ASSIGNMENT: HW8

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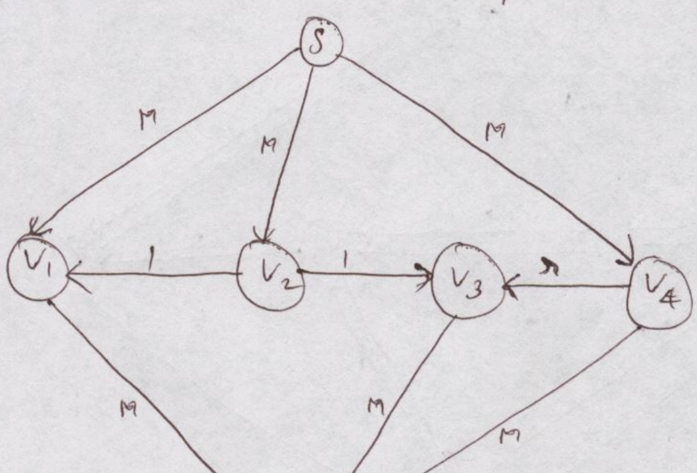
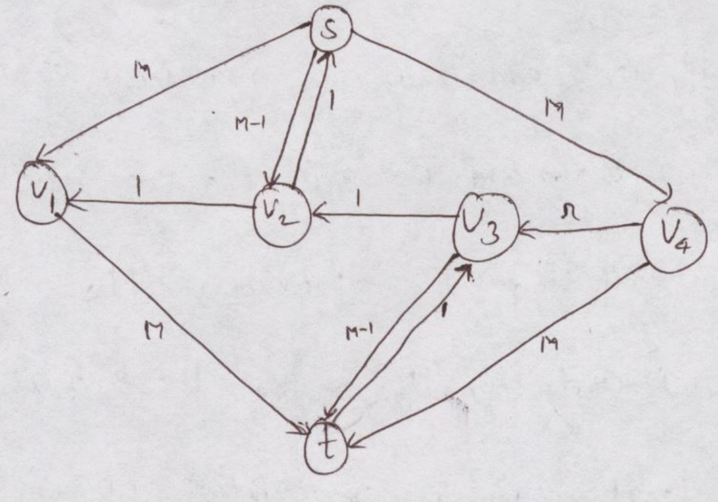
(1) If the capacities are rational, then we can ~~exp~~ express the capacities ~~as~~ in the form a/b, where 'a' and 'b' integers and $b \neq 0$. Now, for all the ~~capacities~~ denominators in the capacity, we find the least common multiplier and then we multiply ~~ea~~ all the cappacities by that number and we get the ~~ca~~ new capacities ~~as~~ integers. So, now we can find the maximum flow for this graph with new capacities and finally divide the maximum flow value by the least common multiplier we found. That will give the maximum flow for the ~~ai~~ graph with original capacities.

Consider the following example with irrational capacities.*



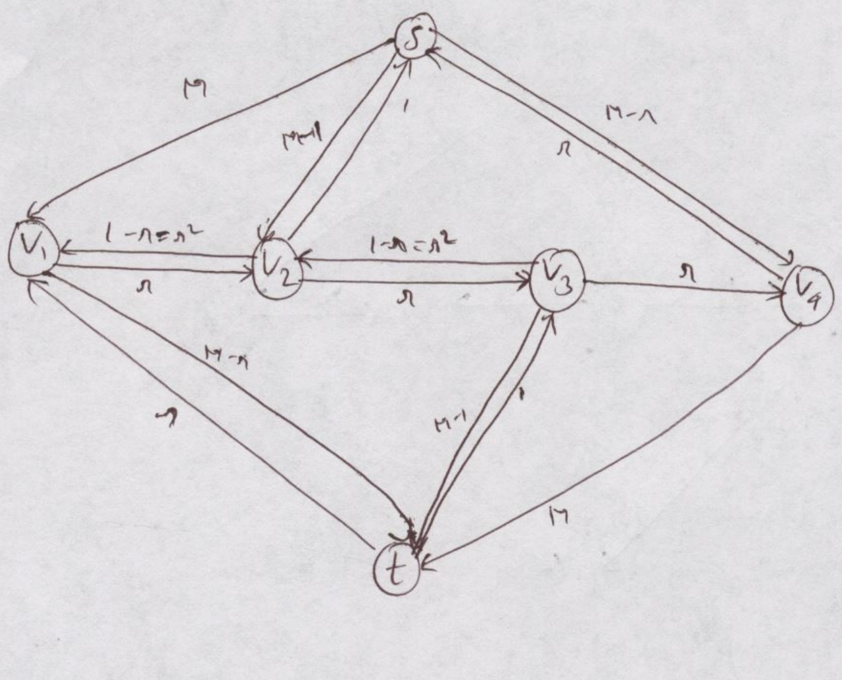
* Referred from Ford-Fulkerson algorithm - Wikipedia.

The capacity 'r' is chosen such that $r^2 = 1 - r$ and $r = (\sqrt{5}-1)/2$ and let the capacity M be a large number. Now, if we choose the augmenting path as s, v_2, v_3, t, the flow sent is 1 and the new residual graph is



Now, if we choose the next augmenting path as $s, v_4, v_3, v_2, v_1, t = P_1$

The flow sent would be 'r' and the new residual graph is



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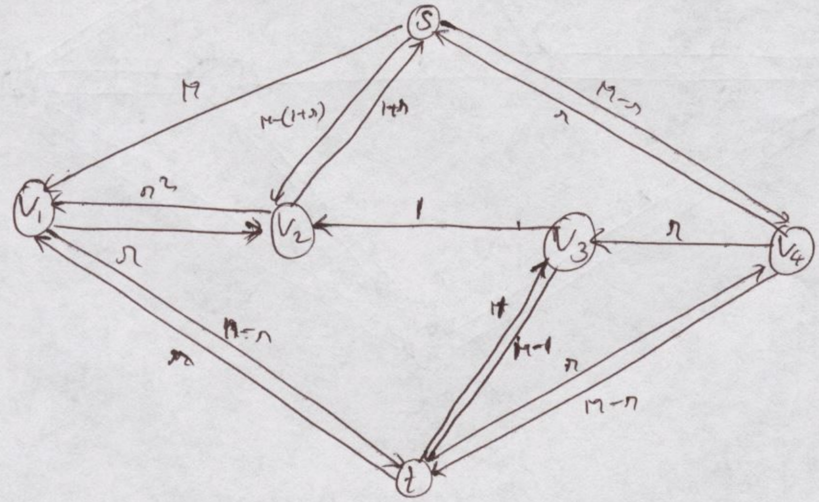
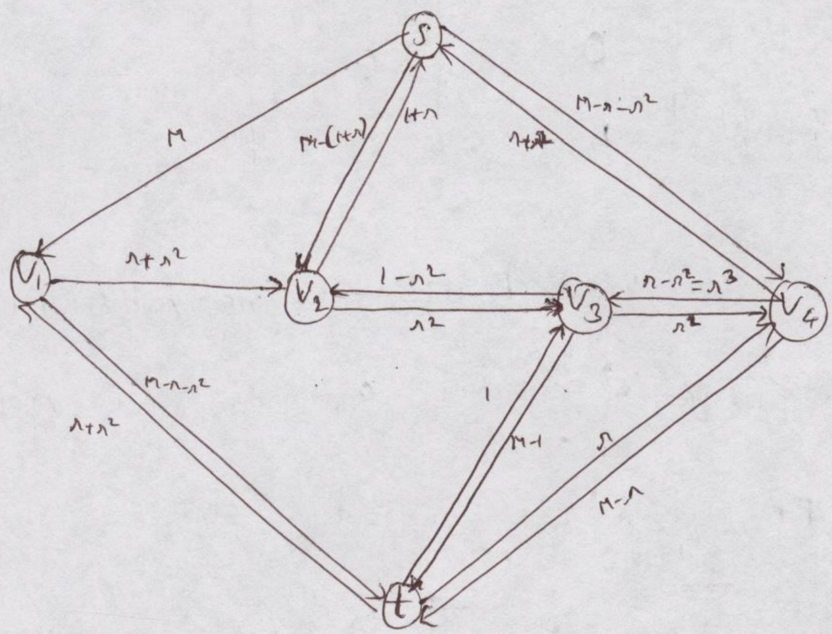
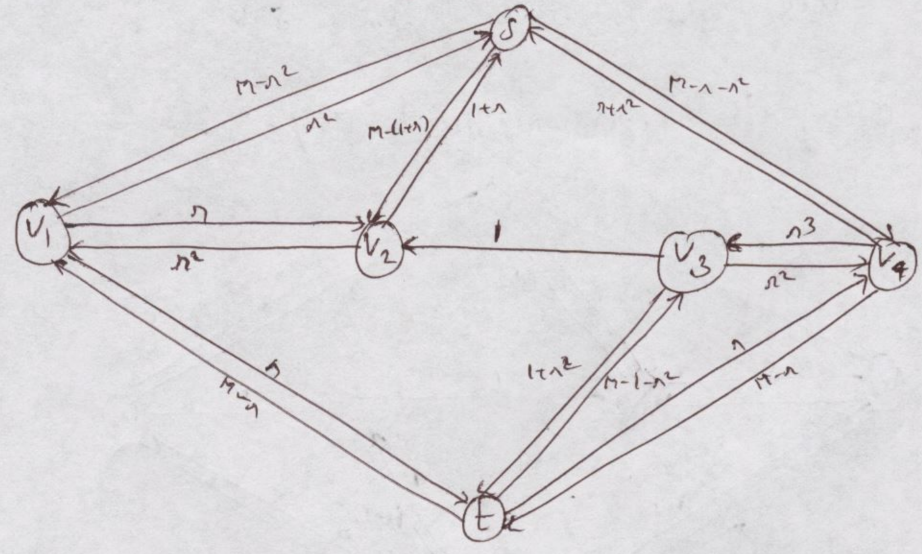
The next augmenting path $p_2 = s, v_2, v_3, v_4, t$, the flow sent would be r and the new residual graph.



The next augmenting path is ~~p~~ again $p_1 = s, v_4, v_3, v_2, v_1, t$; As you can see $r^2 < r$; the ~~sent~~ flow sent is r^2 and the new residual graph is



The next augumenting path ~~p_2~~ $p_3 = s, v_1, v_2, v_3, t$. The flow sent would be r^2 and the new residual graph is



If we look at the capacities before applying the first p_1 and the after applying the above p_3

	Before	After
$c(v_2, v_1)$	$1 = r^0$	r^2
$c(v_2, v_3)$	0	0
$c(v_4, v_3)$	r	r^3

So, for each set of p_4 augumentiong paths p_1, p_2, p_1, p_3, the capacities ~~decrease by~~ decreasing and the flow sent would be

$$1 + 2\sum_{i=1}^{\infty} r^i$$

which will converge to $3 + 2r$, but the maximum flow is $2M + 1$. If we choose, those augmenting paths mentioned we will never converge to maximum flow.

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② When we find the maximum flow and we make the cut to that flow network, we will surely saturate one edge on the minimum cut that we make to the network. Similarly repeating the process again, we will saturate another edge and so, for each time we ~~find~~ make a min-cut, and apply the path, we saturate one edge; So, the upper bound to the number of augmenting paths is simply the number of edges |E|.

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(3) In a bipartite graph $G = (V, E)$ and ~~∀~~ $V = L \cup R$ and $L \cap R = \phi$, we know for sure that all edges in E will be between ~~a~~ two vertices, one vertex belonging to 'L' and the other belonging to 'R'. So, to find an augmenting path, we have to travel back and forth between L and R and so, ~~this can be done in 2~~ the number of edges will be $2 * \min(|L| + |R|)$. ~~Now~~, ~~then~~ Finally we need to travel to 't' and so, we add one more edge and so, the upper bound is $2 * \min(|L| + |R|) + 1$