ASSIGNMENT : HW3

NAME : PRASANTH BHAGAVATULA

CWID : A20355611

FALL 2015

1. Considering the potential function as mentioned below,

$$\phi_i = \begin{cases} 2\cdot num_i - size_i & \text{if } \alpha_i \geq 1/2 \\ size_i - 2\cdot num_i & \text{if } \alpha_i < 1/2 \end{cases}$$

we will calculate the amortized costs for insertions and deletion costs for all possible combinations.

Amortized costs for insertions :-

For all the below cases $m = num_{i-1}$ and $s = size_{i-1}$

(a) $\alpha_{i-1} = 1$: Here $m = s$

c_i	ϕ_i	ϕ_{i-1}	$\hat{c}_i$
$m+1$	$2(m+1) - 2s$	$2m - s$	$= (m+1) + (2(m+1) - 2s) - (2m-s)$

$= m+1+2m+2-2s-2m+s$

$= 3 + m - s = \underline{\underline{3}}$ (since $m = s$)

(b) $\frac{1}{2} \leq \alpha_{i-1} < 1$:

c_i	ϕ_i	ϕ_{i-1}	$\hat{c}_i$
1	$2(m+1) - s$	$2m - s$	$= 1 + (2(m+1) - s) - (2m-s)$

$= 1 + 2m + 2 - s - 2m + s$

$= \underline{\underline{3}}$

ASSIGNMENT : HW3
NAME : PRASANTH BHAGAVATULA
CWID : A20355611

(c) $\alpha_i = 1/2$: Here $m+1 = \frac{S}{2}$.

c_i	ϕ_i	ϕ_{i-1}	$\hat{c}_i$
1	$2(m+1)-S$	$S-2m$	$= 1 + (2(m+1)-S) - (S-2m)$

$= 1 + 2m + 2 - S - S + 2m$
$= 3 + 4m - 2S$
$= 3 + 4m - 2(2(m+1))$
$= 3 + 4m - 4m - 4$
$= -1$

(d) $\alpha_i < 1/2$

c_i	ϕ_i	ϕ_{i-1}	$\hat{c}_i$
1	$S-2(m+1)$	$S-2m$	$= 1 + (S-2(m+1)) - (S-2m)$

$= 1 + S - 2m - 2 - S + 2m$
$= -1$

The amortized cost for insertion operation in all cases is a constant and so it is $O(1)$.

Now, we consider the deletion operation.

(a) $\alpha_i \geq 1/2$

c_i	ϕ_i	ϕ_{i-1}	$\hat{c}_i$
1	$2(m-1)-S$	$2m-S$	$= 1 + (2(m-1)-S) - (2m-S)$

$= 1 + 2m - 2 - S - 2m + S$
$= -1$

ASSIGNMENT: HW3 ③

NAME: PRASANTH BHAGAVATULA

CWID: A20355611

(b) $\alpha_{i-1} = 1/2$: Here $2m = s$

c_i	ϕ_i	ϕ_{i-1}	$\hat{c}_i$
1	$s-2(m-1)$	$2m-s$	$= 1+(s-2(m-1))-(2m-s)$

$= 1+s-2m+2-2m+s$

$= 3+2s-4m$

$= 3+2(2m)-4m$

$= 3+4m-4m$

$= \underline{\underline{3}}$

(c) $1/4 < \alpha_{i-1} \leq 1/2$:

c_i	ϕ_i	ϕ_{i-1}	$\hat{c}_i$
1	$s-2(m-1)$	$s-2m$	$= 1+(s-2(m-1))-(s-2m)$

$= 1+s-2m+2-s+2m$

$= \underline{\underline{3}}$

(d) $\alpha_{i-1} = 1/4$: $m = \frac{s}{4}$ and $\alpha_i < 1/2$

c_i	ϕ_i	ϕ_{i-1}	$\hat{c}_i$
m	$\frac{s}{2} - 2(m-1)$	$s-2m$	$= m+\frac{s}{2}-2m+2-s+2m$

$= 2+m-\frac{s}{2}$

$= \underline{\underline{2-\frac{s}{4}}}$

The deletion operation is not of $O(1)$

ASSIGNMENT : HW3

NAME: PRASANTH BHAGAVATULA

CWID: A20355611

FALL 2015

2.(a) The amortized cost of ORDER is O(1) because, the operation will either return TRUE if label(x) < label(y), otherwise it returns false. There is no change in the ~~label~~ number of entries or the values of the labels and so it takes only a constant time. Hence the amortized cost is O(1).

2.(b) The amortized cost of DELETE is O(1) because, the operation will only ~~rede~~ delete an element and so the overall potential decreases depending only on ~~by~~ the label of the deleted element and that will be some constant number only. Hence the amortized cost is O(1).

ASSIGNMENT: HW3 ①

2. FALL 2015

NAME: PRASANTH BHAGAVATULA

CWID: A20355611

2.(c) Consider, the scenario, where in during the INSERT operation rebelling is not required. So the actual cost will be just only for labelling the new inserted element. Now, if we look at change in potential, lets say we have inserted element y after x.

Now ϕ_i is defined as below.

$$\phi_i = \sum_{0 \le k < n} -c\log\left(\frac{g_k}{M}\right)$$

$$= \sum_{0 \le k < x} -c\log\left(\frac{g_k}{M}\right) - c\log\left(\frac{g_x}{M}\right) - c\log\left(\frac{g_y}{M}\right) - c\log\left(\frac{g_{y+1}}{M}\right) - \sum_{y+2 \le k < n} -c\log\left(\frac{g_k}{M}\right)$$

Similarly, ϕ_{i-1} will be

$$\phi_{i-1} = \sum_{0 \le k < n-1} -c\log\left(\frac{g_k}{M}\right)$$

$$= \sum_{0 \le k < x} -c\log\left(\frac{g_k}{M}\right) - c\log\left(\frac{g_x}{M}\right) - c\log\left(\frac{g_{x+1}}{M}\right) - \sum_{x+2 \le k < n-1} -c\log\left(\frac{g_k}{M}\right)$$

The change in potential would be at element 'y' and previous successor of 'x', which is

$$\phi_i - \phi_{i-1} = -c\left[\log\left(\frac{g_y}{M}\right) + \log\left(\frac{g_{y+1}}{M}\right) - \log\left(\frac{g_{x+1}}{M}\right)\right]$$

$$= c\left[\log M - \log g_y + \log M - \log g_{y+1} + \log g_{x+1} - \log M\right]$$

$$= c\left[\log M - \log g_y - \log g_{y+1} + \log g_{x+1}\right]$$

g_y, g_{y+1} and g_{x+1} are the differences in the values of labels and they will ~~be~~ can be treated as constants and so, the ~~order~~ ~~is~~ amortized cost in this case is $O(\log M)$