Illinois Institute of Technology
Department of Computer Science

Homework Assignment 6

CS 535 Design and Analysis of Algorithms
Fall Semester, 2015

Rules for Homework

Remember, the rules listed on the first homework assignment apply to all assignments.

Due: Thursday, October 8, 2015

1. Pages 546–547 in early printings of CLRS3 say "we treat *min* and *max* differently: the element stored in *min* does not appear in any of the clusters, but the element stored in *max* does." This is not really true (they probably corrected it in later printings)—what that sentence should say is "we treat *min* and *max* differently: the element stored in *min* does not appear in any of the clusters, but unless the vEB tree contains just one element (so that the minimum and maximum elements are the same), the element stored in *max* does." Rewrite the code for vEB-Empty-Tree-Insert, vEB-Tree-Insert, and vEB-Tree-Delete so that indeed, *max* does not appear in any of the clusters, just as *min* does not appear in any of the clusters.

2. Problem 20-1, parts (a) and (b) only, on page 557. In part (b), be careful that you do not get snagged by the pitfall described in the middle of page 86 of CLRS3. (*Hint*: For part (b), can you prove, under reasonable assumptions, that $P(u) = u - 2$?) Do not do parts (c)–(g); instead, do the following replacement for part (c):

   (c) We can store all of the array-of-pointers substructures in a single array outside the vEB tree itself. This would change the recurrence (20.5) to

   $$P(u) = (\sqrt{u} + 1)P(\sqrt{u}) + O(1).$$

   Solve this recurrence. Does this idea improve the vEB structure?

3. Consider the "dynamic range minimum query problem." Let $S \subseteq U = \{0, 1, 2, \dots, u-1\}$. Each $s \in S$ is labeled with a real number $v(s)$. We need to maintain a data structure on S that supports the following operations:

   - initialize(S): Construct and initialize the data structure for S.
   - decreaseKey(s, x): If $x < v(s)$, change $v(s)$ to x; otherwise, do nothing.
   - minimum(x): Return $\min\{v(s) | s \in S, s \leq x\}$.

   Show how a vEB tree can be used to support these three operations. Analyze the time/space complexity of your algorithms.