Illinois Institute of Technology
Department of Computer Science

# Homework Assignment 13

CS 535 Design and Analysis of Algorithms
Fall Semester, 2015

## Rules for Homework

Remember, the rules listed on the first homework assignment apply to all assignments.

## Due: Monday, November 30, 2015

1. Exercise 35.2-2 on page 1116 of CLRS3.
2. Prove inequality (3) on page 1 of the notes with the analysis of the nearest-neighbor heuristic for the TSP. (*Hint*: For $n$ even it is a special case of inequality (2). What do you do for odd $n$?)
3. The *cheapest insertion* algorithm for the TSP starts with an arbitrary city and repeatedly inserts another city into the existing tour, just like the closest-insertion algorithm analyzed in class on November 19. However, instead of selecting the closest city to the tour to be inserted, it inserts the city whose insertion is cheapest—that is, city $k$ (not on the current tour) is inserted between adjacent cities $i$ and $j$ (on the current tour) with $i$, $j$, and $k$ chosen to minimize $C_{ik} + C_{kj} - C_{ij}$.
   (a) Prove that if $C$ is a symmetric cost matrix that satisfies the triangle inequality,
   $$\frac{\text{cost of cheapest insertion tour}}{\text{cost of optimal tour}} < 2.$$
   (b) How many operations, as a function of number of cities, are needed to implement this algorithm?
   (c) How far from optimal can the cheapest insertion tour actually be?