Thinking Efficiently

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Claim

Modern languages provide useful slick tools. However, if you are concerned about how long it takes your programs to run, then you should use these features with care and knowledge.

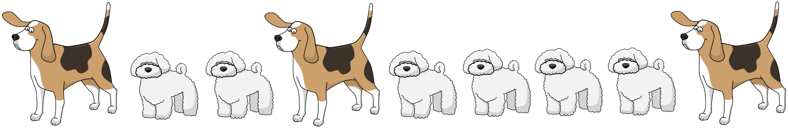
This Afternoon

1. warm-up problem(s)
2. case study: nine different algorithms solving the same problem
   - illustrated with Python 3.7
3. cool-down problem(s)

Problem A

Swapping Dogs

Two types of dogs are standing as shown below.



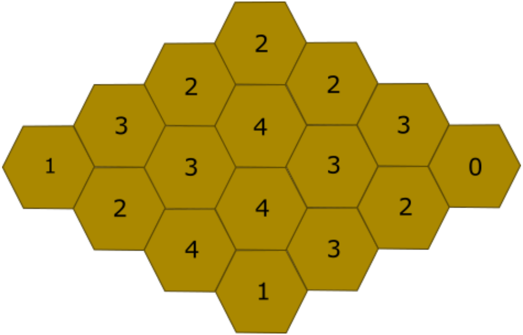
A *swap* occurs when two dogs that are beside each other exchange positions. After some swaps, the three large dogs end up in three consecutive positions.

What is the fewest number of swaps that could have occurred?

Problem B

Beehive

For each hexagon below, a bear records how many *other* hexagons touching this hexagon contain honey. So this number could be 0, 1, 2, 3, 4, 5 or 6. How many hexagons contain honey?



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Beehive Solution

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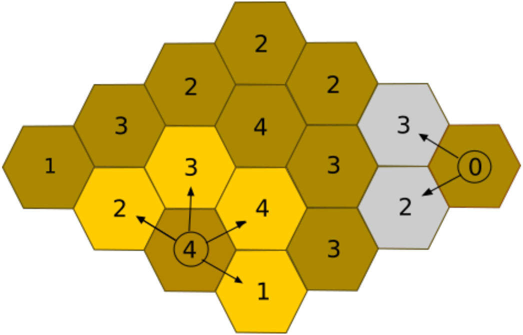

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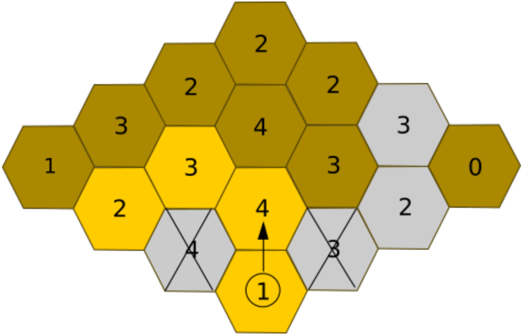

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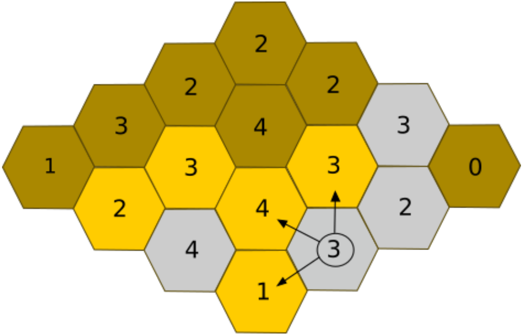

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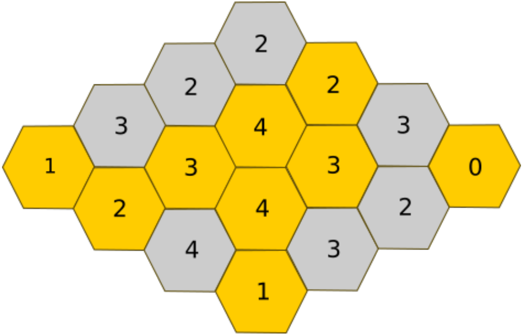

Our Case Study

The problem

Find the maximum among a collection of numbers.

Notes

- unbounded integers
- stored in a non-empty Python list
- duplicates allowed
- do not use the built-in function `max` on a list

Possible solutions

What different algorithms can you come up with?

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Scan the list keeping track of the largest integer seen so far.

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Our model

- each basic operation takes constant time
- ignore constants and low-order terms using big-O notation
  - e.g $O(1), O(n), O(\log n), O(n^2), \ldots$

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$O(2^n)$

When the input size doubled, the runtime is squared.

`max1`: Max So Far – Standard

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Scan the list keeping track of the largest integer seen so far.

- `for v in L`

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- arguably natural
- simple and straightforward

Analysis

- $O(n)$

Our Sample Input

Three Lists

```
L1 = list(range(10))
L2 = list(range(10**3))
L3 = list(range(10**6))
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The items of each list are then scrambled using `random.shuffle`.

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It is impossible to come up with a correct $O(1)$ algorithm.

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Scan the list keeping track of the largest integer seen so far.

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`max4`: Natural Recursion

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Find the maximum of the first element and the maximum of the rest of the list.

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- $O(2^n)$

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max6: Even Faster Natural Recursion

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max7: Scan Until You Find It

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Analysis

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Idea

Find the maximums of the left and right halves and then the maximum of these two maximums.

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Challenge

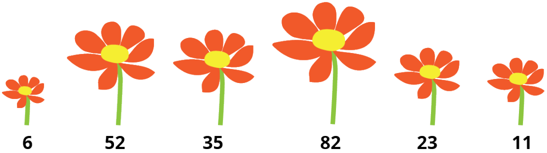
Find the *second* largest item among 100 integers using at most 105 comparisons.

Problem C

Collecting Pollen

Beever the bee makes 20 flights to a field of flowers. On each flight, it visits only one flower and can collect up to 10mg of pollen. It can return to the same flower more than once.

The initial amount of pollen in each flower (in mg) is shown below.



What is the maximum total amount of pollen Beever can collect?

Problem D

Find the Prize

Your friend is thinking of an integer between 1 and 63 (inclusive). They offer to give you money if you guess the integer.

If you guess the number on your first guess, you win \$1000. Every time you guess incorrectly, your friend will take \$10 away from the prize money, but also tell you whether your guess was above or below the integer they were thinking of.

You find a strategy that *guarantees* you win at least \$N, regardless of the number your friend is thinking of.

What is the largest possible value of N?

Beaver Computing Challenge

A CEMC fall contest you may not have heard of

- gentle introduction to logical and algorithmic thinking
- appropriate for students with no background in computer science
- intended for students in Grades 5 to 10
- weeks of November 4 and 11, 2019
- 45 minute online multiple-choice contest
- no programming required
- held in schools and supervised by teachers

We publish answers, explanations and connections to CS.

CEMC Courseware and Online Resources

CS Circles

- free interactive lessons teaching the basics of writing computer programs in Python requiring only a browser

Courseware

1. gentle introduction to programming designed with the beginner in mind
2. collection of videos teaching basic programming concepts in a language-independent manner
3. introduces the main ideas behind the specification of a web page in HTML5 and CSS3
4. presents the use of basic programming concepts as applied to web pages, using the language JavaScript

Canadian Computing Competition

A programming contest

- online grader for training and competition
- tests algorithmic thinking and implementation
- can begin practicing early with real-time feedback
- Pascal, C/C++, Python, Java are permitted
- offered for high school students at Junior and Senior levels
- top performers invited to the Canadian Computing Olympiad at UWaterloo

Sample problem

Compute the second largest item in a list of integers.

Thank you for Listening and Participating

Questions

- Now?
- Later: `jpretti@uwaterloo.ca` (don't be shy!)