Politecnico di Milano

Master of Science in
Space Engineering

Orbital Mechanics Project

Group 33

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Nomenclature

Δv_{GA}	cost of the hyperbola perigee burn manoeuvre
$\mu_{\odot}$	Gravitational Parameter of the Sun
$\mu_{\oplus}$	Gravitational Parameter of Earth
Ω	Right ascension of the ascending node
ω	periapsis anomaly
$\mathbf{V_T^i}$	velocity vector of transfer arc i
a	semi-major axis
AU	Astronomical Unit ($1.495978707 10^8$ km)
e	eccentricity
f	true anomaly
i	inclination
p	semi-latus rectum
r	radius
ToF_{ij}	Parabolic time of flight from i to j
h	specific angular momentum per unit mass
mjd2000	modified julian date 2000

Contents

Assignment 1: Interplanetary Explorer Mission

Introduction

The aim of this assignment is to design an interplanetary trajectory from Neptune to Mercury with a flyby of Venus. The mission is to be completed between 1 February 2031 and 1 February 2071.

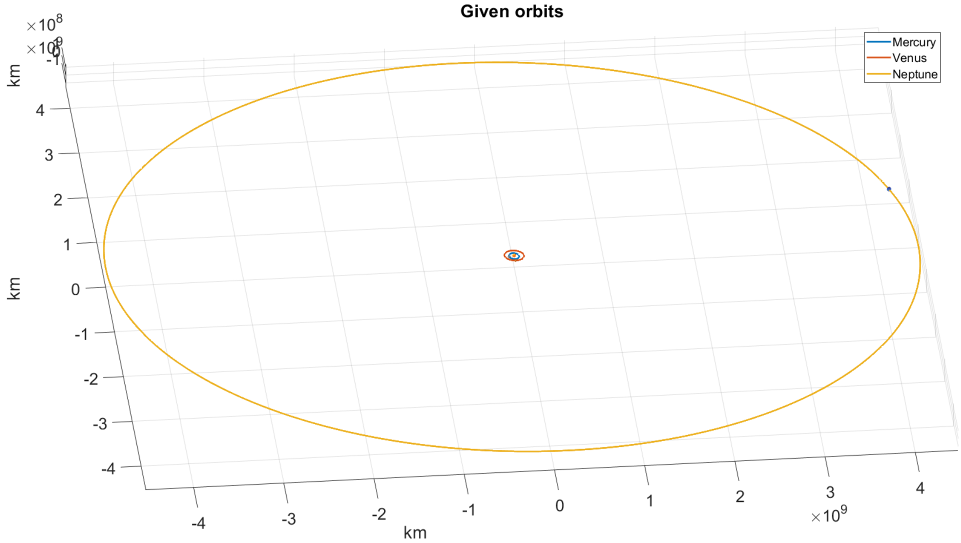


Figure 1.1: Given orbits

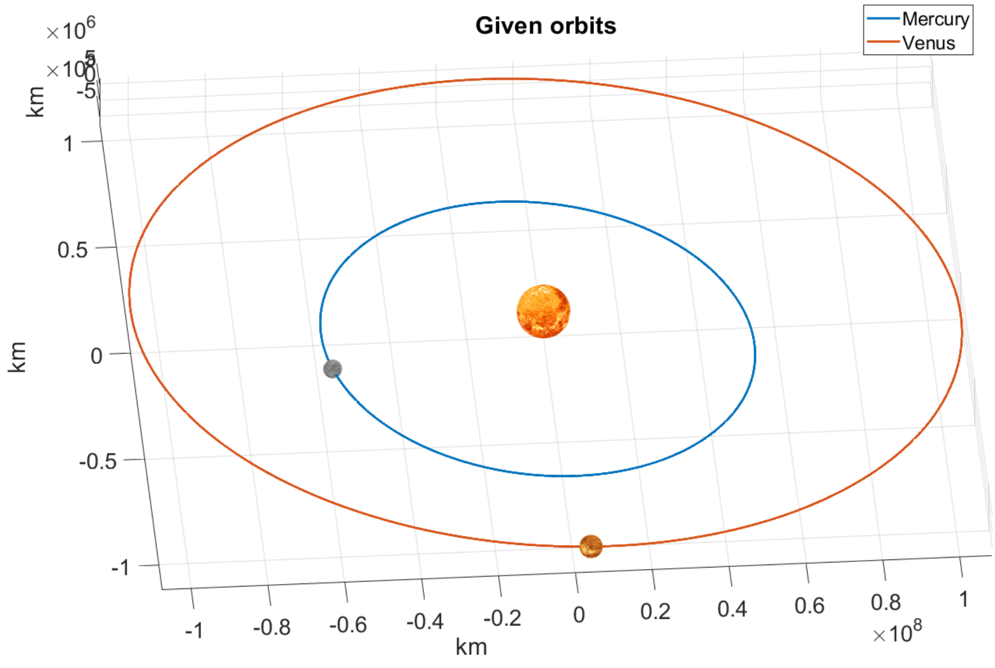


Figure 1.2: Given orbits close-up

The design process requires the following consecutive steps:

1. Define the time windows for possible transfer phases (departure, gravity assist and arrival);
2. Set known parameters and compute the orbits of the three planets of interest;
3. Set and solve the Lambert's problem between first Neptune and Venus and then Venus and Mercury at different times inside the possible time windows;
4. Evaluate the powered gravity assist for every trajectory previously computed;
5. Selection of the most efficient transfer strategy.

1.1 Method of patched conics

The movement of a body inside the interplanetary space cannot be described by the two-body problem since the body itself is subject to the influence of different celestial bodies at once. In order to study such movement, an approximation is introduced with the method of the patched conics. With this approximation, the influence of a celestial body is limited to a sphere named Sphere of Influence (SOI from now on) which radius is calculated using the following equation:

$$R_{SOI} = R_{CB} \left(\frac{m_{CB}}{m_{\odot}} \right)^{\frac{2}{5}}$$

Where R_{CB} is the distance between the celestial body and the sun, m_{CB} is the mass of the central body and $m_{\odot}$ is the mass of the sun.

If the body is outside any celestial body's SOI, then it is considered to be subject only to the Sun's gravity attraction.

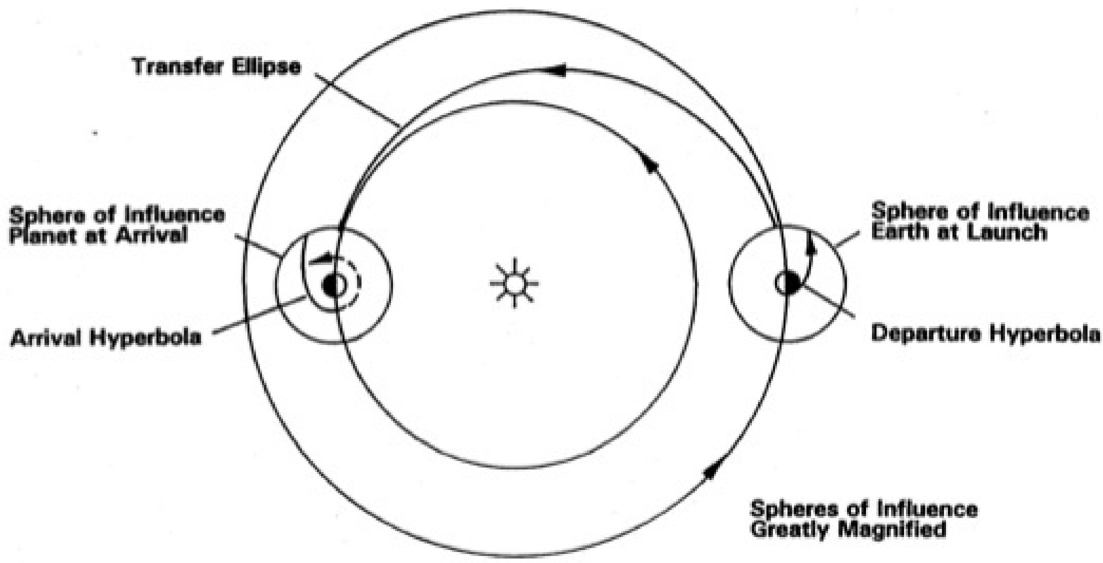


Figure 1.3: Patched conics

In this report, the spacecraft is considered to be already outside of Neptune's SOI at departure and it does not enter Mercury's SOI at arrival. The spacecraft only enters Venus' SOI during the flyby. Therefore, the cost of the mission is only due to the following burns: powered flyby, transfer trajectory insertion and planetary rendezvous burn.

Mission analysis

1.2 Design process

Time windows

The Lambert's problem solution is a section of an elliptical orbit. Furthermore, the minimum time needed to complete a Lambert transfer is the time required to run across an elliptical orbit with eccentricity $e \to 1$, which therefore is a parabola.

Consequently, given the parabolic times of flight from Neptune to Venus and from Venus to Mercury, it is possible to define the earliest possible date of flyby and arrival starting from the departure date. In particular the earliest possible flyby and arrival dates can be defined as follows:

$$Fly_{min} = Dep_{min} + ToF_{NV}$$

$$Arr_{min} = Fly_{min} + ToF_{VM}$$

Where ToF_{ij} is the parabolic time of flight between the planets i and j. Similarly, given the latest arrival date, it is possible to define the latest flyby and departure dates:

$$Fly_{max} = Arr_{max} - ToF_{VM}$$

$$Dep_{max} = Fly_{max} - ToF_{NV}$$

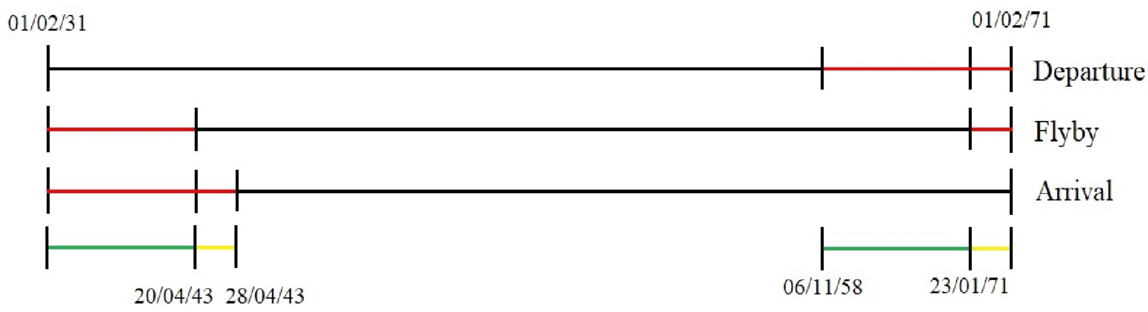


Figure 1.4: time windows

Said dates are:

Fly_{min}	20-04-2043
Arr_{min}	28-04-2043
Dep_{max}	06-11-2058
Fly_{max}	23-01-2071

Additional constraints considered

One additional constraint that was to be considered is the minimum altitude of the pericentre for the planetocentric hyperbola at Venus flyby. We chose this as 300 km, giving us a minimum radius of pericentre of 6351.8 km. For the MATLAB *fmincon* optimisation function this was set using an additional constraint function and also time constraints based on the parabolic time of flights were set using the inequality matrices and simple checks in the coarse grid search.

Transfer options exploration, analysis and comparison

For the selection of an optimum transfer we employed different strategies together. To analyse the solutions we built an objective function giving us the ΔV needed for the transfer given the *arrival*, *gravity assist* and *departure*. The function computes the two arcs connecting planets at departure and flyby and the flyby planet to the arrival planet. The outputs consist of the total ΔV required, then the ΔV's for the separate legs and the information for each transfer arc as an object with various fields useful for the rest of the analysis and plotting.

```
compute and store first arc
compute and store second arc
join them with flyby
output total DV
```

Listing 1.1: GAtransfer objective function structure

Coarse grid search The first step is a coarse grid search using three nested for loops:

```
parfor i = 1 :dep_size
    for j = 1:ga_size
        tof_1 = gravityassist(j) - departure(i);
        for k = 1:arr_size
            tof_2 = arrival(k) - gravityassist(j);
            %% Set NaN for TOFs<parabolic TOF and rp < rpmin
            if  tof_2 < t_p_VM || tof_1 < t_p_NV
                DV(i,j,k) = NaN;

            else
                [DV(i,j,k), ~, ~, ~, FLYBY,~,~]= GAtransfer(Neptune,Venus,Mercury,departure(i),gravityassist(j),arrival(k));
                if FLYBY.rp < rpmin
                    DV(i,j,k) = 1000000;       % Set an impossibly high value
                end

            end
        end
     end
end

[dv_min_grid,loc] = min(DV(:));
[ii,jj,kk] = ind2sub(size(DV),loc);

x_GRID = [departure(ii); gravityassist(jj); arrival(kk)];
```

Listing 1.2: coarse grid search

For these loops we employed a *parfor* outer loop to take advantage of the parallel processing capabilities of MATLAB, using all the available CPU cores. This allowed us to speed up the grid search substantially, now taking only 247s with a size of $2.7e6$ elements.

fmincon refinement The objective function *GAtransfer* can also be used for the *fmincon* MATLAB optimisation function, which finds the minimum for a constrained nonlinear function using a gradient-based optimiser. This function takes as initial guess the minimum point found from the coarse grid search as a vector of 3 mjd2000 dates, the linear constraint on the windows with two **ub** and **lb** vectors, the time of flight constraints with the **A** and **b** inequality matrices and the *nonlinear constraint* on the minimum pericentre radius of the hyperbola as a function flyby_CONSTR

Other strategies explored We also explored using the *genetic algorithm* ("*ga*" MATLAB function) to find an initial guess to then refine further using *fmincon*, making use of the parallel computing toolbox to us all the cores of our CPU simultaneously; however due to the heuristic algorithm this process gave us worse and often inconsistent results between consecutive runs of the program.

Selection of the final solution

The final solution was selected as the one given from the coarse grid search plus the *fmincon* refinement as this gave us a lower total ΔV. By narrowing the windows we also found other shorter transfers but with a higher total ΔV required, so we decided to discard them.

1.2 Final solution

Heliocentric trajectory

The final transfer strategy is composed of two arcs, which connect inside Venus' SOI. The transfer strategy is completed between the times 02:42:52 on 16 February 2033 and 19:54:49 on 10 July 2070. The flyby burn is performed at 13:36:07 on 30 September 2064.

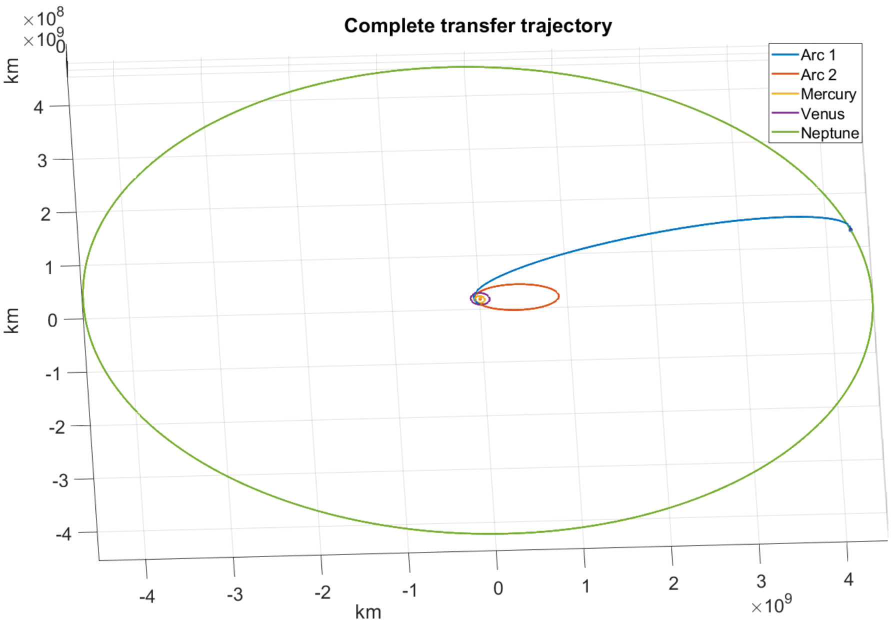


Figure 1.5: Complete strategy

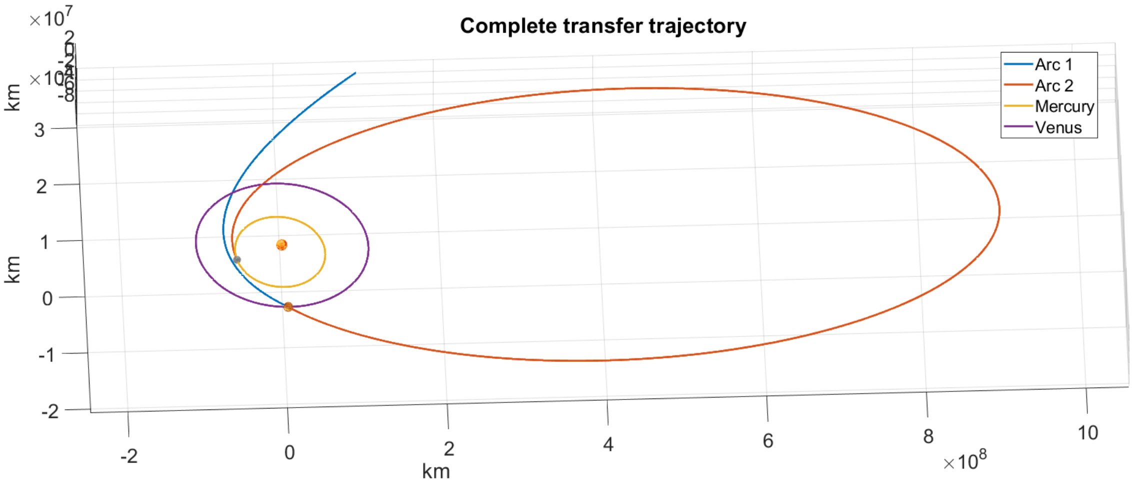


Figure 1.6: Zoom of complete strategy

The first arc connects Neptune's and Venus' orbits. It is characterised by the following data:

a [AU]	e [-]	i [°]	RAAN [°]	ω [°]
15.2090	0.9696	2.07	67.09	130.25

The initial position is characterised by a true anomaly (f) of 179.27°. In Cartesian coordinates, the initial point is identified by the following data:

R_x [AU]	R_y [AU]	R_z [AU]
28.6178	8.5487	-0.8313
v_x [km/s]	v_y [km/s]	v_z [km/s]
0.10	1.02	0.01

The initial position's distance from the Sun is 29.8789AU and the velocity at the initial point is 1.03 km/s. The final position is characterised by a true anomaly (f) of 74.92°. In Cartesian coordinates, the final point is identified by the following data:

R_x [AU]	R_y [AU]	R_z [AU]
0.0285	-0.7264	-0.0112
v_x [km/s]	v_y [km/s]	v_z [km/s]
40.18	-27.65	-1.72

The final position's distance from the Sun is 0.7270AU and the velocity at the final point is 48.81km/s.

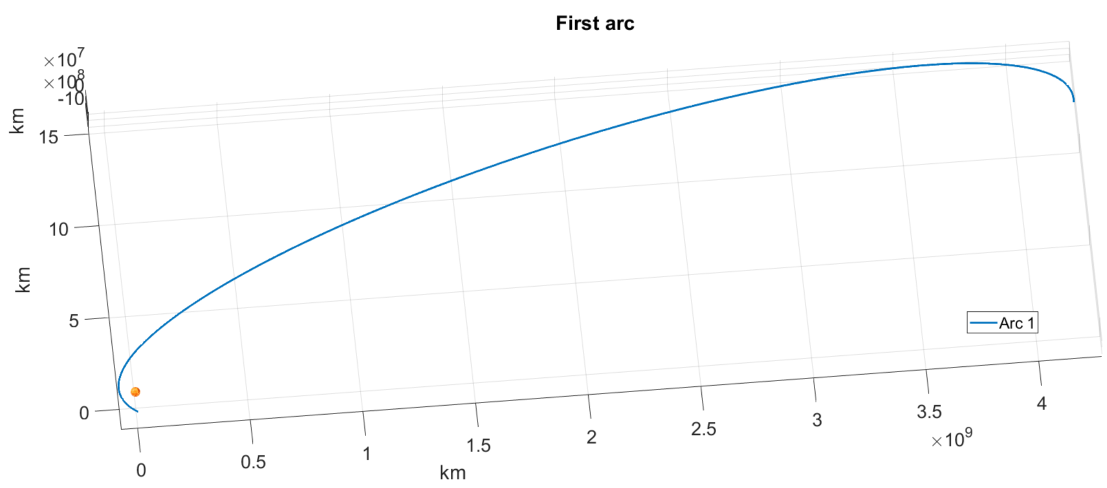


Figure 1.7: First arc

The second arc goes from Venus to Mercury. It is characterised by the following data:

a [AU]	e [-]	i [°]	RAAN [°]	ω [°]
3.2410	0.8729	3.54	77.87	108.43

The initial position is characterised by a true anomaly (f) of 85.98°. In Cartesian coordinates, the initial point is identified by the following data:

$\mathbf{R}_x$ [AU]	$\mathbf{R}_y$ [AU]	$\mathbf{R}_z$ [AU]
0.0285	-0.7264	-0.0112
$\mathbf{v}_x$ [km/s]	$\mathbf{v}_y$ [km/s]	$\mathbf{v}_z$ [km/s]
37.05	-28.06	-2.60

The initial position's distance from the Sun is 0.7270AU and the velocity at the initial point is 46.55km/s. The final position is characterised by a true anomaly (f) of 19.46°. In Cartesian coordinates the final point is identified by the following data:

$\mathbf{R}_x$ [AU]	$\mathbf{R}_y$ [AU]	$\mathbf{R}_z$ [AU]
-0.3806	-0.1841	0.0206
$\mathbf{v}_x$ [km/s]	$\mathbf{v}_y$ [km/s]	$\mathbf{v}_z$ [km/s]
17.93	-59.95	-1.86

The final position's distance from the Sun is 0.4232AU and the velocity at the final point is 62.60 km/s.

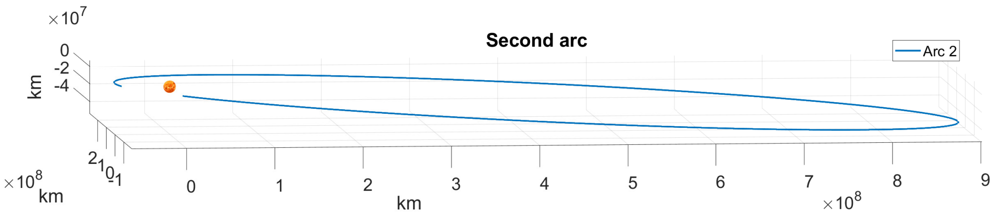


Figure 1.8: Second arc

Powered gravity assist

Venus' SOI radius is calculated to be $6.1935 \cdot 10^5$ km (0.0041AU). The approximation of stationary central body is introduced: while the spacecraft is inside Venus' SOI , the planet itself is considered fixed at the position it has at the moment of the flyby burn. The probe's trajectory inside Venus' SOI is composed of two arcs of two hyperbolas connected at their pericentre where the flyby burn is performed. Those hyperbolas are defined by their escape velocities which are:

	$\mathbf{v}_x$ [km/s]	$\mathbf{v}_y$ [km/s]	$\mathbf{v}_z$ [km/s]	$\|\mathbf{v}\|$ [km/s]
v_∞^-	5.42	-28.89	0.27	29.40
v_∞^+	2.29	-29.30	-0.61	29.40

The incoming hyperbola is characterised by the following data:

a [km]	e [-]	δ [°]	Δ [km]	$\mathbf{v}_p$ [km/s]	θ_∞ [°]	β [°]
-375.956	17.92	6.3967	6728	31.08	93.20	86.80

The outgoing hyperbola is characterised by the following data:

a [km]	e [-]	δ [°]	Δ [km]	$\mathbf{v}_p$ [km/s]	θ_∞ [°]	β [°]
-375.95	17.92	6.3967	6728	31.08	93.20	86.80

As Δs and the hyperbola's pericentre velocities are almost the same in before and after the burn, the resulting flyby manoeuvre cost is trifling. The spacecraft passes at a minimum distance of 6352 km from Venus' centre.

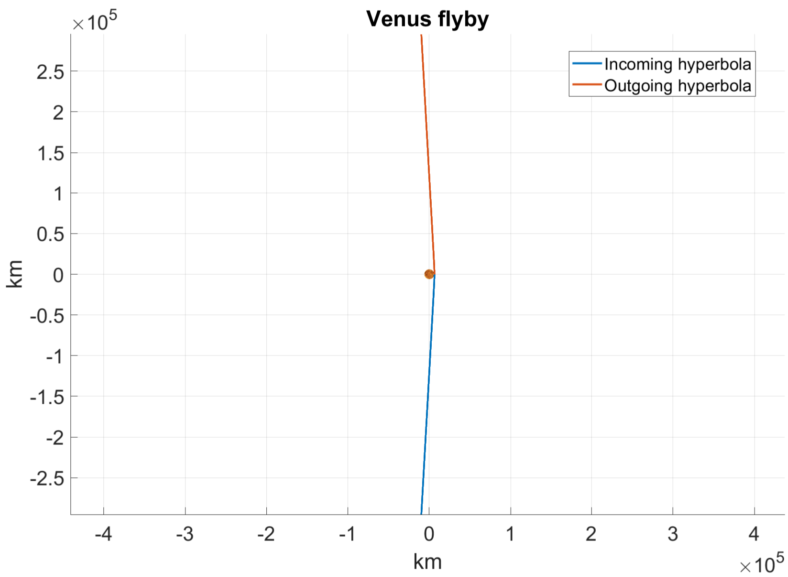


Figure 1.9: Venus Flyby

Cost of the mission

The total cost of the mission is defined by adding the cost of three separate burns. The first one is the one needed to exit Neptune's orbit and initiate the transfer. The cost of this burn is computed by determining the norm of the difference between Neptune's velocity vector at departure date and the transfer initial velocity vector.

$$\Delta V_1 = \| \mathbf{V_T^1} - \mathbf{V_N} \|_{@DEP}$$

In particular, the first burn requires 4.54 km/s to be completed. The second burn is executed at flyby. Since the difference in potential energy between the Venus' and Neptune's orbits is very high, the spacecraft reaches Venus with high kinetic energy. Therefore, the second manoeuvre represents only the 0.00016% of the total transfer cost. Specifically, the flyby manoeuvre requires 0.00004 km/s. Consequently, the flyby can be considered to be unpowered. The final burn is performed at the end of the transfer trajectory in order to rendezvous with Mercury. The cost of this burn is represented by the norm of the difference between Mercury's velocity vector evaluated on the arrival date and the velocity vector at the end of the transfer.

$$\Delta V_2 = \| \mathbf{V_T^2} - \mathbf{V_M} \|_{@ARR}$$

$\mathbf{v}_x$ [km/s]	$\mathbf{v}_y$ [km/s]	$\mathbf{v}_z$ [km/s]
11.12	-41.92	-4.40

The cost of the last burn is 19.43 km/s. From the flyby we get a very high amount of "free" ΔV, as the Δv_{GA} given by the spacecraft at perigee is just 0.0012% of the $\Delta V_{flyby} = 3.2801 km/s$ change in heliocentric velocity. It can be observed that the cost of the last burn is higher than the previous two. The reason for this is that Mercury's orbit is characterised by a higher inclination than Venus' and the final burn is responsible for completing the change of plane. Such fact can be observed by comparing Mercury's velocity vector at arrival (see table above) and the second arc final position velocity vector.

The total cost of the mission is 23.97 km/s.

Assignment 2: Planetary Explorer Mission

Mission requirements

In this chapter we are going to explain how we have designed a planetary explorer mission to perform Earth observation: we were in fact required to analyse the Earth-centred orbit characterised by the values in the table below and estimate its ground tracks, studying the effects of the assigned orbit perturbations by integrating both Gauß's planetary equations and Cartesian equations and subsequently comparing the results of the two methods. After the characterisation of the ground tracks, we propose a modification of the orbit aimed to get a ground track which repeats itself once per sidereal day.
We have chosen a periapsis anomaly of 40 degrees and, for the sake of simplicity, a RAAN and an initial true anomaly of 0 degrees.

a [$10^4 km$]	**e**	**i** [deg]	**hp** [km]
4.0718	0.6177	78.2195	15566.491
Repeating GT ratio k:m^{note}	**Perturbations**	**Parameters**	
1:1	J2 and SRP	cR = 1.2	A/m = 4.000 m^2/kg

Table 2.1: Mission requirements

Note: the ground track repeats itself **k** times every **m** revolutions of the planet.

The orbit we've been assigned, represented in the following images, is sub-geosynchronous: in fact it has a period of $22h42min49s$, which is very close to a sidereal day. With a perigee height of 9195.5 km and an apogee height of 59498.5 km, the orbit is very eccentric and, since its inclination is very high, it would be suitable for an Earth observation satellite designed to service austral regions (e.g. for magnetosphere investigation) as the satellite spends most of the time in the southern hemisphere (apogee dwell). Due to the fact that even at the perigee the atmospheric drag is negligible, this perturbation can be ignored without significantly reducing the accuracy.

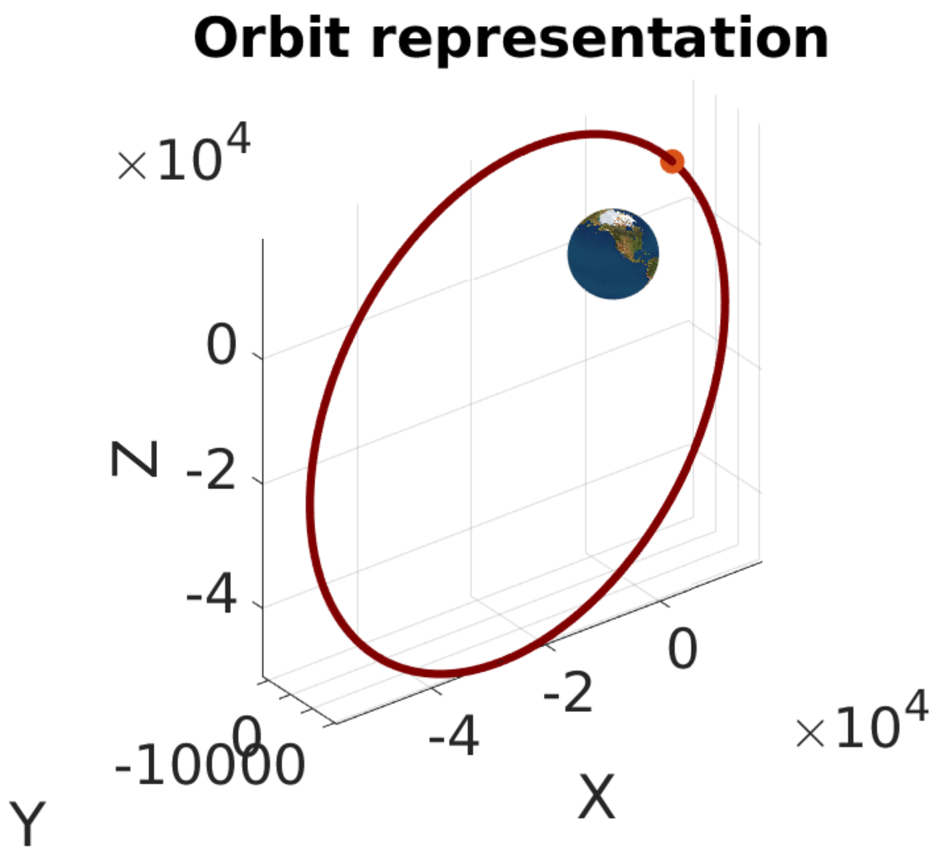


Figure 2.1: Orbit representation

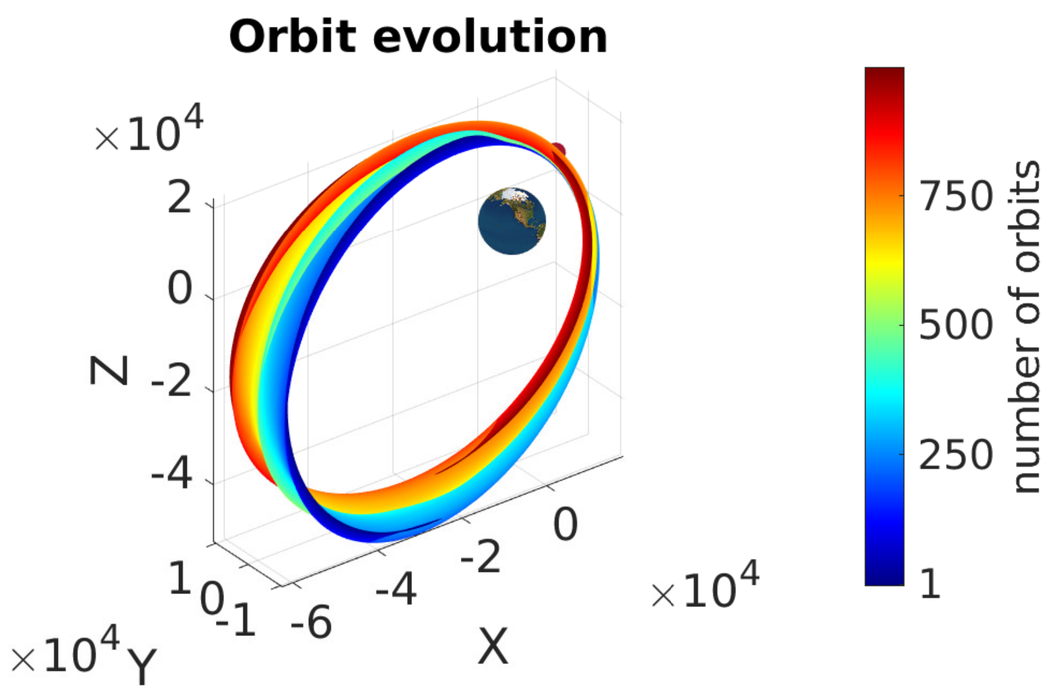


Figure 2.2: Orbit evolution with J2 and SRP perturbations

Mission analysis outputs

2.2 Ground track

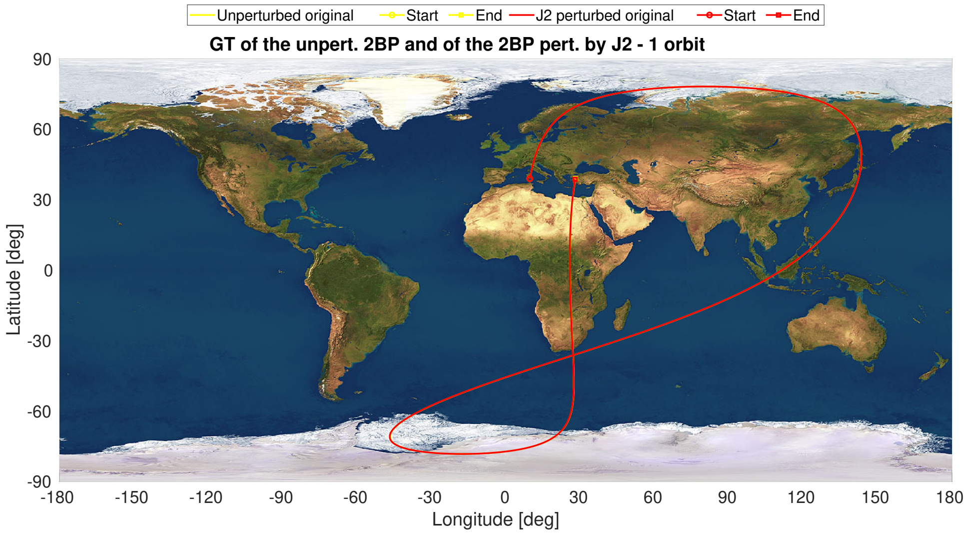


Figure 2.3: GT of the unpert. 2BP and of the 2BP pert. by J2 - 1 period

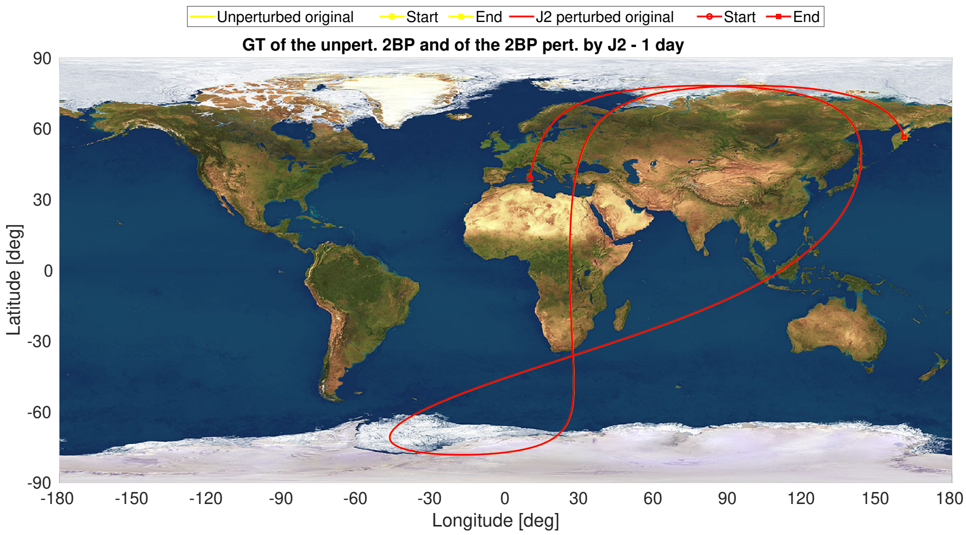


Figure 2.4: GT of the unpert. 2BP and of the 2BP pert. by J2 - 1 day

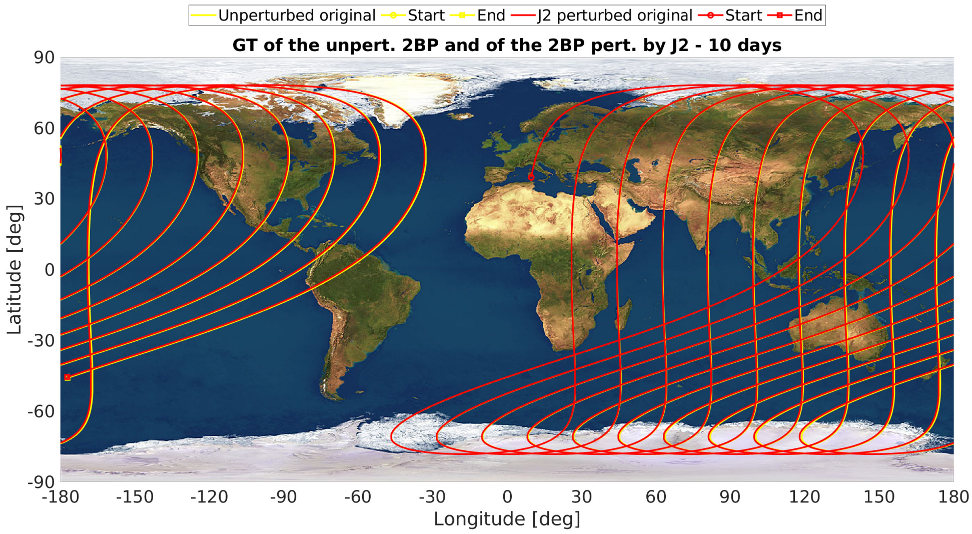


Figure 2.5: GT of the unpert. 2BP and of the 2BP pert. by J2 - 10 days

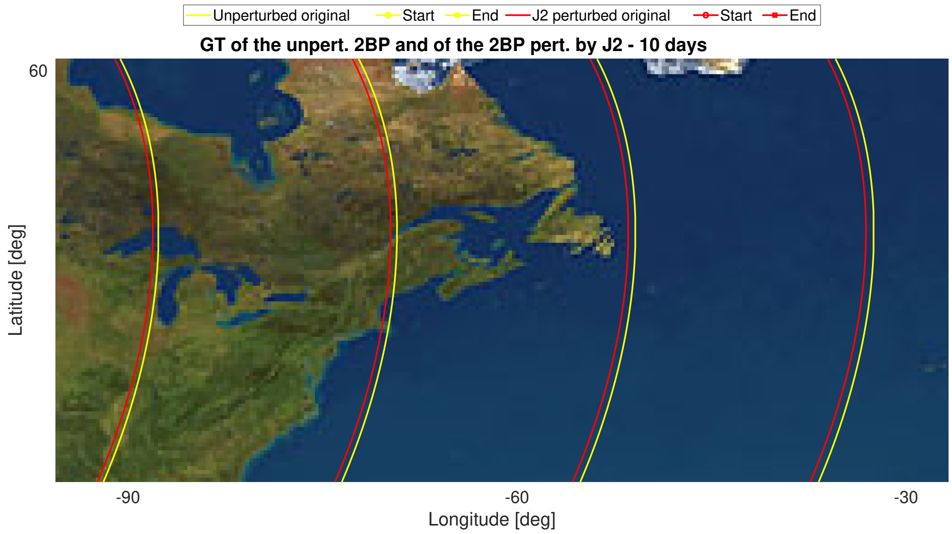


Figure 2.6: GT of the unpert. 2BP and of the 2BP pert. by J2 - 10 days, close-up

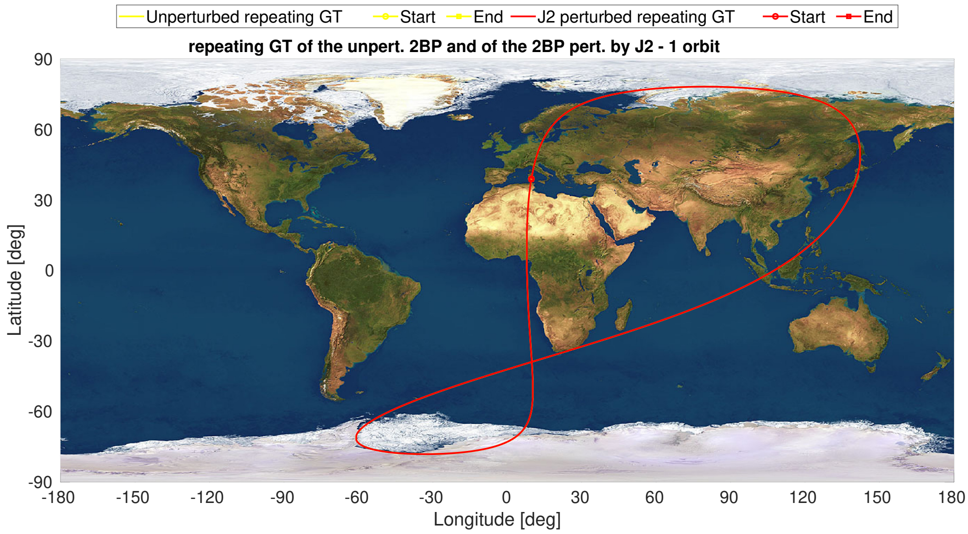


Figure 2.7: repeating GT of the unpert. 2BP and of the 2BP pert. by J2

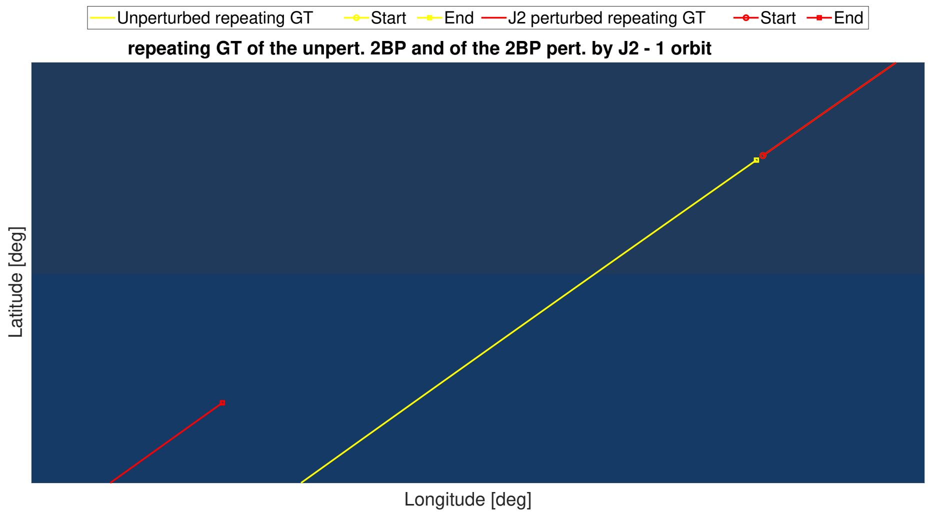


Figure 2.8: repeating GT of the unpert. 2BP and of the 2BP pert. by J2, close-up

To modify the semi-major axis in order to obtain a geosynchronous orbit, for the unperturbed case, we simply calculated the semi-major axis which corresponds to a period of a sidereal day. Whereas, when taking into account the second zonal harmonic perturbation, we included the modification of the nodal periods of satellite and Earth due to the Secular effects of J2: the regression of the nodes in the short term is therefore visible when comparing the ground tracks. In the first case we obtained a semi-major axis of 42166.2 km, while in the second one we got $a = 42163.6$ km

2.2 Orbit Perturbations

In our model we included perturbations due to two effects:

- **Second Zonal Harmonic *J2***

Models the Earth Oblateness using the spherical geopotentials model, truncated to the second term.

- **Solar Radiation Pressure *SRP***

  A force given by the impact of momentum-carrying photons on the surfaces of the spacecraft. For this perturbation we have written a function that, based on the initial date, gets the position vector of the Earth from the ephemerides function and uses it to calculate the direction of the disturbing acceleration. Given the large difference in scale we approximated the position vector of the spacecraft with respect to the Sun with the heliocentric position vector of the Earth. A simple algorithm to check for the eclipse condition using the position vector of the spacecraft found in [4] has been implemented.

$$\mathbf{a_{SRP}} = -P_{SR@1AU} \frac{AU^2}{||\mathbf{r}_{sc-Sun}||^3} c_R \frac{A_{Sun}}{m} \mathbf{r}_{sc-Sun} \tag{2.1}$$

These forces give an acceleration to the spacecraft so that the total acceleration it perceives in orbit is

$$\mathbf{a} = -\frac{\mu_\oplus}{r^3}\mathbf{r} + \mathbf{a}_{SRP} + \mathbf{a}_{J2} \tag{2.2}$$

2.2 Orbit Propagation

Methods

To propagate the orbit we used two different methods:

- **Gauß Planetary Equations**

$$\begin{aligned}
\frac{da}{dt} &= \frac{2a^2}{h}\Big(e \sin f \; a_r + \frac{p}{r} a_s\Big) \\
\frac{de}{dt} &= \frac{1}{h}\Big(p \sin(f) \; a_r + \Big((p+r)\cos f + re\Big) a_s\Big) \\
\frac{di}{dt} &= \frac{r\cos(f+\omega)}{h} a_w \\
\frac{d\Omega}{dt} &= \frac{r\sin(f+\omega)}{h \sin i} a_w \\
\frac{d\omega}{dt} &= \frac{1}{he}\Big(\cos f \; a_r + (p+r)\sin f \; a_s\Big) - \frac{r\sin(f+\omega)\cos i}{h \sin i} a_w \\
\frac{df}{dt} &= \frac{h}{r^2} + \frac{1}{eh}\Big(p \cos f \; a_r - (p+r)\sin f \; a_s
\end{aligned}$$

  We numerically integrate these equations with the ode113 solver, by using them to set the derivatives of the state. The reference frame for this equations is the RSW frame so a_r, a_s, a_w are respectively the radial, transversal and out-of-plane components. [1] [2] [3]

- **Numerical integration of Cartesian equations** This method consists in directly integrating the Cartesian equations of motion

$$\ddot{\mathbf{r}} = -\frac{\mu_\oplus}{r^3}\mathbf{r} + \sum a_p \tag{2.3}$$

  where a_p is the acceleration of each perturbation.

Comparison

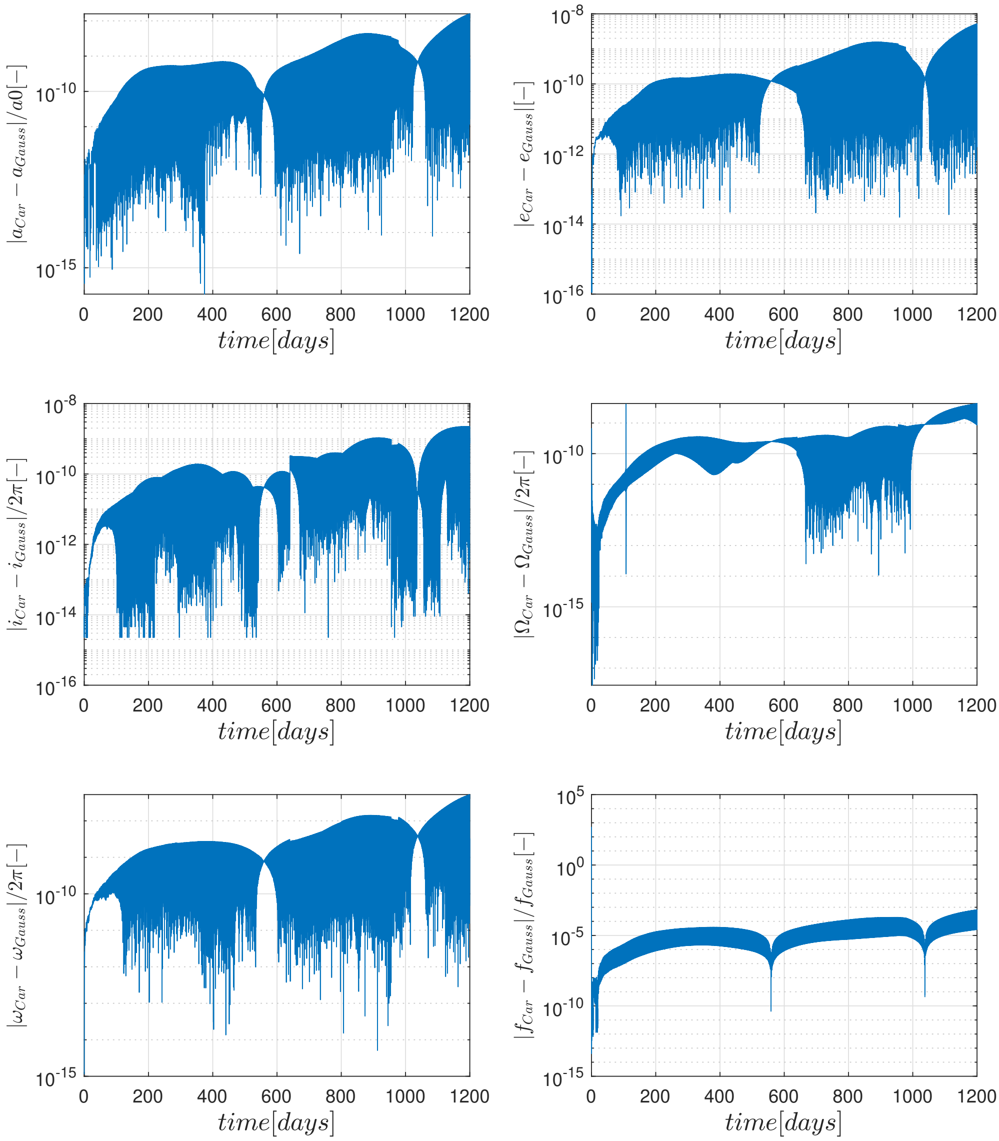
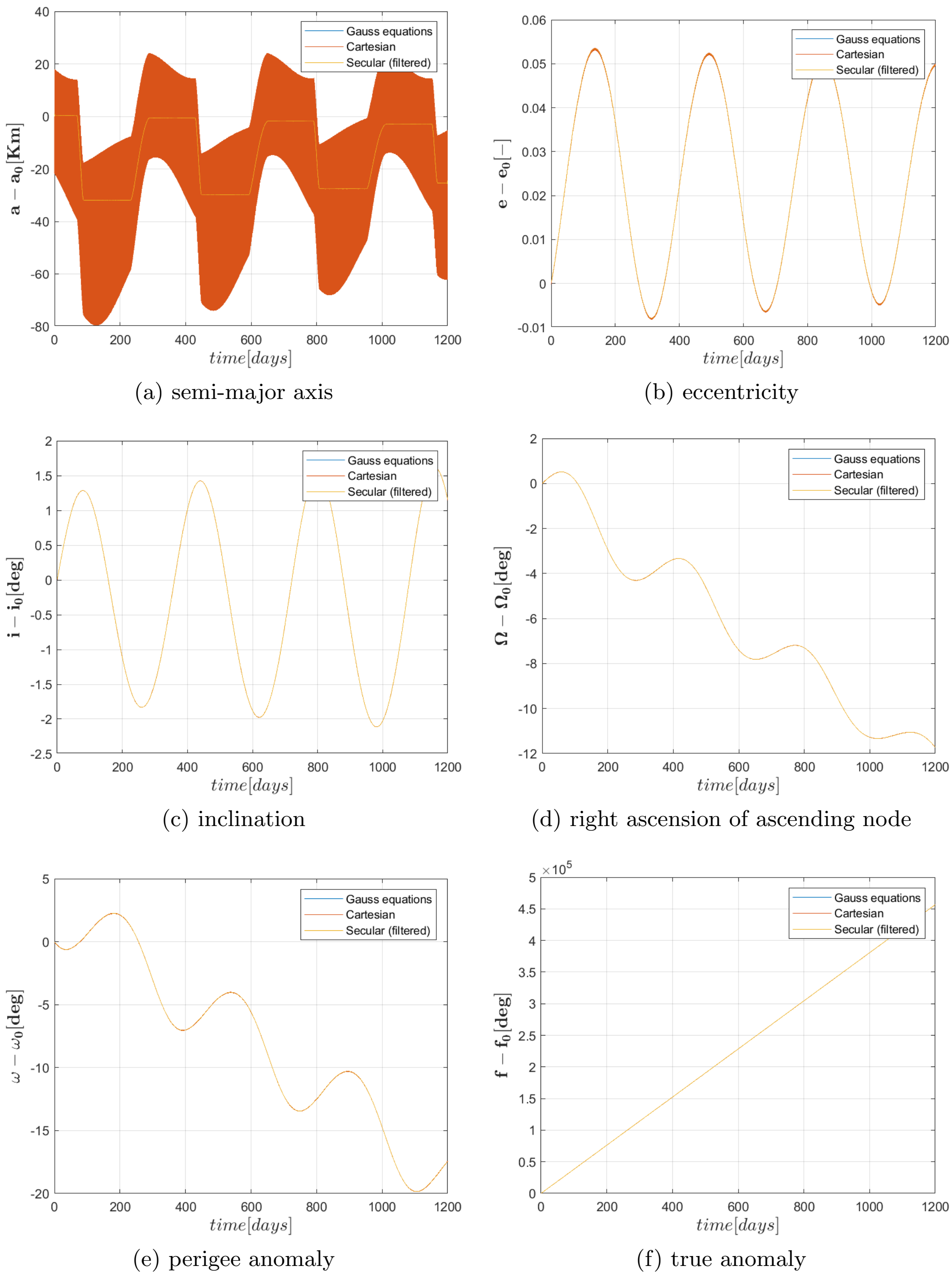


Figure 2.9: error between Gauss and Cartesian propagation methods (log scale)

There are numerical errors introducing differences between the two methods, but the error only grows up to an order between 10^{-10} and 10^{-8} for a propagation of 1200 days, except for the true anomaly which grows to a still small error of 10^{-3}.

2.2 History of the Keplerian elements

The Keplerian elements are plotted here for 1200 days from 2021-01-01



(a) semi-major axis

(b) eccentricity

(c) inclination

(d) right ascension of ascending node

(e) perigee anomaly

(f) true anomaly

Figure 2.10: Evolution of keplerian elements

2.2 Orbit evolution representation

2.2 HF filtering

For the filtering we have used the simple *movmean* function integrated in MATLAB which computes the average at each time instant between the current point and the neighbouring ones with a cutoff period of $3T_{orbit}$.

We can see in the *secular evolution* that there is a concurrency of effects from the J2 perturbation and the SRP:

- a : there is a slight secular decrease in the semi-major axis due to the effect of the SRP
- Ω : there is a *nodal regression* due to the inclination i being less than 90° for the J2 effect
- ω : there is a negative *perigee precession* for the J2 effect due to the inclination $i > 63.4°$ giving a so-called *"Hula-hooping effect"*
- e : there are periodic oscillations with a secular increase caused by ovalisation of the orbit by the SRP

2.2 Comparison with real data

Satellite selection

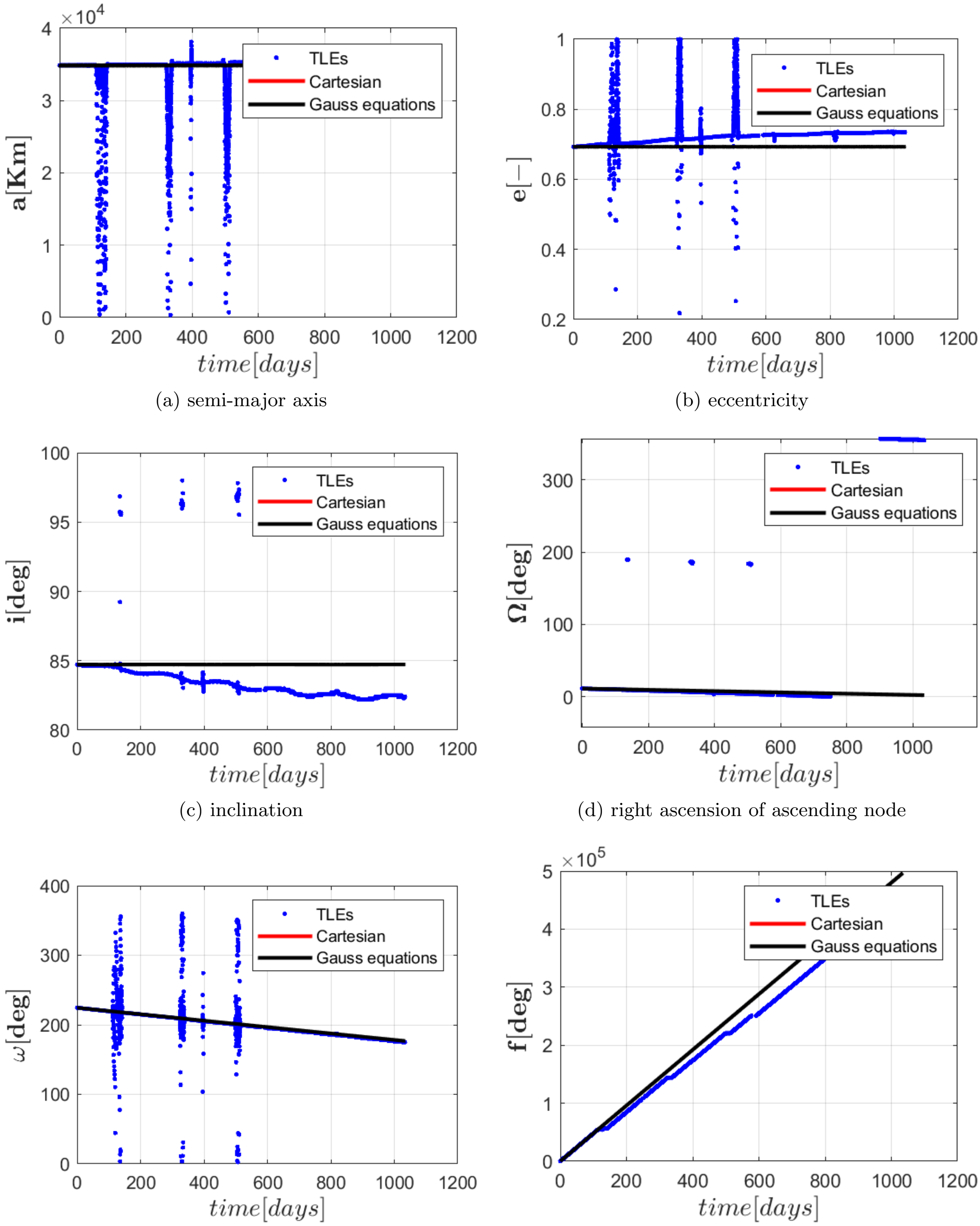
We searched for a satellite in the same orbital region on celestrack.org and we found a satellite with the same inclination as ours. The satellite is NASA's Global Geospace Science (GSS) Polar satellite. This satellite was operative from 24 February 1996 to 28 April 2008.

Comparison with our model

To get a significant time span, we resembled operative years from 2 January 2000 to 1 November 2002 in our model, a propagation of 1035 days with the following initial parameters[5]:

a [km]	e [-]	i [deg]	RAAN [deg]	ω [deg]	f_0 [deg]
34875	0.6932	84.73	11.2375	224.43	122.61

After obtaining the TLEs from celestrack.org, we entered them in the JPL Horizon [6] SGP4 propagator on its website in a text file and loaded it in MATLAB. After extracting the Keplerian elements from the text file, we plotted and compared them with Keplerian elements obtained from our models:



(a) semi-major axis

(b) eccentricity

(c) inclination

(d) right ascension of ascending node

(e) perigee anomaly

(f) true anomaly

Figure 2.11: Evolution of keplerian elements

Bibliography

[1] Curtis, H.D. *Orbital mechanics for engineering students*. Butterworth-Heinemann , 2014. Chapter 12

[2] Vallado, D.A. *Fundamental of Astrodynamics and Applications, 4th Ed*, Microcosm Press, 2013. Chapters 8 and 9

[3] Battin, R, *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA Education Series, 1999. Chapter 10

[4] Curtis, H.D. *Orbital mechanics for engineering students*. Butterworth-Heinemann , 2019. Chapter 10.9

[5] Eoportal website, https://directory.eoportal.org/web/eoportal/satellite-missions/p/polar

[6] NASA Horizons, https://ssd.jpl.nasa.gov/horizons.cgi#top