Handwritten digit recognition with Nonlinear Fisher Discriminant Analysis

Robert Indiana,
From Numbers,
1968, screenprint

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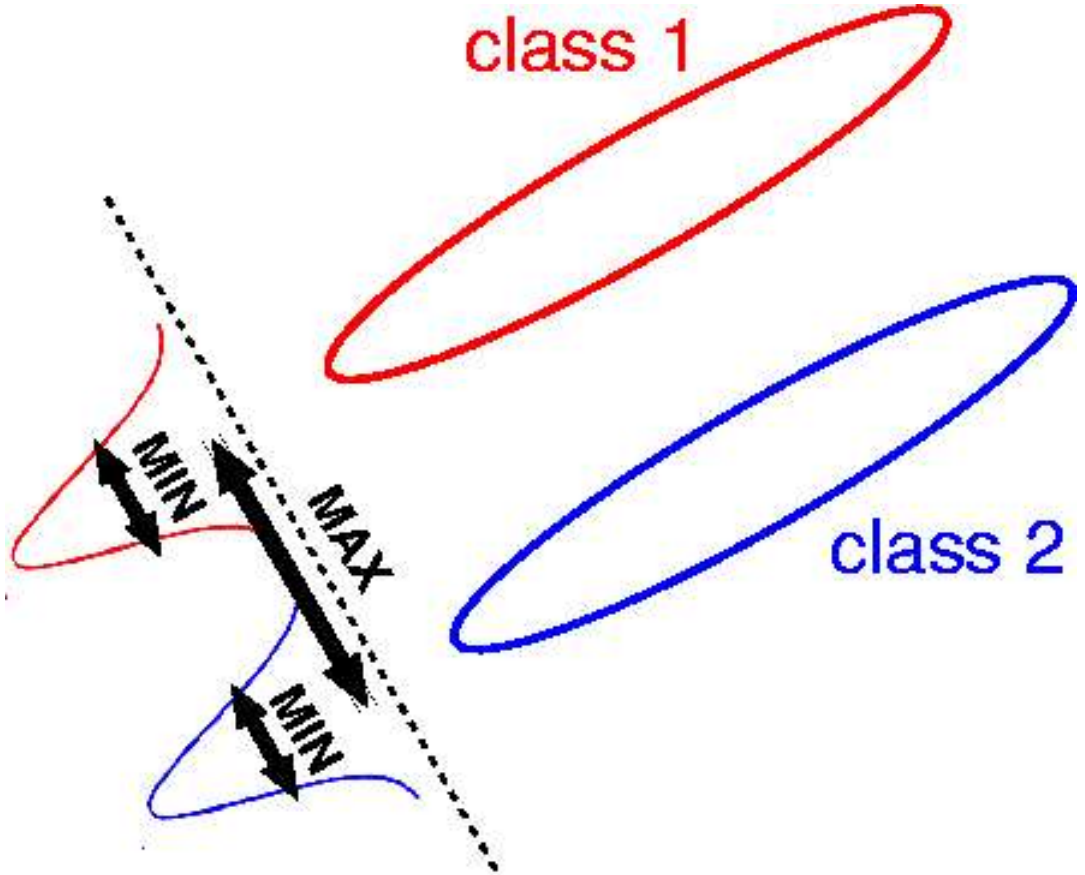
Fisher Discriminant Analysis



Projections can be found solving a generalized eigenvalue problem:

$$\mathbf{S_W V} = \mathbf{S_T V \Lambda}$$

Nonlinear Fisher Discriminant Analysis

- Natural nonlinear extension: expand the input data using a basis of the considered nonlinear function space, then use the linear method.

$$\mathbf{h}(\mathbf{x}) = (x_1^2, x_1x_2, x_1x_3, x_2^2, x_2x_3, x_3^2, x_1, x_2, x_3)^T$$

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- It is possible to formulate NFDA using the kernel trick (Mika et al., 1999; Baudat and Anouar, 2000; Mika et al., 2000; Mika et al., 2001). Limiting factor: number of training patterns, unrealistic for this application.

Properties of NFDA

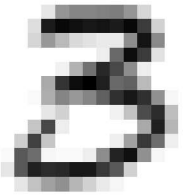
- Given *C* class, only *C-1* projections are relevant
- The data forms clusters corresponding to the single classes
- Classification should be easy
  (e.g., Gaussian classifiers)
- One-step, global solution
- No parameters (given the function space)

Digit recognition

MNIST database:
- handwritten digits
- standard
- freely available
- 28x28 images, centered
- 60,000 training images,
  10,000 test images

(LeCun et al. 1998)

Architecture



dimensionality reduction by PCA
(*N* dimensions)

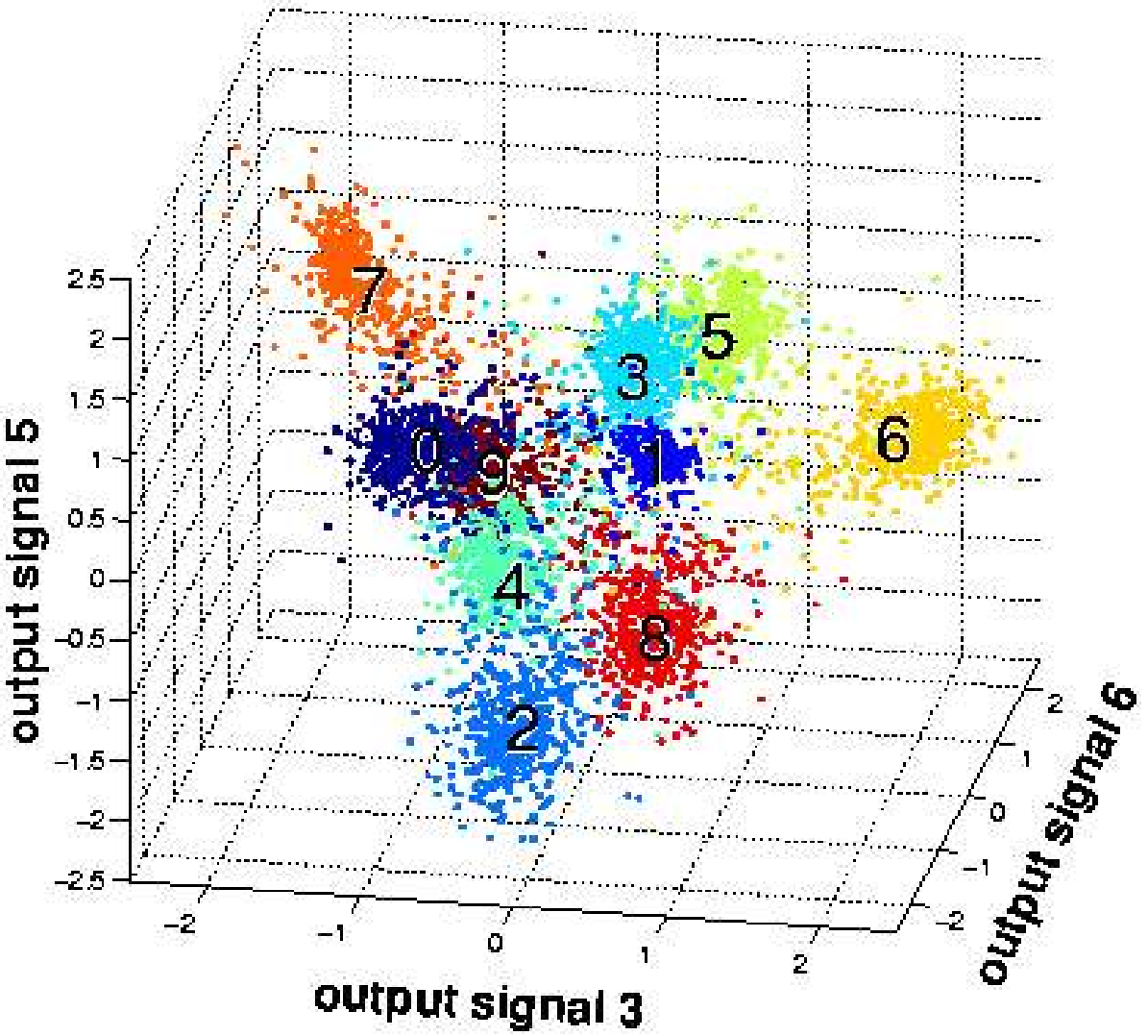
nonlinear expansion
(space of polynomials of degree *d*)

FDA

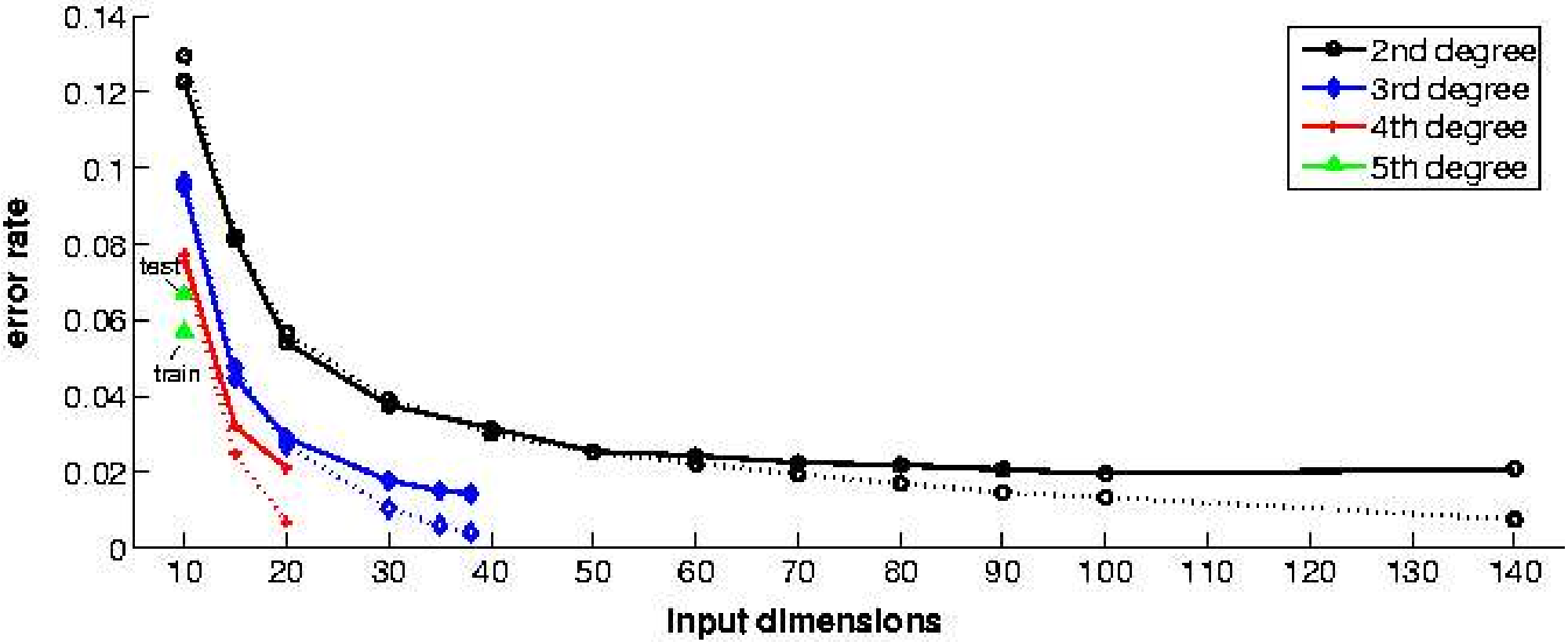
Gaussian classifier
on the 9-dim feature space

Two parameters: *N*: nr. PCA dimensions, *d*: degree of the function space

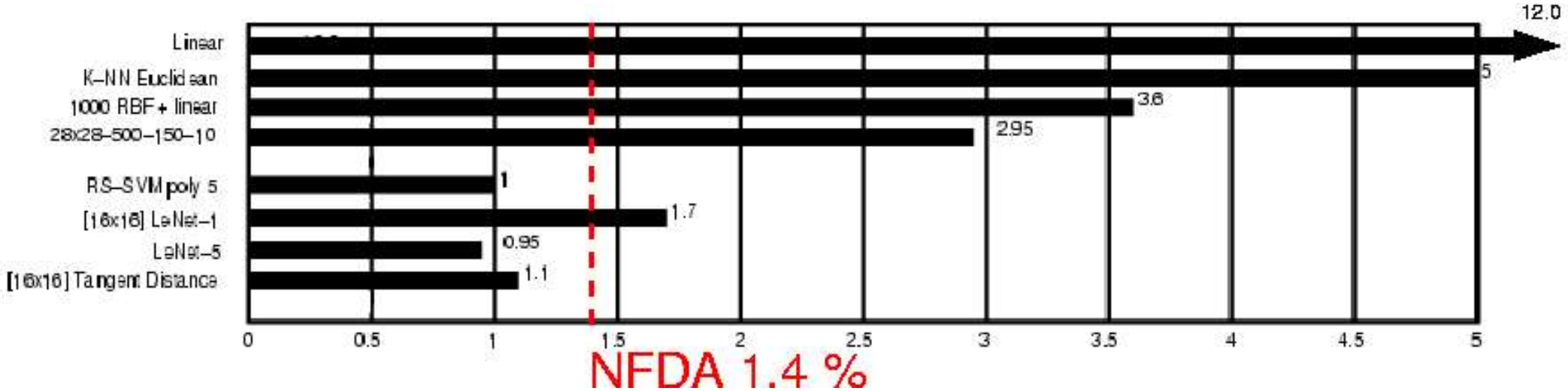
Feature space structure



Error rates

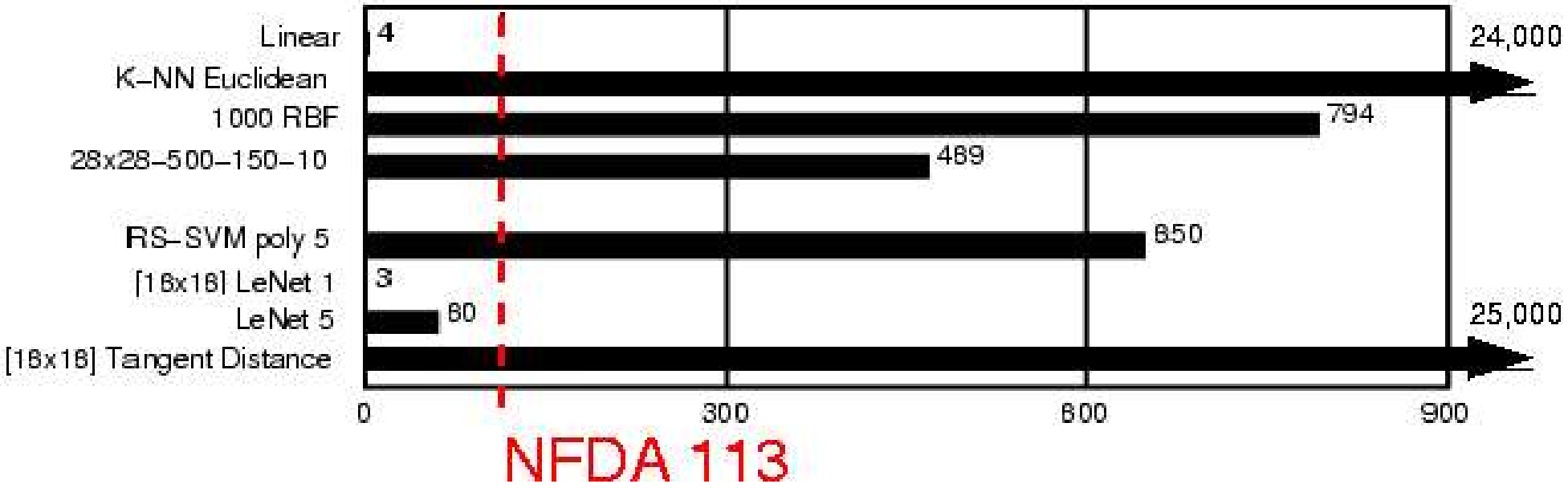

0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
error rate
test
train
10
20
30
40
50
60
70
80
90
100
110
120
130
140
Input dimensions
2nd degree
3rd degree
4th degree
5th degree

Error rates comparison



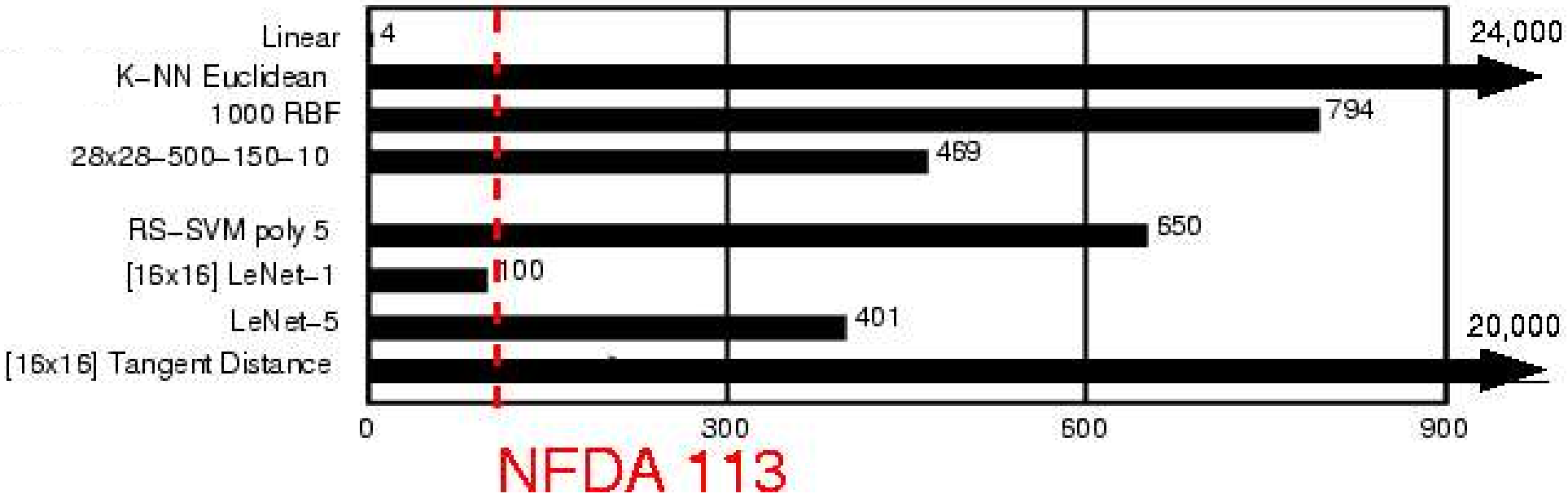
(adapted from
LeCun et al., 1998)

Memory



(adapted from
LeCun et al., 1998)

Speed



(adapted from
LeCun et al., 1998)

Conclusion

- **NFDA provides a simple, problem-independent and parameter-free way to perform pattern recognition**
- Efficient in terms of recognition speed and memory
- Results could be further improved with standard methods or including a-priori knowledge

Acknowledgments

Tiziano Zito (ITB) :

Modular Toolkit for Data Processing
(`mdp-toolkit.sourceforge.net`)

Laurenz Wiskott (ITB)