Lecture 9: Inference and Hypothesis Testing

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Last time

- We reviewed how sample estimators are random variables
- We discussed how information on the distribution of sample estimators can help us make inferences about the true population parameters, focusing on the example of the sample mean
- We showed how we can transform the sample mean into two statistics with known distributions:

$$Z = \frac{\bar{X} - \mu}{\sigma_x / \sqrt{n}} \sim N(0,1) \qquad t = \frac{\bar{X} - \mu}{s_x / \sqrt{n}} \sim t_{n-1} \tag{1}$$

- Because we often don't know σ_x, we said we would often have to use the t statistic
- We examined how to construct Confidence Intervals using critical values associated with particular confidence levels for each distribution

Formalizing our understanding of confidence intervals

To Jupyter!

- To recap: With a 95% confidence level, μ will be inside the calculated CI for 95% of samples.
- Show on board.
- The estimated CI for a given sample is useful for identifying likely values of μ, particularly if the interval is small, but it is still just an estimator for the true CI.

Calculating Z vs t statistic

- If we know σ_x we use standard normal distribution
- If we don't, we use the t distribution with $n - 1$ degrees of freedom (converges to standard normal for large n)
- When do we know σ_x?
- When we have a binary random variable

When do we know σ_x?

- Binary Random Variables take on 2 values (usually, 0 or 1)
  - $X = 1$ often indicates success in a trial, for example
- If X is binary, $E[X] = Pr(X = 1) * 1 + (1 - Pr(X = 1)) * 0 = Pr(X = 1) = p = \mu_X$
- $\sigma_X^2 = p(1 - p) = \mu_X(1 - \mu_X)$
  - In this case, if we know μ_X then we know σ_X^2
- We will say that $X \sim Bernoulli(p)$
- While Normal Random variables are defined by 2 parameters (μ, σ), Bernoulli random variables are defined by one (p)

Binary random variables

- For a binary random variable, If we know μ, we can construct

$$Z = \frac{\bar{X} - \mu}{\sigma_X / \sqrt{n}} = \frac{\bar{X} - \mu}{\sqrt{\mu(1-\mu)/n}} \sim N(0,1) \tag{2}$$

- So, for binary random variables we can construct a confidence interval using the Z statistic.
- We'll take critical values from the Standard Normal distribution
- For non-binary random variables, need to use the t statistic and t_{n-1} distribution

Hypothesis testing

- Our Goal: Use Econometrics to test economic hypotheses
  - e.g., Education leads to higher wages
  - More *GDP* leads to more CO_2
- We've estimated $\hat{\beta}$ for these relationships and want to use that to learn about the true β.
- What does it mean to test a hypothesis? Let's continue to focus on inference about population means.
- We have a statistical environment:
  - A random variable X and n observations from a random sample $(X_1, X_2, ..., X_n)$.
  - $E[X] = \mu$
  - We can construct $\bar{X}$ in the sample and estimate its variance.
  - We don't know what μ is, but we can use the distribution of $\bar{X}$ to test a hypothesis about μ.

Statistical hypothesis testing

- Statistical hypothesis testing evaluates whether our sample estimate is consistent with a particular parameter value in the population.
- The null hypothesis proposes what parameter value we want to test.
- Example: is μ_0 the true population mean of X? We define the null hypothesis:

$$H_0 : \mu = \mu_0 \tag{3}$$

- Example: is 67 inches the true population mean for height in the classroom?

$$H_0 : \mu_{height} = 67 \tag{4}$$

We test a null hypothesis against an alternate hypothesis

$$H_1 : \mu \neq \mu_0 \tag{5}$$

- The alternative is what must be true if the null hypothesis is false
- For example, suppose X is a coin flip
  - $X = 1$ if the coin is heads up; $X = 0$ if tails
- A natural null hypothesis: Is the coin fair?

$$H_0 : \mu = 0.5$$
$$H_1 : \mu \neq 0.5$$

- For mean classroom height:

$$H_0 : \mu_{height} = 67$$
$$H_1 : \mu_{height} \neq 67$$

To test a null hypothesis, we need a test statistic

- We have defined two test statistics

$$Z = \frac{\bar{X} - \mu}{\sigma_X / \sqrt{n}} \sim N(0,1) \qquad t = \frac{\bar{X} - \mu}{s_x / \sqrt{n}} \sim t_{n-1} \tag{6}$$

- We substitute in our null hypothesis for μ, in this case $\mu = \mu_0$:
  - Our test statistic distribution is centered on the value of μ under the null
- The value of the test statistic will depend of the value of $\bar{X}$ for a particular sample
- We use information about the distribution to calculate "If the null is true, what is the probability that we would observe the particular test statistic we obtained with this sample?"
  - For example, if the true mean classroom height is 67 inches, what is the probability we would observe $\bar{X} = 64.5$?

Goal of hypothesis testing

- We want to ask "Do we have evidence to reject the null hypothesis?"
  - Note: as in other sciences we can never "accept" the null, only reject or fail to reject
- Our test statistic is usually Normally-distributed or t-distributed
- In principle, Normally and t-distributed variables can take on *any* value, no matter how large or small
  - Could sample the n shortest students (by random chance) and get a very small $\bar{X}$, or sample the n tallest students and get a very large $\bar{X}$
  - Large (in absolute terms) values are just not very likely when drawing a random sample since the distribution of sample values should follow the population distribution
- Our standard of proof will be probabilistic: is our test statistic *sufficiently unlikely* if the null is true to reject the null hypothesis?

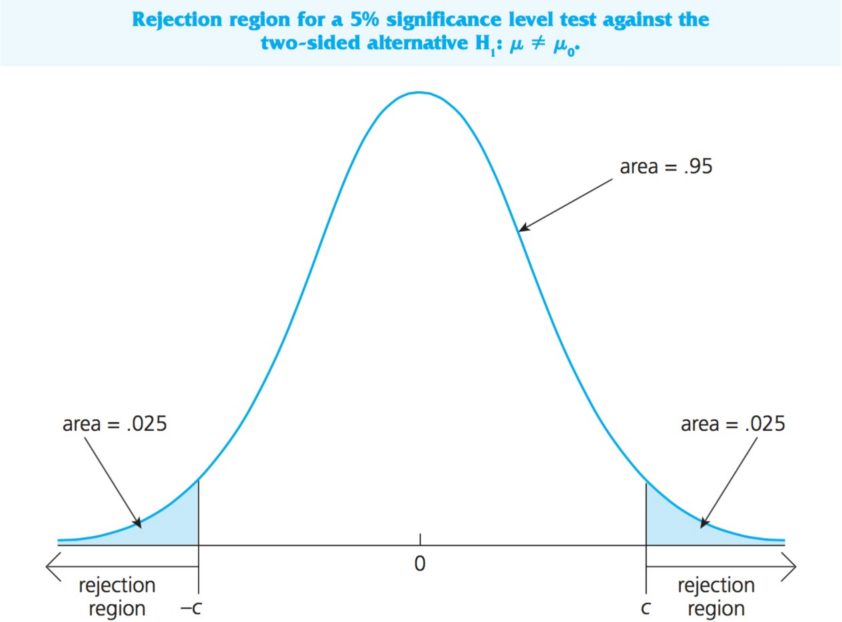
We use critical values to set a standard of 'sufficiently unlikely'

- Suppose observing a test statistic *as large in magnitude* as what we observe with a 5% probability under the null is 'sufficiently unlikely'.
- We want to identify a *critical value* c such that if H_0 is true, then $Pr(Z > c) = Pr(Z < -c) = 0.025$
- We can identify c since we know the distributions of the test statistics:

$$Z = \frac{\bar{X} - \mu}{\sigma_X / \sqrt{n}} \sim N(0, 1) \qquad t = \frac{\bar{X} - \mu}{s_x / \sqrt{n}} \sim t_{n-1} \tag{7}$$

- For a standard normal distribution, there is a 5% probability of observing $Z > 1.96$ or $Z < -1.96$
- Can find critical values for other confidence levels or for t

Visualizing *critical values*



Critical values for t and Z

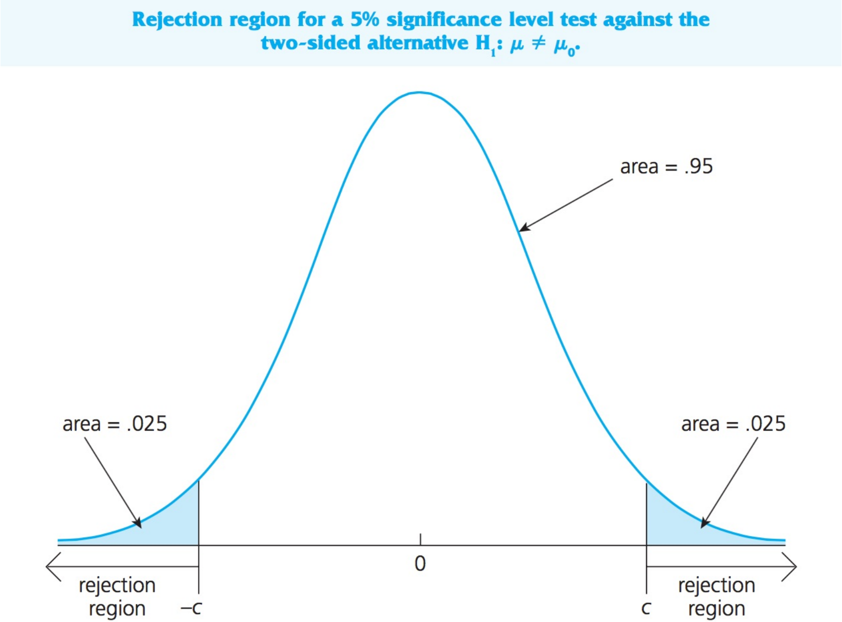
t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

5 steps to hypothesis testing

1. Determine the null and alternate hypotheses
   - Two-sided test: $H_0 : \mu = \mu_0$; $\quad H_1 : \mu \neq \mu_0$
   - One-sided test: $H_0 : \mu \leq \mu_0$ (or $\geq$); $\quad H_1 : \mu > \mu_0$ (or $<$)
2. Identify a test statistic with a known distribution (Z or t)
   - In the example of testing a population mean $t = \frac{\bar{X} - \mu_0}{s_x / \sqrt{n}}$
   - Note that the test statistic is the same for a one-sided or two-sided test
3. Identify a significance level α and critical values
   - For a 95% confidence level, the significance level is $\alpha = 1 - 0.95 = 0.05$ (or 5%)
   - Use $c_{\alpha/2}$ for a two-sided test and c_α for a one-sided test

$c_{\alpha/2}$ vs. c_{α}



5 steps to hypothesis testing

4. Set a rejection rule
   - For a two-sided test, reject if $|t| > c_{\alpha/2}$, i.e., if your test statistic is either very large or very small
   - For a $H_0 : \mu \leq \mu_0$, reject if $t > c_\alpha$, i.e., for large t
   - For a $H_0 : \mu \geq \mu_0$, reject if $t < -c_\alpha$, i.e., for small t
5. Determine the outcome of our test
   - We can either *reject the null* in favor of the alternative at a given confidence level
   - Or we *fail to reject the null*.
   - We *never* accept the null (since our test is probabilistic, not certain).

Hypothesis testing: Intuition

- We started from a population parameter that we don't know (i.e., μ).
- We said: let's pick a meaningful value of this parameter (μ_0).
- Our null hypothesis: Suppose $\mu = \mu_0$, how likely is it that we would obtain this particular sample estimate for $\bar{X}$?
- Using a test statistic with a known distribution, we can quantify *exactly* how likely that is.

Example: Coin flip

- Suppose we are betting on a coin flip.
- We observe the fraction of coin flips that are heads.
- I flip the coin 36 times, and it is heads for 32 of them.
- That *seems* unlikely if the coin is fair; can we *reject* the hypothesis that $Pr(Heads) = 0.5$?

Formalizing the coin flip hypothesis test

- Step 1: State null and alternate hypotheses

$$H_0 : \mu = 0.5$$
$$H_1 : \mu \neq 0.5$$

- Step 2: Identify a test statistic
  - Since coin flips are binary (Heads = 1, Tails = 0):

$$Z = \frac{\bar{X} - \mu}{\sigma_X / \sqrt{n}} = \frac{\bar{X} - \mu}{\sqrt{\mu(1 - \mu)/n}} \sim N(0, 1) \tag{8}$$

- Step 3: Select a confidence level and critical value
  - The 95% confidence level is the most commonly used threshold (90% and 99% are also common)
  - 95% confidence implies α=0.05%
  - This is a two-sided test, so $c_{\alpha/2} = 1.96$

Step 4: Define rejection rule

- If H_0 is true, then

$$Z_0 = \frac{\bar{X} - \mu_0}{\sqrt{\mu_0(1-\mu_0)/n}} \sim N(0,1) \tag{9}$$

- With 36 coin flips, we reject H_0 if

$$|Z_0| = |\frac{\bar{X} - 0.5}{\sqrt{0.5(1-0.5)/36}}| > c_{\frac{\alpha}{2}} = 1.96 \tag{10}$$

$$Z_0 = \frac{\bar{X} - 0.5}{0.5/6} \tag{11}$$

Step 5: Implement the test

$$
\begin{aligned}
\bar{X} &= \frac{32}{36} = 0.89 \\
Z_0 &= \frac{\bar{X} - 0.5}{0.5/6} = \frac{0.39}{0.5/6} = 4.68 \\
4.68 &> c_{\frac{\alpha}{2}} = 1.96
\end{aligned}
$$

- We reject H_0 at the 95% confidence level in favor of H_1.

We might fail to reject if the data were less extreme

- Suppose 22/36 were heads - still $> 1/2$

$$\bar{X} = \frac{22}{36} = 0.61$$
$$Z_0 = \frac{\bar{X} - 0.5}{0.5/6} = \frac{0.61}{0.5/6} = \frac{0.11}{0.5/6} = 1.32 < 1.96$$

- We would *fail to reject the null* that the coin was fair at the 95% confidence level.

Example: Student heights

- Suppose we want to test whether the mean height in the class is at least 67 inches.
- Step 1: State null and alternate hypotheses

$$H_0 : \mu \geq 67$$
$$H_1 : \mu < 67$$

- Step 2: Identify a test statistic

$$t = \frac{\bar{X} - \mu}{s_X / \sqrt{n}} \sim t_{n-1}$$

- Step 3: Select a confidence level and critical value
  - With a sample size of 25, we have 24 degrees of freedom
  - What is c at a 95% confidence level?

Find c for this test at 95% confidence level with 24 d.f.

t Table

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80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
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z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
						Confidence Level					

Example: Student heights

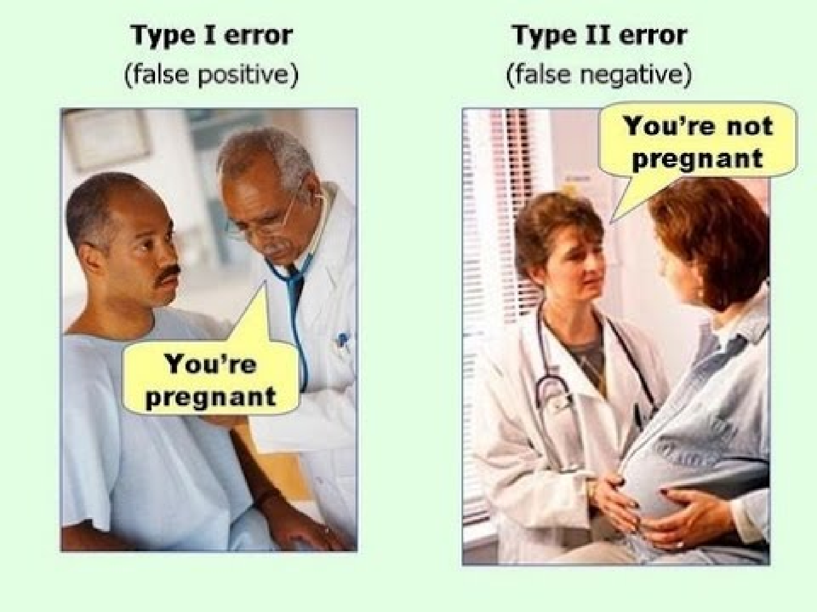
- Step 4: Define rejection rule
  - Reject H_0 if $t = \frac{\bar{X} - \mu}{s_X / \sqrt{n}} < -c_\alpha = -1.711$
- Step 5: Implement the test

To Jupyter!

2 types of errors in hypothesis testing

1. Our rejection rule is *probabilistic*: it can return errors when low-probability events occur
2. *Type 1 error*: Reject the null (H_0) when the null is true.
3. *Type 2 error*: Fail to reject the null when the null is false.

Suppose H_0 is 'you are not pregnant'

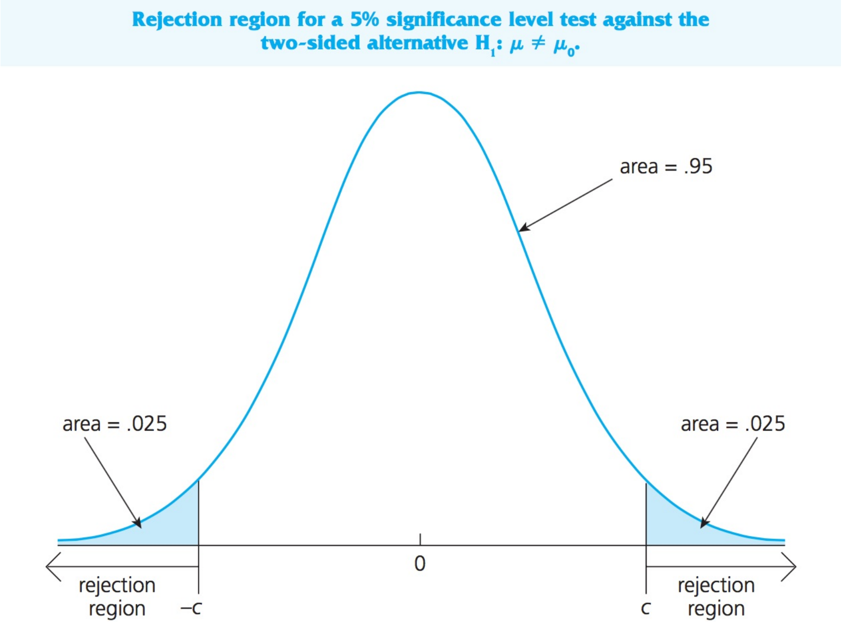


Type 1 error

- In a 2-sided test, we set critical values $c_{\alpha/2}$ where:
  - If the null is true, then there was an $\alpha/2$ probability of $t > c_{\alpha/2}$ and an $\alpha/2$ probability of $t < -c_{\alpha/2}$
- In a 1-sided test, we set critical values c_α where:
  - If the null is true, there is an α% chance of $t > c_\alpha$ (or $t < -c_\alpha$)
- In both cases, we set the probability of type 1 error $= \alpha$.
- We did not place any controls on type 2 error.
- This is because type 1 error is usually seen as more serious.

To Jupyter!

Low-probability observations *do* occur



More complicated hypotheses

- We can also test more complicated hypotheses, such as testing a difference in means
- For example, consider a project on farm irrigation in Rwanda
- The government has constructed a number of irrigation schemes (at great cost)
- A natural question: Does irrigation increase crop yields?

Irrigation context

Building Irrigation is a ton of work

Formalizing the hypothesis

- Suppose we have data on mean yields $\bar{Y}$ from a random sample of N_I farmers with irrigation and N_C farmers without.
- We want to test whether $\mu_I > \mu_C$, but we need to set up the null hypothesis as something we can reject.
- We therefore formally test

$$H_0 : \mu_I \leq \mu_C \tag{12}$$

$$H_1 : \mu_I > \mu_C \tag{13}$$

- This is a one-sided test.
- Note: testing $H_0 : \mu_I \leq \mu_C$ is equivalent to testing $H_0 : \mu_I = \mu_C$, because if we can reject that they are equal in a one-side test (in favor of $H_1 : \mu_I > \mu_C$), we can obviously reject that $\mu_I < \mu_C$.

Determining the test statistic

- We observe the difference in means between the two groups.

$$d = \bar{Y_I} - \bar{Y_C} \tag{14}$$

- Can we use this to construct a test statistic? What is the distribution of d?
- $\bar{Y_I} \sim N(\mu_I, \frac{\sigma_I^2}{N_I}) \quad \bar{Y_C} \sim N(\mu_C, \frac{\sigma_C^2}{N_C})$
- d is the sum of two normally distributed random variables.

Property of Normal variables

- If $X_1 \sim N(\mu_{X_1}, \sigma^2_{X_1}), \quad X_2 \sim N(\mu_{X_2}, \sigma^2_{X_2})$
- then $X_1 + X_2 \sim N(\mu_{X_1} + \mu_{X_2}, \sigma^2_{X_1} + \sigma^2_{X_2})$
- So $d = \bar{Y_I} - \bar{Y_C} \sim N(\mu_{Y_I} - \mu_{Y_C}, \frac{\sigma^2_I}{N_I} + \frac{\sigma^2_C}{N_C})$
- Since d has a known distribution, we can create a test statistic.

Test statistic

$$Z = \frac{\bar{Y_I} - \bar{Y_C} - (\mu_I - \mu_C)}{\sqrt{\frac{\sigma_I^2}{N_I} + \frac{\sigma_C^2}{N_C}}} \sim N(0,1) \tag{15}$$

- Of course, don't observe σ_I^2, σ_C^2, so we use a t-distribution (which has the same additive property as the Normal distribution)

$$t = \frac{\bar{Y_I} - \bar{Y_C} - (\mu_I - \mu_C)}{\sqrt{\frac{s_I^2}{N_I} + \frac{s_C^2}{N_C}}} \approx\sim t_{N_I + N_C - 2} \tag{16}$$

- (The approximation is because the d.f. isn't exactly right if the variances are different. Can calculate distribution exactly, but this will be fine for our purposes.)

Steps 3 and 4

- At a 95% confidence level for a one-sided test, we want $c_{0.05}$
  - Need to know our d.f. to identify c
- Rejection rule: under our null, $\mu_I = \mu_C$, so we reject the null if

$$t = \frac{\bar{Y}_I - \bar{Y}_C}{\sqrt{\frac{s_I^2}{N_I} + \frac{s_C^2}{N_C}}} > c_{0.05} \tag{17}$$

- Let's look at the actual data

To Jupyter!

Jupyter Results

- Our rejection rule is

$$t = \frac{\bar{Y_I} - \bar{Y_C}}{\sqrt{\frac{s_I^2}{N_I} + \frac{s_C^2}{N_C}}} > c_\alpha \tag{18}$$

- From Jupyter:
  $\bar{Y_I} = 7.55$, $\bar{Y_C} = 7.18, s_I^2 = 2.29, N_I = 1309, s_c^2 = 2.46, N_C = 778$

$$t = \frac{\bar{Y_I} - \bar{Y_C}}{\sqrt{\frac{2.29}{1309} + \frac{2.46}{778}}} = \frac{0.37}{\sqrt{0.0017 + 0.0032}} = \frac{0.37}{0.07} = 5.29 \tag{19}$$

- Is $t = 5.29 > c_\alpha$? Consult a t-table for the row with $1309 + 778 - 2 = 2085$ d.f. and column with one-tail $p = 0.05$

T-table

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%

Confidence Level

Result

- Since 5.29>1.645 we *reject the null* at a 95% confidence level
- We conclude yields under irrigation are greater than yields without irrigation