Illinois Institute of Technology
Department of Computer Science

Homework Assignment 12

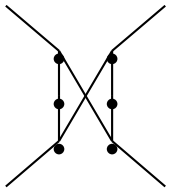
CS 535 Design and Analysis of Algorithms
Fall Semester, 2015

Rules for Homework

Remember, the rules listed on the first homework assignment apply to all assignments.

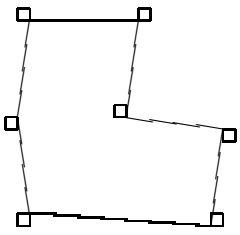
Due: Thursday, November 19, 2015

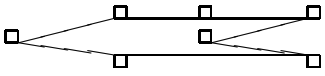
1. In Figure 34.16 (page 1092 of CLRS3), explain why the widget cannot be replaced with the simpler graph



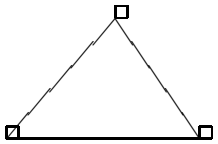
2. You are given a directed graph $G = (V, E)$ with weights w_e on its edges $e \in E$. The weights can be negative or positive. The ZERO-WEIGHT-CYCLE problem is to decide if there is a non-empty simple cycle in G so that the sum of edge weights on this cycle is exactly 0. Prove that ZERO-WEIGHT-CYCLE is NP-complete. You may only use problems proved NP-hard in class or in the text for your reduction.

3. You are given a chain of n rigid struts of lengths $l_1, l_2, l_3, \ldots, l_n$ linked together into a cycle by n hinges. The CHAIN-FOLDING problem is to determine if a given chain can be laid out flat on a single line. We assume that hinges and struts are infinitesimally thin. For example, the following chain can be so positioned:





whereas the following chain cannot:



(a) Prove CHAIN-FOLDING is in NP.

(b) Prove CHAIN-FOLDING is NP-hard by reduction from PARTITION, which you may assume to be NP-complete:

PARTITION($\{x_1, x_2, \ldots x_n\}$) is true if and only if there exists $a_i \in \{-1, +1\}$ such that

$$\sum_{i=0}^{n} a_i x_i = 0.$$

4. **Extra Credit** Prove PARTITION (see previous problem) is NP-hard using the following reduction to SUBSET-SUM (CLRS3, page 1097):

$$f(\langle S, t\rangle) = \begin{cases} \{1,2\} & \text{if } t > \alpha \\ \{1,2,3\} & \text{if } t = 0, \alpha \\ S & \text{if } t = \alpha/2 \\ S \cup \{\alpha + t, 2\alpha - t\} & \text{otherwise} \end{cases}$$

where

$$\alpha = \sum_{x \in S} x.$$