Illinois Institute of Technology
Department of Computer Science

Homework Assignment 4

CS 535 Design and Analyis of Algorithms
Fall Semester, 2015

Rules for Homework

Remember, the rules listed on the first homework assignment apply to all assignments.

Due: Saturday, September 26, 2015

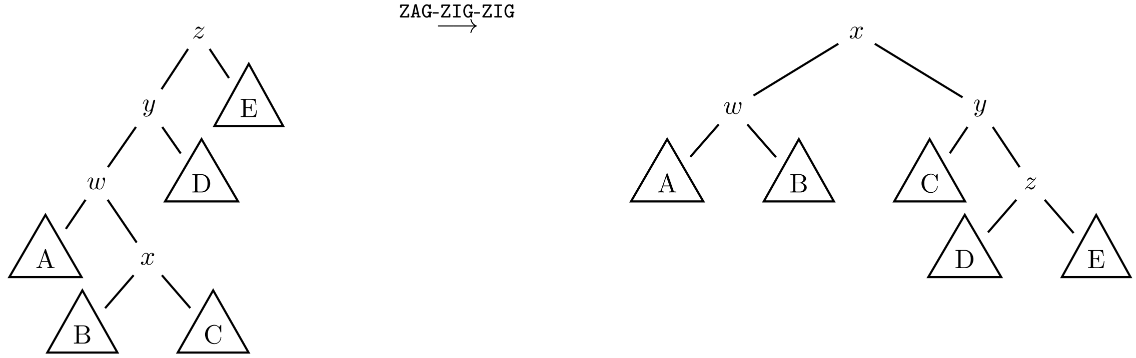
1. We could get the same effect as splaying (moving the accessed item to the root) using only `ZIG` and `ZAG` steps, not bothering with the more complicated `ZIG-ZIG`, `ZIG-ZAG`, `ZAG-ZIG`, and `ZAG-ZAG` steps. If we do that, we only need Case 1 on pages 4–5 in the notes. Compute the amortized cost of an access with such simple-minded splaying.

Near the top of page 667 of the Sleator-Tarjan splay-tree paper, they say that M. D. McIlroy suggested "simple splaying" in which we omit the second rotation in the `ZIG-ZAG` case (the `ZIG`, or right rotation—see the figure at the top of page 2 in the notes). McIlroy claimed to obtain an amortized analysis in which Lemma 1 holds with a constant factor of 3.16^{+} in place of 3. Problems 3–5 comprise an analysis of simple splaying, giving a constant of 4.27^{-}, not 3.16^{+}, a weaker result than McIlroy's.

2. Explain why all that we need consider in analyzing simple splaying (compared to "normal" splaying) are two sub-cases in the `ZIG-ZAG` case: (i) when the operation following the modified `ZIG-ZAG` is a `ZIG` and (ii) when that following operation is a `ZIG-ZIG`. That is, explain why (i) follows immediately from the analysis in the notes and why we need not consider a following operation that is a `ZAG` or a `ZAG-ZAG`.

The remaining problems deal with case (ii).

3. Suppose we have case (ii), in which the simplified `ZIG-ZAG` (now just a `ZAG`) is on a node x and is followed by a `ZIG-ZIG`. In other words w, the parent of x, and y, the grandparent of x, are like-sided children of their parents, but opposite-sided from x; the two operations combine to a `ZAG-ZIG-ZIG`:



Explain why, if $\hat{c}$ is the amortized cost of this `ZAG-ZIG-ZIG` operation, then (in the notation of the notes)

$$\hat{c} = 3 + r'(x) + r'(y) + r'(w) + r'(z) - r(x) - r(y) - r(w) - r(z),$$

and prove that

$$\hat{c} \leq 3 + r'(x) + 2r'(y) + r'(w) - 4r(x).$$

4. Consider

$$2r'(y) + r'(w) - 3r'(x) = \lg \frac{S'(y)}{S'(x)} + \lg \frac{S'(w)S'(y)}{S'(x)^2}. \tag{1}$$

Explain why $S'(x) > S'(w) + S'(y)$ and use this to prove that the expression (**??**) is no greater than when $S'(w)/S'(x) = 1/3$, and hence that

$$2r'(y) + r'(w) - 3r'(x) \leq \alpha = \lg \frac{2}{3} + \lg \frac{2}{9} \approx -2.755.$$

5. Let $\beta = -3/\alpha \approx 1.089 > 1$; prove that

$$\hat{c} \leq (1 + 3\beta)(r'(x) - r(x)),$$

and hence that the amortized cost terms still telescope, but with the increased coefficient of

$$1 + 3\beta = 1 + \frac{-9}{\lg \frac{2}{3} + \lg \frac{2}{9}} \approx 4.267.$$

6. Derive the bound for `delete` given in the table at the top of page 11 of the splay tree notes.