CBPV course: exercises

P. B. Levy

University of Birmingham

1. Give introduction rules, elimination rules, β-laws and η-laws for the types $\boxed{\sum}\{^0 A, A';\ ^1 B, B', B''\}$ and $\boxed{\prod}\{^0 A, A' \vdash B;\ ^1 C, C', C' \vdash D\}$.
2. For these connectives, give reversible rules. Define the up-operation and down-operation, and show that each inverse is identity modulo $=_{\beta\eta}$.
3. Take the term

   $\mathtt{f} : (\mathtt{int}+\mathtt{bool}) \to (\mathtt{int}+\mathtt{bool}) \vdash \mathtt{f} : (\mathtt{int}+\mathtt{bool}) \to (\mathtt{int}+\mathtt{bool})$

   Apply an η-expansion for $\to$, then for $+$, then for $\mathtt{bool}$.
4. Suppose that $\Gamma \vdash M : \mathtt{bool}$ and $\Gamma \vdash N_0, N_1, N_2, N_3 : C$. Show that

   $$\begin{array}{l}\Gamma \vdash \mathtt{case}\ M\ \mathtt{of}\ \{\\ \quad \mathtt{true}.\ \mathtt{case}\ M\ \mathtt{of}\ \{\mathtt{true}.N_0, \mathtt{false}.N_1\},\\ \quad \mathtt{false}.\ \mathtt{case}\ M\ \mathtt{of}\ \{\mathtt{true}.N_2, \mathtt{false}.N_3\}\\ \}\\ =_{\beta\eta} \mathtt{case}\ M\ \mathtt{of}\ \{\mathtt{true}.N_0, \mathtt{false}.N_3\} : C\end{array}$$
5. Show that $\mathtt{inl}\ -$ is injective, i.e. if $\Gamma \vdash M, M' : A$ and $\Gamma \vdash \mathtt{inl}\ M =_{\beta\eta} \mathtt{inl}\ M' : A + B$ then $\Gamma \vdash M =_{\beta\eta} M' : A$.
6. Write down the η-law for the 0 type.
7. A typing context Γ is *inconsistent* if there is a term $\Gamma \vdash M : 0$. Show that if Γ is inconsistent then for every type A there is a unique (up to $=_{\beta\eta}$) term $\Gamma \vdash N : A$.
8. Given a term $\Gamma, \mathtt{x} : A \vdash M : 0$, show that it is an "isomorphism" in the sense that there is a term $\Gamma, \mathtt{y} : 0 \vdash N : A$ satisfying

   $$\Gamma, \mathtt{y} : 0 \vdash M[N/\mathtt{x}] =_{\beta\eta} \mathtt{y} : 0$$
   $$\Gamma, \mathtt{x} : A \vdash N[M/\mathtt{y}] =_{\beta\eta} \mathtt{x} : A$$
9. In CBV jumbo λ-calculus with errors, and (separately) with printing, give an interpreter and big-step semantics and denotational semantics for the term constructors.
10. Deduce the semantics of pattern-match and project product.
11. Translate these term constructors into fine-grain CBV.