Illinois Institute of Technology
Department of Computer Science

Homework Assignment 5

CS 535 Design and Analysis of Algorithms
Fall Semester, 2015

Rules for Homework

Remember, the rules listed on the first homework assignment apply to all assignments.

Due: Thursday, October 1, 2015

1. At the top of page 4 in the notes on lazy weight-balanced trees, it is claimed that "after rebuilding all imbalances are zero in the subtree". Prove this statement.

2. What happens in lazy weight-balanced trees if β is chosen to be just slightly above 1, say $1+\epsilon$? What happens in lazy weight-balanced trees if β is chosen to be very large, say several billion? Describe how one would determine the *optimal* value (or range of values) of β for lazy weight-balanced trees.

3. Redo the amortized analysis of insertion/deletion in lazy weight-balanced trees with

$$I(x) = |\texttt{size}(\texttt{left}(x)) - \texttt{size}(\texttt{right}(x))|\}$$

   in the potential function.

4. **Extra credit.** We can concatenate two lazy weight-balanced trees T_1 and T_2 (all nodes of T_1 precede all nodes of T_2), so that the result is a lazy weight-balanced tree, as follows: Delete the largest-value node from T_1, which might be a deleted value; call the deleted node the "patch node". Now go down the right edge of the modified T_1 to a certain point, say T_3. In place of the root of T_3, put the patch node with left tree T_3 and right subtree T_2. Fill in the details of this algorithm, including the choice of T_3 and any necessary rebuilding. Analyze the amortized running time of the algorithm.

5. **Extra credit.** In the last section of the notes on lazy weight-balanced trees the possibility of coping without the size and height fields is discussed. Give detailed algorithms and amortized analyses for the method described there.

6. Problem 19.4-2 on page 526 (don't forget the case $k = 1$). Give amortized time bounds for all Fibonacci heap operations as a function of k.

Remember, students taking the section of the course in lieu of the theory qualifying exam must do all extra credit problems.