Politecnico di Milano

Master of Science in
Space Engineering

Orbital Mechanics Project

Group 33

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Nomenclature

a	semi-major axis
e	eccentricity
f	true anomaly
i	inclination
Ω	Right ascension of the ascending node
ω	periapsis anomaly

Contents

Assignment 1: Interplanetary Explorer Mission

Mission requirements

In this report different transfer strategies between the planets Neptune and Mercury are analysed. Such strategies are constrained by two time limits: earliest departure from Neptune 1^{st} January 2031 and latest arrival at Mercury 1^{st} February 2071. Furthermore, they must include a flyby of the planet Venus. Once possible transfers are identified, the least expensive in terms of required is selected.

Mission analysis outputs

2.1 Design process

2.1.1 Initial choice for the time windows

2.1.2 Additional constraints considered

2.1.3 Transfer options exploration, analysis and comparison

2.1.4 Selection of the final solution

2.2 Final solution

2.2.1 Heliocentric trajectory

(The transfer is studied by placing the system of reference at the centre of the Sun.) The final transfer strategy is composed of four arcs: the first and the last one lie inside the Sun's sphere of influence; the second and third one are inside Venus' sphere of influence (and are analysed in a later paragraph). The transfer is executed between the times 00:00:36 on 01/02/2031 (dd/mm/yyyy) and 12:43:52 on 05/07/2057. The powered flyby is performed at 08:29:31 on 02/07/2050. Figure with complete strategy The first arc connects Neptune and Venus orbits. It is characterised by the following data:

a [km]	**e [-]**	**i [rad]**	**RAAN [rad]**	**[rad]**
$2.7167 \cdot 10^9$	0.9738	0.0267	1.7574	1.4913

The initial position is characterised by a true anomaly (f) of 3.2451 rad (, which means that the probe enters its orbit around the sun after the aphelion). In Cartesian coordinates the initial point is identified by the following data:

$\mathbf{R}_x$ [km]	$\mathbf{R}_y$ [km]	$\mathbf{R}_z$ [km]
$43.712 \cdot 10^8$	$9.348 \cdot 10^8$	$-1.193 \cdot 10^8$
$\mathbf{v}_x$ [km/s]	$\mathbf{v}_y$ [km/s]	$\mathbf{v}_z$ [km/s]
-3.2267	0.2969	0.0832

The initial position's distance from the Sun is $4.4716{\cdot}10^8$ km and the velocity at the initial point is 3.2414 km/s. The fourth arc connects the point where the spacecraft exits the Venus' sphere of influence with Mercury.

2.2.2 Powered gravity assist

2.2.3 Cost of the mission

Assignment 2: Planetary Explorer Mission

Mission requirements

In this chapter we are going to explain how we have designed a planetary explorer mission to perform Earth observation: we were in fact required to analyse the Earth-centred orbit characterised by the values in the table below and estimate its ground track, studying the effects of the assigned orbit perturbations by integrating both Gauß's planetary equations and Cartesian equations and subsequently comparing the results of the two methods. After the characterisation of the ground track, we propose a modification of the orbit aimed to get a ground track which repeats itself once a sidereal day.

a [$10^4 km$]	**e**	**i** [deg]	**hp** [km]
4.0718	0.6177	78.2195	15566.491
Repeating GT ratio k:m^{note}	**Perturbations**	**Parameters**	
1:1	J2 and SRP	cR = 1.2	A/m = 4.000 m^2/kg

Note: the groun-track repeats itself **k** times every **m** revolutions of the planet.

Table 3.1: Mission requirements

Mission analysis outputs

4.1 Nominal orbit

4.2 Ground track

4.3 Orbit Perturbations

In our model we included perturbations due to two effects:

- **Second Zonal Harmonic *J2***

  Models the Earth Oblateness using the spherical geopotentials model, truncated to the second term.

- **Solar Radiation Pressure *SRP***

  A force given by the impact of momentum-carrying photons on the surfaces of the spacecraft. For this perturbation we have written a function that, based on the initial date gets the position vector of the Earth from the ephemerides function and uses that together with the position vector of the spacecraft with respect to the Earth to calculate the direction of the disturbing acceleration. A simple algorithm for the eclipse condition found in [4] has been implemented.

$$\mathbf{a_{SRP}} = -P_{SR@1AU} \frac{AU^2}{||\mathbf{r}_{sc-Sun}||^3} c_R \frac{A_{Sun}}{m} \mathbf{r}_{sc-Sun} \tag{4.1}$$

These forces give an acceleration to the spacecraft so that the total acceleration it perceives in orbit is

$$\mathbf{a} = -\frac{\mu_{\oplus}}{r^3}\mathbf{r} + \mathbf{a}_{SRP} + \mathbf{a}_{J2} \tag{4.2}$$

4.4 Orbit Propagation

4.4.1 Methods

To propagate the orbit we used two different methods:

- **Gauss Planetary Equations**

$$\begin{aligned}
\frac{da}{dt} &= \frac{2a^2}{h}\Big(e\sin f\ a_r + \frac{p}{r}a_s\Big)\\
\frac{de}{dt} &= \frac{1}{h}\Big(p\sin(f)\ a_r + \Big((p+r)\cos f + re\Big)a_s\Big)\\
\frac{di}{dt} &= \frac{r\cos(f+\omega)}{h}a_w\\
\frac{d\Omega}{dt} &= \frac{r\sin(f+\omega)}{h\sin i}a_w\\
\frac{d\omega}{dt} &= \frac{1}{he}\Big(\cos f\ a_r + (p+r)\sin f\ a_s\Big) - \frac{r\sin(f+\omega)\cos i}{h\sin i}a_w\\
\frac{df}{dt} &= \frac{h}{r^2} + \frac{1}{eh}\Big(p\cos f\ a_r - (p+r)\sin f\ a_s
\end{aligned}$$

  We numerically integrate these equations with the ode113 solver, by using them to set the derivatives of the state. The reference frame for this equations is the RSW frame so a_r, a_s, a_w are respectively the radial, transversal and out-of-plane components. [1] [2] [3]

- **Numeric integration of cartesian equations** This method consists in directly integrating the Cartesian equations of motion

$$\ddot{\mathbf{r}} = -\frac{\mu_\oplus}{r^3}\mathbf{r} + \sum a_p \tag{4.3}$$

4.4.2 Comparison

There is a marked difference in the computational time between the two methods as the Gaussian elements propagation for a timespan of 50 T_{orbit} has a computation time $t_{GAUSS} = 5.4660 * 10^{-4}s$ that is much faster than the Cartesian $t_{CART} = 62.2s$ one.

There are numerical error introducing differences between the two methods, but the error only grows up to an order of 10^{-5} for a propagation of 200 periods.

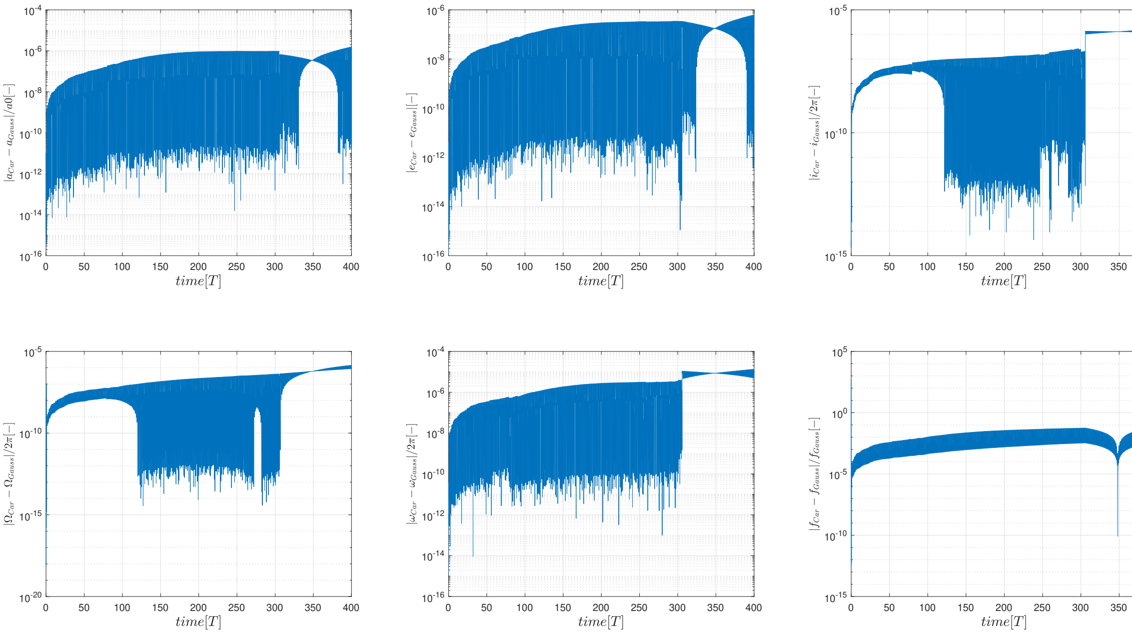

Figure 4.1: Evolution of the errors

4.5 History of the Keplerian elements

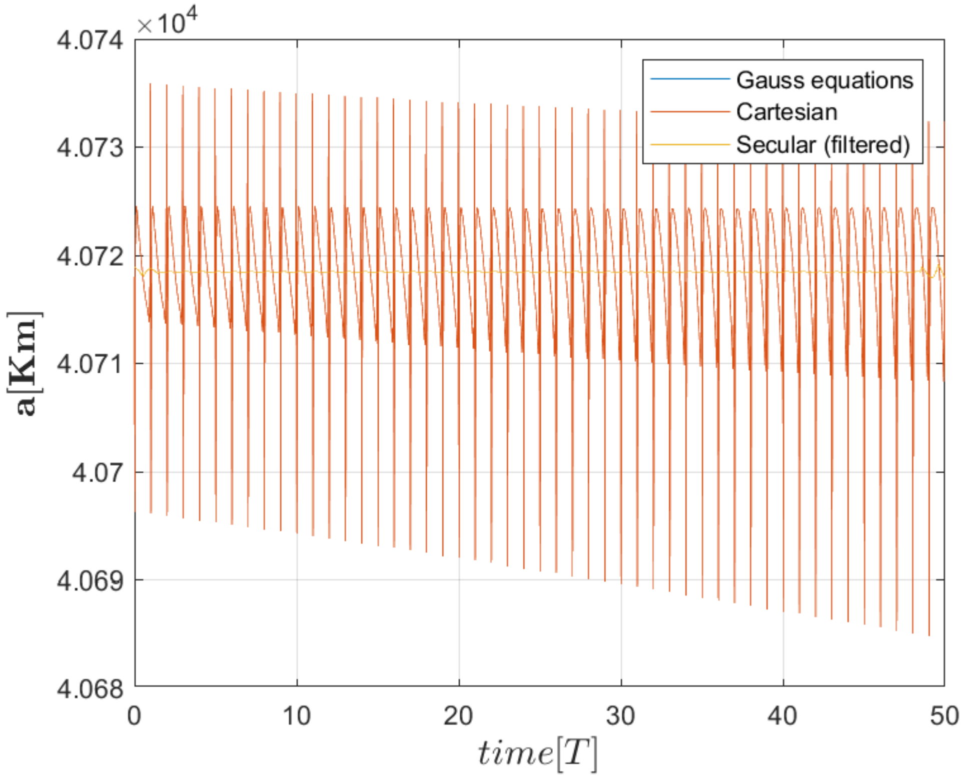

Figure 4.2: Evolution of the semi-major axis

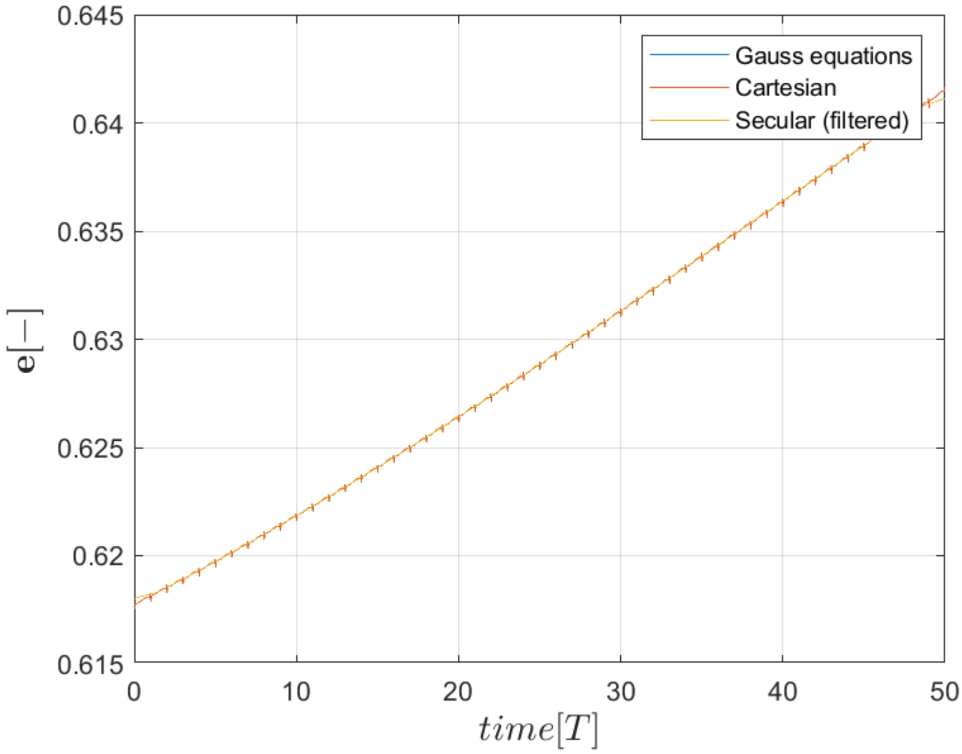

Figure 4.3: Evolution of the eccentricity

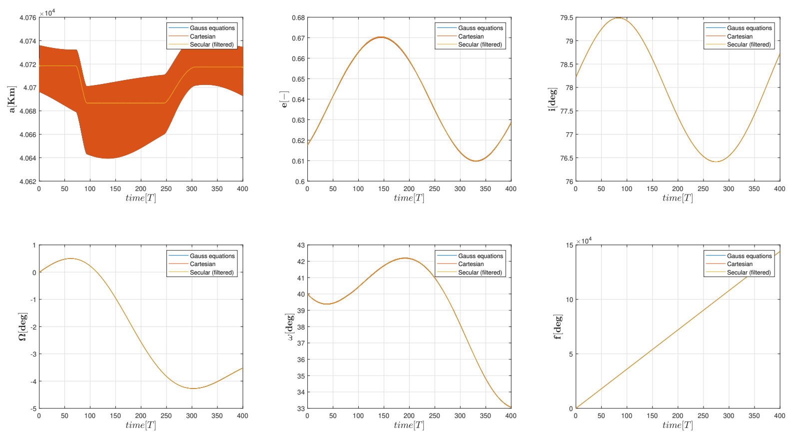

Figure 4.4: Evolution of keplerian elements

4.6 Orbit evolution representation

4.7 HF filtering

For the filtering we have used the simple *movmean* function integrated in MATLAB which computes the average at each time instant between the current point and the neighboring ones with a cutoff period of $3T_{orbit}$.

We can see in the *secular evolution* that there is a superposition of effects from the J2 perturbation and the SRP:

- $\dot{\Omega}$: there is a *nodal regression* due to the inclination i being less than $90°$
- $\dot{\omega}$: there is a negative *perigee precession* due to the inclination $i > 63.4°$ giving a so-called *"Hula-hooping effect"*
- $\dot{e}$: there are periodic oscillations with a slight secular decrease

4.8 Comparison with real data

4.8.1 Satellite selection

4.8.2 Comparison with our model

Bibliography

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[3] Battin, R, *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA Education Series, 1999. Chapter 10

[4] Curtis,H.D. *Orbital mechanics for engineering students.* Butterworth-Heinemann , 2019. Chapter 10.9