

Homework Assignment 7

CS 535 Design and Analysis of Algorithms
Fall Semester, 2015

Rules for Homework

Remember, the rules listed on the first homework assignment apply to all assignments.

Due: Thursday, October 15, 2015

1. Problem 21.4-5 on pages 581–582.
2. Prove that $A_3(j) > \text{tower}(j)$, where

$$\text{tower}(n) = \begin{cases} 2^{\text{tower}(n-1)} & \text{if } n > 0, \\ 1 & \text{if } n = 0. \end{cases}$$

3. We can add the operation of $\text{DELETE}(x)$ to the Union/Find data structure of Chapter 21 in CLRS3 as follows: We augment each set with two counters (pointed to from the root of a tree): One counter gives the total number of elements in the tree, and the other gives the number of those elements that have been deleted (“ghost elements”). Each element also has a “mark bit” that indicates if it is a ghost. The MAKESET operation is done as before, but also sets up the counters and mark bit. UNION (by rank) is done as before, but also maintains the counters. FIND (with path compression) is done as before.

$\text{DELETE}(x)$ is done in a lazy way as follows: First, mark the node containing x as a ghost. Then, perform a $\text{FIND}(x)$ to determine the (root of) the set containing x and increment its ghost count. If the number of ghost elements in that set S is at least $\lfloor |S|/2 \rfloor$, then rebuild S , picking any one of its non-ghost elements as the root, setting its rank to 1, making all other non-ghost elements children of the root, and updating the two counters; ghost elements in S thus disappear.

Do an amortized analysis of the operations MAKESET , UNION , FIND , and DELETE , proving that any sequence of m operation that includes n MAKESET operations takes amortized time $O(m + m\alpha(n))$.