Ministry of Education

Saskatchewan Curriculum

Mathematics

9

2009

Mathematics 9
ISBN 978-1-926631-47-9
1. Study and teaching (Middle school) - Saskatchewan - Curricula. 2. Competency-based education - Saskatchewan.
Saskatchewan. Ministry of Education. Curriculum and E-Learning. Science and Technology Unit.

Table of Contents

Acknowledgements

The Ministry of Education wishes to acknowledge the professional contributions and advice of the provincial curriculum reference committee members:

Daryl Bangsund
Good Spirit School Division
LEADS

Dr. Murray Bremner
Department of Mathematics and Statistics
University of Saskatchewan

Linda Goulet
Associate Professor
First Nations University of Canada

Angie Harding
Regina Roman Catholic Separate School Division
Saskatchewan Teachers' Federation

Susan Jeske
Prairie Spirit School Division
Saskatchewan Teachers' Federation

Wendy Lang
St. Paul's Roman Catholic Separate School Division
Saskatchewan Teachers' Federation

George McHenry
Division Board Trustee, Saskatchewan Rivers School Division
Saskatchewan School Boards Association

Dr. Shaun Murphy
College of Education
University of Saskatchewan

Dr. Kathy Nolan
Faculty of Education
University of Regina

Kathi Sandbeck
Sun West School Division
Saskatchewan Teachers' Federation

Doug Sthamann
Regina Public School Division
Saskatchewan Teachers' Federation

Rodney White
North East School Division
Saskatchewan Teachers' Federation

A special thank you is extended to the Elders who reviewed and provided advice to strengthen the curriculum outcomes:

- Jonas Bird
- Albert Scott
- Darlene Spiedel
- Allan Adams.

In addition, the Ministry of Education acknowledges the guidance of:

- Dr. Edward Doolittle, Department of Mathematics and Statistics, First Nations University
- program team members
- focus groups of teachers
- other educators and reviewers.

Introduction

As a Required Area of Study, mathematics is to be allocated 200 minutes per week for the entire school year at Grade 9. It is important that students receive the full amount of time allocated to their mathematical learning and that the learning be focused upon students attaining the understanding and skills as defined by the outcomes and indicators stated in this curriculum.

The outcomes in Grade 9 Mathematics build upon students' prior learnings and continue to develop their number sense, spatial sense, logical thinking, and understanding of mathematics as a human endeavour. These continuing learnings prepare students to be confident, flexible, and capable with their mathematical knowledge in new contexts.

Outcomes describe the knowledge, skills, and understandings that students are expected to attain by the end of a particular grade level.

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. These indicators are a representative list of the kinds of things a student needs to know and/or be able to do in order to achieve the learnings intended by the outcome. New and combined indicators, which remain within the breadth and depth of the outcome, can be created by teachers to meet the needs and circumstances of their students and communities.

Indicators are a representative list of the types of things a student should know or be able to do if they have attained the outcome.

Within the outcomes and indicators in this curriculum, the terms "including" and "such as", as well as the abbreviation "e.g.," occur. The use of each term serves a specific purpose. The term "including" prescribes content, contexts, or strategies that students must experience in their learning, without excluding other possibilities. For example, an indicator might say that students are to sketch graphs for given linear relations, including horizontal and vertical lines. This would mean that, within their study of linear relations, it is expected that horizontal and vertical lines would be included.

The term "such as" provides examples of possible broad categories of content, contexts, or strategies that teachers or students may choose, without excluding other possibilities. For example, an indicator might include the phrase "such as painting or dance" as examples of the types of art works that might be referenced with reference to line and rotational symmetry. This statement provides teachers and students with possible categories of art works to consider, while not excluding other forms.

Finally, the abbreviation "e.g.," offers specific examples of what a term, concept, or strategy might look like. For example, an indicator might include the phrase "e.g., $(-2)^4$, (-2^4), and -2^4" which are specific illustrations of the positioning of a negative sign with respect to a power.

This curriculum's outcomes and indicators have been designed to address current research in mathematics education as well as the needs of Saskatchewan students. The Grade 9 Mathematics outcomes are based on the Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K-9 Mathematics* outcomes (2006). Changes throughout all of the grades have been made for a number of reasons including:

- decreasing content in each grade to allow for more depth of understanding
- rearranging concepts to allow for greater depth of learning in one year and to align related mathematical concepts
- increasing the focus on numeracy (i.e., understanding numbers and their relationship to each other) beginning in Kindergarten
- introducing algebraic thinking earlier.

In Grade 9, students begin developing their understanding of polynomials including operations on polynomials.

Also included in this curriculum is information regarding how Grade 9 Mathematics connects to the K-12 goals for mathematics. These goals define the purpose of mathematics education for Saskatchewan students.

In addition, teachers will find discussions of the critical characteristics of mathematics education, assessment and evaluation of student learning in mathematics, inquiry in mathematics, questioning in mathematics, and connections between Grade 9 Mathematics and other Grade 9 areas of study within this curriculum.

Finally, the Glossary provides explanations of some of the mathematical terminology used in this curriculum.

Core Curriculum

The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater.

(NCTM, 2000, p. 4)

Core Curriculum is intended to provide all Saskatchewan students with an education that will serve them well regardless of their choices after leaving school. Through its various components and initiatives, Core Curriculum supports the achievement of the Goals of Education for Saskatchewan. For current information regarding Core Curriculum, please refer to *Core Curriculum: Principles, Time Allocations, and Credit Policy* on the Ministry of Education website.

Broad Areas of Learning

There are three Broad Areas of Learning that reflect Saskatchewan's Goals of Education. K-12 mathematics contributes to the Goals of Education through helping students achieve knowledge, skills, and attitudes related to these Broad Areas of Learning.

Developing Lifelong Learners

Students who are engaged in constructing and applying mathematical knowledge naturally build a positive disposition towards learning. Throughout their study of mathematics, students should be learning the skills (including reasoning strategies) and developing the attitudes that will enable the successful use of mathematics in daily life. Moreover, students should be developing understandings of mathematics that will support their learning of new mathematical concepts and applications that may be encountered within both career and personal interest choices. Students who successfully complete their study of K-12 mathematics should feel confident about their mathematical abilities and have developed the knowledge, understandings, and abilities necessary to make future use and/or studies of mathematics meaningful and attainable.

Developing lifelong learners is related to the following Goals of Education:

- *Basic Skills*
- *Lifelong Learning*
- *Self Concept Development*
- *Positive Lifestyle.*

In order for mathematics to contribute to this Broad Area of Learning, students must actively learn the mathematical content in the outcomes through using and developing logical thinking, number sense, spatial sense, and understanding of mathematics as a human endeavour (the four goals of K-12 Mathematics). It is crucial that the students discover the mathematics outlined in the curriculum rather than the teacher covering it.

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

(NCTM, 2000, p. 20)

Developing a Sense of Self and Community

Developing a sense of self and community is related to the following Goals of Education:

- *Understanding & Relating to Others*
- *Self Concept Development*
- *Positive Lifestyle*
- *Spiritual Development.*

To learn mathematics with deep understanding, students not only need to interact with the mathematical content, but with each other as well. Mathematics needs to be taught in a dynamic environment where students work together to share and evaluate strategies and understandings. Students who are involved in a supportive mathematics learning environment that is rich in dialogue are exposed to a wide variety of perspectives and strategies from which to construct a sense of the mathematical content. In such an environment, students also learn and come to value how they, as individuals and as members of a group or community, can contribute to understanding and social well-being through a sense of accomplishment, confidence, and relevance. When encouraged to present ideas representing different perspectives and ways of knowing, students in mathematics classrooms develop a deeper understanding of the mathematics. At the same time, students also learn to respect and value the contributions of others.

Many of the topics and problems in a mathematics classroom can be initiated by the children themselves. In a classroom focused on working mathematically, teachers and children work together as a community of learners; they explore ideas together and share what they find. It is very different to the traditional method of mathematics teaching, which begins with a demonstration by a teacher and continues with children practicing what has been demonstrated.
(Skinner, 1999, p. 7)

Mathematics provides many opportunities for students to enter into communities beyond the classroom by engaging with people in the neighbourhood or around the world. By working towards developing a deeper understanding of mathematics and its role in the world, students develop their personal and social identity, and learn healthy and positive ways of interacting and working together with others.

Developing Engaged Citizens

Developing engaged citizens is related to the following Goals of Education:
- *Understanding & Relating to Others*
- *Positive Lifestyle*
- *Career and Consumer Decisions*
- *Membership in Society*
- *Growing with Change*

Mathematics brings a unique perspective and way of knowing to the analysis of social impact and interdependence. Doing mathematics requires students to "leave their emotions at the door" and to engage in different situations for the purpose of understanding what is really happening and what can be done. Mathematical analysis of topics that interest students such as trends in climate change, homelessness, health issues (hearing loss, carpal tunnel syndrome, diabetes), and discrimination can be used to engage the students in interacting and contributing positively to their classroom, school, community, and world. With the understandings that students derive through mathematical analysis, they become better informed and have a greater respect for and understanding of differing opinions and possible options. With these understandings, students can make better informed and more personalized decisions regarding roles within, and contributions to, the various communities in which students are members.

Cross-curricular Competencies

The Cross-curricular Competencies are four interrelated areas containing understandings, values, skills, and processes which are considered important for learning in all areas of study. These competencies reflect the Common Essential Learnings and are intended to be addressed in each area of study at each grade level.

Developing Thinking

K-12 Goals for Developing Thinking:
- *thinking and learning contextually*
- *thinking and learning creatively*
- *thinking and learning critically.*

It is important that, within their study of mathematics, students are engaged in personal construction and understanding of mathematical knowledge. This most effectively occurs through student engagement in inquiry and problem solving when students are challenged to think critically and creatively. Moreover, students need to experience mathematics in a variety of contexts – both real world applications and mathematical contexts – in which students are asked to consider questions such as "What would happen if …", "Could we find …", and "What does this tell us?" Students need to be engaged in the social construction of mathematics to develop an understanding and appreciation of mathematics as a tool which can be used to consider different perspectives, connections, and relationships. Mathematics is a subject that depends upon the effective incorporation of independent work and reflection with interactive contemplation, discussion, and resolution.

Developing Identity and Interdependence

Given an appropriate learning environment in mathematics, students can develop both their self-confidence and self-worth. An interactive mathematics classroom in which the ideas, strategies, and abilities of individual students are valued supports the development of personal and mathematical confidence. It can also help students take an active role in defining and maintaining the classroom environment and accept responsibility for the consequences of their choices, decisions, and actions. A positive learning environment combined with strong pedagogical choices that engage students in learning serves to support students in behaving respectfully towards themselves and others.

K-12 Goals for Developing Identity and Interdependence:
- *understanding, valuing, and caring for oneself*
- *understanding, valuing, and caring for others*
- *understanding and valuing social and environmental interdependence and sustainability.*

Developing Literacies

Through their mathematics learning experiences, students should be engaged in developing their understandings of the language of mathematics and their ability to use mathematics as a language and representation system. Students should be regularly engaged in exploring a variety of representations for mathematical concepts and should be expected to communicate in a variety of ways about the mathematics being learned. Important aspects of learning mathematical language is to make sense of mathematics, communicate one's own understandings, and develop strategies to explore what and how others know about mathematics. The study of mathematics should encourage the appropriate use of technology. Moreover, students should be aware of and able to make the appropriate use of technology in mathematics and mathematics learning. It is important to encourage students to use a variety of forms of representation (concrete manipulatives, physical movement, oral, written, visual, and symbolic) when exploring mathematical ideas, solving problems, and communicating understandings.

K-12 Goals for Developing Literacies:
- *developing knowledge related to various literacies*
- *exploring and interpreting the world through various literacies*
- *expressing understanding and communicating meaning using various literacies.*

All too often, it is assumed that symbolic representation is the only way to communicate mathematically. The more flexible students are in using a variety of representations to explain and work with the mathematics being learned, the deeper students' understanding becomes.

Students gain insights into their thinking when they present their methods for solving problems, when they justify their reasoning to a classmate or teacher, or when they formulate a question about something that is puzzling to them. Communication can support students' learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols. Misconceptions can be identified and addressed. A side benefit is that it reminds students that they share responsibility with the teacher for the learning that occurs in the lesson.

(NCTM, 2000, pp. 60 – 61)

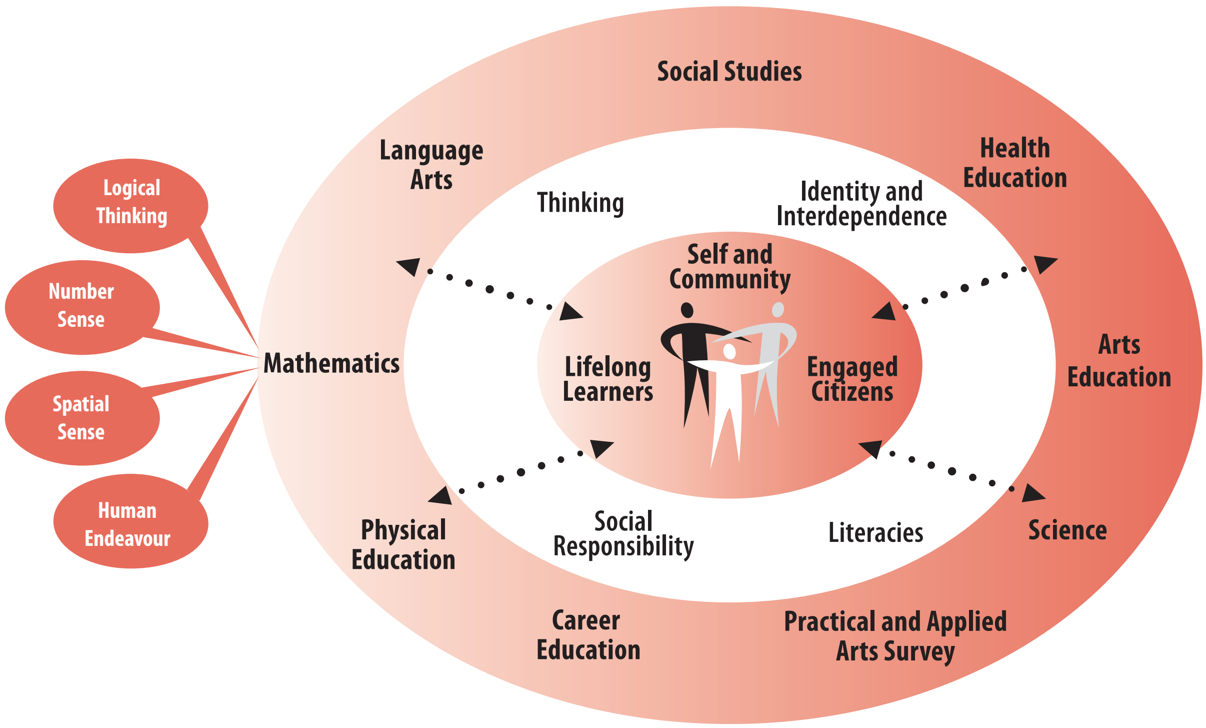
Developing Social Responsibility

K-12 Goals for Developing Social Responsibility:
- *using moral reasoning*
- *engaging in communitarian thinking and dialogue*
- *taking social action.*

As students progress in their mathematical learning, they need to experience opportunities to share and consider ideas, and resolve conflicts between themselves and others. This requires that the learning environment be constructed by the teacher and students to support respectful, independent, and interdependent behaviours. Every student should feel empowered to help others in developing their understanding, while finding respectful ways to seek help from others. By encouraging students to explore mathematics in social contexts, students can be engaged in understanding the situation, concern, or issue and then in planning for responsible reactions or responses. Mathematics is a subject dependent upon social interaction and, as a result, social construction of ideas. Through the study of mathematics, students learn to become reflective and positively contributing members of their communities. Mathematics also allows for different perspectives and approaches to be considered, assessed for situational validity, and strengthened.

Aim and Goals of K-12 Mathematics

The aim of Saskatchewan's K-12 mathematics program is to help students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities and ongoing learnings and work experiences. The mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.



Defined below are four goals for K-12 mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are directly related to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes must, therefore, also promote student achievement with respect to the goals.

Logical Thinking

Through their learning of K-12 Mathematics, students will **develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems.**

This goal encompasses processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- observation
- inductive and deductive thinking

A … feature of the social culture of [mathematics] classrooms is the recognition that the authority of reasonability and correctness lies in the logic and structure of the subject, rather than in the social status of the participants. The persuasiveness of an explanation, or the correctness of a solution depends on the mathematical sense it makes, not on the popularity of the presenter.

(Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, Human, 1997, p. 10)

- proportional reasoning
- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proving
- exploring, identifying, and describing relationships
- modeling and representing (including concrete, oral, physical, pictorial, and symbolic representations)
- conjecturing and asking "what if" (mathematical play).

In order to develop logical thinking, students need to be actively involved in constructing their mathematical knowledge using the above strategies and processes. Inherent in each of these strategies and processes is student communication and the use of, and connections between, multiple representations.

Number Sense

In Grade 9, students move from working with fractions to understanding rational numbers. What are the connections between the students' prior number sense development and their new learnings?

Through their learning of K-12 mathematics, students will **develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers and apply this understanding to new situations and problems.**

Foundational to students developing number sense is having ongoing experiences with:

- decomposing and composing of numbers
- relating different operations to each other
- modeling and representing numbers and operations (including concrete, oral, physical, pictorial, and symbolic representations)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- developing and understanding algebraic representations and manipulations as an extension of numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Students also develop understanding of place value through the strategies they invent to compute.
(NCTM, 2000, p. 82)

Number sense goes well beyond being able to carry out calculations. In fact, in order for students to become flexible and confident in their calculation abilities, and to transfer those abilities to more abstract contexts, students must first develop a strong understanding of numbers in general. A deep understanding of the meaning, roles, comparison, and relationship between numbers is critical to the development of students' number sense and their computational fluency.

Spatial Sense

Through their learning of K-12 mathematics, students will **develop an understanding of 2-D shapes and 3-D objects, and the relationships between geometrical shapes and objects and numbers, and apply this understanding to new situations and problems.**

2-D shapes are abstract ideas because they only really exist as parts of 3-D objects. How can you assess to see if students really understand this connection?

Development of a strong spatial sense requires students to have ongoing experiences with:

- construction and deconstruction of 2-D shapes and 3-D objects
- investigations and generalizations about relationships between 2-D shapes and 3-D objects
- explorations and abstractions related to how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- explorations and generalizations about the movement of 2-D shapes and 3-D objects
- explorations and generalizations regarding the dimensions of 2-D shapes and 3-D objects
- explorations, generalizations, and abstractions about different forms of measurement and their meaning.

Being able to communicate about 2-D shapes and 3-D objects is foundational to students' geometrical and measurement understandings and abilities. Hands-on exploration of 3-D objects and the creation of conjectures based upon patterns that are discovered and tested should drive the students' development of spatial sense, with formulas and definitions resulting from the students' mathematical learnings.

As students sort, build, draw, model, trace, measure, and construct, their capacity to visualize geometric relationships will develop.
(NCTM, 2000, p. 165)

Mathematics as a Human Endeavour

Through their learning of K-12 mathematics, students will **develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs.**

Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:

- value place-based knowledge and learning
- value learning from and with community
- encourage and value varying perspectives and approaches to mathematics
- recognize and value one's evolving strengths and knowledge in learning and doing mathematics
- recognize and value the strengths and knowledge of others in doing mathematics
- value and honour reflection and sharing in the construction of mathematical understanding

What types of instructional strategies support student attainment of the K-12 mathematics goals?

How can student attainment of these goals be assessed and the results be reported?

- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematically and personally)
- build self-confidence related to mathematical insights and abilities
- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics.

Meaning does not reside in tools; it is constructed by students as they use tools.
(Hiebert et al., 1997, p. 10)

Students should be encouraged to challenge the boundaries of their experiences, and to view mathematics as a set of tools and ways of thinking that every society develops to meet their particular needs. This means that mathematics is a dynamic discipline in which logical thinking, number sense, and spatial sense form the backbone of all developments and those developments are determined by the contexts and needs of the time, place, and people.

The content found within the grade level outcomes for the K-12 mathematics program, and its applications, is first and foremost the vehicle through which students can achieve the four goals of K-12 mathematics. Attainment of these four goals will result in students with the mathematical confidence and tools necessary to succeed in future mathematical endeavours.

Teaching Mathematics

At the National Council of Teachers of Mathematics (NCTM) Canadian Regional Conference in Halifax (2000), Marilyn Burns said in her keynote address, "When it comes to mathematics curricula there is very little to cover, but an awful lot to uncover [discover]." This statement captures the essence of the ongoing call for change in the teaching of mathematics. Mathematics is a dynamic and logic-based language that students need to explore and make sense of for themselves. For many teachers, parents, and former students, this is a marked change from the way mathematics was taught to them. Research and experience indicate there is a complex, interrelated set of characteristics that teachers need to be aware of in order to provide an effective mathematics program.

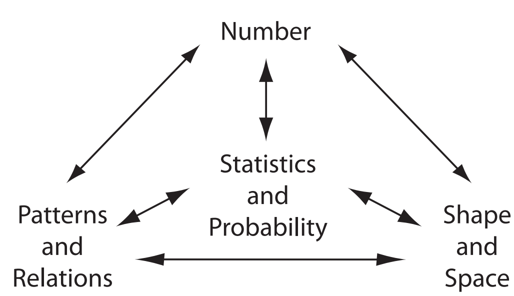
Critical Characteristics of Mathematics Education

The following sections in this curriculum highlight some of the different facets for teachers to consider in the process of changing from covering to supporting students in discovering mathematical concepts. These facets include:

- organization of the outcomes into strands
- seven mathematical processes
- the difference between covering and discovering mathematics
- development of mathematical terminology
- First Nations and Métis learners and mathematics
- critiqueing statements
- continuum of understanding from concrete to abstract
- modelling and making connections
- role of homework
- importance of ongoing feedback and reflection.

Strands

The content of K-12 mathematics can be organized in a variety of ways. In this curriculum, the outcomes and indicators are grouped according to four strands: **Number, Patterns and Relations, Shape and Space, and Statistics and Probability.** Although this organization implies a relatedness among the outcomes identified in each of the strands, it should be noted the mathematical concepts are interrelated across the strands as well as within strands. Teachers are encouraged to design learning activities that integrate outcomes both within a strand and across the strands so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate strands.



Mathematical Processes

This Grade 9 Mathematics curriculum recognizes seven processes inherent in the teaching, learning, and doing of mathematics. These processes focus on: communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing along with using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding.

The outcomes in K-12 mathematics should be addressed through the appropriate mathematical processes as indicated by the bracketed letters following each outcome. Teachers should consider carefully in their planning those processes indicated as being important to supporting student achievement of the respective outcomes.

Communication [C]

Communication works together with reflection to produce new relationships and connections. Students who reflect on what they do and communicate with others about it are in the best position to build useful connections in mathematics.
(Hiebert et al., 1997, p. 6)

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas using both personal and mathematical language and symbols. These opportunities allow students to create links among their own language, ideas, and prior knowledge, the formal language and symbols of mathematics, and new learnings.

Communication is important in clarifying, reinforcing, and adjusting ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology, but only when they have had sufficient experience to develop an understanding for that terminology.

Concrete, pictorial, symbolic, physical, verbal, written, and mental representations of mathematical ideas should be encouraged and used to help students make connections and strengthen their understandings.

Connections [CN]

Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding …. Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching.
(Caine & Caine, 1991, p.5)

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to other real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

The brain is constantly looking for and making connections. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and prior knowledge, and increase student willingness to participate and be actively engaged.

Mental Mathematics and Estimation [ME]

What strategies should Grade 9 students be using in their mental mathematics and estimation tasks? When would you expect the students to use mental mathematics and estimation?

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally and reasoning about the relative size of quantities without the use of external memory aids. Mental mathematics enables students to determine answers and propose strategies without paper and pencil. It improves computational fluency and problem solving by developing efficiency, accuracy, and flexibility.

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you …?", "Can you …?", or "What if …?", the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

Mathematical problem-solving often involves moving backwards and forwards between numerical/algebraic representations and pictorial representations of the problem.
(Haylock & Cockburn, 2003, p. 203)

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not problem solving but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students actively look for, and engage in finding, a variety of strategies for solving problems empowers students to explore alternatives and develops confidence, reasoning, and mathematical creativity.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. High-order inquiry challenges students to think and develop a sense of wonder about mathematics.

A Grade 9 student might conjecture that the students in Grade 9 have more pet cats than dogs. What experiences and tasks will that student need to have to reach and verify such conjectures?

Mathematical experiences in and out of the classroom should provide opportunities for students to engage in inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Posing conjectures and trying to justify them is an expected part of students' mathematical activity.
(NCTM, 2000, p. 191)

[Visualization] involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.
(Armstrong, 1993, p.10)

Visualization [V]

The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number sense, spatial sense, and logical thinking. Number visualization occurs when students create mental representations of numbers and visual ways to compare those numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes including aspects such as dimensions and measurements.

Visualization is also important in the students' development of abstraction and abstract thinking and reasoning. Visualization provides a connection between the concrete, physical, and pictorial to the abstract symbolic. Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations as well as the use of communication to develop connections among different contexts, content, and representations.

There are many opportunities for the use of technology in Grade 9 mathematics. What are some of the patterns that students could discover through the use of technology that would support the development of their new mathematical understandings?

Technology [T]

Technology tools contribute to student achievement of a wide range of mathematical outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators, computers, and other forms of technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense
- develop spatial sense
- develop and test conjectures.

Technology should not be used as a replacement for basic understandings and intuition.
(NCTM, 2000, p. 25)

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. It is important for students to understand and appreciate the appropriate use of technology in a mathematics classroom. It is also important that students learn to distinguish between when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding, but should be used to enhance it.

Discovering versus Covering

Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content. Knowing what needs to be covered and what can be discovered is crucial in planning for mathematical instruction and learning. The content that needs to be covered (what the teacher needs to explicitly tell the students) is the social conventions or customs of mathematics. This content includes things such as what the symbol for an operation looks like, mathematical terminology, and conventions regarding recording of symbols.

The content that can and should be discovered by students is the content that can be constructed by students based on their prior mathematical knowledge. This content includes things such as strategies and procedures, rules, and problem solving. Any learning in mathematics that is a result of the logical structure of mathematics can and should be constructed by students.

What mathematical content in Grade 9 can students discover (through the careful planning of a teacher) and what does a teacher need to tell the students?

For example, in Grade 9, the students encounter similarity of 2-D shapes for the first time in outcome SS9.3 :

> Demonstrate understanding of similarity of 2-D shapes.
>
> [C, CN, PS, R, V]

In this outcome, the term "similarity" and the symbol "~" are both social conventions of the mathematics the students are learning and, as such, both are something that the teacher must tell the student. Identifying and describing patterns of proportionality between the side lengths of 2-D shapes as well as the relationship between interior angles is the foundation of the students' construction of understanding. This type of learning requires students to work concretely, pictorially, orally, in writing, and symbolically. It also requires that students share their ideas with classmates and reflect upon how the ideas and understandings of others relate to, inform, and clarify what students individually understand. In this type of learning, the teacher does not tell the students how to do the mathematics but, rather, invites the students to explore and develop an understanding of the logical structures inherent in the

The plus sign, for example, and the symbols for subtraction, multiplication, and division are all arbitrary convention. … Learning most of mathematics, however, relies on understanding its logical structures. The source of logical understanding is internal, requiring a child to process information, make sense of it, and figure out how to apply it.
(Burns & Silbey, 2000, p. 19)

mathematics in increasing patterns. Thus, the teacher's role is to create inviting and rich inquiring tasks and to use questioning to effectively probe and further students' learning.

Development of Mathematical Terminology

Teachers should model appropriate conventional vocabulary.
(NCTM, 2000, p. 131)

Part of learning mathematics is learning how to speak mathematically. Teaching students mathematical terminology when they are learning for deep understanding requires that the students connect the new terminology with their developing mathematical understanding. As a result, it is important that students first linguistically engage with new mathematical concepts using words that students already know or that make sense to them.

For example, in outcome SS9.1:

> Demonstrate understanding of circle properties including:
>
> - perpendicular line segments from the centre of a circle to a chord bisect the chord
> - inscribed angles subtended by the same arc have the same measure
> - the measure of a central angle is twice the measure of an inscribed angle subtending the same arc
> - tangents to a circle are perpendicular to the radius ending at the point of tangency.
>
> [C, CN, PS, R, T, V]

the terminology of "chord", "subtended", "inscribed angles", "arc", "central angle", and "point of tangency" will likely be new. Before being formally introduced to this terminology, it is important that students be concretely and pictorially, including through the use of technology, representing and analyzing different relationships involving lines, angles, and circles. As the students discover different relationships, their need for specific terminology emerges and it is at that time the specific terms should be introduced. Students should be encouraged to use their own personal vocabulary as they develop their understandings and to then demonstrate the new terminology as they summarize their discoveries and learnings for themselves and for others in the classroom.

In helping students develop their working mathematical language, it is also important for the teacher to recognize that for many students, including First Nations and Métis, that just because a student doesn't recognize a specific term or procedure, the student may in fact have a deep understanding of the overall mathematical topic. Many perceived learning difficulties in mathematics are the result of students' cultural and personal ways of knowing not being connected to the mathematical language.

In addition, the English language often allows for multiple interpretations of the same sentence, depending upon where the emphasis is placed. For example, consider the sentence "The shooting of the hunters was terrible" (Paulos, 1980, p. 65). Were the hunters that bad of a shot, was it terrible that the hunters got shot, was it terrible that they were shooting, or is this all about the photographs that were taken of the hunters? It is important that students be engaged in dialogue through which they explore possible meanings and interpretations of mathematical statements and problems.

First Nations and Métis Learners and Mathematics

It is important for teachers to realize that First Nations and Métis students, like all students, come to mathematics classes with a wealth of mathematical understandings. Within these mathematics classes, some First Nations and Métis students may develop a negative sense of their ability in mathematics and, in turn, do poorly on mathematics assessments. Such students may become alienated from mathematics because it is not taught to their schema, cultural and environmental content, or real life experiences. A first step in actualization of mathematics from First Nations and Métis perspectives is to empower teachers to understand that mathematics is not acultural. As a result, teachers then realize that the traditional ways of teaching the mathematics are also culturally-biased. These understandings will support the teacher in developing First Nations and Metis students' personal mathematical understandings and mathematical self-confidence and ability through a more holistic and constructivist approach to learning. Teachers need to consider factors that impact the success of First Nations and Métis students in mathematics: cultural contexts and pedagogy.

It is important for teachers to recognize the influence of cultural contexts on mathematical learning. Educators need to be sensitive to the cultures of others, as well as to how their own cultural background influences their current perspective and practice. Mathematics instruction focuses on the individual parts of the whole understanding and, as a result, the contexts presented tend to be compartmentalized and treated discretely. This focus on parts may be challenging for students who rely on whole contexts to support understanding.

For some First Nations and Métis students, the word "equal" may carry the cultural understanding of being "for the good of the community". For example, "equal" sharing of the meat from a hunt may not mean that everyone gets the same amount.

Mathematical ideas are valued, viewed, contextualized, and expressed differently by cultures and communities. Translation of these mathematical ideas between cultural groups cannot be assumed to be a direct link. Consider, for example, the concept of "equal", which is a key understanding in this curriculum. The Western understanding of "equal" is 'the same'. In many First Nations and Métis communities, however, "equal" is understood as meaning 'for the good of the community'. Teachers need to support students in uncovering these differences in ways of knowing and understanding within the mathematic classroom. Various ways of knowing need to be celebrated to support the learning of all students.

Along with an awareness of students' cultural context, pedagogical practices also influence the success of First Nations and Métis students in the mathematics classroom. Mathematical learning opportunities need to be holistic, occurring within social and cultural interactions through dialogue, language, and the negotiation of meanings. Constructivism, ethnomathematics, and teaching through an inquiry approach are supportive of a holistic perspective to learning. Constructivism, inquiry learning, and ethnomathematics allow students to enter the learning process according to their ways of knowing, prior knowledge, and learning styles. Ethnomathematics also shows the relationship between mathematics and cultural anthropology. It is used to translate earlier forms of thinking into modern-day understandings. Individually, and as a class, teachers and students need to explore the big ideas that are foundational to this curriculum and investigate how those ideas relate to them personally and as a learning community. Mathematics learned within contexts that focus on the day-to-day activities found in students' communities support learning by providing a holistic focus. Mathematics needs to be taught using the expertise of elders and the local environment as educational resources. The variety of interactions that occur among students, teachers, and the community strengthen the learning experiences for all.

Critiquing Statements

One way to assess students' depth of understanding of an outcome is to have the students critique a general statement which, on first reading, may seem to be true or false. By having students critique such statements, the teacher is able to identify strengths and deficiencies in their understanding. Some indicators in this curriculum are examples of statements that students can analyze for accuracy. For example, for outcome P9.3, one of the indicators reads:

Critique the statement: "For any linear equality, there are two related linear inequalities".

The purpose of this indicator is for teachers to assess the depth of understanding students have about the relationship between equality and inequality in a mathematical, and more specifically variable, context. Although it may be true that one quantity can always be described as being equal to, less than, or greater than another quantity, the same is not true of variable expressions. In such cases, two additional possibilities exist in the relationship, that of being less than or equal to and greater than or equal to. Students often find this conceptually difficult if they have not grasped the relationship, and also the fundamental difference between quantities and variable expressions. Asking students to critique statements like the one given above will give teachers insight into the students' understandings, and also provide guidance regarding further experiences that the student may need to have.

Critiquing statements is an effective way to assess students individually or as a small or large group. When engaged as a group, the discussion and strategies that emerge not only inform the teacher, but also engage all of the students in a deeper understanding of the topic.

The Concrete to Abstract Continuum

It is important that, in learning mathematics, students be allowed to explore and develop understandings by moving along a concrete to abstract continuum. As understanding develops, this movement along the continuum is not necessarily linear. Students may at one point be working abstractly but when a new idea or context arises, they need to return to a concrete starting point. Therefore, the teacher must be prepared to engage students at different points along the continuum.

It is important for students to use representations that are meaningful to them.
(NCTM, 2000, p. 140)

In addition, what is concrete and what is abstract is not always obvious and can vary according to the thinking processes of the individual. For example, when considering a problem about the total number of pencils, some students might find it more concrete to use pictures of pencils as a means of representing the situation. Other students might find coins more concrete because they directly associate money with the purchasing or having of a pencil.

As well, teachers need to be aware that different aspects of a task might involve different levels of concreteness or abstractness. Consider the following situational question involving surface area:

What is the surface area of your computer?

Depending upon how the question is expected to be solved (or if there is any specific expectation), this question can be approached abstractly (using symbolic number statements), concretely (e.g., using manipulatives, pictures), or both.

Models and Connections

New mathematics is continuously developed by creating new models as well as combining and expanding existing models. Although the final products of mathematics are most frequently represented by symbolic models, their meaning and purpose is often found in the concrete, physical, pictorial, and oral models and the connections between them.

A major responsibility of teachers is to create a learning environment in which students' use of multiple representations is encouraged.
(NCTM, 2000, pp. 139)

To develop a deep and meaningful understanding of mathematical concepts, students need to represent their ideas and strategies using a variety of models (concrete, physical, pictorial, oral, and symbolic). In addition, students need to make connections between the different representations. These connections are made by having the students try to move from one type of representation to another (how could you write what you've done here using mathematical symbols?) or by having students compare their representations with others in the class.

In making these connections, students should be asked to reflect upon the mathematical ideas and concepts that are being used in their new models (e.g., I know that addition means to put things together into a group, so I'm going to move the two sets of algetiles together to determine the sum of the polynomials).

Making connections also involves looking for patterns. For example, in outcome P9.1:

Demonstrate understanding of linear relations including:

- graphing
- analyzing
- interpolating and extrapolating
- solving situational questions.

[C, CN, PS, R, T, V]

the students' exploration and recognition of patterns and relationships between graphs and equations of linear relations is key to the development of a deep understanding of linear relations. Students need to build strong connections between the characteristics and properties of the graphs and the characteristics of the equations.

Role of Homework

The role of homework in teaching for deep understanding is important. Students should be given unique problems and tasks that help students to consolidate new learnings with prior knowledge, explore possible solutions, and apply learnings to new situations. Although drill and practice does serve a purpose in learning for deep understanding, the amount and timing of the drill will vary among different learners. In addition, when used as homework, drill and practice frequently serves to cause frustration, misconceptions, and boredom to arise in students.

As an example of the type or style of homework that can be used to help students develop deep understanding of Grade 9 Mathematics, consider outcome SP9.3:

> SP9.3 Demonstrate an understanding of the role of probability in society.
>
> [C, CN, R, T]

As a homework task, students might be asked to collect examples from their homes, interests, and personal lives that involve or include statements of probability. From there, those examples could be shared in class and would provide the foundation for the students' learning about the impact of probability theory on their everyday lives. Some of the questions that students might discuss related to their examples include:

- Would you make the same decision as the person/people in your example? Why or why not?
- Are there factors other than the known probability that you would consider before making such a decision?
- Is an 80% chance of an event happening equally influential in different contexts?

By first engaging the students in contexts that are personally relevant, students can explore and bring to light different factors to consider when making a decision regarding known probability. From these personal contexts, students can expand into community and socially relevant contexts and bring a better understanding of the role of emotions, values, and probability in decision making.

Characteristics of Good Math Homework

- *It is accessible to children at many levels.*
- *It is interesting both to children and to any adults who may be helping.*
- *It is designed to provoke deep thinking.*
- *It is able to use concepts and mechanics as means to an end rather than as ends in themselves.*
- *It has problem solving, communication, number sense, and data collection at its core.*
- *It can be recorded in many ways.*
- *It is open to a variety of ways of thinking about the problem although there may be one right answer.*
- *It touches upon multiple strands of mathematics, not just number.*
- *It is part of a variety of approaches to and types of math homework offered to children throughout the year.*

(Raphel, 2000, p. 75)

> *Feedback can take many different forms. Instead of saying, "This is what you did wrong," or "This is what you need to do," we can ask questions: "What do you think you need to do? What other strategy choices could you make? Have you thought about …?"*
> *(Stiff, 2001, p. 70)*

> *Not all feedback has to come from outside – it can come from within. When adults assume that they must be the ones who tell students whether their work is good enough, they leave them handicapped, not only in testing situations (such as standardized tests) in which they must perform without guidance, but in life itself.*
> *(NCTM, 2000, p. 72)*

> *A simple model for talking about understanding is that to understand something is to connect it with previous learning or other experiences… A mathematical concept can be thought of as a network of connections between symbols, language, concrete experiences, and pictures.*
> *(Haylock & Cockburn, 2003, p. 18)*

Ongoing Feedback and Reflection

Ongoing feedback and reflection, both for students and teachers, are crucial in classrooms when learning for deep understanding. Deep understanding requires that both the teacher and students need to be aware of their own thinking as well as the thinking of others.

Feedback from peers and the teacher helps students rethink and solidify their understandings. Feedback from students to the teacher gives much needed information in the teacher's planning for further and future learnings.

Self-reflection, both shared and private, is foundational to students developing a deep understanding of mathematics. Through reflection tasks, students and teachers come to know what it is that students do and do not know. It is through such reflections that not only can a teacher make better informed instructional decisions, but also that a student can set personal goals and make plans to reach those goals.

Teaching for Deep Understanding

For deep understanding, it is vital that students learn by constructing knowledge, with very few ideas being relayed directly by the teacher. As an example, the addition sign (+) is something which the teacher must introduce and ensure that students know. It is the symbol used to show the combination or addition of two quantities. The process of adding, however, and the development of addition and subtraction facts should be discovered through the students' investigation of patterns, relationships, abstractions, and generalizations.

It is important for teachers to analyze the outcomes to identify what students need to know, understand, and be able to do. Teachers also need to consider opportunities they can provide for students to explain, apply, and transfer understanding to new situations. This reflection supports professional decision making and planning effective strategies to promote students' deeper understanding of mathematical ideas.

It is important that a mathematics learning environment include effective interplay of:

- reflection
- exploration of patterns and relationships
- sharing of ideas and problems
- consideration of different perspectives
- decision making
- generalizing

- verifying and proving
- modeling and representing.

Mathematics is learned when students are engaged in strategic play with mathematical concepts and differing perspectives. When students learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong connections necessary for learning to be meaningful, easily accessible, and transferable. The learning environment must be respectful of individuals and groups, fostering discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the construction of meaning.

What types of things might you hear or see in a Grade 9 classroom that would indicate to you that students were learning for deep understanding?

Inquiry

Inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience. The inquiry process focuses on the development of compelling questions, formulated by teachers and students, to motivate and guide inquiries into topics, problems, and issues related to curriculum content and outcomes.

Inquiry is more than a simple instructional method. It is a philosophical approach to teaching and learning, grounded in constructivist research and methods, which engages students in investigations that lead to disciplinary and transdisciplinary understanding.

Inquiry is a philosophical stance rather than a set of strategies, activities, or a particular teaching method. As such, inquiry promotes intentional and thoughtful learning for teachers and children.
(Mills & Donnelly, 2001, p. xviii)

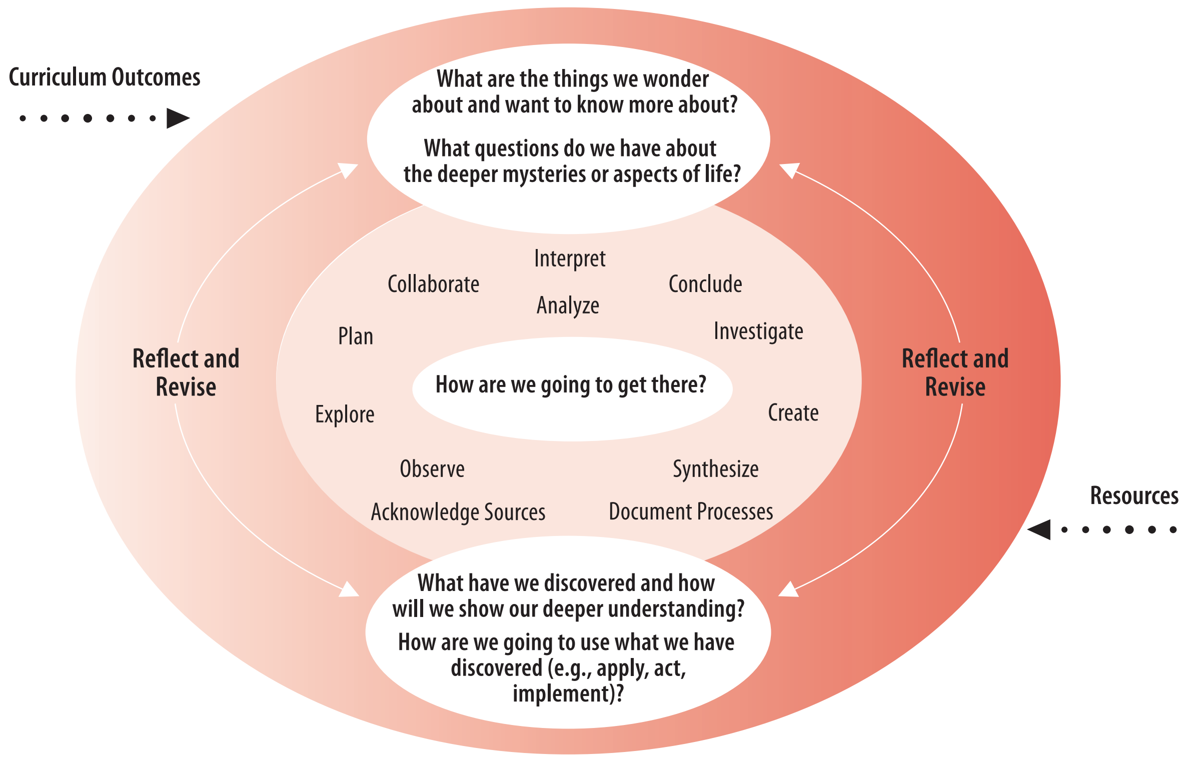
Inquiry builds on students' inherent sense of curiosity and wonder, drawing on their diverse backgrounds, interests, and experiences. The process provides opportunities for students to become active participants in a collaborative search for meaning and understanding. Students who are engaged in inquiry:

- construct deep knowledge and deep understanding rather than passively receiving it
- are directly involved and engaged in the discovery of new knowledge
- encounter alternative perspectives and conflicting ideas that transform prior knowledge and experience into deep understanding
- transfer new knowledge and skills to new circumstances
- take ownership and responsibility for their ongoing learning and mastery of curriculum content and skills.

(Adapted from Kuhlthau & Todd, 2008, p. 1)

Inquiry learning is not a step-by-step process, but rather a cyclical process, with various phases of the process being revisited and rethought as a result of students' discoveries, insights, and construction of new knowledge. The following graphic shows the cyclical inquiry process.

Constructing Understanding Through Inquiry



Inquiry prompts and motivates students to investigate topics within meaningful contexts. The inquiry process is not linear or lock-step, but is flexible and recursive. Experienced inquirers move back and forth through the cyclical process as new questions arise and as students become more comfortable with the process.

Well-formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, conclude, document, reflect on learning, and develop new questions for further inquiry.

In mathematics, inquiry encompasses problem solving. Problem solving includes processes to get from what is known to discover what is unknown. When teachers show students how to solve a problem and then assign additional problems that are similar, the students are not problem solving but practising. Both are necessary in mathematics, but one should not be confused with the other. If the path for getting to the end situation has already been determined, it is no longer problem solving. Students too must understand this difference.

Creating Questions for Inquiry in Mathematics

Teachers and students can begin their inquiry at one or more curriculum entry points; however, the process may evolve into transdisciplinary integrated learning opportunities, as reflective of the holistic nature of our lives and interdependent global environment. It is essential to develop questions that are evoked by students' interests and have potential for rich and deep learning. Compelling questions are used to initiate and guide the inquiry and give students direction for discovering deep understandings about a topic or issue under study.

The process of constructing inquiry questions can help students to grasp the important disciplinary or transdisciplinary ideas that are situated at the core of a particular curricular focus or context. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school.

Effective questions in mathematics are the key to initiating and guiding students' investigations, critical thinking, problem solving, and reflection on their own learning. Questions such as:

- "When would you want to add two numbers less than 100?"
- "How do you know you have an answer?"
- "Will this work with every number? Every similar situation?"
- "How does your representation compare to that of your partner?"

are examples of questions that will move students' inquiry towards deeper understanding. Effective questioning is essential for teaching and student learning, and should be an integral part of planning in mathematics. Questioning should also be used to encourage students to reflect on the inquiry process and the documentation and assessment of their own learning.

Questions should invite students to explore mathematical concepts within a variety of contexts and for a variety of purposes. When questioning students, teachers should choose questions that:

Effective questions:
- *cause genuine and relevant inquiry into the important ideas and core content.*
- *provide for thoughtful, lively discussion, sustained inquiry, and new understanding as well as more questions.*
- *require students to consider alternatives, weigh evidence, support their ideas, and justify their answers.*
- *stimulate vital, ongoing rethinking of key ideas, assumptions, and prior lessons.*
- *spark meaningful connections with prior learning and personal experiences.*
- *naturally recur, creating opportunities for transfer to other situations and subjects.*

(Wiggins & McTighe, 2005, p. 110)

- *help students make sense of the mathematics.*
- *are open-ended, whether in answer or approach. There may be multiple answers or multiple approaches.*
- *empower students to unravel their misconceptions.*
- *not only require the application of facts and procedures but encourage students to make connections and generalizations.*
- *are accessible to all students in their language and offer an entry point for all students.*
- *lead students to wonder more about a topic and to perhaps construct new questions themselves as they investigate this newly found interest.*

(Schuster & Canavan Anderson, 2005, p. 3)

As teachers of mathematics, we want our students not only to understand what they think but also to be able to articulate how they arrived at those understandings.
(Schuster & Canavan Anderson, 2005, p. 1

Reflection and Documentation of Inquiry

An important part of any inquiry process is student reflection on their learning and the documentation needed to assess the learning and make it visible. Student documentation of the inquiry process in mathematics may take the form of reflective journals, notes, drafts, models, and works of art, photographs, or video footage. This documentation should illustrate the students' strategies and thinking processes that led to new insights and conclusions. Inquiry-based documentation can be a source of rich assessment materials through which teachers can gain a more in-depth look into their students' mathematical understandings.

It is important that students are required to engage in the communication and representation of their progress within a mathematical inquiry. A wide variety of forms of communication and representation should be encouraged and, most importantly, have links made between them. In this way, student inquiry into mathematical concepts and contexts can develop and strengthen student understanding.

Outcomes and Indicators

Number

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes (What students are expected to know and be able to do.)	**Indicators** (Students who have achieved this outcome should be able to:)
N9.1 Demonstrate (concretely, pictorially, and symbolically) understanding of powers with integral bases (excluding base 0) and whole number exponents including: • ***representing using powers*** • ***evaluating powers*** • ***powers with an exponent of zero*** • ***solving situational questions.*** ***[C, CN, PS, R, T]***	a. Demonstrate the difference between the exponent and base of a power by representing two powers with exponent and base interchanged (e.g., 2^3 and 3^2 or 10^3 and 3^{10}) using repeated multiplication or concrete models and describe the result. b. Predict which of two powers represents the greater quantity, explain the reasoning, and verify using technology. c. Analyze the role of brackets in powers by using repeated multiplication [e.g., $(-2)^4$, (-2^4), and -2^4] and generalize strategies for evaluating powers involving brackets. d. Justify why a^0, $a \neq 0$, must equal to 1. e. Predict whether the value of a given power will be positive or negative (e.g., what will the sign of -7^{15} be?). f. Evaluate powers with integral bases (excluding base 0) and whole number exponents, with or without the use of technology. g. Generalize, using repeated multiplication to represent powers, the exponent laws of powers with integral bases (excluding base 0) and whole number exponents: • $(a^m)(a^n) = a^{m+n}$ • $\frac{a^m}{a^n} = a^{m-n}, m > n$ • $(a^m)^n = a^{mn}$ • $(ab)^m = a^m b^m$. h. Apply the exponent laws to expressions involving powers, and determine the quantity represented by the expression, with or without the use of technology. i. Prove by contradiction that $a^m + a^n \neq a^{mn}$, $a^m - a^n \neq a^{m-n}$, and $a^m - a^n \neq a^{\frac{m}{n}}$. j. Describe and apply strategies for evaluating sums or differences of powers. k. Analyze a simplification of an expression involving powers for errors.

Mathematics 9

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
N9.2 Demonstrate understanding of rational numbers including: • ***comparing and ordering*** • ***relating to other types of numbers*** • ***solving situational questions.*** ***[C, CN, PS, R, T, V]***	a. Order a given set of rational numbers, in fraction and decimal form, by placing them on a number line and explaining the reasoning used (e.g., $\frac{3}{5}$, - 0.666, 4,… , 0.5, $\frac{-5}{8}$). b. Determine a rational number between two given rational numbers and describe the strategy used. c. Create a representation depicting how whole numbers, fractions, decimals, integers, square roots, and rational numbers are related to each other. d. Provide examples to explain how knowing about how to add, subtract, multiply, and divide integers and positive rational numbers informs knowing how to add, subtract, multiply, and divide rational numbers. e. Provide examples to demonstrate how the order of operations can be extended to rational numbers. f. Solve situational questions involving operations on rational numbers, with or without the use of technology. g. Analyze a simplification of an expression involving rational numbers for errors.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
N9.3 Extend understanding of square roots to include the square root of positive rational numbers. ***[CN, ME, R, T, V]***	a. Develop a generalization about what type of number results from the squaring of a rational number. b. Describe strategies for determining if a rational number is a perfect square. c. Determine the square root of a rational number that is a perfect square. d. Determine the rational number for which a given rational number is its square root (e.g., $\frac{4}{3}$ is the square root of what rational number?). e. Explain and apply strategies involving benchmarks for determining an estimate of the square root of a rational number that is not a perfect square. f. Determine, with the use of technology, an approximate value for the square root of a rational number that is not a perfect square.

Outcomes	Indicators
N9.3 (continued)	g. Explain why the value shown by technology may only be an approximation of the square root of a rational number. h. Describe a strategy that, if applied to writing a decimal number, would result in an irrational number (e.g., students describe a strategy in which they repeatedly write the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 but separate each group of these digits by an increasing number of repeats of the digit 7 or 0.0123456789701234567897701234567897770123…). i. Determine a rational number whose square root would be between two given rational numbers and explain the reasoning used (e.g., a rational number whose square root is between $\frac{1}{2}$ and $\frac{1}{3}$ would be between $\frac{1}{4}$ and $\frac{1}{9}$ because those are $\frac{1}{2}$ and $\frac{1}{3}$ squared. I need to find a number between $\frac{1}{4}$ and $\frac{1}{9}$. I can do this by making the two fractions into fractions of the same type: $\frac{9}{36}$ and $\frac{4}{36}$. One number between these is $\frac{6}{36}$ or $\frac{4}{36}$).

Patterns and Relations

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes (What students are expected to know and be able to do.)	**Indicators** (Students who have achieved this outcome should be able to:)
P9.1 Demonstrate understanding of linear relations including: • ***graphing*** • ***analyzing*** • ***interpolating and extrapolating*** • ***solving situational questions.*** *[C, CN, PS, R, T, V]*	a. Observe and describe a situation relevant to self, family, or community that a given graph might represent and explain the meaning conveyed by the graph. b. Sort a set of graphs into representations of linear and non-linear relations. c. Sketch graphs for given linear relations, including horizontal and vertical lines, with and without the use of technology. d. Generalize strategies for determining if a given linear relation will have a graph that is horizontal, vertical, increasing, or decreasing. e. Extrapolate to determine a value for either variable in a linear relation beyond the shown graph. f. Verify an extrapolated value from a graph by using substitution in the related linear relation. g. Interpolate to determine a value for either variable in a linear relation within the shown graph. h. Verify an interpolated value from a graph by using substitution in the related linear relation.

Outcomes	Indicators
P9.1 (continued)	i. Solve *- questions by graphing linear relations and interpreting the resulting graphs.

Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P9.2. Model and solve situational questions using linear equations of the form: • $ax = b$ • $\frac{x}{a} = b, a \neq 0$ • $ax + b = c$ • $\frac{x}{a} + b = c, a \neq 0$ • $ax = b + cx$ • $a(x + b) = c$ • $ax + b = cx + d$ • $a(bx + c) = d(ex + f)$ • $\frac{a}{x} = b, x \neq 0$ ***where a, b, c, d, e, and f are rational numbers.*** ***[C, CN, PS, V]***	a. Explain why the equation $\frac{a}{x} = b$, cannot have a solution of $x = 0$. b. Write a linear expression representing a given pictorial, oral, or written pattern. c. Write a linear equation to represent a particular situation. d. Observe and describe a situation relevant to self, family, or community which could be represented by a linear equation. e. Write a linear equation representing the pattern in a given table of values and verify the equation by substituting values from the table. f. Model the solution of a linear equation using concrete or pictorial representations, and explain how to record the process symbolically. g. Explain how the preservation of equality is involved in the solving of linear equations. h. Verify, by substitution, whether or not a given rational number is a solution to a given linear equation. i. Solve a linear equation symbolically. j. Analyze the given solution for a linear equation that has resulted in an incorrect solution, and identify and explain the error(s) made. k. Provide examples from the modern world in which linear equations are used and solved.

Goals: Spatial Sense, Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P9.3 Demonstrate understanding of single variable linear inequalities with rational coefficients including: • ***solving inequalities*** • ***verifying*** • ***comparing*** • ***graphing.*** *[C, CN, PS, R, V]*	a. Observe and describe situations relevant to self, family, or community, including First Nations and Métis communities, that involve inequalities and classify the inequality as being less than, greater than, less than or equal to, or greater than or equal to. b. Verify whether or not a given rational number is part of the solution set for a linear inequality. c. Generalize and apply rules for adding or subtracting a positive or negative number to determine the solution of an inequality. d. Generalize and apply a rule for multiplying or dividing by a positive or negative number to determine the solution of an inequality. e. Solve a linear inequality algebraically and explain the strategies used. f. Compare and explain the process for solving a linear equation to the process for solving a linear inequality. g. Explain how knowing the solution to a linear equality can be used to determine the solution of a related linear inequality, and provide an example. h. Critique the statement: "For any linear equality, there are two related linear inequalities". i. Graph the solution of a linear inequality on a number line. j. Explain why there is more than one solution to a linear inequality. k. Verify the solution of a given linear inequality using substitution for multiple elements, in the solution and outside of the solution. l. Solve a situational question involving a single variable linear inequality and graph the solution.

Goals: Spatial Sense, Logical Thinking, Number Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
P9.4 Demonstrate understanding of polynomials (limited to polynomials of degree less than or equal to 2) including: • ***modeling*** • ***generalizing strategies for addition, subtraction, multiplication, and division*** • ***analyzing*** • ***relating to context*** • ***comparing for equivalency.*** ***[C, CN, R, V]***	a. Model (concretely or pictorially) and describe the relationship between x and x^2. b. Represent polynomials concretely or pictorially, and describe how the concrete or pictorial model reflects the symbolic form. c. Write a polynomial for a given concrete or pictorial representation. d. Identify the variables, degree, number of terms, and coefficients, including the constant term, of a given simplified polynomial expression and explain the role or significance of each. e. Identify the type of expression that is represented by a polynomial of degree 1. f. Sort a set of polynomials into monomials, binomials, and trinomials. g. Critique the statement "A binomial can never be a degree 2 polynomial". h. Write equivalent forms of a polynomial expression by interchanging terms or by decomposing terms, and justify the equivalence. i. Explain why terms with different variable exponents cannot be added or subtracted. j. Generalize, from concrete and pictorial models, and apply strategies for adding and subtracting polynomials symbolically. k. Verify whether or not the simplification of the addition or subtraction of two polynomials is correct and explain. l. Describe the relationship between multiplication of a polynomial and a monomial, and determining the area of a rectangular region. m. Generalize, from concrete and pictorial models, and apply strategies for multiplying a polynomial by a monomial. n. Generalize, from concrete and pictorial models, and apply strategies for dividing a polynomial by a monomial. o. Verify whether or not the simplification of the multiplication or division of a polynomial by a monomial is correct.

Shape and Space

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes (What students are expected to know and be able to do.)	**Indicators** (Students who have achieved this outcome should be able to:)
SS9.1 Demonstrate understanding of circle properties including: • ***perpendicular line segments from the centre of a circle to a chord bisect the chord*** • ***inscribed angles subtended by the same arc have the same measure*** • ***the measure of a central angle is twice the measure of an inscribed angle subtending the same arc*** • ***tangents to a circle are perpendicular to the radius ending at the point of tangency.*** ***[C, CN, PS, R, T, V]***	a. Observe and describe situations relevant to self, family, or community that involve circles, chords, central angles, inscribed angles, radii, arcs, and/or points of tangency. b. Construct a tangent line to a circle by applying the knowledge that a tangent line to the circle is perpendicular to a radius of the circle. c. Generalize, from personal explorations, the relationship between the measures of inscribed angles subtended by the same arc. d. Generalize, from personal explorations, the relationship between the measure of a central angle and the measure of inscribed angles subtended by the same arc. e. Generalize, from personal explorations, the relationship between a perpendicular line segment from the centre of a circle to a chord and the chord. f. Model how to find the diameter of a circle using an inscribed angle of 90° and explain why the strategy works. g. Describe examples of where First Nations and Métis, past and present, lifestyles and worldviews demonstrate one or more of the circle properties (e.g., tipi and medicine wheel). h. Solve a situational question involving the application of one or more of the circle properties.

Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS9.2 Extend understanding of area to surface area of right rectangular prisms, right cylinders, right triangular prisms, to composite 3-D objects. [CN, PS, R, V]	a. Describe 3-D composite objects from the natural and constructed world, including objects relevant to First Nations and Métis people (e.g., Mesoamerican pyramids). b. Analyze a composite 3-D object to identify areas of overlap and explain the impact of these areas on determining the surface area of the composite 3-D object. c. Critique the statement "To find the surface area of a composite 3-D object, add together the surface areas of the individual 3-D objects from which the composite 3-D object is comprised". d. Determine the surface area of composite 3-D objects. e. Solve situational questions involving the surface area of composite 3-D objects. f. Give dimensions for a single 3-D object that will have the same surface area as a composite 3-D object. g. Approximate the surface area of a 3-D object from the natural environment using composites of standard 3-D objects such as right rectangular prisms, right cylinders, and right triangular prisms.

Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS9.3. Demonstrate understanding of similarity of 2-D shapes. [C, CN, PS, R, V]	a. Observe and describe 2-D shapes, relevant to self, family, or community, that are similar. b. Explain the difference between similarity and congruence of polygons. c. Verify whether or not two polygons are similar. d. Explain how ratios and proportionality are related to similarity of polygons. e. Draw a polygon similar to a given polygon and explain the strategies used. f. Solve situational questions involving the similarity of polygons.

Outcomes	**Indicators**
SS9.3 *(continued)*	g. Identify and describe situations relevant to self, family, or community that involve scale diagrams and explain the meaning of the scale factor involved. h. Explain how scale diagrams are related to similarity, ratios, and proportionality. i. Draw a diagram to scale that represents an enlargement or reduction of a given 2-D shape and explain the strategies used. j. Explain how to determine the scale factor for a given 2-D shape and an enlargement or reduction of the shape. k. Verify whether or not a given diagram is a scale diagram of a 2-D shape and, if it is, identify the scale factor for the diagram. l. Solve situational questions involving scale diagrams and scale factors.

Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	**Indicators**
SS9.4 Demonstrate understanding of line and rotation symmetry. [C, CN, PS, V]	a. Observe and describe examples of line and rotation symmetry in situations relevant to self, family, or community. b. Classify different 2-D shapes or designs made of 2-D shapes, according to the number of lines of symmetry. c. Complete a 2-D shape or design given part of a shape or design and one or more lines of symmetry. d. Determine, with justification, if a given 2-D shape or design has rotation symmetry about the point at the centre of the shape or design and, if it does, state the order and angle of rotation. e. Identify a line of symmetry, or the order and angle of rotation symmetry, in a given tessellation. f. Describe examples of the use and significance of line and rotation symmetry in First Nations and Métis art. g. Analyze different transformations of 2-D shapes on the Cartesian plane and describe the type of symmetry, if any, that results. h. Determine whether or not two 2-D shapes on the Cartesian plane are related by either rotation or line symmetry and explain.

Outcomes	Indicators
SS9.4 (continued)	i. Create or provide an art work (such as a painting or dance) that demonstrates line and rotation symmetry, and identify the line(s) of symmetry and the order and angle of rotation.

Statistics and Probability

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes (What students are expected to know and be able to do.)	**Indicators** (Students who have achieved this outcome should be able to:)
SP9.1 Demonstrate understanding of the effect of: • ***bias*** • ***use of language*** • ***ethics*** • ***cost*** • ***time and timing*** • ***privacy*** • ***cultural sensitivity and*** • ***population or sample*** ***on data collection.*** *[C, PS, R, T]*	a. Analyze given case studies of data collection, including data pertaining to First Nations and Métis peoples, and identify potential problems related to bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity. b. Provide examples to illustrate how bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity may influence the data collected. c. Identify situations relevant to self, family, or community where a set of data was collected and classify each situation as involving a sample or the population. d. Provide an example of a situation in which a population may be used to answer a question, and justify the choice. e. Provide an example of a question where a limitation precludes the use of a population and describe the limitation (e.g., too costly, not enough time, limited resources). f. Identify and critique given examples in which a generalization from a sample of a population, including from First Nations and Métis data, may or may not be valid for the population. g. Explain different strategies for trying to minimize negative effects on data collection. h. Explain the importance of protocols for respectful data collection and information sharing.

Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SP9.2 Demonstrate an understanding of the collection, display, and analysis of data through a project. [C, PS, R, T, V]	a. Devise a project plan related to a situation relevant to self, family, or community, that involves: • formulating a question for investigation • choosing a data collection method that includes social considerations • electing a population or a sample, and justifying the choice • collecting the data • displaying the collected data in an appropriate manner • drawing conclusions to answer the question. b. Create and apply a rubric to assess a project that includes the assessment of all requirements for the project. c. Complete the project according to the plan, draw conclusions, and communicate findings to an audience.

Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SP9.3 Demonstrate an understanding of the role of probability in society. [C, CN, R, T]	a. Observe examples of probabilities that impact or influence aspects of one's self, family, community, or environment and describe those impacts or influences. b. Analyze the meaningfulness of a probability against the limitations of assumptions associated with that probability. c. Provide examples of how a single probability could be used to support opposing positions. d. Explain, using examples, how decisions based on probability may be a combination of theoretical probability, experimental probability, and subjective judgement.

Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SP9.4 Research and present how First Nations and Métis peoples, past and present, envision, represent, and make use of probability and statistics.	a. Gather and document information regarding the significance and use of probability and statistics for at least one First Nation or Métis peoples from a variety of sources such as Elders and traditional knowledge keepers. b. Compare the significance, representation, and use of probability and statistics for different First Nations and Métis peoples, and other cultures. c. Communicate concretely, pictorially, orally, visually, physically, and/or in writing, what has been learned about the envisioning, representing, and use of probability and statistics by First Nations and Métis peoples and how these understandings parallel, differ from, and enhance one's own mathematical understandings about probability and statistics.

Assessment and Evaluation of Student Learning

Assessment and evaluation require thoughtful planning and implementation to support the learning process and to inform teaching. All assessment and evaluation of student achievement must be based on the outcomes in the provincial curriculum.

Assessment involves the systematic collection of information about student learning with respect to:

- ☑ Achievement of provincial curriculum outcomes
- ☑ Effectiveness of teaching strategies employed
- ☑ Student self-reflection on learning.

Evaluation compares assessment information against criteria based on curriculum outcomes for the purpose of communicating to students, teachers, parents/caregivers, and others about student progress and to make informed decisions about the teaching and learning process.

Assembling evidence from a variety of sources is more likely to yield an accurate picture.
(NCTM, 2000, p. 24)

Reporting of student achievement must be in relation to curriculum outcomes. Assessment information which is not related to outcomes can be gathered and reported (e.g., attendance, behaviour, general attitude, completion of homework, effort) to complement the reported achievement related to the outcomes of Grade 9 Mathematics. There are three interrelated purposes of assessment. Each type of assessment, systematically implemented, contributes to an overall picture of an individual student's achievement.

Assessment for learning involves the use of information about student progress to support and improve student learning and inform instructional practices and:

- is teacher-driven for student, teacher, and parent use
- occurs throughout the teaching and learning process, using a variety of tools
- engages teachers in providing differentiated instruction, feedback to students to enhance their learning, and information to parents in support of learning.

Assessment should not merely be done to students; rather it should be done for students.
(NCTM, 2000, p. 22)

Assessment as learning involves student reflection on and monitoring of her/his own progress related to curriculum outcomes and:

- is student-driven with teacher guidance for personal use
- occurs throughout the learning process
- engages students in reflecting on learning, future learning, and thought processes (metacognition).

What are examples of assessments as learning that could be used in Grade 9 Mathematics and what would be the purpose of those assessments?

> *Assessment should become a routine part of the ongoing classroom activity rather than an interruption.*
> *(NCTM, 2000, p. 23)*

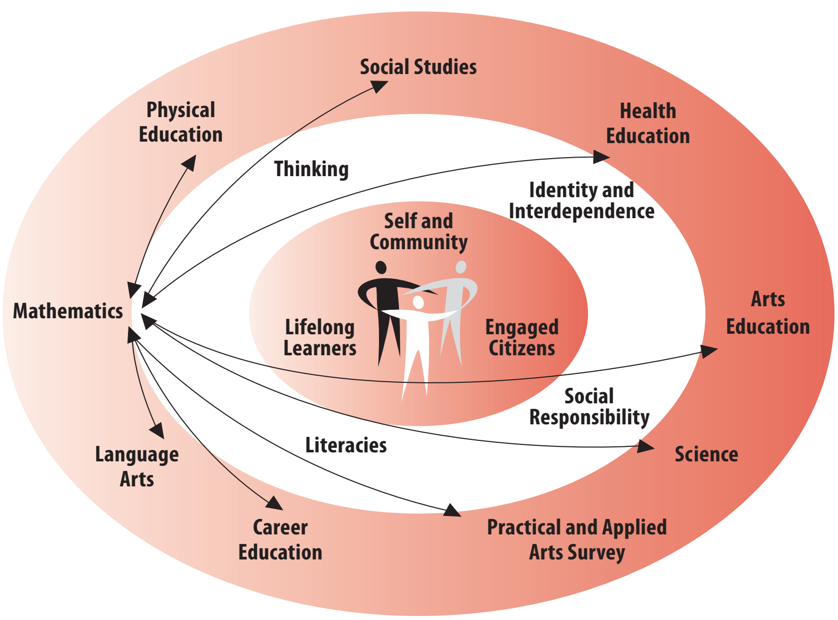
Assessment of learning involves teachers' use of evidence of student learning to make judgements about student achievement and:

- provides opportunity to report evidence of achievement related to curricular outcomes
- occurs at the end of a learning cycle, using a variety of tools
- provides the foundation for discussion on placement or promotion.

In mathematics, students need to be regularly engaged in assessment as learning. The assessments used should flow from the learning tasks and provide direct feedback to the students regarding their progress in attaining the desired learnings as well as opportunities for the students to set and assess personal learning goals related to the mathematical content for Grade 9.

Connections with Other Areas of Study

There are many possibilities for connecting Grade 9 mathematical learning with the learning occurring in other subject areas. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learnings. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. Following are just a few of the ways in which mathematics can be connected to other subject areas (and other subject areas connected to mathematics) at Grade 9.



Arts Education – There are many strong connections between arts education and mathematics at the Grade 9 level. Specifically, these connections can be made to outcome SP9.3:

> Demonstrate an understanding of the role of probability in society.
>
> [C, CN, R, T]

In Grade 9 Arts Education, students investigate how today's arts expressions can inspire change and challenge thinking related to values, ideas, and beliefs. Such arts expressions often relate to contexts where probabilities are used to argue opposing or differing perspectives or views. By linking the arts expressions to the exploration of the influences and occurences of probability in daily lives, students can be engaged in a rich personal and social exploration of beliefs, values, and ideas.

As well, students in Grade 9 Arts Education are to create dance, drama, art, and music expressions to express their perspectives and raise awareness about a topic of concern. Many such topics will also have an abundance of data and related probabilites that can be incorporated within the students' artistic creations. Consideration of such probabilities within their creations will give students greater meaning and richer contexts in which they can ponder, debate, and make decisions about the role of probability in their lives.

Career Education - In Grade 9 Career Education, students are appraising and analyzing many of their own abilities, including those related to their ability to respond positively to change and growth. Students could do an individual or class project in which they collect personal or class data related to their responses to change and growth over a period of time and then use this data within the data analysis project found in SP9.2:

> Demonstrate an understanding of the collection, display, and analysis of data through a project.

Students could set targets or goals related to these abilities and then use the data analysis project to assess their relative success. In addition, students could be considering the outside factors that affect their ability to grow in these abilities, thus also contributing to outcome SP9.1 in Grade 9 Mathematics.

English Language Arts (ELA) – ELA and mathematics share a common interest in students developing their abilities to reflect upon and communicate about their learnings through viewing, listening, reading, representing, speaking, and writing. As an example of how mathematics involves these strands of language, consider outcome P9.4:

> Demonstrate understanding of polynomials (limited to polynomials of degree less than or equal to 2) including:
>
> - modeling
> - generalizing strategies for addition, subtraction, multiplication, and division
> - analyzing
> - relating to context
> - comparing for equivalency.
>
> [C, CN, R, V]

To achieve this learning outcome, students are to be explicitly engaged in understanding the role and meaning of polynomials. In developing such understandings, students need to create and relate different representations of polynomial expressions. Students can share and discuss those representations with their classmates. Students can also be involved in reading and writing, as well as solving problems related to polynomials.

Through the use of language, both orally and in writing, students can communicate with their classmates and others about their developing understandings and vocabulary related to polynomials and seek clarification of the ideas presented by other students. Such activity requires students to effectively speak, listen, and show appreciation for the ideas communicated by other students. By actively engaging in the use of language and other ways of representing their understanding, students reflect deeply upon their learnings, leading to a combination of affirmations, changes, and extensions to each student's understandings of polynomials.

Health Education – At Grade 9, connections between mathematics and health education can be made in relation to outcome SP9.2:

> Demonstrate an understanding of the collection, display, and analysis of data through a project.
>
> [C, PS, R, T, V]

With the overarching perspective of Promoting Health for Grade 9 Health Education, many of the analyses that students are required to do can become the context in which students define and carry out their data analysis project for mathematics. For example, students could collect and analyze data related to the importance of leadership skills and health promotion in healthy decision making, the norms and expectations associated with romantic relationships, or the health, economic, and social supports and challenges for addictions. Each of these Grade 9 Health Education contexts provides rich and diverse situations that students could effectively explore and demonstrate their comprehensive understanding of data collection and analysis. As well, outcome SP9.1 could also be addressed through the students' discussions of the factors that affect or bias data that are already collected and available on these and other related topics.

Physical Education – As with health education, there are many possible topics for the data analysis project of mathematics found within the Grade 9 Physical Education outcomes. Two such examples include the students applying their data analysis understandings as a way of monitoring and assessing the effectiveness or success of their personal action plans for health-related fitness or skill-related fitness.

As well, outcome SP9.1:

> Demonstrate understanding of the effect of:
> - bias
> - use of language
> - ethics
> - cost
> - time and timing
> - privacy
> - cultural sensitivity and
> - population or sample
>
> on data collection.
>
> [C, PS, R, T]

can support the students' investigation of the influence of mass media on body image and body composition. Students can consider the effects of bias, use of language, cost, ethics, and other factors on the body images that are presented by the media and the perceived and/or real impact each of these have on society's values and beliefs.

Science – In Grade 9 Science, two of the units of study have very strong ties to outcome SP9.3:

> Demonstrate an understanding of the role of probability in society.

In the Reproduction unit of Grade 9 Science, students are to describe the role of DNA, genes, and chromosones in storing and transferring genetic material. Students' natural curiosity will raise questions such as "How likely is it that a particular characteristic will be passed on to the offspring?" or "What are the options knowing that a specific outcome has a certain likelihood of occuring?". These types of questions support the students' engagement in understanding the role of probability in society.

Similarly, the Characteristics of Electricity unit has students explore the impacts of past, current and possible future methods of small and large scale electrical energy production and distribution in Saskatchewan. In exploring these ideas, many ecological and social questions arise that are often prefaced with statements of probability. As the students will quickly realize though, probability alone is rarely the deciding factor when people's health, lives, convenience, and happiness are involved.

Social Studies – Social studies and mathematics often connect through the investigation of patterns and trends, and in the representation of data. In Grade 9, students in social studies are learning about worldviews of past societies and connections between differing worldviews. Such a study of worldviews opens students up to awareness of their and others' biases that can impact how different societies are perceived and understood. This study also helps to inform students about the effect of different factors on data analysis, collection, and interpretation or outcome SP9.1:

Demonstrate understanding of the effect of:

- bias
- use of language
- ethics
- cost
- time and timing
- privacy
- cultural sensitivity and
- population or sample

on data collection.

[C, PS, R, T]

Students may then choose to explore further some of the different factors that affect data collection by specifically using examples related to social studies as the foundation for their data analysis project described in outcome SP9.2.

Glossary

Central Angle: The angle formed by joining the centre of a circle to each of the end points of an arc on the circle.

Coefficient: Any constant factor of a term is the coefficient of the rest of the term. For example, in the term 3 x^2, the coefficient is 3.

Ethnomathematics: The study of the relationship between mathematics and culture.

Inscribed Angle: The angle formed by joining one point on a circle to each of the end points of an arc on the circle, where the point is not on the arc.

Interdisciplinary: Disciplines connected by common concepts and skills embedded in disciplinary outcomes.

Line Symmetry: Also known as reflection symmetry, a 2-D shape or 3-D object has line symmetry if at least one line can be drawn that reflects a similar portion of the shape or object across the line.

Linear Equation: A statement showing that two linear expressions are equal for all variable values. For example, $3x - 7 = 8$.

Linear Expression: An expression in which the variables are not multiplied by one another or raised to any power, they are just multiplied by constants and added together.

Linear Inequality: Two linear expressions which are not equal to each other. Symbolic representation of the relationship between two such expressions is based upon the ordering of the two expressions and can be < (less than), > (greater than), ≤ (less than or equal to), or ≥ (greater than equal to). For example, $12 + 3y < 17$ can also be written as $17 > 12 + 3y$.

Linear Relation: A linear equation involving two variables which results in a straight line graph. A linear relation with only one variable will result in either a horizontal or vertical line for a graph.

Multidisciplinary: Discipline outcomes organized around a theme and learned through the structure of the disciplines.

Number, Numeral, Digit: A number is the name that we give to quantities. For example, there are seven days in a week, or I have three brothers – both seven and three are numbers in these situations because they are defining a quantity. The symbolic representation of a number, such as 287, is called the numeral. If 287 is not being used to define a quantity, we call it a numeral.

Numerals, as the symbolic representation of numbers, are made up of a series of digits. The Hindu-Arabic number system that we use has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. (Note: sometimes students are confused between these digits and their finger digits – this is because they count their fingers starting at one and get to ten rather than zero to nine.) These digits are also numerals and can be numbers (representing a quantity), but all numbers and all numerals are combinations of digits. The placement of a digit in a number or numeral affects the place value of the digit and, hence, how much of the quantity that it represents. For example, in 326, the 2 is contributing 20 to the total, while in 236 the 2 contributes 200 to the total quantity.

Object: In this curriculum, object is used to refer to a three-dimensional geometrical figure. To distinguish this meaning from that of shape, the word "object" is preceded by the descriptor "3-D".

Personal Strategies: Personal strategies are strategies that the students have constructed and understand. Outcomes and indicators that specify the use of personal strategies convey the message that there is not a single procedure that is correct. Students should be encouraged to explore, share, and make decisions about what strategies to use in different contexts. Development of personal strategies is an indicator of the attainment of deeper understanding.

Polynomial: A polynomial expression is an expression in which there are whole-numbered powers of the variable multiplied by numerical coefficients, and added together. The powers of the variable must be positive integers, or zero.

Power: A simplified expression of a repeated multiplication. In the power a^b, a is referred to as the base of the power (the value or variable that is repeatedly being multiplied) and b is referred to as the exponent (the number of times the base is being multiplied).

Square Root: The square root of a given number is that number which, when multiplied by itself, gives the original number. The square root of a given quantity could be positive or negative. The principal square root is the name given to the positive of these values.

Rational Number: A number is rational if it can be written as the ratio of two whole numbers.

Referents: A concrete approximation of a quantity or unit of measurement. For example, seeing what 25 beans in a container looks like makes it possible to estimate the number of beans the same container will hold when it is full of the same kind of beans. Compensation must be made if the container is filled with smaller or larger beans than the referent or if the size or shape of the container is changed.

Representations: Mathematical ideas can be represented and manipulated in a variety of forms including concrete manipulatives, visual designs, sounds and speech, physical movements, and symbolic notations (such as numerals and operation signs). Students need experience in working with many different types of representations, and in transferring and translating knowledge between the different forms of representations.

Rotational Symmetry: When a figure can be rotated so that it fits exactly onto its outline more than once in a complete turn, it has rotational symmetry.

Shape: In this curriculum, shape is used to refer to two-dimensional geometric figures and is thus preceded by "2-D". The term shape is sometimes also used in mathematics resources and conversations to refer to three-dimensional geometric figures. It is important that students learn to be clear in identifying whether their use of the term shape is in reference to a 2-D or 3-D geometrical figure.

Similar: Two or more 2-D shapes are similar if they have the same shape, but are not necessarily the same size. The corresponding sides are in proportion and the corresponding angles are equal.

Tangent Line: A straight line which touches a given curve (such as a circle) exactly once, at a given point.

Term: One of the parts of an algebraic expression that is added to, or subtracted from, the rest of the expression.

Transdisciplinary: All knowledge interconnected and interdependent; real-life contexts emphasized and investigated through student questions.

Variable: A quantity, represented by a letter or other symbol, whose size we do not know, or whose size can sometimes change.

References

Armstrong, Thomas. (1993). *Seven kinds of smart: Identifying and developing your many intelligences*. New York NY: NAL-Dutton.

Burns, Marilyn & Silbey, Robyn. (2000). *So you have to teach math? Sound advice for K-6 teachers*. Sausalito CA: Math Solutions Publications.

Caine, Renate Numella & Caine, Geoffrey. (1991). *Making connections: Teaching and the human brain*. Menlo Park CA: Addison-Wesley Publishing Company.

Haylock, Derek & Cockburn, Anne. (2003). *Understanding mathematics in the lower primary years: A guide for teachers of children 3 -8*. (Second Edition). London UK: Paul Chapman Publishing.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth NH: Heinemann.

Kuhlthau, C.C. & Todd, R. J. (2008). *Guided inquiry: A framework for learning through school libraries in 21st century schools*. Newark NJ: Rutgers University.

Mills, H. & Donnelly, A. (2001). *From the ground up: Creating a culture of inquiry*. Portsmouth NH: Heinemann Educational Books, Ltd.

NCTM (2000). *Principles and standards for school mathematics*. Reston VA: NCTM.

Paulos, John Allen. (1980). *Mathematics and humor: A study of the logic of humor.* Chicago Il: The University of Chicago Press.

Raphel, Annette. (2000). *Math homework that counts: Grades 4 – 6*. Sausalito CA: Math Solutions Publications.

Schuster, Lainie & Canavan Anderson, Nancy. (2005). *Good questions for math teaching: Why ask them and what to ask, Grades 5 – 8*. Sausalito CA: Math Solutions Publications.

Skinner, Penny. (1999). *It all adds up! Engaging 8-to-12-year-olds in math investigations*. Sausalito CA: Math Solutions Publications.

Stiff, Lee. (2001). *Constructivist mathematics and unicorns (President's Message)*. NCTM News Bulletin. Reston VA: NCTM.

Wiggins, G. & McTighe, J. (2005) *Understanding by design*. Alexandria VA: Association for Supervision and Curriculum Development.

WNCP (2006). *The common curriculum framework for K-9 mathematics*. Edmonton AB: WNCP.

Feedback Form

The Ministry of Education welcomes your response to this curriculum and invites you to complete and return this feedback form.

Document Title: **Mathematics Grade 9 Curriculum**

1. Please indicate your role in the learning community

   ☐ parent ☐ teacher ☐ resource teacher

   ☐ guidance counsellor ☐ school administrator ☐ school board trustee

   ☐ teacher librarian ☐ school community council member

   ☐ other ____

   What was your purpose for looking at or using this curriculum?

2. a) Please indicate which format(s) of the curriculum you used:

   ☐ print

   ☐ online

   b) Please indicate which format(s) of the curriculum you prefer:

   ☐ print

   ☐ online

3. How does this curriculum address the needs of your learning community or organization? Please explain.

4. Please respond to each of the following statements by circling the applicable number.

The curriculum content is:	Strongly Agree	Agree	Disagree	Strongly Disagree
a. appropriate for its intended purpose	1	2	3	4
b. suitable for your learning style (e.g., visuals, graphics, texts)	1	2	3	4
c. clear and well organized	1	2	3	4
d. visually appealing	1	2	3	4
e. informative	1	2	3	4

5. Explain which aspects you found to be:

   Most useful:

   Least useful:

6. Additional comments:

7. Optional:

   Name: ______________________________

   School: ______________________________

   Phone:_______________ Fax: _______________

Thank you for taking the time to provide this valuable feedback.

Please return the completed feedback form to:

Executive Director
Curriculum and E-Learning Branch
Ministry of Education
2220 College Avenue
Regina SK S4P 4V9
Fax: 306-787-2223