

QMM Goal Programming

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2022-10-31

Maximize $Z = P - 6C - 3D$, where P = total (discounted) profit over the life of the new products.

C = change (in either direction) in the current level of employment.

D = decrease (if any) in next year's earnings from the current year's level.

Profit P is defined as: $P = 20x_1 + 15x_2 + 25x_3$

Employment level is defined as : $6x_1 + 4x_2 + 5x_3 = 50$

Next year Earnings goal is defined as: $8x_1 + 7x_2 + 5x_3 \geq 75$

1) Model_Formulation:

Let us consider y_1 - Employment Level minus the target.

y_2 - Next Year Earnings minus the Target.

y_{1+} - Penalty for employment level goal exceeding 50.

y_{1-} - Penalty for employment level goal decreasing below 50.

y_{2+} - Exceed the next year earnings.

y_{2-} - Penalty for not reaching the next year earnings.

$$y_1 = 6x_1 + 4x_2 + 5x_3 - 50.$$

$$y_2 = 8x_1 + 7x_2 + 5x_3 - 75$$

For Employment level goal

$$y_1 = y_{1+} - y_{1-} \text{ where } y_{1+}, y_{1-} \geq 0.$$

$$y_{1+} - y_{1-} = 6x_1 + 4x_2 + 5x_3 - 50$$

For Next year earnings goal

$$y_2 = y_{2+} - y_{2-} \text{ where } y_{2+}, y_{2-} \geq 0.$$

$$y_{2+} - y_{2-} = 8x_1 + 7x_2 + 5x_3 - 75$$

Final Formulation is expressed as

$$\text{Max } P = 20x_1 + 15x_2 + 25x_3.$$

$$6x_1 + 4x_2 + 5x_3 - (y_{1+} - y_{1-}) = 50.$$

$$8x_1 + 7x_2 + 5x_3 - (y_{2+} - y_{2-}) = 75$$

Where, $x_j \geq 0$, where $j=1,2,3$.

$$y_i \geq 0, \text{ where } i=1,2.$$

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2)Managements objective function Objective Function

$$\text{Maximize } Z = P - 6C - 3D$$

Objective function in terms of $x_1, x_2, x_3, y_{1+}, y_{1-}, y_{2+}$ and y_{2-} .

$$\text{Max } Z = 20x_1 + 15x_2 + 25x_3 - 6y_{1+} - 6y_{1-} - 3y_{2-}.$$

$$6x_1 + 4x_2 + 5x_3 - (y_{1+} + y_{1-}) = 50.$$

$$8x_1 + 7x_2 + 5x_3 - (y_{2+} + y_{2-}) = 75.$$

Where, $x_j \geq 0$ where $j=1,2,3$

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3) Formulate and solve the linear programming model

```
library(lpSolveAPI)
```

```
GoalProgram<- read.lp("C:/Users/Pavan Chaitanya/Documents/Emax1.lp")
```

```
GoalProgram
```

```
## Model name:
```

	x_1	x_2	x_3	y_{1p}	y_{1m}	y_{2m}	y_{2p}	
## Maximize	20	15	25	-6	6	-3	0	
## EmploymentLevelGoal	6	4	5	-1	1	0	0	= 50
## NextYearEarningsGoal	8	7	5	0	0	1	-1	= 75
## Kind	Std	Std	Std	Std	Std	Std	Std	
## Type	Real	Real	Real	Real	Real	Real	Real	
## Upper	Inf	Inf	Inf	Inf	Inf	Inf	Inf	
## Lower	0	0	0	0	0	0	0	

```
solve(GoalProgram)
```

```
## [1] 0
```

```
get.objective(GoalProgram)
## [1] 225
get.variables(GoalProgram)
## [1] 0 0 15 25 0 0 0
get.constraints(GoalProgram)
## [1] 50 75
```

Interpretation :

The penalty is 225 if you are not satisfying the goals on the objective function. The results show that $x_1 = 0$.

$$x_2 = 0.$$

$$x_3 = 15.$$

$$y_{1+} = 25.$$

$$y_{1-} = 0.$$

$$y_{2+} = 0.$$

$$y_{2-} = 0.$$

which explains the Next years Earnings (y_2) expectations are fully satisfied.

Emax need to produce 15 units of product 3 and none of product 1 and 2 to achieve 225 millions in profit.