Problem set 6

Due: 12/13/2013

Problem 1 [1]

Consider the following maximization problem

$$\max_{x,y} xy$$

$$s.t \;\; x^2 + y^2 \leq 1$$

a) Form the Lagrangian

b) Write out the first order conditions

c) Solve for the pair(s) (x, y) that solve the problem

Problem 2 [2]

Find the maximum value of the function

$$r(a, b) = 2a^2 + b^2$$

subject to the constraints

$$2a + b \leq 9$$

$$a^2 + b^2 \geq 16$$

Problem 3

What is the probability of a rolling pair of dice and getting a sum of 3?

[1] Taken from *Carl P. Simon and Lawrence E. Blume textbook (Mathematics for Economists)*

[2] Taken from *Michael W. Klein textbook (Mathematical Methods for Economics)*

Problem 4

Three people, A, B, and C, are running for the same office, and we assume that one and only one of them wins. The sample space may be taken as the 3-element set $\Omega = \{A, B, C\}$ where each element corresponds to the outcome of that candidate's winning. Suppose that A and B have the same chance of winning, but that C has only 1/2 the chance of A or B. Determine the probability that each individual has to win.

Problem 5

Suppose you roll a die 3 times. What is the probability of rolling a 2 in the third trial if you rolled 5 and 6 in the first and second trial respectively? Why?

Problem 6

Suppose you randomly and sequentially select two cards from a deck.

a. What is the probability that the two cards are Heart if you select the cards with replacement (you select the first card, you put it back to the deck and then you select second card)

b. What is the probability that the two cards are Heart if you select the cards with no replacement (you select the first and then the second without replacing the first)?

c. What is the probability that the second card is Red given that the first is Black and the selection is done with replacement?

d. What is the probability that the second card is Red given that the first is Black and the selection is done with no replacement? Compare this result with the if there is replacement. Explain.

Problem 7

Suppose you flip a coin 3 times.

a. What is the probability that Face turns up exactly three times?

b. What is the probability that Face turns up exactly twice?

c. What is the probability that at least one Face turns up?

Problem 8

A hospital has two rooms for surgery. The probability that a given room is free is 0.5. Show your work in answering the following questions:

1. What is the probability that both rooms are free?
2. What is the probability that both rooms are occupied?
3. What is the probability that at least one room is free?

Problem 9

Among the students registered in Econ 999, 30% are social science majors, 20% are business majors, and the remaining 50% are other majors. Among each of these groups, 40% are first year students, 30% are second year students and the remaining 30% are upper year students.

1. Define which events are mutually exclusive.
2. What is the probability that a randomly chosen student is a second year business major?
3. What is the probability that a randomly chose student is either a second year, or a business major (he/she could be both)?

Problem 10

A student must choose exactly two out of three electives: Art(A), French(F), and Mathematics(M). He chooses art with probability 5/8, French with probability 5/8, and Art and French together with probability 1/4.

a) What is the probability that he chooses Mathematics?

b) What is the probability that he chooses Mathematics and French?

c) What is the probability that he chooses either Art or French?