Lecture 16: More on Categorical Variables

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Agenda

1. Hypothesis testing with categorical variables
2. Categorical variables and policy analysis
3. Bad controls
4. Interactions with categorical variables
5. Chow tests

Hypothesis testing with the interactive model: practice

$$wage_i = \delta_0 + \delta_1 married_i + \delta_2 female_i + \delta_3 female_i * married_i + \beta_1 educ_i + \beta_2 exper_i + \beta_3 tenure_i + u_i$$

1. How to test if the impact of marriage on wages is different for females than non-females?
2. How to test if married females earn a different amount than single females?
3. How to test if females earn a different amount than non-females?

Hypothesis testing with the interactive model: practice

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1. How to test if the impact of marriage on wages is different for females than non-females?
   - $H_0 : \delta_3 = 0$
2. How to test if married females earn a different amount than single females?
   - $H_0 : \delta_1 + \delta_3 = 0 \; (\delta_1 + \delta_2 + \delta_3 = \delta_2)$
3. How to test if females earn a different amount than non-females?
   - $H_0 : \delta_2 = 0 \; and \; \delta_3 = 0$

Hypothesis testing with the categorical model: practice

$$\begin{aligned} wage_i = \gamma_0 + \gamma_1 marrmale_i + \gamma_2 singfem_i + \gamma_3 marrfem_i \\ + \beta_1 educ_i + \beta_2 exper_i + \beta_3 tenure_i + u_i \end{aligned}$$

Excluded category: single non-females

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2. How to test if females earn a different amount than non-females?

Hypothesis testing with the categorical model: practice

$$wage_i = \gamma_0 + \gamma_1 marrmale_i + \gamma_2 singfem_i + \gamma_3 marrfem_i$$
$$+ \beta_1 educ_i + \beta_2 exper_i + \beta_3 tenure_i + u_i$$

Excluded category: single non-females

1. How to test if the impact of marriage on wages is different for females than non-females?
   - $H_0 : \gamma_3 - \gamma_2 = \gamma_1$
2. How to test if females earn a different amount than non-females?
   - $H_0 : \gamma_2 = 0$ *and* $\gamma_3 - \gamma_1 = 0$

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Categorical variables and policy analysis

It is often useful to test whether groups experience different outcomes. Examples:

1. Test for discrimination: do differences in mean wages between females and non-females (the female wage penalty) reflects discrimination or other underlying differences.
   - If underlying differences (e.g. in education or childcare responsibilities), target those. If discrimination, enact anti-discrimination policy.
2. Test for program impacts: do mean outcomes differ between treatment and control units?
   - If yes, program has an impact! May be relevant for policy.
3. Test for heterogeneity: does the impact of a treatment differ by level of education?
   - If yes, implementation should account for this.

Example: the female wage penalty

- May want to test whether differences in mean wages between females and non-females (the female wage penalty) reflects discrimination or other underlying differences.
- Could estimate

$$log(wage_i) = \beta_0 + \beta_1 female_i + \beta_2 educ_i + \beta_3 exper_i + \beta_4 tenure_i + u_i \quad (1)$$

- Possible test for gendered wage discrimination is $H_0 : \beta_1 = 0$ against $H_1 : \beta_1 \neq 0$.
- In policy analysis, will often want to go farther.

What about potential omitted variables?

- Wage differences by sex could be due to differences in other factors: omitted variable concerns.
- If so, could have $\hat{\delta}_1 \neq 0$ even when $\delta_1 = 0$.
- Solution: test the sensitivity of $\hat{\delta}_1$ to additional controls.
- For example, what if wage differences result from different preferred occupations?

$$\begin{aligned} log(wage_i) = \beta_0 + \beta_1 female_i + \beta_2 educ_i + \beta_3 exper_i + \beta_4 tenure_i \\ + \beta_5 profocc_i + \beta_6 services_i + u_i \end{aligned} \tag{2}$$

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Concern about "bad controls"

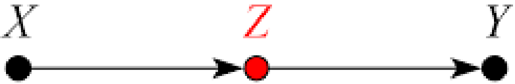
- Occupational choice matters for wages, but doesn't account for a large share of the female wage penalty.
  - β_1 changes from -0.30 without occupation controls to -0.27.
  - Could say occupational sorting accounts for around 10% of the gender wage gap.

Concern about "bad controls"

- Occupational choice matters for wages, but doesn't account for a large share of the female wage penalty.
  - β_1 changes from -0.30 without occupation controls to -0.27.
  - Could say occupational sorting accounts for around 10% of the gender wage gap.
- But what if occupational sorting is not an independent choice? What if discrimination leads women to choose certain occupations?
- In this case we have a concern about "bad controls."
  - If part of the effect of discrimination is through job choice, then including those controls prevents us from estimating the full effect of discrimination.
  - They are "bad controls" in the sense that *Female* has a causal effect on them.
- Including controls removes possible sources of omitted variable bias, but also removes certain pathways through which *Female* might affect wages.

Bad controls

On the board: illustrating "bad control" relationships.



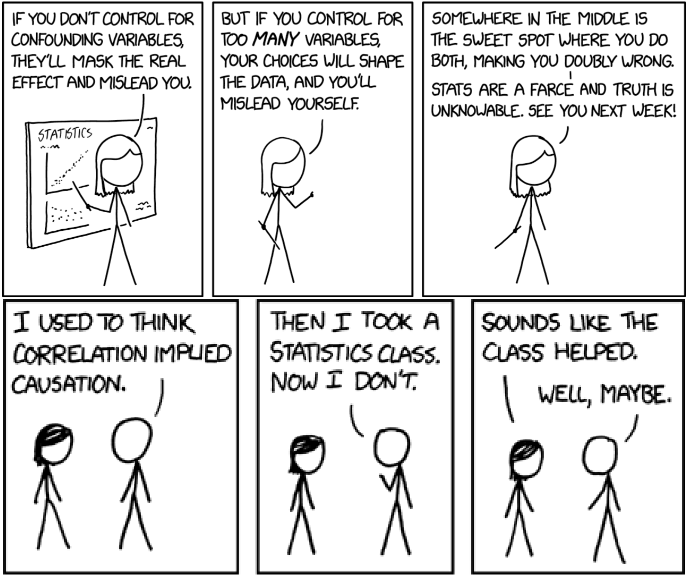
(11)

Goal: block spurious paths between X and Y, but don't perturb causal paths between them.

What to do about bad controls?

- Suppose we want to identify the effect of discrimination in the female wage penalty.
- Run a regression *without* controls.
  - Estimate the effect of being female on wages across all potential pathways.
  - Discuss concerns about likely sources of OVB and directions of bias.
- Then run regression(s) *with* controls.
  - Removes some possible sources of OVB.
  - Identifies residual effect of being female through pathways other than what you've controlled for. Discuss if any controls could be shutting off some mechanisms/pathways.
  - Discuss MLR interpretation of *Female* coefficient.
- Takeaway: be careful about how independent variables might be related and how interpretation changes with controls.
  - Be especially careful of whether one variable causally affects another.

Break



Source: xkcd

Deeper policy analysis: *why* do we observe this wage penalty

- Earlier, we looked at the marriage premium for wages.

$$
\begin{aligned}
log(wage_i) = \delta_0 + \delta_1 married_i + \delta_2 female_i + \delta_3 female_i * married_i + \\
\beta_1 educ_i + \beta_2 exper_i + \beta_3 tenure_i + u_i
\end{aligned} \quad (3)
$$

- We find $\hat{\delta_1} = 0.292$ $(p = 0.09)$, $\hat{\delta_2} = -0.097$ $(p < 0.01)$, and $\hat{\delta_3} = -0.316$ $(p < 0.01)$.
- How do we interpret δ_3?

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- How do we interpret δ_3?
- For women, the marriage "premium" increases hourly wages by 31.6% less than for men (making it negative overall), all else equal.
- Why? Research suggests marriage is associated with stability and responsibility for men, but concerns about availability (due to pregnancy, childcare, and the possibility of dropping out of the workforce) for women.
- This seems to explain a large share of the female wage penalty.

Other explanations for wage penalty

- We can also evaluate whether females and non-females have different returns to other attributes.
- What if females and non-females face different returns to education?
- How do we separate these?

Interaction terms help us explore differences by sex

$$log(wage_i) = \beta_0 + \delta_0 female_i + \beta_2 educ_i + \delta_1 female_i * educ_i + u_i \quad (4)$$

- $\Delta log(wage) = (\beta_2 + \delta_1 female)\Delta educ$
- For non-females

$$\frac{\Delta log(wage)}{\Delta educ} = \beta_2 \quad (5)$$

- While for females, the slope is different by δ_1

$$\frac{\Delta log(wage)}{\Delta educ} = \beta_2 + \delta_1 \quad (6)$$

- Graph on board

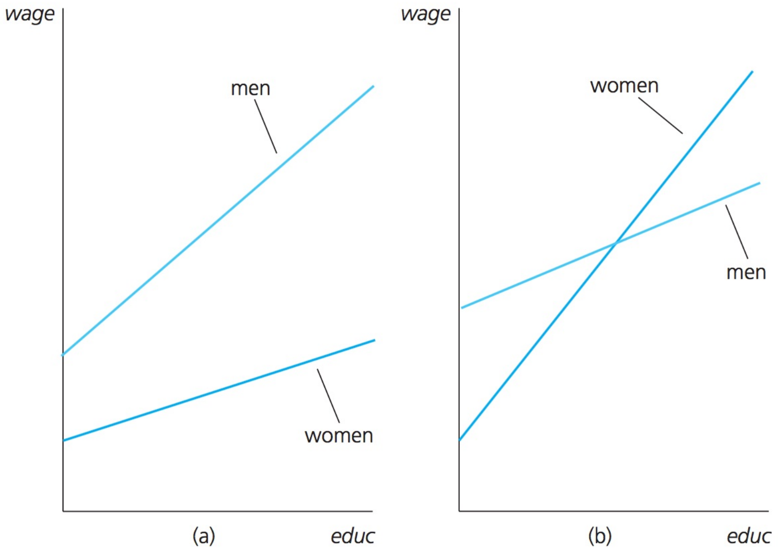
Interpretations on gender gaps

$$log(wage_i) = \beta_0 + \delta_0 female_i + \beta_2 educ_i + \delta_1 female_i * educ_i + u_i \quad (7)$$

$$\begin{aligned}
E[log(wage)|female = 1, educ] &= \beta_0 + \delta_0 + \beta_2 educ_i + \delta_1 educ_i \\
E[log(wage)|female = 0, educ] &= \beta_0 + \beta_2 educ_i \\
E[log(wage)|female = 1, educ] &- E[log(wage)|female = 0, educ] \\
&= \delta_0 + \delta_1 educ
\end{aligned}$$

- So the gender gap depends on education (if $\delta_1 \neq 0$).
    - Helpful to evaluate at the median.
    - In these data, $p50(Educ) = 12$.
    - So at the median value of education, the gender wage gap = $\delta_0 + 12 * \delta_1$

Graphs of equation (7.16): (a) $\delta_0 < 0$, $\delta_1 < 0$; (b) $\delta_0 < 0$, $\delta_1 > 0$.



Does the gender gap widen or close with education?

$$log(wage_i) = \beta_0 + \delta_0 female_i + \beta_2 educ_i + \delta_1 female_i * educ_i + u_i \quad (8)$$

- We've already shown $\delta_0 < 0$.
- Changes in gender gap with education depends on the sign of δ_1.

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Interpretation

- In these data there is a large and robust difference in wages between genders.
- It doesn't appear strongly related to differences in the returns to education.
  - Note that there could be differences in the returns to education even if there were *no* average difference in wages.
- Can we test whether there are *any* important differences in returns between genders?

Testing for any differences 1: many interactions

$$log(wage_i) = \beta_0 + \delta_1 female_i + \beta_2 educ_i + \beta_3 exper_i + \beta_4 tenure_i \tag{9}$$
$$+\delta_2 female_i * educ_i + \delta_3 female_i * exper_i + \delta_4 female_i * tenure_i + u_i$$

- How to test whether there are no differences by gender?

Testing for any differences 1: many interactions

$$
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+\delta_2 female_i * educ_i + \delta_3 female_i * exper_i + \delta_4 female_i * tenure_i + u_i
\end{aligned} \tag{9}
$$

- How to test whether there are no differences by gender?
- Run an F test for $H_0 : \delta_1, \delta_2, \delta_3, \delta_4 = 0$.
- Tests whether any of the parameters differs by gender.

Testing for any differences 2: Chow test

A way to do the same test without specifying all the interaction terms.

1. Take the full (P=pooled) sample. Estimate

$$log(wage_i) = \beta_0 + \beta_1 educ_i + \beta_2 exper_i + \beta_3 tenure_i + u_i \qquad (10)$$

- This is your "restricted" model: no differences by gender.

2. Run the same regression twice more.
   - First on the subsample $female_i = 0$ (F).
   - Then on the subsample $female_i = 1$ (NF).
   - These are your "unrestricted" regressions: allow the parameters to vary across models (by *female*).
3. Test whether these separate subsample analyses do a significantly better job explaining the variation in wages than the pooled model.
   - Observe that $SSR_U = SSR_F + SSR_N F$ (Chow's insight).

Chow test

- Run an F-test, modifying the test statistics as follows:

$$F = \frac{\frac{SSR_r - SSR_u}{q}}{\frac{SSR_u}{n-k-1}} = \frac{\frac{SSR_P - (SSR_F + SSR_{NF})}{k+1}}{\frac{SSR_F + SSR_{NF}}{n-2(k+1)}} \tag{11}$$

- The hypothesis that the βs are the same across subsamples involved $q = k + 1$ restrictions.
- The unrestricted model has $n - 2(k + 1)$ degrees of freedom because each subsample estimates $k + 1$ parameters.

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- Interpretation: $F = 18.8$, $p < 0.001$: strongly reject that there are no differences in returns between genders.
- Even though the interaction term is only significant for the interaction with experience at a 10% level.