EEP/IAS 118 - Introductory Applied Econometrics, Section 9

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Agenda

- Review of regression output interpretation
- Chow tests (a special kind of *F*-test)
- Assignent 3 review

Review: Functional Forms

Model	DepVar	IndepVar	How does Δy ?	Interpretation
			relate to Δx	
Linear	у	×	$\Delta y = \beta_1 \Delta x$	$\Delta y = \beta_1 \Delta x$
Logarithmic	у	$\log(x)$	$\Delta y = \beta_1 \frac{\Delta x}{x}$	$\Delta y = (\beta_1/100)\%\Delta x$
Exponential	$\log(y)$	x	$\frac{\Delta y}{y} = \beta_1 \Delta x$	$\%\Delta y = (100\beta_1)\Delta x$
Log-Log	$\log(y)$	$\log(x)$	$\frac{\Delta y}{y} = \beta_1 \frac{\Delta x}{x}$	$\%\Delta y = \beta_1\%\Delta x$
Standardized	$ ilde{y} = \left(rac{y - \overline{y}}{\sigma_y} ight)$	$\tilde{x} = \left(\frac{x - \bar{x}}{\sigma_x}\right)$	$\Delta \tilde{y} = \beta_1 \Delta \tilde{x}$	$\Delta s.d.(y) = \beta_1 \Delta s.d.(x)$
LPM	$y(y \in \{0,1\})$	x	$\Delta y = \beta_1 \Delta x$	$\Delta P(Y=1) = \beta \Delta x$

Questions: Consider the following regression results analyzing the relationship between sleep and work. Interpret the coefficient on sleep and its statistical significance in the following models (you can assume N is very large).

1
$$sleep = 8.538 - 0.151 work$$
; $SE(\beta_{work}) = 0.0168$

2
$$sleep = 8.753 - 0.578log(work)$$
; $SE(\beta_{log(work)}) = 0.0727$

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- 1 sleep = 8.538 0.151work; $SE(\beta_{work}) = 0.0168$
 - An increase of 1 hour of work per day is associated with a decrease of 0.151 hours of sleep per day. $t = \beta/SE(\beta) = -0.151/0.0168 = -9.005.$ This is larger in

absolute value than any reasonable critical value. We therefore reject the null that $\beta_{work} = 0$ with a high level of confidence.

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- 2 sleep = 8.753 0.578log(work); $SE(\beta_{log(work)}) = 0.0727$
 - An increase of 1% in hours of work per day is associated with a decrease of 0.0058 hours (0.35 minutes) of sleep per day. $t = \beta/SE(\beta) = -0.578/0.0727 = -7.918.$ This is larger in absolute value than any reasonable critical value. We therefore reject the null that $\beta_{log(work)} = 0$ with a high level of confidence.

3
$$log(sleep) = 2.144 - 0.024work; SE(\beta_{work}) = 0.0024$$

4
$$log(sleep) = 2.169 - 0.076log(work);$$

 $SE(\beta_{log(work)}) = 0.0104$

- 3 log(sleep) = 2.144 0.024work; $SE(\beta_{work}) = 0.0024$
 - An increase of 1 hour of work per day is associated with a decrease of 2.4% in hours of sleep per day. $t = \beta/SE(\beta) = -0.024/0.0024 = -8.522.$ This is larger in absolute value than any reasonable critical value. We therefore reject the null that $\beta_{work} = 0$ with a high level of confidence.
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 - An increase of 1% in hours of work per day is associated with a decrease of 0.076% in hours of sleep per day. $t = \beta/SE(\beta) = -0.076/0.0104 = -7.302.$ This is larger in absolute value than any reasonable critical value. We therefore reject the null that $\beta_{log(work)} = 0$ with a high level of confidence.

5 We standardized the sleep and work variables (recall that standardizing means subtracting the mean and dividing by the standard deviation) and obtained $scale(sleep) = -2.6e^{-16} - 0.321scale(work);$ $SE(\beta_{scale(work)}) = 0.0357$

6 We created a dummy variable indicating whether an individual sleeps at least 8 hours per day (sleep8up). We estimate the following linear probability model sleep8up = 0.687 - 0.055work; $SE(\beta_{work}) = 0.00794$

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 - An increase of 1 standard deviation of hours of work per day decreases hours of sleep per day by 0.321 standard deviations. $t = \beta/SE(\beta) = -0.321/0.0357 = -9.005$.
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- 6 We created a dummy variable indicating whether an individual sleeps at least 8 hours per day (sleep8up). We estimate the following linear probability model sleep8up = 0.687 0.055work; $SE(\beta_{work}) = 0.00794$
 - An increase of 1 hour of work per day is associated with a decrease of 0.055 (or 5.5%) in the probability that an individual sleeps at least 8 hours per day. $t = \beta/SE(\beta) = -0.055/0.00794 = -6.979.$

Interpreting Qualitative Variables

When an explanatory variable doesn't have a clear numerical ordering (e.g. colors, industries, cities, etc.) we create g-1 dummies for the g possible values it takes on.

 Need to leave out one dummy, otherwise we will have perfect multicollinearity

$$y_i = \beta_0 + \beta_1 group 1_i + \ldots + \beta_{g-1} group gminus 1_i + u_i$$

Interpretation of Coefficients

- groupk_i is a dummy for whether i is a member of group k.
 Simplest case is a binary variable.
- β_0 is the mean of y for the reference (left-out) group when any other included variables are 0.
- β_k coefficients on variables groupk are the difference in means of y between group k and the reference group.

Review Questions: Interpreting Qualitative Variables

Suppose we estimate $(R^2 = 0.1047)$

$$sleep = .8553 - .151 wrkhrs - .029 yngkid - .173 black$$
 (.094) (.0167) (.112) (.174)

where *wrkhrs* is the number of hours worked per day, *yngkid* is a dummy for whether a person has young kids and *black* is a dummy for whether the person is Black.

1 Interpret the coefficient for the intercept (ignore significance).

2 Interpret the coefficient estimate for yngkid (including significance).

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- 1 Interpret the coefficient for the intercept (ignore significance).
 - Individuals who work 0 hours per day, have no young kids, and are not Black sleep 8.55 hours per day on average.
- 2 Interpret the coefficient estimate for *yngkid* (including significance).

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- 1 Interpret the coefficient for the intercept (ignore significance).
 - Individuals who work 0 hours per day, have no young kids, and are not Black sleep 8.55 hours per day on average.
- 2 Interpret the coefficient estimate for *yngkid* (including significance).
 - Individuals with young children sleep 0.029 hours (1.78 minutes) less per day on average, holding hours of work and whether they are Black constant. We have t=-0.263, indicating that we cannot reject the null that the coefficient is equal to 0 (there is no association between having a young child and sleep hours) at any reasonable level of confidence.

3 What is the average hours of sleep per day for individuals who work 5 hours per day, have young children, and are not Black?

Suppose we estimate
$$(R^2 = 0.1033)$$

 $sleep = 8.538 + .151wrkhrs$
 $(.092)$ $(.016)$

4 Test the joint null hypothesis that neither having young children nor being Black affect sleep.

- 3 What is the average hours of sleep per day for individuals who work 5 hours per day, have young children, and are not Black?
 - 8.553 + 5*(-0.151) 0.0296*1 0.174*0 = 7.765 hours per day.

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Suppose we estimate (
$$R^2 = 0.1033$$
)
 $sleep = 8.538 + .151 wrkhrs$
(.092) (.016)

- 4 Test the joint null hypothesis that neither having young children nor being Black affect sleep.
 - Formula: $F = \frac{\left(R_{UR}^2 R_R^2\right)/q}{\left(1 R_{UR}^2\right)/(n k_{UR} 1)} = \frac{(0.1047 0.1033)/2}{(1 0.1047)/(706 3 1)} = 0.549$ With q = 2 and dof = 702, we can look up $c_{.05} \sim 3$. Since 0.549 < 3, we cannot reject the null hypothesis that neither having young children nor being Black affect sleep at a 5% significance level (or even a 10% level).

Interactions and total marginal effects

When we believe the effect of one variable depends on the value of another, we use an interaction term. Example: Does the effect of education on wages vary by sex?

$$wage = \beta_0 + \beta_1 female + \beta_2 educ + \beta_3 female \times educ + u$$

Interpreting β_0 : This parameter reflects the intercept (value of wages) for males with no education.

Interpreting β_1 : This parameter reflects the difference in the intercepts between women and men with no education.

Interpreting β_2 : This parameter reflects the effect of an additional year of education for males.

Interpreting β_3 : This parameter reflects the difference in the returns to education (differential effect of education on wage) for males and females.

Interactions and total marginal effects

$$wage = \beta_0 + \beta_1 female + \beta_2 educ + \beta_3 female \times educ + u$$

Note that because we have an interaction term, the total marginal effect for each variable depends on the level of the other variable. For *educ*, we can calculate two different effects, one for females and one for males. For *female*, we would need to choose a level of *educ* to be able to estimate a total effect - for example, the total effect of being female on wages at the mean years of education. We calculate the total marginal effect of a variable by taking the derivative of the regression equation w.r.t to that variable.

- Total effect of $female = \beta_1 + \beta_3 \times educ$
- Total effect of $educ = \beta_2 + \beta_3 \times female$

Interactions and total marginal effects

The following regression output analyzes whether the effect of age (in years) on sleep hours per day varies by age and by work hours per day.

• Note that including a quadratic term (in this case $age2 = age^2$) is essentially including an interaction of a variable with itself

```
Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) 8.5059657 0.6132099 13.871 <2e-16 ***

wrkhrs -0.1043907 0.0580052 -1.800 0.0723 .

age -0.0118602 0.0285583 -0.415 0.6781

age2 0.0002884 0.0003228 0.894 0.3719

wrkhrs:age -0.0010902 0.0014329 -0.761 0.4470

---

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 1.001 on 701 degrees of freedom Multiple R-squared: 0.1108, Adjusted R-squared: 0.1057 F-statistic: 21.84 on 4 and 701 DF, p-value: < 2.2e-16

12 / 24

1 Interpret the coefficient for the intercept (ignore significance).

2 Interpret the coefficient estimate for *wrkhrs* (including significance).

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 - Individuals who work 0 hours per day and are 0 years old sleep 8.51 hours per day on average.
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- 1 Interpret the coefficient for the intercept (ignore significance).
 - Individuals who work 0 hours per day and are 0 years old sleep 8.51 hours per day on average.
- 2 Interpret the coefficient estimate for wrkhrs (including significance).
 - One additional hour of work per day is associated with 0.104 fewer hours of sleep for individuals at age 0. We observe p=0.0723, suggesting we can reject the null that this effect is 0 at the 10% significance level, but not at the 5% significance level.

3 What is the total marginal effect of age on sleep? What is the effect of age on sleep at age 25 for individuals who work 8 hours per day?

For individuals who work 8 hours per day, at what age does the sign of the relationship between age and sleep change, and how does it change?

- 3 What is the total marginal effect of age on sleep? What is the effect of age on sleep at age 25 for individuals who work 8 hours per day?
 - Taking the derivative with respect to age gives -0.012 + 0.00029 * 2age 0.0011wrkhrs. One additional year of age at age 25 for individuals who work 8 hours per day will decrease sleep by -0.012 + 0.00029 * 2 * 25 0.0011 * 8 = -0.0063 hours per day.
- 4 For individuals who work 8 hours per day, at what age does the sign of the relationship between age and sleep change, and how does it change?

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- 4 For individuals who work 8 hours per day, at what age does the sign of the relationship between age and sleep change, and how does it change?
 - Set -0.012 + 0.00058age .0011 * 8 = 0 and solve for age = 35.86. Sleep hours decrease with age up until age 36, and increase with age afterward.

```
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 8.53899 0.09265 92.165 <2e-16 ***
wrkhrs -0.15075 0.01674 -9.005 <2e-16 ***
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 1.003 on 704 degrees of freedom
Multiple R-squared: 0.1033, Adjusted R-squared: 0.102
F-statistic: 81.09 on 1 and 704 DF, p-value: < 2.2e-16
```

5 Test the joint null hypothesis that age does not affect sleep, controlling for total hours of work.

```
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Residual standard error: 1.003 on 704 degrees of freedom Multiple R-squared: 0.1033, Adjusted R-squared: 0.102
F-statistic: 81.09 on 1 and 704 DF, p-value: < 2.2e-16
```

- 5 Test the joint null hypothesis that age does not affect sleep, controlling for total hours of work.
 - The null hypothesis here is that $\beta_{age}=0$ & $\beta_{age2}=0$ & $\beta_{age:wrkhrs}=0$ against the alternative that any of these coefficients are not 0.

$$F = \frac{\left(R_{UR}^2 - R_R^2\right)/q}{\left(1 - R_{UR}^2\right)/(n - k_{UR} - 1)} = \frac{(0.1108 - 0.1033)/3}{(1 - 0.1108)/(706 - 4 - 1)} = 1.971$$

With q=3 and dof=701, we can look up $c_{.05}\sim 2.6$. Since 1.971<2.6, we cannot reject the null hypothesis that age does not affect sleep at a 5% significance level (it would be close at a 10% level, where $c_{.10}\sim 2.08$).

```
Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) 8.61512 0.09864 87.339 <2e-16 ***
wrkhrs -0.16499 0.01784 -9.251 <2e-16 ***
yngkid -0.62474 0.28252 -2.211 0.0273 *
wrkhrs:yngkid 0.11607 0.05084 2.283 0.0227 *
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1 on 702 degrees of freedom
Multiple R-squared: 0.11, Adjusted R-squared: 0.1062
F-statistic: 28.92 on 3 and 702 DF, p-value: < 2.2e-16
```

- 6 What would be the null and alternative hypothesis to test whether the association between work hours and sleep does not vary by whether the individual has young children? Implement this test.
- What would be the null and alternative hypothesis to test whether average sleep hours are the same for individuals with and without young children who work the same number of hours per week?

6 What would be the null and alternative hypothesis to test whether the association between work hours and sleep does not vary by whether the individual has young children? Implement this test.

What would be the null and alternative hypothesis to test whether average sleep hours are the same for individuals with and without young children who work the same number of hours per week?

- 6 What would be the null and alternative hypothesis to test whether the association between work hours and sleep does not vary by whether the individual has young children? Implement this test.
 - The null hypothesis is that $\beta_{wrkhrs:yngkid}=0$, and the alternative is that $\beta_{wrkhrs:yngkid}\neq 0$. We can test this hypothesis by looking at the t and p-values for this coefficient. With p=0.0227 we can reject the null at the 5% significance level, but not at the 1% significance level.
- What would be the null and alternative hypothesis to test whether average sleep hours are the same for individuals with and without young children who work the same number of hours per week?

- 6 What would be the null and alternative hypothesis to test whether the association between work hours and sleep does not vary by whether the individual has young children? Implement this test.
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- What would be the null and alternative hypothesis to test whether average sleep hours are the same for individuals with and without young children who work the same number of hours per week?

$$H_0: \beta_{yngkid} = 0$$
 & $\beta_{wrkhrs:yngkid} = 0$

vs.
$$H_1: \beta_{yngkid} \neq 0$$
 &/or $\beta_{wrkhrs:yngkid} \neq 0$

Chow test

- We have seen that interacting dummy variables with a) other dummies and b) continuous variables allows us to test whether different groups have different intercepts and different slopes, respectively.
- We may also wish to test the null that two groups follow the same regression function, against the alternative that one or more of the slopes or intercepts differ across groups.
- In other words, we may want to test that there are *any* differences in model parameters between two groups.
- Can do this with F test, but sometimes easier to use a Chow test.

Chow test: Example using sleep75 data set

Suppose you have the following model of sleep (in minutes per week):

$$sleep = \beta_0 + \beta_2 age + \beta_4 totwrk + u$$

BUT you suspect that the relationship between sleep and age and totwrk is different if you have young kids vs not. There are two ways you could formally test this hypothesis:

- F-test: compare unrestricted and restricted regressions
- Chow test: compare restricted regression and regressions for subsamples of interest

F-test approach

If you suspect that this whole regression might be different if we ran it for only people with young kids, that's equivalent to saying that each of the β s is different depending on whether someone has young kids. What would the unrestricted regression be in this case?

F-test approach

We can rewrite restricted and unrestricted regressions as:

Unrestricted :
$$sleep = \beta_0 + \beta_1 yngkids + \beta_2 age + \beta_3 yngkids * age + \beta_4 totwrk + \beta_5 yngkids * totwork + e$$

Restricted : $sleep = \beta_0 + \beta_2 age + \beta_4 totwrk + e$

The F-test that will tell us whether there is a significant difference between these two models:

$$H_0: \beta_1, \ \beta_3, \ \beta_5 = 0$$

 $H_1: \text{not } H_0$

Chow test approach

We get everything we need to compute the F-stat from running the following regressions (A and B) on different subsamples:

- $sleep = \beta_0 + \beta_1 age + \beta_2 totwrk(A : Have young kids only)$
- $sleep = \beta_0 + \beta_1 age + \beta_2 totwrk(B : No young kids only)$

Noting that:

- $SS = SSR_A + SSR_B$
- q = k + 1 the hypothesis that each beta is the same across the two groups involves k + 1 restrictions.
- ullet The unrestricted model, which we can think of as having a group dummy variable and k interaction terms in addition to the intercept and variables themselves, has n-2(k+1) degrees of freedom

Chow test approach

We can use these three facts to rewrite our F-statistic in a way so that we only need to run (1) Restricted Model, (2) Model A and (3) Model B instead of the usual restricted and unrestricted regressions:

$$F = \frac{\left(SSR_{pooled} - (SSR_A + SSR_B)\right)/q}{\left(SSR_A + SSR_B\right)/(n - 2(k+1))}$$

What this means is that we can calculate the F-statistic that tests whether or not each parameter in our original model (1) is different for household with and without young kids without actually running the unrestricted model.

Note: R example in notes

Assignment 3 review

To Jupyter!