

INTRODUCTION

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1. Overview

Besides the book by Laumon and Moret-Bailly, see [LMB00], and the work (in progress) by Fulton et al, we think there is a place for an open source textbook on algebraic stacks and the algebraic geometry that is needed to define them. The Stacks Project attempts to do this by building the foundations starting with commutative algebra and proceeding via the theory of schemes and algebraic spaces to a comprehensive foundation for the theory of algebraic stacks.

We expect this material to be read online as a key feature are the hyperlinks giving quick access to internal references spread over many different pages. If you use an embedded pdf or dvi viewer in your browser, the cross file links should work.

This project is a collaborative effort and we encourage you to help out. Please email any typos or errors you find while reading or any suggestions, additional material, or examples you have to stacks.project@gmail.com. You can download a tarball containing all source files, extract, run make, and use a dvi or pdf viewer locally. Please feel free to edit the LaTeX files and email your improvements.

2. Attribution

The scope of this work is such that it is a daunting task to attribute correctly and succinctly all of those mathematicians whose work has led to the development of the theory we try to explain here. We hope eventually to generate enough community interest to find contributors willing to write sections with historical remarks for each and every chapter.

Those who contributed to this work are listed on the title page of the book version of this work and online. Here we would like to name a selection of major contributions:

- (1) Jarod Alper contributed a chapter discussing the literature on algebraic stacks, see Guide to Literature, Section 1.
- (2) Bhargav Bhatt wrote the initial version of a chapter on étale morphisms of schemes, see Étale Morphisms, Section 1.
- (3) Bhargav Bhatt wrote the initial version of More on Algebra, Section 89.
- (4) Kiran Kedlaya contributed the initial writeup of Descent, Section 4.
- (5) The initial versions of
 - (a) Algebra, Section 28,
 - (b) Injectives, Section 2, and
 - (c) the chapter on fields, see Fields, Section 1.
 are from The CRing Project, courtesy of Akhil Mathew et al.
- (6) Alex Perry wrote the material on projective modules, Mittag-Leffler modules, including the proof of Algebra, Theorem 95.6.
- (7) Alex Perry wrote the chapter on deformation theory à la Schlessinger and Rim, see Formal Deformation Theory, Section 1.
- (8) Thibaut Pugin, Zachary Maddock and Min Lee took notes for a course which formed the basis for a chapter on étale cohomology and a chapter on the trace formula. See Étale Cohomology, Section 1 and The Trace Formula, Section 1.
- (9) David Rydh has contributed many helpful comments, pointed out several mistakes, helped out in an essential way with the material on residual gerbes, and was the originator for the material in More on Groupoids in Spaces, Sections 12 and 15.
- (10) Burt Totaro contributed Examples, Sections 64, 65, and Properties of Stacks, Section 12.
- (11) The chapter on pro-étale cohomology, see Pro-étale Cohomology, Section 1, is taken from a paper by Bhargav Bhatt and Peter Scholze.
- (12) Bhargav Bhatt contributed Examples, Sections 71 and 75.
- (13) Ofer Gabber found mistakes, contributed corrections and he contributed Varieties, Lemma 7.17, Formal Spaces, Lemma 14.7, the material in More on Groupoids, Section 15, the main result of Properties of Spaces, Section 17, and the proof of More on Flatness, Proposition 25.13.
- (14) János Kollár contributed Algebra, Lemma 119.2 and Local Cohomology, Proposition 8.7.
- (15) Kiran Kedlaya wrote the initial version of More on Algebra, Section 90.
- (16) Matthew Emerton, Toby Gee, and Brandon Levin contributed some results on thickenings, in particular More on Morphisms of Stacks, Lemmas 3.7, 3.8, and 3.9.
- (17) Lena Min Ji wrote the initial version of More on Algebra, Section 125.
- (18) Matthew Emerton and Toby Gee wrote the initial versions of Geometry of Stacks, Sections 3 and 5.

3. Other chapters

Preliminaries

- | | |
|------------------|-----------------------|
| (1) Introduction | (4) Categories |
| (2) Conventions | (5) Topology |
| (3) Set Theory | (6) Sheaves on Spaces |
| | (7) Sites and Sheaves |
| | (8) Stacks |

- (9) Fields
- (10) Commutative Algebra
- (11) Brauer Groups
- (12) Homological Algebra
- (13) Derived Categories
- (14) Simplicial Methods
- (15) More on Algebra
- (16) Smoothing Ring Maps
- (17) Sheaves of Modules
- (18) Modules on Sites
- (19) Injectives
- (20) Cohomology of Sheaves
- (21) Cohomology on Sites
- (22) Differential Graded Algebra
- (23) Divided Power Algebra
- (24) Differential Graded Sheaves
- (25) Hypercoverings
- Schemes
 - (26) Schemes
 - (27) Constructions of Schemes
 - (28) Properties of Schemes
 - (29) Morphisms of Schemes
 - (30) Cohomology of Schemes
 - (31) Divisors
 - (32) Limits of Schemes
 - (33) Varieties
 - (34) Topologies on Schemes
 - (35) Descent
 - (36) Derived Categories of Schemes
 - (37) More on Morphisms
 - (38) More on Flatness
 - (39) Groupoid Schemes
 - (40) More on Groupoid Schemes
 - (41) Étale Morphisms of Schemes
- Topics in Scheme Theory
 - (42) Chow Homology
 - (43) Intersection Theory
 - (44) Picard Schemes of Curves
 - (45) Weil Cohomology Theories
 - (46) Adequate Modules
 - (47) Dualizing Complexes
 - (48) Duality for Schemes
 - (49) Discriminants and Differents
 - (50) de Rham Cohomology
 - (51) Local Cohomology
 - (52) Algebraic and Formal Geometry
 - (53) Algebraic Curves
 - (54) Resolution of Surfaces
- (55) Semistable Reduction
- (56) Functors and Morphisms
- (57) Derived Categories of Varieties
- (58) Fundamental Groups of Schemes
- (59) Étale Cohomology
- (60) Crystalline Cohomology
- (61) Pro-étale Cohomology
- (62) Relative Cycles
- (63) More Étale Cohomology
- (64) The Trace Formula
- Algebraic Spaces
 - (65) Algebraic Spaces
 - (66) Properties of Algebraic Spaces
 - (67) Morphisms of Algebraic Spaces
 - (68) Decent Algebraic Spaces
 - (69) Cohomology of Algebraic Spaces
 - (70) Limits of Algebraic Spaces
 - (71) Divisors on Algebraic Spaces
 - (72) Algebraic Spaces over Fields
 - (73) Topologies on Algebraic Spaces
 - (74) Descent and Algebraic Spaces
 - (75) Derived Categories of Spaces
 - (76) More on Morphisms of Spaces
 - (77) Flatness on Algebraic Spaces
 - (78) Groupoids in Algebraic Spaces
 - (79) More on Groupoids in Spaces
 - (80) Bootstrap
 - (81) Pushouts of Algebraic Spaces
- Topics in Geometry
 - (82) Chow Groups of Spaces
 - (83) Quotients of Groupoids
 - (84) More on Cohomology of Spaces
 - (85) Simplicial Spaces
 - (86) Duality for Spaces
 - (87) Formal Algebraic Spaces
 - (88) Algebraization of Formal Spaces
 - (89) Resolution of Surfaces Revisited
- Deformation Theory
 - (90) Formal Deformation Theory
 - (91) Deformation Theory
 - (92) The Cotangent Complex
 - (93) Deformation Problems
- Algebraic Stacks
 - (94) Algebraic Stacks
 - (95) Examples of Stacks
 - (96) Sheaves on Algebraic Stacks
 - (97) Criteria for Representability

(98) Artin's Axioms	(109) Moduli of Curves
(99) Quot and Hilbert Spaces	
(100) Properties of Algebraic Stacks	Miscellany
(101) Morphisms of Algebraic Stacks	(110) Examples
(102) Limits of Algebraic Stacks	(111) Exercises
(103) Cohomology of Algebraic Stacks	(112) Guide to Literature
(104) Derived Categories of Stacks	(113) Desirables
(105) Introducing Algebraic Stacks	(114) Coding Style
(106) More on Morphisms of Stacks	(115) Obsolete
(107) The Geometry of Stacks	(116) GNU Free Documentation License
Topics in Moduli Theory	
(108) Moduli Stacks	(117) Auto Generated Index

References

- [LMB00] Gérard Laumon and Laurent Moret-Bailly, *Champs algébriques*, Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge., vol. 39, Springer-Verlag, 2000.