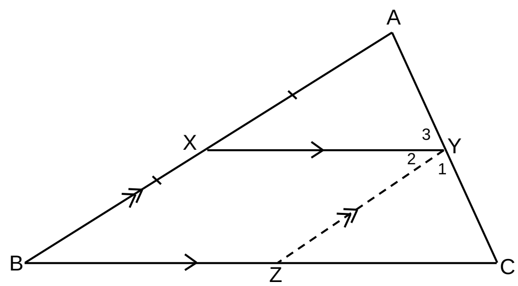
Q Given the information in the diagram below, prove that $AB \parallel DC$



Answer:

$\angle A = \angle B$ Isoceles Triangle
$\angle ACB = 180° - 2(\angle A)$ Sum of angles of a triangle
$\angle ACB = 180° - 2(x)$ Sum of angles of a triangle
$\therefore 2x = 2\angle A$ Transitive Property
$x = \angle A = \angle B$ Alternate Angles, Corresponding Angles
$\therefore AB \parallel DC$

Q In the diagram below, $\parallel BC$, $YZ \parallel AB$, and $AX = BX$. Prove that Y is the mid-point of AC.



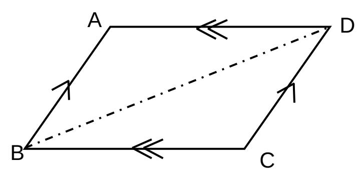
Answer:

$\angle XYZ = \angle YZC = \angle 2$ Alternate Angles
$\angle AXY = \angle XYZ = \angle 2$ Alternate Angles
$\angle AYX = \angle YCZ = \angle 3$ Corresponding Angles
$\angle XAY = \angle ZYC = \angle 1$
$= 180° - \angle 2$
$- \angle 3$ Sum of angles of a triangle
$\overline{BX} = \overline{ZY}$ BXYZ is a parallelogram
$\overline{XA} = \overline{ZY}$ Given
$\therefore \triangle AXY \cong \triangle YZC$ ASA

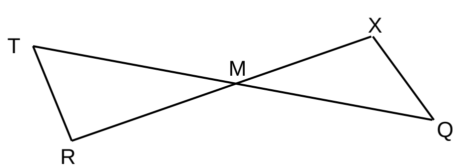
Q Prove that the opposite sides of a parallelogram are equal.

Answer:

$AB \parallel DC$ Given
$AD \parallel BC$ Given
$\angle ADB = \angle CBD$ Alternate Angles
$\angle ABD = \angle CDB$ Alternate Angles
$\overline{BD} = \overline{DB}$ Common side
$\therefore \triangle ADB \cong \triangle CBD$ ASA
$\therefore \overline{AB} = \overline{CD}$ *and* $\overline{DA} = \overline{BC}$ Congruency



Q In the diagram below $QM = TM$ and $XM = RM$. Prove that $XQ \parallel RT$.

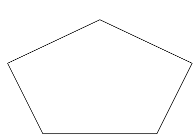


Answer:

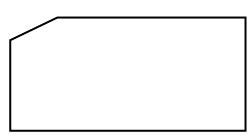
$QM = TM$ Given
$XM = RM$ Given
$\angle TMR = \angle QMX$ Opposite Angles
$\therefore \triangle TMR \cong \triangle QMX$ ASA
$\angle TRM = \angle QXM$ Congruent Triangles
$\therefore XQ \parallel RT$

Q Find the sum of the interior angles of the following shapes.

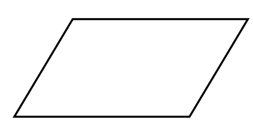
a)



b)



c)



Answers: *a)* $180°(5-2) = 540°$ *b)* $540°$ *c)* $180°(4-2) = 360°$

SCO	**G2: Solve problems that involve the properties of angles and triangles.** [CN, PS, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

G2: Solve problems that involve the properties of angles and triangles.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
G4: Solve problems that involve angle relationships between parallel, perpendicular and transversal lines. *(GMF10)*	**G2:** Solve problems that involve the properties of angles and triangles.	

ELABORATION

In the previous outcome students have used deductive reasoning to prove properties of angles that involve parallel lines, transversals and triangles.

In this outcome student will learn to construct parallel lines using only a compass or a protractor and explain how this method ensures lines are parallel.

Students will practice using the properties of angles to determine angle measures of transversals and triangles, and conversely to determine if lines are parallel given the angle measurements. They will use these angle relationships to solve contextual problems.

ACHIEVEMENT INDICATORS

- Determine the measures of angles in a diagram that involves parallel lines, angles and triangles, and justify the reasoning.
- Identify and correct errors in a given solution to a problem that involves the measures of angles.
- Solve a contextual problem that involves angles or triangles.
- Construct parallel lines, using only a compass or a protractor, and explain the strategy used.
- Determine if lines are parallel, given the measure of an angle at each intersection formed by the lines and a transversal.

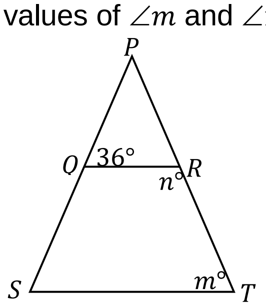
SCO	**G2: Solve problems that involve the properties of angles and triangles.** [CN, PS, V]

Suggested Instructional Strategies

- Have students construct parallel lines using a straight edge and a compass (instructions given in the core resource p.73-74). Using the properties of transversals of parallel lines and congruent triangles, have them prove that they have drawn parallel lines.
- Having proven angle properties in the last outcome, time should be allowed for students to practice applying these properties to a variety of theoretical and contextual problems.
- Online sites such as the following can be helpful:
  This site provides an excellent review of terminology, and interactive questions:
  http://www.mathsisfun.com/geometry/parallel-lines.html
  More interactive questions:
  http://regentsprep.org/Regents/math/geometry/GP8/PracParallel.htm
- Challenge students to create tiling designs using 1, 2, 3 or more regular polygons and calculate the angles for each shape, to demonstrate that the angles at the intersection add up to $360°$ (to lie flat) http://en.wikipedia.org/wiki/Tiling_by_regular_polygons .
- As an extension have students research and create the platonic solids. With reference to measures of internal angles of each regular polygon, have them explain why there are only five possible solids.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Calculate the values of $\angle m$ and $\angle \mathrm{n}$, if $PQ = PR$ and $QR \parallel ST$.

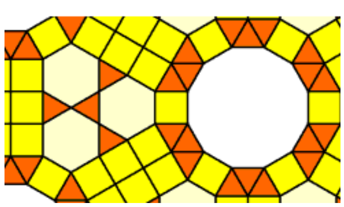


Answer: $\angle n = 144°, \angle m = 36°$

Q The following tiling has been created using regular (all sides and angles equal) triangles, squares, hexagons and dodecahedrons.

a) How many different combinations of regular polygons are found at the vertices in the tiling design?

b) Determine the angle measures for each polygon at each vertex combination and demonstrate that the sum of the angles is always the same. Why does this make sense?



Answers:
$\boldsymbol{a})$ 2 *triangles*, 1 *square*, 1 *dodecahedron* $\boldsymbol{b})$ $2(60°) + 90° + 150° = 360°$
$\boldsymbol{a})$ 1 *triangle*, 2 *squares*, 1 *hexagon* (2 *types*) $\boldsymbol{b})$ $60° + 2(90°) + 120° = 360°$
$\boldsymbol{a})$ 2 *triangles*, 2 *hexagons* $\boldsymbol{b})$ $2(60°) + 2(120°) = 360°$
$\boldsymbol{a})$ **3** *triangles*, 2 *squares* $\boldsymbol{b})$ $3(60°) + 2(90°) = 360°$
$\boldsymbol{a})$ 4 *squares* $\boldsymbol{b})$ $4(90°) = 360°$
The sum must be 360° *so that the tiles meet and lie flat.*

SCO **G3: Solve problems that involve the cosine law and the sine law, including the ambiguous case**. [CN, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

G3: Solve problems that involve the cosine law and the sine law, including the ambiguous case.

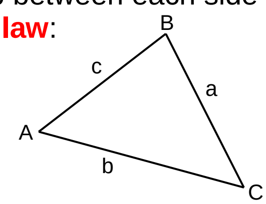
Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
A1: Solve problems that require the manipulation and application of formulas related to: perimeter, area, volume, capacity, the Pythagorean theorem, primary trigonometric ratios, income. currency exchange, interest and finance charges. **G2:** Demonstrate an understanding of the Pythagorean theorem by: identifying situations that involve right triangles, verifying the formula, applying the formula, and solving problems. *(GMF 10)* **G3:** Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems. *(GMF10)*	**G3:** Solve problems that involve the cosine law and the sine law, including the ambiguous case.	

ELABORATION

In Grade 10, students studied the primary trigonometric ratios and solved problems involving right triangles. In Grade 11, the angles and side lengths of **oblique triangles** will be explored. This will include both **acute** and **obtuse** triangles.

For acute and right triangles, students will explore the relationship between each side and the sine of its opposite angle to discover and then prove the **sine law**:



$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ *(more convenient when determining side lengths)*

or $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$ *(more convenient when determining angles).*

Capital letters are used for the angles, and small letters for the side opposite the named angle. Students will then apply the sine law to calculate unknown side lengths and angle measures and will explain their reasoning. They will use the sine law to solve a variety of contextual problems. In their explorations they should be led to the realization that the sine law is useful only when three of the four measurements of two sides and their opposite angles are known.

When there are two unknowns in each pair of equivalent ratios for an acute triangle, the sine law cannot be used, and another relationship is needed. The cosine law can be used to solve an acute triangle when two sides and the contained angle, or all three sides are known. Students will use the Pythagorean theorem to derive and prove the **cosine law**:

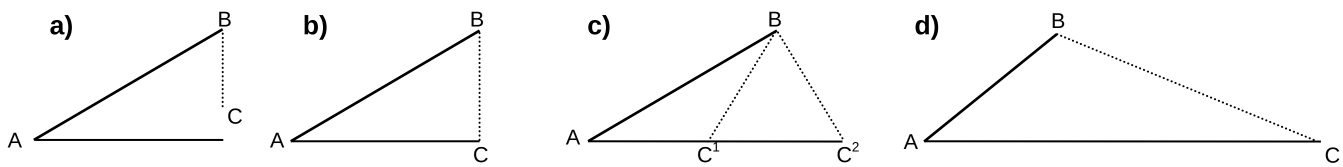
$a^2=b^2+c^2-2bc\ \cos A$
$b^2=a^2+c^2-2ac\ \cos B$
$c^2=a^2+b^2-2ab\ \cos C.$

SCO	**G3: Solve problems that involve the cosine law and the sine law, including the ambiguous case**. [CN, PS, R]

Students will apply the cosine law to calculate unknown side lengths and angle measures, explaining their reasoning, and then use the cosine law to solve contextual problems.

Students will explore the relationship between primary trigonometric ratios of acute and obtuse angles and how these relate to the sine and cosine laws for obtuse triangles: $\sin\theta = \sin\ (180° - \theta)$; $\cos\theta = -\cos\ (180° - \theta)$; $\tan\theta = -\tan\ (180° - \theta)$.

When the sine law is used to determine the measure of an angle, there are always two possibilities because the angles that give equal sine ratios are supplementary. The **ambiguous case** of the sine law may occur when two side lengths and the measure of an angle that is opposite one of these sides are given (SSA). In non-contextual situations, when given SSA students will determine whether zero, one, or two triangles exist and explain their reasoning in a variety of ways. For example. when given SSA, BC(S), AB(S), $\angle$A(A), for the triangle ABC;



a) If BC is too short to reach AC, no triangle will be possible.
b) If BC is just long enough to reach AC at $90°$, this right angle triangle will be the only possible triangle.
c) If BC is longer than in b) but shorter that AB, this will be the ambiguous case, in which there are two possible triangles, one in which $\angle C$ is acute and one in which $\angle C$ is obtuse.
d) If BC is longer than AB, there will only be one possible triangle.

In contextual problems where the given information leads to two possible triangles, students must decide whether the acute or obtuse situation applies and justify their decision. The diagram or problem needs to be considered to determine which is the correct angle, the acute angle θ, or the obtuse angle $(180° - \theta)$.

ACHIEVEMENT INDICATORS

- Draw a diagram to represent a problem that involves the cosine law or sine law.
- Explain the steps in a given proof of the sine law or cosine law.
- Solve a problem involving the cosine law that requires the manipulation of the formula.
- Explain, concretely, pictorially or symbolically, whether zero, one or two triangles exist, given the SSA situation. (side, side, angle)
- Solve a problem involving the sine law that requires the manipulation of the formula.
- Solve a contextual problem that involves the cosine law or the sine law.

SCO	**G3: Solve problems that involve the cosine law and the sine law, including the ambiguous case**. [CN, PS, R]

Suggested Instructional Strategies

- Math Warehouse is a good website with many useful resources. The following is an interactive site using law of cosines where students can solve a problem and then verify their solution.
http://www.mathwarehouse.com/trigonometry/law-of-cosines-formula-examples.php
- The following is a good site to derive the law of Sines and Cosines.
http://www.regentsprep.org/Regents/math/algtrig/ATT12/derivelawofsines.htm
- The following site provides a good worksheet for the ambiguous case.
http://www.mathworksheetsgo.com/sheets/trigonometry/advanced/law-of-sines-and-cosines/law-of-sines/law-of-sines-ambiguous-case.php
- Have students create a problem where a side length can be determined using Law of Sines, and then where an angle measure can be determined using Law of Sines. Have them show the solution.

Questions (Q) and Activities (Act) for Instruction and Assessment

Q Matthew enjoys swimming in the ocean. One day Matthew decides to swim $9.2\ km$ from Island A to Island B. After resting a few moments, he swam $8.6\ km$ to Island C. If Island C to Island A to Island B forms a $52°$ angle, determine how much further Matthew swam by swimming to Island B first, than by simply swimming straight from Island A to Island C.

Answer:

$$\frac{\sin C}{9.2\ km} = \frac{\sin 52°}{8.6\ km}$$

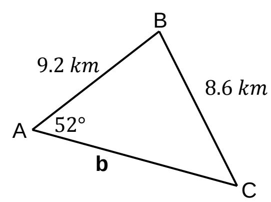
$$\sin C = 0.8430$$

$$\angle C = 57.5°$$

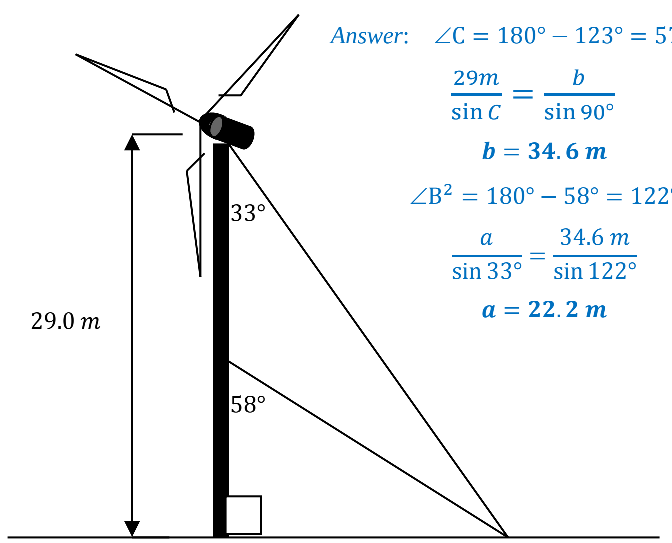
$$\angle B = 70.5°$$

$$\frac{b}{\sin 70.5°} = \frac{8.6\ km}{\sin 52°}$$

$$\boldsymbol{b = 10.3\ km}$$



Q A windmill on a farm is supported by two guy wires, as shown below. Find the length of the two guy wires



Answer: $\angle C = 180° - 123° = 57°$

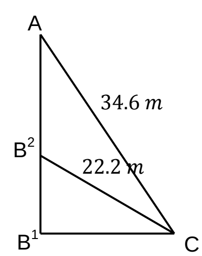
$$\frac{29m}{\sin C} = \frac{b}{\sin 90°}$$

$$\boldsymbol{b = 34.6\ m}$$

$$\angle B^2 = 180° - 58° = 122°$$

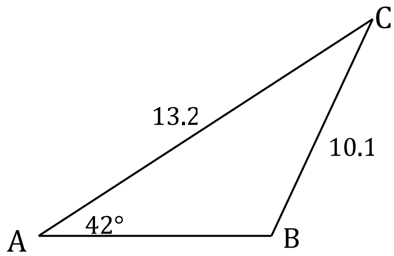
$$\frac{a}{\sin 33°} = \frac{34.6\ m}{\sin 122°}$$

$$\boldsymbol{a = 22.2\ m}$$



SCO	**G3: Solve problems that involve the cosine law and the sine law, including the ambiguous case**. [CN, PS, R]

Q An engineer is asked to build a support frame for an airplane wing with $\angle A = 42°$ and with sides $b = 13.2$ cm and $a = 10.1$ cm respectively. Based on this drawing, supplied to the engineer, what is the measure of C?



Answer: $$\frac{\sin B}{13.2\ cm} = \frac{\sin 42°}{10.1\ cm}$$

$$\sin B = 0.8745$$

$$\angle B = 61° \ \textbf{or} \ \angle B = 180° - 61° = 119°$$

$$\angle B \text{ is obtuse } \therefore \angle C = 180° - 42° - 119° = 19°$$

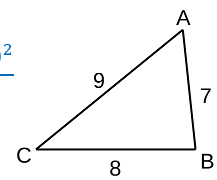
Q For triangle ABC, for which $a = 8$, $b = 9$ and $c = 7$, what is the measure of angle C?

Answer: $$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{(8)^2 + (9)^2 - (7)^2}{2(8)(9)}$$

$$\cos C = 0.6\overline{6}$$

$$\angle C = 48.2°$$



SCO	**RF1: Model and solve problems that involve systems of linear inequalities in two variables.** [CN, PS, T, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Relations and Functions

RF1: Model and solve problems that involve systems of linear inequalities in two variables.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Eleven/Twelve
RF1: Interpret and explain the relationships among data, graphs and situations. *(NRF10)* **RF6**: Relate linear relations expressed in slope-intercept form ($y = mx + b$), general form ($Ax + By + C = 0$), and slope-point form ($(y - y_1) = m(x - x_1)$), to their graphs. *(NRF10)* **RF9:** Represent a linear function, using function notation. *(NRF10)* **RF10:** Solve problems that involve systems of linear equations in two variables, graphically and algebraically. *(NRF10)*	**RF1:** Model and solve problems that involve systems of linear inequalities in two variables.	**RF7:** Solve problems that involve linear and quadratic inequalities in two variables. *(PC11)* **RF8:** Solve problems that involve quadratic inequalities in one variable. *(PC11)*

ELABORATION

In Grade 10 students expressed word problems as systems of linear equations and then solved the system. To solve systems graphically, the slope-intercept method, the slope-point form, and the x and y intercept method were used, and the solution was identified as the point at which the two lines intersect. To solve systems of linear equations algebraically, students used methods of substitution and elimination.

For this outcome students will build on their Grade 10 work. They will be introduced to **linear inequalities in two variables**, extending the graphical model of a linear inequality from a one-dimensional linear number line explored in Grade 9, to a two dimensional coordinate plane. The solution set is extended from points along a linear number line to points in a solution region.

Terms such as greater than, less than, greater than or equal to, less than or equal to, at most, no more than, no less than, at least, more than, fewer than, maximum, minimum, and optimal are used with reference to inequalities.

Students will need considerable practice to develop an understanding of graphing solution sets for linear inequalities, before going on to solving **systems of linear inequalities**. They will learn when to use a solid or broken line, and will use test points to confirm the solution set.

The outcome will finish with applications of these graphing skills to simple **linear optimization** problems, also referred to as **linear programming.** This is a widely used mathematical method that helps to make the most efficient use of limited resources - like money, time, materials, and machinery.

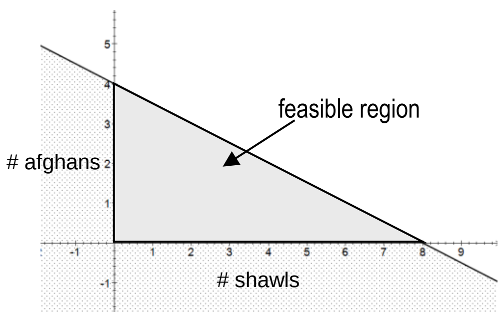
SCO	**RF1: Model and solve problems that involve systems of linear inequalities in two variables.** [CN, PS, T, V]

It is important to realize that the intention of this outcome is to introduce linear optimization problems as an example of an application for the students' newly learned skills of solving systems of linear inequalities. Problems should be kept at an appropriate level and should focus on applications and conceptual understanding.

Factors that limit or restrict resources required for production are referred to as **constraints**. These constraints can be expressed as inequalities and graphed to illustrate a solution set. For example, in the following question, the time available to do spinning is limited to 8 hours, so time is a constraint in the production of shawls.

A shawl requires 1 hour of spinning; an afghan 2 hours of spinning. If the spinning wheel is only available for 8 hours, what is the maximum number of shawls and afghans that can be spun?

This can be expressed as the inequality: $1s + 2a \leq 8$. Graphing $1s + 2a = 8$ will give the boundary line, and everything including and below that (solid) line is the solution set to the inequality, also called the **feasible region**. Students can verify the feasible region using graphing technology.



The implied constraints, that the number of shawls or afghans cannot be negative, $s \geq 0$, and $a \geq 0$, must be taken into consideration as well, limiting the solution to those values equal to or greater than zero.

The **feasible region**, includes all the values that fall within the constraints of a problem. Or, put another way, any combination of number of shawls and afghans that fall within this feasible region would be possible within 8 hours. Students will verify their solution by testing points on the border, at the vertices, from within, and from outside the feasible region, to determine if a given ordered pair satisfies the inequality. They should be able to explain what each means in terms of the feasibility of spinning varying numbers of shawls and afghans in an 8 hour day. They might find it helpful to organize their thinking in a chart similar to the following:

# shawls	# afghans	# hours total (shawls 1 hour, afghans 2 hours)	In feasibility region?	Total # hours > or ≤ 8?
1	3	7	yes	≤ 8
8	0	8	yes	≤ 8
4	3	10	no	> 8
2	3	8	yes	≤ 8
0	5	10	no	> 8

SCO	**RF1: Model and solve problems that involve systems of linear inequalities in two variables.** [CN, PS, T, V]

In their explorations students should try a variety of points to find when profit is optimized. For example, if the profit for spinning a shawl is $\$20$, and for spinning an afghan is $\$30$, the profit at $(2,3)$ would be $\$100$, at $(8,0)$ would be $\$160$, and at $(1,3)$ would be $\$110$. They will discover that optimal values are found at the vertices, although not all vertices give optimal values.

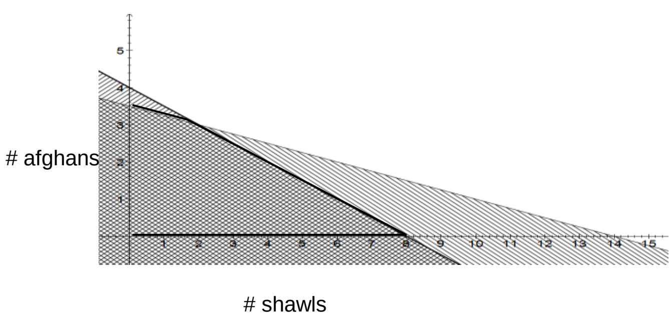
The number of constraints can be increased once students have mastered simpler questions. This will change the position and number of vertices, and the corresponding profit at each point. For example, the following adds the constraint of limited time available for weaving to the shawls and afghans problem, changing the feasible region. Students can be provided with the chart, to allow a focus on solving the optimization problem.

Carmella and Walt spin the yarn and then weave it to produce handmade shawls and afghans. A shawl requires 1 hour of spinning and 1 hour of weaving; an afghan 2 hours of spinning and 4 hours of weaving. The spinning wheel is only available for 8 hours and the weaving machine for only 14 hours. The profit for each shawl is $\$20$*, and for each afghan is* $\$30$*.*

Express this situation as a system of inequalities. Graph the system of inequalities and determine how many shawls and afghans should be produced to maximize profit.

	SPINNING	**WEAVING**
Shawl	1 hour	1 hour
Afghan	2 hours	4 hours
TOTAL # hours	8 hours	14 hours

$Answer$: $1s + 2a \leq 8\ (spinning)\quad and \quad 1s + 4a \leq 14\ (weaving)$ The vertices for the new feasible region are at $(0,3.5)$, $(2,3)$ and $(8,0)$, earning a profit of $\$115$, $\$130$, and $\$160$, respectively. At this profit level, the producer should make only shawls, a total of 8 in 8 hours.



SCO **RF1: Model and solve problems that involve systems of linear inequalities in two variables.** [CN, PS, T, V]

ACHIEVEMENT INDICATORS

- Graph, justifying the choice of a solid or broken line, and explain the solution region that satisfies a linear inequality, using a test point when given a boundary line.
- Model a problem, using a system of linear inequalities in two variables.
- Graph the boundary line between two half planes for each inequality in a system of linear inequalities.
- Determine, graphically, the solution region for a system of linear inequalities, and verify the solution.
- Explain, using examples, the significance of the shaded region in the graphical solution of a system of linear inequalities.
- After in-class demonstration, solve linear optimization problems.

Suggested Instructional Strategies

- For students with net books or access to the computer lab, use internet sites such as the following for practice:
  http://teachers.henrico.k12.va.us/math/hcpsalgebra2/3-4.htm which has a PowerPoint lesson on how to solve systems of inequalities.
  http://www.algebra-class.com/systems-of-inequalities.html which has a good example of an inequality question with the solution worked out. At the bottom of the page there is a link to systems of inequalities practice problems.
- Graph, justifying the choice of a solid or broken line, and explain the solution region that satisfies a linear inequality, using a test point when given a boundary line.
  http://www.purplemath.com/modules/ineqgrph.htm
- Develop skills for graphing systems of linear inequalities gradually, beginning with graphing each linear inequality separately, before graphing on the same graph to identify the feasible region.
- Begin with problems in which students develop a single inequality from a problem, to more complicated examples which introduce more than a single constraint, as is shown in the *Elaborations*.

SCO	**RF1: Model and solve problems that involve systems of linear inequalities in two variables.** [CN, PS, T, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Graph the solution set for each inequality:

$a)\ \ y > 2x + 3$ $\qquad b)\ \ 5y + 2 > x$ $\qquad c)\ \ 2x + 4y \geq -10$

$d)\ \ -2y \leq 5x + 12$ $\qquad e)\ \ y < 5x$ $\qquad f)\ y \geq 3$

$g)\ \ -6 < -2x + 2$ $\qquad h)\ 10x - 5 < -y$ $\qquad i)\ \ 4x + 4 > y$

Q Sketch the solution sets for the following systems of linear inequalities.

$a)\ \ y \leq 6 - x\ \text{ and }\ y \geq x - 2$ $\qquad b)\ \ 3x - y < 4\ \text{ and }\ x - 2y > 3$

$c)\ \ y > 1 - 2x\ \ and\ y \leq x + 3$ $\qquad d)\ \ 2x + y > 3\ \ and\ \ \ x - y \leq 4$

Q For $y < 3x + 5$ which of the following points fall within the solution set?

$(-1,-3), (-1,2), (-4,3), (-2,-3), (3,1), (1,5), (0,5), (-1,3)$

$Answer: (-1,-3), (-2,-3), (3,1), (1,5), (-1,3)$

Q The following chart describe the length of time it takes on each machine, to manufacture a unit of plastic plates and or plastic cups. Each machine is operated for a maximum of 15 hours per day.

	Machine A	***Machine B***
plates	1 hour	2 hours
cups	3 hours	1 hour
Max # hours	15 hours	15 hours

From this chart, develop two inequalities and solve as a system of inequalities, to determine the maximum number of plates and cups the company can produce each day.

$Answer:\ 1p + 3c \leq 15 \quad and \quad 2p + 1c \leq 15$

$Vertices:$

$(p,c) = (6,3)$

y-intercept $(p,c) = (0,5)$

x-intercept $(p,c) = (7.5,0)$

$Maximum\ number\ = P + C\ = 6\ plates\ + 3\ cups\ = 9$

Q John's company repairs tennis and jogging shoes. Tennis shoes take 16 minutes to strip and 12 minutes to re-sew. Jogging shoes take 8 minutes to strip and 16 minutes to re-sew. Each machine is available for 8 hours a day.

The profit on a tennis shoe repair is \$3 and the on a jogging shoe is \$5. How many of each type should be repaired daily to maximize profit?

$Answer:$ $\quad 8\ hours = 480\ minutes$

$16j + 12t \leq 480 \quad and \quad 8j + 16t \leq 480$

$vertices\ at\ (t,j) = (30,0), (0,30), (24,12)$

\$3 *Tennis*	**\$5 *Jogging***	***Profit***
30	0	\$90
0	30	\$150
24	12	\$132

$\therefore He\ should\ repair\ only\ jogging\ shoes - 30\ a\ day\ for\ a\ profit\ of\ \$150.$

SCO	**RF2: Demonstrate an understanding of the characteristics of quadratic functions, including: vertex, intercepts, domain and range, axis of symmetry.** [CN, PS, T, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

RF2: Demonstrate an understanding of the characteristics of quadratic functions, including: vertex, intercepts, domain and range, axis of symmetry.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Eleven
RF2: Demonstrate an understanding of relations and functions *(NRF10)* **RF4:** Describe and represent linear relations using words, ordered pairs, tables of values, graphs, and equations. *(NRF10)* **RF5:** Determine the characteristics of the graphs of linear relations, including the intercepts, slope, domain and range. *(NRF10)* RF6: Relate linear relations expressed in slope-intercept form ($y = mx + b$), general form ($Ax + By + C = 0$), and slope-point form ($y - y_1) = m(x - x_1)$, to their graphs. *(NRF10)* **AN1:** Demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root. *(NRF10)* **AN5:** Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically. *(NRF10)*	**RF2:** Demonstrate an understanding of the characteristics of quadratic functions, including: vertex, intercepts, domain and range, axis of symmetry.	**RF3:** Analyze quadratic functions of the form $y = a(x-p)^2 + q$ and determine the vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts. *(PC11)* **RF4:** Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts, and to solve problems. *(PC11)* **RF5:** Solve problems that involve quadratic equations. *(PC11)*

ELABORATION

This outcome will build on many of the outcomes addressed in Grade 10 when students examined linear relations and their graphs, including ordered pairs, table of values, intercepts, slope, domain and range. They also examined slope-intercept, general and slope-point forms of linear equations and how each form related to graphs.

Also in grade 10, students were first introduced to factoring of expressions. Students used algebra tiles, diagrams, and symbols to multiply binomials and factor trinomials. Trinomials were factored as the inverse of multiplication, and other methods such as common factors, perfect squares, and difference of squares were also used.

In *Foundations of Mathematics 110*, students will extend their understanding of functions to include quadratic functions and how these functions can model real life situations.

It is important to note that this outcome is an introduction to quadratic functions and should not include material that will be covered in later courses. In this course student explorations should be **limited to the general and the factored forms, and to factoring methods first explored in Grade 10 (with the exception of partial factoring which will be new), and the Quadratic Formula.**

In *Pre-Calculus 110* students will build on what is learned in this course to analyze quadratic functions, and to solve quadratic equations. They will explore the use of the vertex form, and will solve quadratic equations that require the use of substitution, SCO **RF2: Demonstrate an understanding of the characteristics of quadratic functions, including: vertex, intercepts, domain and range, axis of symmetry.** [CN, PS, T, V]

factoring by decomposition, and completing the square. **These topics are beyond the scope of *Foundations of Mathematics 110*.**

For this outcome, students will begin by graphing quadratic functions from the general form $f(x) = ax^2 + bx + c$, with or without technology, to determine the **vertex**, **axis of symmetry**, the **maximum** and **minimum** values, and the **domain** and **range**. As students vary values of $a, b,$ and c they will discover how the shape and orientation of the graph is affected. If $a > 0$, the parabola opens up, if $a < 0$ the parabola opens down. The value c gives the y-intercept of the parabola.

Students should understand that the axis of symmetry passes vertically through the vertex and this x-value is the x-coordinate of the vertex. By substitution they can then determine the y-coordinate of the vertex which is the function's maximum (if $a \leq 0$) or minimum (if $a \geq 0$) value. Based on the location of the vertex and the direction the parabola opens, they will then be able to determine if a parabola has $0, 1$ or 2 x-intercepts

The equation of the axis of symmetry can be found by averaging the x-coordinates of two points that have the same y-coordinate, based on the understanding that two points with the same y-coordinate on a parabola are equidistant from the axis of symmetry,

Once students demonstrate an understanding of the relationship between quadratic equations and their graphs, they will use graphs to find solutions to the quadratic equations (with and without technology). These solutions or **roots**, are the values of the variables that make an equation in standard form equal to zero. These values are also the **zeros** of the corresponding function, and the **x-intercepts** of the graph.

Students should examine and discuss the advantages and disadvantages of using a graphical approach. One disadvantage is that the solutions can only be estimated. By factoring the general form to the factored form, $f(x) = a(x - r)(x - s)$, an alternate algebraic approach can be used to calculate exact solutions.

For the factored form, each element (a, r, s) relates in specific ways to the graph of the function. The zeros of the function can be determined by setting each factor equal to zero and solving. The x-intercepts of the graph of the function are $x = r$ and $x = s$. The linear equation of the axis of symmetry is $x = \frac{r+s}{2}$. The y-intercept c is $c = a \times r \times s$.

Students now have a chance to practice factoring methods introduced in Grade 10, in the context of solving quadratic functions. Some questions in which $a \neq 1$, can be included as long as the solution(s) can be determined using one of the methods listed.

Starting with a graph, the equation of the parabola can be written in factored form using the x-intercept(s) and the coordinates of one other point on the parabola. However, quadratic functions without any zeros cannot be written in factored form.

To factor $f(x) = ax^2 + bx + c$, methods could include, removing the common factor, by inspection (find two numbers whose sum is b and product is c), modelling with algebra tiles, identifying perfect squares, or identifying difference of squares.

SCO	**RF2: Demonstrate an understanding of the characteristics of quadratic functions, including: vertex, intercepts, domain and range, axis of symmetry.** [CN, PS, T, V]

If the roots are not whole numbers, and the expression cannot be factored with the above methods, **partial factoring** can be used to sketch the graph of a quadratic function. For example:

$f(x) = -x^2 + 4x + 8$

Using partial factoring: $f(x) = -x(x - 4) + 8$

If one or the other factor equals 0, then $f(x) = 8$.

$$\begin{array}{rr} -x = 0 & x - 4 = 0 \\ x = 0 & x = 4 \\ f(0) = 8 & f(4) = 8 \end{array}$$

Therefore two points of the function are $(0,8)$ and $(4,8)$. If students understand that points sharing a y-value must be equidistant from the axis of symmetry, it will follow that the axis of symmetry is given by, $x = \frac{0+4}{2} = 2$. Substituting, $f(2) = 12$, so the vertex is $(2,12)$.

The **Quadratic Formula**, $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, which can be used for any quadratic function, should also be introduced to students in this outcome. Students should know how to use the formula, but are not responsible for deriving it themselves or for memorizing it.

By the end of this outcome students will demonstrate their understanding by sketching the graph of a quadratic function, and by using both graphing and algebraic methods to solve a contextual problem that involves the characteristics of a quadratic function. They will choose the method most suitable for solving a given situation and justify the approach they used. Some simple optimization problems should be examined where the function is given in the form $f(x) = ax^2 + bx + c$ as part of the problem.

SCO **RF2: Demonstrate an understanding of the characteristics of quadratic functions, including: vertex, intercepts, domain and range, axis of symmetry.** [CN, PS, T, V]

ACHIEVEMENT INDICATORS

- Determine, with or without technology, the intercepts of the graph of a quadratic function.
- Determine, by factoring, the roots of a quadratic equation, and verify by substitution. Limit factoring methods to: removing the common factor, factoring by inspection, modeling with algebra tiles, identifying perfect squares, identifying difference of squares, and partial factoring.
- Determine, using the quadratic formula, the roots of a quadratic equation.
- Explain the relationships among the roots of an equation, the zeros of the corresponding function, and the x-intercepts of the graph of the function.
- Explain, using examples, why the graph of a quadratic function may have zero, one or two x-intercepts.
- Express a quadratic equation in factored form, using the zeros of a corresponding function or the x-intercepts of its graph.
- Determine, with or without technology, the coordinates of the vertex of the graph of a quadratic function.
- Determine the equation of the axis of symmetry of the graph of a quadratic function, given the x-intercepts of the graph.
- Determine the coordinates of the vertex of the graph of a quadratic function, given the equation of the function and the axis of symmetry, and determine if the y-coordinate of the vertex is a maximum or a minimum.
- Determine the domain and range of a quadratic function.
- Sketch the graph of a quadratic function.
- Solve a contextual problem that involves the characteristics of a quadratic function

Suggested Instructional Strategies

- Students should be given opportunity to fully explore quadratic functions graphically, algebraically through factoring and the quadratic formula, and with the use of graphing technology.
- If technology is available the following websites can be used for student practice.

  http://nlvm.usu.edu/en/nav/vlibrary.html This website allows students to review factoring using algebra tiles.

  http://www.algebra-class.com/quadratic-equation.html This website has good examples for solving quadratic equations using different methods.

  http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html This website contains an online graphing calculator.

SCO	**RF2: Demonstrate an understanding of the characteristics of quadratic functions, including: vertex, intercepts, domain and range, axis of symmetry.** [CN, PS, T, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Write the following quadratic function in the form $y = ax^2 + bx + c$ using algebra and showing the work that leads to your solution.

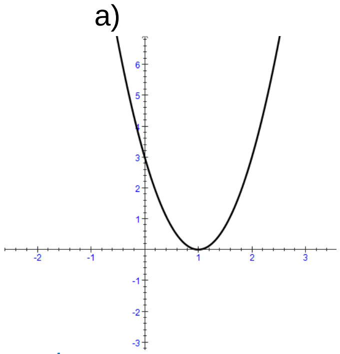
$y = 5(x-1)^2 - 8$

$Answer$: $y = 5(x-1)(x-1) - 8$
$y = 5(x^2 - 2x + 1) - 8$
$y = 5x^2 - 10x - 3$

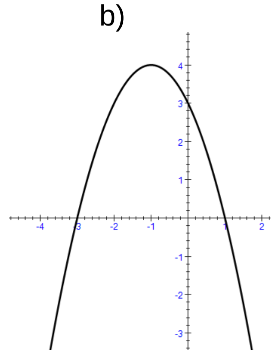
Q List five characteristics of the function: $f(x) = -3x^2 + 24x - 60$.
$Answers$:
- $1)\ opens\ downward$
- $2)\ y\text{-}intercept\ is\ (0,-60)$
- $3)\ has\ imaginary\ roots$
- $4)\ has\ zero\ x\text{-}intercepts$
- $5)\ 2\ points\ on\ graph\ are\ (0,-60)\ and\ (8,-60)\ \ (partial\ factoring)$
- $6)\ \frac{0+8}{2} = 4,\ \ x = 4\ is\ the\ axis\ of\ symmetry$
- $7)\ f(4) = -12,\ \ vertex\ is\ \ (4,-12)$
- $8)\ vertex\ is\ a\ maximum$
- $9)\ Domain\ \ \{x \in R\}, Range\ \ \{y \leq -12, y \in R\}$

Q List five characteristics of each function:

a)



b)



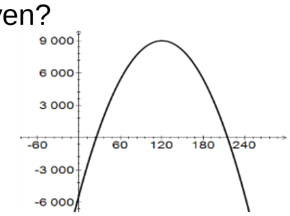
$Answers$:

$\mathbf{a})$
- $1)\ opens\ upward$
- $2)\ x = 1\ \ is\ the\ axis\ of\ symetry$
- $3)\ y\text{-}intercept\ is\ (0,3)$
- $4)\ one\ Real\ equal\ root\ (1,0)$
- $5)\ has\ one\ x\text{-}intercept$
- $6)\ vertex\ (1,0)\ is\ a\ minimum$
- $7)\ Domain\ \ \{x \in R\}, Range\ \ \{y \geq 0, y \in R\}$

$\mathbf{b})$
- $1)\ opens\ downward$
- $2)\ \ x = -1\ is\ the\ axis\ of\ symetry$
- $3)\ \ y\text{-}intercept\ \ is\ (0,3)$
- $4)\ two\ Real\ distinct\ roots\ (-3,0), (1,0)$
- $5)\ has\ two\ x\text{-}intercepts$
- $6)\ vertex\ (-1,4) is\ a\ maximum$
- $7)\ Domain\ \ \{x \in R\}, Range\ \ \{y \leq 4, y \in R\}$

Q The daily profit, P, (dollars) for a company that makes tennis racquets is given by $P = -n^2 + 240n - 5400$ where n is the number of racquets produced per day.
a) How many tennis racquets must be produced per day to have a maximum profit?
b) What is the maximum profit?
c) What profit is made when 75 racquets per day are produced
d) How many tennis racquets must be produced to break even?

$Answers$: $Vertex\ \ (120, 900)\ (partial\ factoring)$
$a)\ \ 120 \quad b)\ \$9000 \quad c)\ P(75) = \$6975$
$d)\ Quad\ formula\ x = 25.13\ or\ x = 214.87$
$\therefore\ the\ company\ must\ make\ at\ least\ 26\ racquets\ per\ day\ to\ break\ even$



SCO	**RF2: Demonstrate an understanding of the characteristics of quadratic functions, including: vertex, intercepts, domain and range, axis of symmetry.** [CN, PS, T, V]

Q Create a quadratic equation that has roots $x = -2$ and $x = 3$. Create an alternate equation with the same roots.

$Answers$: $y = (x + 2)(x - 3)$
$y = (3x + 6)(x - 3)$
$y = 2x^2 - 2x - 12$

Q What is the minimum value of $y = x^2 - 8x - 9$?

$Answer$: $y = (x - 9)(x + 1)$

$x = \frac{+9-1}{2} = 4$, $\quad y = 4^2 - 8(4) - 9 = -25 \quad vertex\ (4, -25)$

$\therefore minimum\ is -25$

Q For $y = x^2 - 2x - 35$.

a) Determine the roots of the function.
b) Find the vertex of the function.
c) State the y-intercept.
d) Sketch a graph of the function on grid paper. Label completely clearly indicating all intercepts, the vertex, axis of symmetry and scales of the axes.
e) State the domain and the range.

$Answers$: $a)\ x = 7\ or\ x = -5$

$b)\ x = \frac{7+(-5)}{2} = 1,\ y = (1)^2 - 2(1) - 35 = -36,\ vertex\ (1, -36)$

$c)\ y = (0)^2 - 2(0) - 35 = -35, \quad y\text{-}intercept\ (0, -35)$

$e)\ \{x | x \in R\} \quad \{y | y \geq 36, y \in R\}$

SCO	**N1: Analyze costs and benefits of renting, leasing and buying.** [CN, PS, R, T]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Number

N1: Analyze costs and benefits of renting, leasing and buying.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
A1: Solve problems that require the manipulation and application of formulas related to: perimeter, area, volume, capacity, the Pythagorean theorem, primary trigonometric ratios, income. currency exchange, interest and finance charges.(GMF10) **N1:** Solve problems that involve unit pricing and currency exchange, using proportional reasoning. (GMF10) **N2:** Demonstrate an understanding of income, including: wages, salary, contracts, commissions, and piecework to calculate gross pay and net pay.(GMF10) **N3:** Demonstrate an understanding of financial institution services used to access and manage finances. (GMF10) **N4:** Demonstrate an understanding of compound interest. (GMF 10) **N5:** Demonstrate an understanding of credit options, including: credit cards, and loans. (GMF 10)	**N1:** Analyze costs and benefits of renting, leasing and buying.	

ELABORATION

In Grade 10, students solved problems involving simple and compound interest, and the computation of finance charges. In this outcome they will use this knowledge to explore the costs of renting, leasing and buying.

Assets or property are items that are owned or partially owned. For example: vehicles, i-phones, laptops or real estate.

Appreciation is the increase in the value of an asset over time. This increase can occur for a number of reasons including increased demand or weakening supply, or as a result of changes in inflation or interest rates. **Depreciation** is the decrease in the value of an asset over time.

Renting and **leasing** are similar, but differ in duration. A **rental** is a short-term agreement or contract under which capital property is rented from one person to another on an hourly, daily, weekly or monthly basis with rates tending to decrease the longer the rental period. On the other hand, a **lease** is a long-term agreement or contract, under which capital property is rented from one person to another for a fixed period of time (usually one year or more) at a specified rate. However, these terms are often used interchangeably.

SCO **N1: Analyze costs and benefits of renting, leasing and buying.** [CN, PS, R, T]

ACHIEVEMENT INDICATORS

- Identify and describe examples of assets that appreciate or depreciate.
- Compare, using examples, renting, leasing and buying.
- Justify, for a specific set of circumstances, if renting, buying or leasing would be advantageous.
- Solve a problem involving renting, leasing or buying that requires the manipulation of a formula.
- Solve, using technology, a contextual problem that involves cost-and-benefit analysis.

Suggested Instructional Strategies

- Bring in a manager from a car dealership or a bank officer to speak to the class about the advantages and disadvantages of renting, leasing and buying.
- Have students browse the classified ads, and make predictions as to which properties might appreciate or depreciate the most in the next few years. Discuss the reasons for these predictions.
- When discussing depreciation, choose examples which are pertinent to teenagers. Get students to choose their favorite vehicle and research the depreciation rate of the car. Use a website such as www.ehow.com/list_6923399_depreciation-rules-canada.html, or www.canadianblackbook.com for research on depreciation.
- Using a graphing calculator, explore the decay curve that is representative of an automobile's depreciation.
- Use www.smbtn.com/books/gb79.pdf as a reference when comparing advantages and disadvantages of renting, leasing and buying.
- The site http://www.handsonbanking.org/en/ provides free curriculum resources for a variety of financial mathematics topics.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Choose three different vehicles that you may want to purchase some day. Research the depreciation on each model of car using a website such as www.ehow.com/list_6923399_depreciation-rules-canada.html, or www.canadianblackbook.com.

a) Which model of car depreciates fastest?
b) Which model of car depreciates slowest?
c) For each model of car determine the depreciation rate after 1, 2, and 3 years.
d) What affects the rate at which a car depreciates?

SCO **N1: Analyze costs and benefits of renting, leasing and buying.** [CN, PS, R, T]

Q Sara is thinking of buying a new TV. She went to one store and found an excellent deal on the lease of a Toshiba 32″ LCD HDTV. The weekly lease payment is $\$12$/week, plus the one-time cost of product of $\$10.93$ and a one-time cost of TPC (total protection coverage) of $\$1.07$. On her brother's suggestion, Sara checked another shop and found the same TV on sale there for $\$349.99$.

a) If Sara decides to lease the TV from the first shop rather than buy it at the other shop, how many months will it be before she exceeds the purchase price?

$Answer:\ Leasing\ = \$24 + \$12x \qquad Buying\ \$349.99 + 13\%\ tax\ = \ \$395.49$

$\therefore 395.49 = \ 24 + 12x \qquad 371.49 = 12x \qquad 30.95 = x$

$After\ 31\ weeks\ or\ {\sim}7\ months\ the\ leasing\ price\ will\ exceed\ the\ purchase\ price.$

b) Do you think this would be the right choice? Why or why not?

Q Rhonda has just graduated from community college and is now wanting to move out of her parents' house. She has worked part time while she studied and has been able to save $5000.00 over the past three years. She has found a nice apartment that is renting for $380.00/month and requires a $400.00 damage deposit. Using her savings how long will Rhonda be able to rent her apartment for?

$Answer:\ She\ will\ be\ able\ to\ rent\ for\ 9\ months.$

Q Joanne is going away to university. She has saved up some money from part-time jobs and plans to buy or lease a new vehicle. She has found a compact car that she likes and is trying to decide whether she should lease or buy, based on the following information from the dealership's website:

Pricing details	**Finance (60 months)**	**Lease (60 months)**	**Cash**
	$13 995	$13 995	$13 995
Selected Savings and Offers	- $750	- $750	- $750
Selected Accessories	$0.00	$0.00	$0.00
	Monthly Payment	**Monthly Payment**	**Cash Price**
	$276.92	**$218.75**	**$13 245**
		Lease end value $4618.35	

a) What is the total price for each?

b) What are the advantages and disadvantages of each option?

c) If you were Joanne, which option would you choose and why?

$Answer:$

Option	Total Price	Advantages	Disadvantages
Financing	$16,615.20	Not paying for extra mileage Own vehicle after 60 months	Higher monthly payments Higher final cost with interest
Leasing	$13,125.00	Cheapest monthly payment	Mileage charge Still owe $4618.35 if wanting to own at end of 60 months
Cash	$13,295.00	Own outright Least expensive option	Need to have money up front

SCO	**N2: Analyze an investment portfolio in terms of: interest rate, rate of return, total return**. [ME, PS, R, T]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

N2: Analyze an investment portfolio in terms of: interest rate, rate of return, total return.

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
N3: Demonstrate an understanding of financial institution services used to access and manage finances. (GMF10). **N4:** Demonstrate an understanding of compound interest. (GMF10). **N5:** Demonstrate an understanding of credit options, including: credit cards, and loans. (GMF10)	**N2:** Analyze an investment portfolio in terms of: interest rate, rate of return, total return.	

ELABORATION

In grade 10, students investigated services offered by financial institutions, and were introduced to simple and compound interest, with a focus on credit options. For this outcome students will apply this knowledge to investment opportunities.

The focus for this outcome is on understanding and comparing the effects of simple and compound interest on future values of investments, and on analyzing, comparing and designing **investment portfolios** to meet specific financial goals.

An investment portfolio is comprised of all the different investments that an individual or organization holds. Investments can include **shares**, **bonds**, or **investment certificates**.

When you buy shares (also called stocks or equities) you become a part owner in a company. This gives you the right to a portion of the company's earnings and may entitle you to vote at the shareholder meetings. Compared to other types of investment, shares can be riskier but can potentially offer higher returns. The value of the shares are dependent on the success of the company.

When you buy a bond you are lending money to a government or company for a certain period of time. In return, you receive a fixed rate of interest and your money back at the end of the term. Company bonds offer better rates of return than investments such as **Guaranteed Investment Certificates** but this is because if the company fails you may not get back all the money you originally paid, so there is more risk involved. Government bonds such as Canada Savings Bonds are more secure. **Mutual funds** are a mix of stocks and bonds.

Investment certificates are offered at banks at a higher rate of interest than a chequing account, and are easier to access that stocks or bonds.

SCO **N2: Analyze an investment portfolio in terms of: interest rate, rate of return, total return**. [ME, PS, R, T]

ACHIEVEMENT INDICATORS

- Determine and compare the strengths and weaknesses of two or more portfolios.
- Determine, using technology, the total value of an investment when there are regular contributions to the principal.
- Graph and compare the total value of an investment with and without regular contributions.
- Apply the *Rule of 72* to solve investment problems, and explain the limitations of the rule.
- Determine, using technology, possible investment strategies to achieve a financial goal.
- Explain the advantages and disadvantages of long-term and short-term investment options.
- Explain, using examples, why smaller investments over a longer term may be better than larger investments over a shorter term.
- Solve an investment problem

Suggested Instructional Strategies

- Go to the *New Brunswick Securities Commission* website http://investinknowingmore.ca/educationprograms.html for links to:
  - Download PDF of *Make it Count: An Instructors Guide for Youth Money Management* (with associated budgeting app, Parent's Guide, Make it Count website) http://csa-acvm.ca/investortools.aspx?id=87
  - the NB Financial Education Network http://investinknowingmore.ca/FinancialEducationNetwork.html which lists other free resources from groups across NB.
- The *Canadian Securities Administrators* site http://csa-acvm.ca/investortools.aspx?id=1005 offers a wealth of information (in English and French) on investing, including free downloadable PDF brochures on investing: *Investing basics: Getting started, Investments at a Glance, Understanding Mutual Funds.*
- Have a competition within the class. Group students and give each a set amount to invest in a portfolio. Every day, give the students an opportunity to check, buy and sell stocks. The group whose portfolio is worth the most at the end of a specified time period wins. You may wish to use the website www.wallstreetsurvivor.com to do this activity.
- Periodically, provide a copy of a newspaper which carries a "World Markets" section (e.g. the Globe and Mail) and have student report on what they read.
- Go to www.getsmarteraboutmoney.ca which is a Canadian website, to access lesson plans, videos for students, and reference materials that support this curriculum.
- Invite local stockbrokers and/or wealth managers to speak to the class.
- Using a T1-83 graphing calculator, have students explore the TVM solver application for various interest rates, amortization periods, principal amounts etc.

SCO **N2: Analyze an investment portfolio in terms of: interest rate, rate of return, total return**. [ME, PS, R, T]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Thomas has two years before he goes off to community college. He has estimated that it will cost about $10,000 to go to college. He invests in a GIC that pays 6% interest per year. He deposits $360 per month for two years. Determine if Thomas will have enough money to go to college or will he have to find another way to supplement his education?

$Answer$: $year\ 1$: $(360 \times 12) \times 1.06 = \$4579.20$

$year\ 2$: $[\$4579.20 + (360 \times 12)] \times 1.06 = \$9433.15$

$He\ will\ be\ short\ by\ \$566.85, so\ will\ need\ to\ supplement\ his\ income.$

Q Richard invested $500 at 4% interest rate per annum. How long will it take for the $500 investment to have a value of approximately $1000?

$Answer$: $A = P + Prt$

$1000 = 500 + 500(0.04)t$

$t = 25 \quad \therefore\ it\ will\ take\ 25\ years\ for\ the\ investment\ to\ increase\ to\ \$1000$

Act Present the following scenario to the class and discuss as a group:

Samantha and Rick are old high school buddies who have met up again after twenty years. After some discussion they find out that they have both created investment portfolios, shown below. After the analysis, list two strengths and two weakness of each portfolio. Remember to consider interest rate, rate of return and total return when analyzing.

Samantha: Back in 1984, Samantha was an eighteen year old student. While she had time left to grow her savings, she didn't want to take a lot of risk. Samantha adopted a moderate investment profile with 50% invested in Canadian stocks (moderate risk), 40% in bonds (low risk), and 10% in cash equivalents.

Rick: In 1984, Rick was a 19-year-old professional hockey player. He thrived on taking risk. He also knew he wouldn't play hockey forever and his income might drop after he retired from the game. Rick adopted a moderately aggressive investment profile with 70% invested in Canadian Stocks (moderate risk), 20% in bonds (low risk), and 10% in cash equivalents.

The results: This table shows sample returns for Samantha and Rick, based on data from the past 20 years.

Investor	Started with (Jan 1984)	After 5 years (Jan 1989)	After 10 years (Jan 1994)	After 20 years (Jan 2004)	Average annual return
Samantha	$100 000	$154 330	$236 103	$424 785	7.5%
Rick	$100 000	$129 503	$236 736	$560 441	9.0%

Many things affected our two investors' results over this 20-year period:

- Interest rates soared in the late 1980s. The stock market on the other hand, went through a major drop in 1987 and a slow recovery. During this time, Rick's investments lagged. Samantha's investments grew faster and continued to do well throughout the ups and downs of the early 1990s.
- By 2000, the picture changed. Interest rates fell and the stock market was at a new high. Samantha's investments soon fell behind the rest.
- In the last five years, interest rates stayed low while the stock market went through another cycle of ups and downs. Rick's investments continued to grow the fastest, while Samantha's fell further behind.

Lesson learned: *In most cases, ultraconservative portfolios will see slower, steady growth. This is the tradeoff for keeping money stable and secure. For more growth potential, a more aggressive asset mix, with a higher level of risk should be selected.*
Losses are more likely when more risk is taken when investing and it is important to ensure that there is enough time and money to recover from those losses. Registered advisers are available to determine the right asset mix for your situation. (adapted from: http://www.getsmarteraboutmoney.ca/managing-your-money/planning/investing-basics/Pages/the-power-of-asset-mix-joan-michel-and-miriams-stories.aspx)

SCO **N3: Solve problems that involve personal budgets**. [CN, PS, R, T]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

N3: Solve problems that involve personal budgets (optional).

Scope and Sequence of Outcomes:

Grade Ten	Grade Eleven	Grade Twelve
N1: Solve problems that involve unit pricing and currency exchange, using proportional reasoning. *(GMF10)* **N2:** Demonstrate an understanding of income, including: wages, salary, contracts, commissions, and piecework to calculate gross pay and net pay. *(GMF10)* **N3:** Demonstrate an understanding of financial institution services used to access and manage finances. *(GMF10)* **N4:** Demonstrate an understanding of compound interest. *(GMF10)* **N5**: Demonstrate an understanding of credit options, including: credit cards, and loans. *(GMF10)*	**N3:** Solve problems that involve personal budgets *(optional).*	

ELABORATION

In grade 10, students gained an understanding of income and calculated gross and net pay including various tax and other deductions. They also explored in depth the services offered by financial institutions, and the impact of compound interest as related to various credit options. In this course (N1, N2) they have considered options for renting leasing and buying, and investment options.

This outcome, N3, is optional. However if time is available in-class or for a student project, this outcome provides an opportunity for students to draw on all of the financial knowledge they have gained, and explore how this might apply in a variety of circumstances, and at a personal level.

As they develop budgets students should explore a variety of scenarios, including short- and long-term goals, recurring expenses, and unexpected small or large expenses such as loss of a roommate, illness, fire, or loss of job.

A **balanced budget** is one in which the total income equals the total expenses. **Fixed expenses** are expenses that are unlikely to change from month to month while **variable expenses** are expenses that are likely to change from week to week or from month to month.

SCO **N3: Solve problems that involve personal budgets**. [CN, PS, R, T]

ACHIEVEMENT INDICATORS

- Identify income and expenses that should be included in a personal budget.
- Explain considerations that must be made when developing a budget; For example, prioritizing, recurring and unexpected expenses.
- Create a personal budget based on given income and expense data.
- Collect income and expense data, and create a budget.
- Modify a budget to achieve a set of personal goals.
- Investigate and analyze, with or without technology, “what if …” questions related to personal budgets.

Suggested Instructional Strategies

- Have students explore various scenarios and develop personal budgets around these scenarios.
- Use some of the free online resources available on financial literacy such as:

  www.getsmarteraboutmoney.ca

  http://www.rbcroyalbank.com/products/personalloans/budget/budget-calculator.html (a resource provided by RBC to explore the financial implications of “What if? situations such as job loss, loss of roommate, illness etc.)

  www.gailvazoxlade.com/articles.html

SCO **N3: Solve problems that involve personal budgets**. [CN, PS, R, T]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Have students use a chart similar to the following, as a guide to list all the expenses they think they might incur living on their own or with one or more roommates (this can be used as a group activity) .

Expense	*Amount ($)*
Getting started costs: One-time costs such as: hook-up fees for phone, cable or internet,; purchase of furniture, dishes, appliances.	
Rent or Mortgage	
Utilities: Electricity, telephone, heat, cable.	
Food: Staples such as flour, spices, condiments, beans; regular groceries. Home cooked meals are cheaper and usually healthier than restaurant food.	
Transportation: Public transit, bicycle, or car. If you have a car you will need to budget form insurance, gas, maintenance and parking.	
Medical/ Dental: Medical plan payments and/or costs such as glasses, contacts, prescriptions and dental care not covered by provincial Medicare or by a medical plan.	
Clothing: Consider clothes required for work, and seasonal clothes such as boots and a winter coat.	
Miscellaneous: This may include laundry, entertainment, toiletries, and cleaning supplies. Also consider purchasing gifts for birthdays and holidays.	
Other: This includes anything else that is not included in other categories such as loan payments, vacations, membership or workshop fees.	
TOTAL OF ALL ESTIMATED COSTS	

Act Present students with various scenarios; living on their own and working, living as a single parent with an infant or school aged child, going to school and working part time, working full time, living as a two income family with two children etc. Have them develop their own budget that includes all of their expenses or complete a budget such as the one found at:
http://moneyandyouth.cfee.org/en/resources/pdf/moneyyouth_chap9.pdf .

This could also be an opportunity for students to interview someone living in various circumstances listed above, to get a true picture of expenses, especially hidden ones.

SUMMARY OF CURRICULUM OUTCOMES

Foundations of Mathematics 11

Mathematical Processes

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Logical Reasoning

General Outcome: Develop logical reasoning.

Specific Outcomes

LR1: Analyze and prove conjectures using logical reasoning, to solve problems. [C, CN, PS, R]

LR2: Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [CN, PS, R, V]

Geometry

General Outcome: Develop spatial sense.

Specific Outcomes

G1: Derive proofs that involve the properties of angles and triangles. [CN, R, V]

G2: Solve problems that involve the properties of angles and triangles. [CN, PS,V]

G3: Solve problems that involve the cosine law and the sine law, including the ambiguous case. [CN, PS, R]

Relations and Functions

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes

RF1: Model and solve problems that involve systems of linear inequalities in two variables. [CN, PS, T, V]

RF2: Demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, axis of symmetry. [CN, PS, T, V]

Number

General Outcome: Develop number sense in financial applications.

Specific Outcomes

N1: Analyze costs and benefits of renting, leasing and buying. [CN, PS, R, T]

N2: Analyze an investment portfolio in terms of interest rate, rate of return, total return. [ME, PS, R, T]

N3: Solve problems that involve personal budgets *(optional).* [CN, PS, R, T]

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