

# Homework Assignment 1

CS 535 Design and Analysis of Algorithms  
Fall Semester, 2015

**Due: Thursday, September 3, 2015**

1. We want to break a string into pieces using a basic operation  $\text{STRING-SPLIT}(s, k, f, b)$  that splits a string  $s$  into two pieces: the first  $k$  characters,  $f$ , and  $b$  the remaining characters; this basic operation requires copying the string, so it costs  $O(|s|)$  time to break string  $s$  characters into two pieces. What we really want to do, however, is to break a string into many pieces using successive applications of  $\text{STRING-SPLIT}$ . The order in which the breaks are made affects the total amount of time used—for example, suppose we need to break a 20-character string after characters 3, 8, and 10 (numbering the characters in ascending order from the left, starting from 1). If the breaks are made in left-to-right order, then the first break costs 20 units of time, the second break costs 17 units of time, and the third break costs 12 units of time, a total of 49 units of time. If the breaks are made in the right-to-left order, then the first break costs 20 units of time, the second break costs 10 units of time, and the third break costs 8 units of time, a total of 38 units of time.
  - (a) Devise and analyze an *un-memoized* dynamic programming algorithm that, when given the numbers of characters after which to break, determines the cheapest cost of those breaks.
  - (b) Show how to memoize your algorithm in (a) and analyze the resulting memoized algorithm. The result should take polynomial time.
2. Find counter examples to the following heuristics for the string-cutting problem above. That is, find a string and places to cut such that when cuts are made in the order given, the cost is higher than the optimal.
  - (a) Start by cutting the string as close to the middle as possible, and then repeat the same thing on the resulting pieces.
  - (b) Start by making (at most) two cuts to separate the smallest substring. Repeat this until finished.
  - (c) Start by making (at most) two cuts to separate the largest substring. Repeat this until finished.