ASSIGNMENT : HW12

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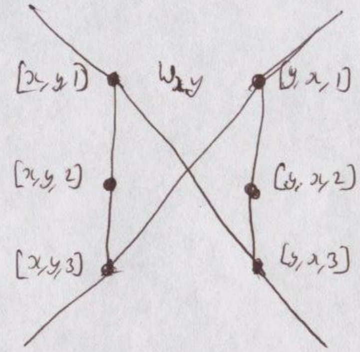
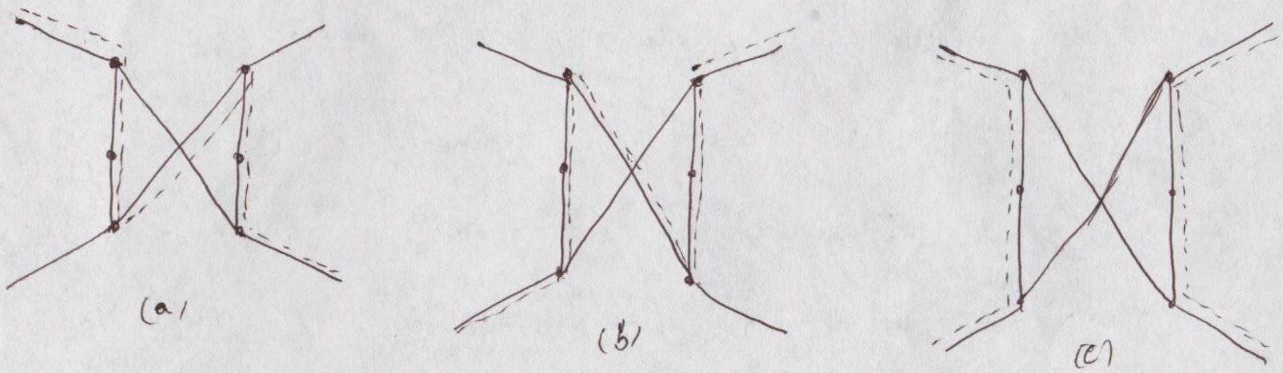


This is the simpler wigdet ~~m~~given in the proble and the below mentioned are the only 3 ~~ways~~ path in which we can cover all the vertices in this widget



The two sides of widget i.e. [x, y, 1] to [x, y, 3] and [y, x, 1] to [y, x, 3] represent the ~~sides~~ edges going from x to y and y to x respectively. ~~As~~ according, if we are going from x to y then we should enter the widget at [x, y, 1] and needs to exit at [x, y, 3]; similarly, if we ~~xx~~ are going from y to x we need to enter widget from [y, x, 1] and exit at [y, x, 3].

~~Although~~ In cases (a) and (b), this is not happenning and the path can occur as mentioned in (c), if and only if 'x' and 'y' are both in the hamiltonian path. Hence, we cannot use this simpler widget.

(2) ~~Given~~ Given a directed graph $G=(V,E)$ with weight w_e on its edges $e \in E$, ~~we can~~ and a ~~s~~ simple ~~path~~ cycle ~~P~~ $Z =$ ~~{~~ a sequence of vertices $v_i \in V$, we can verify within polynomial time by adding the weights on the edges connecting between the sequence of vertices. and finally determine if the sum is zero or not. If zero, the given ~~path~~ cycle is "ZERO-WEIGHT-CYCLE. Hence the problem belongs to class NP, since it is verifiable in polynomial time.

Consider a subset-sum problem of set $S = \{e_1, e_2, e_3 \cdots e_n\}$ and we need to find a subset $S' \subseteq S$ whose sum of elements is 0. But i.e our target value is '0'. ~~Infact this is what we want to find, a set of edges within the given set 'E' whose sum is '0'~~ Now, we construct a graph such that, there are two vertices for each of the value in set x_i and y_i and ~~the~~ a directed ~~an~~ edge from x_i to y_i whose weight is e_i. So, x_i is connected to y_i and has weight e_i. Now connect all y_i to ~~any~~ all other x_j, where $j \neq i$. Keep the weight as '0'. Now, if we are able to find a ZERO-WEIGHT-CYCLE, it includes ~~t~~ ~~th~~ atleast one of the weighted edges in the set ~~E~~ $S = \{e_1, e_2 \cdots e_n\}$. Conversely, if we have a subset-~~sum~~ of ~~E~~ S, S', whose sum is "0", then we will find ~~the~~ a cycle connecting the corresponding vertices of the edges to the values in the subset. ~~Hen~~

Now, there are a total 'n' numbers in the set 'S' and so, there will be '2n' vertices in the graph. and one half of vertices have only one edge ($x_i \to y_i$) and the other half of vertices have 'n-1' edges connecting to the remaining weighted edges and so there a total of 'n^2' edges. Hence constructing this graph from the given set ~~is~~ takes only polynomial time.

~~Hence~~ So, the ZERO-WEIGHT-CYCLE is as hard as SUBSET-SUM problem and so, this problem is NP-complete.

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(3) (a) Given a chain of n rigit struts of lengths $l_1, l_2, l_3, \cdots l_n$ linked together into a cycle of n hinges, we can determine whether the given chain can be laid out in a single line or not in polynomial time. First, we calculate the sum of lengths of ~~chain~~ each struts of chain and starting with the longest strut, we lay the chain and 'n' h the following hinges on the ground. By the time we reach the half the entire length of chain, we should see that the chain is already in single line.

Computationally, we need to find out the largest strut which can be done in linear time and from there on we sum up the lengths till we reach the half the length exactly. If we dont reach, half the value of total length, then the chain cannot be laid in single line. If we reach, the half length value exactly, from then on, whatever the lengths follow we subtract it from the total length being calculated and finally, when we reach starting point, we should have the total value as 0. This verification can be done in polynomial time and so, the ~~chain~~ CHAIN-FOLDING problem belongs to class 'NP'.

5 Consider a set $S = \{x_1, x_2, x_3, \ldots x_n\}$ such that

$$\sum_{i=0}^{n} a_i x_i = t \quad \text{where } a_i \in \{-1, +1\}$$

Since a_i is either '+1' or '-1', we can divide the entire summation into two parts; one part having '+1' co-efficient and other part having '-1' co-efficient. Lets assume that there are 'k' values having positive co-efficient and ~~k-1~~ 'n-k' values having negative coefficient. Now, we consider chain ~~the~~ struts each of length same as the values present in the set S i.e. $x_1, x_2, x_3 \ldots x_n$.

For the 'k' values having ~~k~~ positive coefficient, we join the corresponding length struts together. ~~and~~ ~~ke~~ For the ~~k~~ next strut, we choose one strut from the (n-k) values ~~set~~ and then join the remaining ~~to~~ one after another. ~~The~~ Finally, the last strut and the first strut are hinged together. So, we created a chain of struts of lengths $x_1, x_2, \ldots x_n$.

Now if $\sum_{i=0}^{n} a_i x_i = 0$, this means the sum of ~~value of~~ ~~val~~ lengths of positive coefficient values is same as the sum of length of negative coefficient values. That means, we can lay the chain in a single line and so if we find a PARTITION satisfying set, we will also have a solution for CHAIN-FOLDING

On the contrary, if we find a solution for the CHAIN-FOLDING, ~~so~~ the we just need to give positive co-efficients for struts laid from left to righ and give negative coefficients for struts laid from right to left and so, we get the equation satisfying

$$\sum_{i=0}^{n} a_i x_i = 0$$

and we have a solution for PARTITION.

Hence CHAIN-FOLDING is as hard as PARTITION and so it is ~~NP~~ ~~complete~~ hard. NP complete.

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4. Consider the SUBSET-SUM problem with a set S and $\alpha = \sum_{x \in S} x$.

~~Given~~ For the partition problem, given a set of elements ~~as~~ $x_1, x_2 \ldots x_n$ along with the constants $a_1, a_2, \ldots a_n$, where $a_i \in \{-1, 1\}$, we can easily verify whether the $\sum_{i=0}^{n} a_i x_i = 0$ (or) not in polynomial time.

For each and every term we need to multiply and sum ~~up~~ as the result with the term before we proceed to the next term. So, the partition problem belongs to class NP.

Given as set ~~the~~ S and $\alpha = \sum_{x \in S} x$, if the SUBSET-SUM there exists a subset with total ~~value~~ sum of elements of that subset as zero, then the partition function produces

$$\{1, 2, 3\} \quad , \text{since } t = 0$$

For this set, there exists another set $\{+1, +1, -1\}$ which will help us satisfy

$$(+1)(1) + (+1)(2) + (-1)(3) = 1 + 2 - 3 = \underline{\underline{0}}$$

Hence ~~Hence~~ for $t = 0$, if SUBSET-SUM is true, then ~~part~~ this PARTITION is also true.

For $t = \alpha'$, the subset is nothing but the whole set S and ~~it~~ this is true, always, and so, the partition function output is

$\{1, 2, 3\}$

~~and 'their has its true~~, ~~which~~

and this set is satisfied by $\{1, 1, -1\}$ as shown previously.

For $t > \alpha$, there is no subset of 's', which can satisfy this condition and so, the SUBSET-SUM is always 'false' and for the partition function the output is $\{1, 2\}$.

~~But there is no satisfying~~ such ~~set~~,

But there are no a_1, a_2 such that $a_1(1) + a_2(2) = 0$, given that $a_1, a_2 \in \{1, -1\}$. Hence PARTITION function is false.

~~For t = 'other values', if SUBSET-SUM is true, then the PARTITION function output is~~ $\cancel{S \cup \{\alpha+t, 2\alpha-t\}}$

~~Now, we know that sum of all elements of S is α. Now if we consider (+1) as coefficient for all elements in S, (+1) co-efficient for '$\alpha+t$'~~ ~~and~~

For $t = \alpha/2$, the PARTITION function gives, 'S'. ~~since~~, Since there exists a subset which has sum 't' $= \alpha/2$'. So the sum of other elements ~~sum~~ is $\alpha - t = \alpha - \alpha/2 = \alpha/2$. Hence, we can use (+1) co-efficients for ~~all elemen~~ first subset and -1 coefficient for second subset. will sati satisfy $\sum_{i=0}^{n} a_i x_i = 0$. and so PARTITION function is true.

For all other values of 't', the partition function output is

$S \cup \{\alpha + t, 2\alpha - t\}$.

We know that there is a subset of S, whose sum of elements is t. Hence, the sum of all other elements is $\alpha - t$. Now, if we apply the co-efficients ~~fact~~, as mentioned below

$$(+1)(t) + (-1)(\alpha - t) + (-1)(\alpha + t) + (+1)(2\alpha - t)$$

$$= t - \alpha + t - \alpha - t + 2\alpha - t$$

$$= 0$$

So, we apply (+1) co-efficients for all elements in 'S' which belongs to the subset giving sum 't' and '+1' co-efficient for $(\alpha + t)$ and '-1' co-efficient for all other elements in S and '-1' co-efficients for $2\alpha - t$. Hence the PARTITION function is true.

So, the given partition function is as hard as subset-sum problem and so, it is NP-HARD.