Problems - Information Retrieval and Recommender Systems

Information Retrieval - an online shop

Your online shop, sells 80 different products. You recently contracted a company to implement a "search" functionality for your shop. The company has just delivered their new retrieval system, and you are now testing it out.

You try the system, with a specific query, and the system displays the 10 products with the following IDs on the first page:

```
import numpy as np

result = np.array([ 2, 20, 36, 41, 44,  6, 71, 79, 78,  9])
```

You know that for the query you tried out, the relevant products are the ones with ids:

```
relevant_idx = np.array([ 2,  3,  5,  9, 14, 20, 36, 38, 39, 44, 48, 54, 58, 71,
print(len(relevant_idx))
```

```
The number of relevant items is: 17
```

How many relevant items did the system return?

```
# YOUR CODE HERE
relevant_returned = np.intersect1d(result, relevant_idx)
print(f"The number of relevant items is: {len(relevant_returned)}")
```

```
The number of relevant items is: 7
```

What is the Precision, Recall and F1-score values for the above query?

Remember:

$P = \frac{\lvert ret \cap rel \rvert}{ret}$

$R = \frac{\lvert ret \cap rel \rvert}{rel}$

$F = \frac{2 P R}{P + R}$

```
precision = len(relevant_returned) / len(result)

recall = len(relevant_returned) / len(relevant_idx)

if precision + recall > 0:
    f1_score = 2 * (precision * recall) / (precision + recall)
else:
    f1_score = 0

print(f"Precision: {precision}")
print(f"Recall: {recall}")
print(f"F1-score: {f1_score}")
```

```
Precision: 0.7
Recall: 0.4117647058823529
F1-score: 0.5185185185185185
```

The retrieval system only displays the ten best products on the first page, but internally it has scored all the products in the database against the query. For the same query as before, the scores for all 80 products that the system has calculated are given in the list below. The higher the number, the better it matches the query.

```
scores = np.array([76.25960904, 37.2712716 , 97.43522538, 85.2135435 , 38.4216(
       59.33358171, 90.84382517, 34.59173848, 53.24126568, 87.80759099,
       71.13477237, 52.24501123, 53.84857359, 72.34145019, 82.60724712,
       60.13854056, 50.02218001, 67.69020948, 49.20740763, 48.19740307,
       97.26734714, 33.43817817, 46.61489   , 45.70630798, 12.03883746,
       43.71217679, 66.03930779, 76.31438734, 10.98192963, 29.80459484,
       14.7397168 , 21.48280983, 39.81730095, 28.18530124, 36.35538331,
       55.10834932, 95.5515258 , 13.10765296, 86.70784949, 81.82826684,
       55.79913761, 94.58702549, 79.3577133 , 72.84811652, 92.9954021 ,
       78.36304059, 62.00806942, 16.43752292, 83.44531762, 82.46218716,
       10.26819206, 64.56747866, 61.1888231 , 11.73771   , 36.15240615,
       63.08775437, 18.5772151 , 15.75830887, 40.75549881, 68.84171716,
       16.72596633, 57.49016213, 51.46845883, 23.56191592, 43.31518323,
       74.93840142, 25.68485212, 26.20211702, 27.50582533, 63.53842237,
       28.30218436, 89.65913426, 31.90662515, 41.68472171, 69.13468994,
       36.35852244, 19.61632789, 79.82716108, 88.39815675, 88.55666924])
```

Can you calculate the Precision and Recall if you displayed 20 or 40 products on the first page?

> Note 1: This is what we would call P@20, R@20, P@40, R@40.

> Note 2: Have a look at the Numpy function `argsort()` to solve this problem (https://numpy.org/doc/1.18/reference/generated/numpy.argsort.html).

```
precision_20 = len(relevant_top_20) / len(top_20)
recall_20 = len(relevant_top_20) / len(relevant_idx)
if precision_20 + recall_20 > 0:
    f1_score_20 = 2 * (precision_20 * recall_20) / (precision_20 + recall_20)
else:
    f1_score_20 = 0

precision_40 = len(relevant_top_40) / len(top_40)
recall_40 = len(relevant_top_40) / len(relevant_idx)
if precision_40 + recall_40 > 0:
    f1_score_40 = 2 * (precision_40 * recall_40) / (precision_40 + recall_40)
else:
    f1_score_40 = 0

print(f"P@20: {precision_20}, R@20: {recall_20}, F1@20: {f1_score_20}")
print(f"P@40: {precision_40}, R@40: {recall_40}, F1@40: {f1_score_40}")
```

```
P@20: 0.65, R@20: 0.7647058823529411, F1@20: 0.7027027027027027
P@40: 0.35, R@40: 0.8235294117647058, F1@40: 0.4912280701754386
```

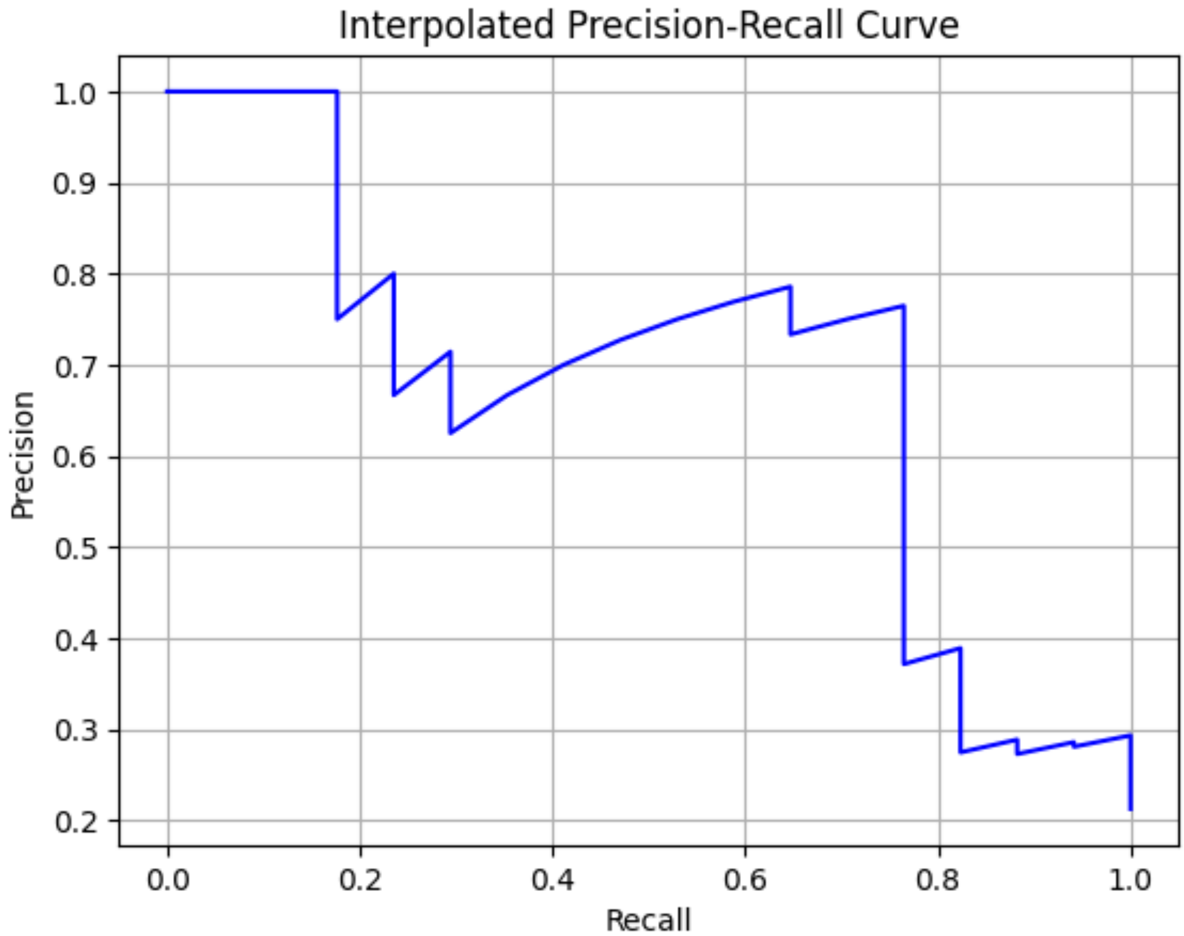
In order to get an overall idea about how good the retrieval is, draw the Precision/Recall plot

```
from sklearn.metrics import precision_recall_curve

y_true = np.zeros_like(scores)
y_true[relevant_idx] = 1

precision_vals, recall_vals, _ = precision_recall_curve(y_true, scores)

plt.plot(recall_vals, precision_vals, color='blue')
plt.xlabel('Recall')
plt.ylabel('Precision')
plt.title('Interpolated Precision-Recall Curve')
plt.grid(True)
plt.show()
```



Now calculate the Average Precision value.

Remember:

$AP = \displaystyle\sum_{k=1}^{N} P(k) \Delta R(k)$$AP = \displaystyle\sum_{k=1}^{N} P(k) \Delta R(k)$

or

$AP = \displaystyle \frac{\sum_{k=1}^{N} P(k) rel(k)}{|rel|}$$AP = \displaystyle \frac{\sum_{k=1}^{N} P(k) rel(k)}{|rel|}$

```
ap_sum = 0
relevant_so_far = 0

for k, idx in enumerate(sorted_indices, start=1):
    if idx in relevant_idx:
        relevant_so_far += 1
        precision_at_k = relevant_so_far / k
        ap_sum += precision_at_k

average_precision = ap_sum / len(relevant_idx)
print(f"Average Precision (AP): {average_precision}")
```

```
Average Precision (AP): 0.6872967180508048
```

Recommender Systems

You are working on a movie recommendation system, and you have been given the following ratings table.

Each row corresponds to a user, and each column to a movie. Ratings are between 1 and 5 stars, while a rating equal to `None` (NaN) means that the user has not rated that particular movie.

```
import pandas as pd
```

```
ratings = pd.read_csv("ratings.csv", index_col = 0)
ratings
```

	Movie 0	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7	Movie 8	Movie 9	Mo
User 0	4.0	1.0	5	2.0	4.0	5	NaN	1.0	3.0	1.0	
User 1	5.0	1.0	3	5.0	1.0	2	4.0	3.0	NaN	NaN	
User 2	1.0	NaN	1	NaN	5.0	1	5.0	4.0	2.0	1.0	
User 3	NaN	1.0	3	1.0	NaN	1	3.0	4.0	4.0	NaN	
User 4	3.0	3.0	4	2.0	NaN	2	4.0	1.0	4.0	NaN	N
User 5	4.0	NaN	1	2.0	2.0	4	2.0	1.0	2.0	1.0	
User											

In addition, you are given the following ratings that another user has made for the same movies.

```
target = pd.read_csv("target.csv", header = 0, index_col=0)
target = target.squeeze()
target
```

```
Movie 0     1.0
Movie 1     NaN
Movie 2     NaN
Movie 3     2.0
Movie 4     4.0
Movie 5     NaN
Movie 6     1.0
Movie 7     4.0
Movie 8     NaN
Movie 9     NaN
Movie 10    NaN
Movie 11    NaN
Movie 12    NaN
Movie 13    3.0
Movie 14    5.0
Name: 0, dtype: float64
```

You are asked to predict the ratings for the movies that the target user has not seen yet, that is for the movies that have a rating of zero in the target user array.

First, we will calculate the similarity between the target user and each of the users in our ratings table. We will then use the 3 most similar users to derive recommendations.

To calculate similarity between users, we will use the Pearson correlation formula:

$sim(a, b) = \frac{\sum_{p \in P} (r_{a,p}-\hat{r_a}) (r_{b,p}-\hat{r_b}) }{\sqrt{\sum_{p \in P} (r_{a,p}-\hat{r_a})^2} \sqrt{\sum_{p \in P} (r_{b,p}-\hat{r_b})^2} }$$sim(a, b) = \frac{\sum_{p \in P} (r_{a,p}-\hat{r_a}) (r_{b,p}-\hat{r_b}) }{\sqrt{\sum_{p \in P} (r_{a,p}-\hat{r_a})^2} \sqrt{\sum_{p \in P} (r_{b,p}-\hat{r_b})^2} }$

where a and b are users, and P is the set of the items for which BOTH users have provided ratings.

Luckily, pandas has a function for calculating correlation: `DataFrame.corrwith()`. This will calculate the correlation between the target series and every row of the ratings dataframe if we specify that `axis = 1` (row-wise).

> See the docs: https://pandas.pydata.org/pandas-docs/stable/reference/api/pandas.DataFrame.corrwith.html?highlight=corrwith#pandas.DataFrame.corrwith

```
similarity = ratings.corrwith(target, axis = 1, method = 'pearson') # This func
similarity
```

```
User 0   -0.333333
User 1   -0.751237
User 2    0.062994
User 3    0.645497
User 4   -0.912871
User 5   -0.040032
User 6   -0.944911
User 7   -0.303697
User 8   -0.762493
User 9    0.023702
dtype: float64
```

In order to predict accurate preferences, we should only take into account users that have a minimum number of items in common to our target user. This minimum is up to us to define (depends on your data and your application).

First, we need to count count how many items each user has in common with our target user.

We can calculate this using boolean logic. First we check for which items we have ratings (True) and for which no (False). Then we check for which items we have True in both the ratings data frame AND the target series. Then it is a matter of summing up how many True values we have in every row. This is summarised below, although it would be easier to understand if you do the calculations step by step and print out the result.

```
# Find how many items do they have in common

commonItems = (~ratings.isna()) & (~target.isna())
nCommon = commonItems.sum(axis = 1)
nCommon
```

```
User 0    4
User 1    6
User 2    6
User 3    4
User 4    4
User 5    7
User 6    3
User 7    7
User 8    4
User 9    6
dtype: int64
```

We can now get the indices of the users that have more than 3 items with the target. This should drop User 6 in this case who only has exactly 3 items in common with the target.

```
minItemsCommon = 3

# Get the indices of users with whom we have more than minItemsCommon items in
idx = nCommon.index[nCommon > minItemsCommon]
print (idx)

#keep only the users that have more than minItemsCommon with the query
similarity = similarity.loc[idx]
similarity
```

```
Index(['User 0', 'User 1', 'User 2', 'User 3', 'User 4', 'User 5', 'User 7'
       'User 8', 'User 9'],
      dtype='object')
User 0   -0.333333
User 1   -0.751237
User 2    0.062994
User 3    0.645497
User 4   -0.912871
User 5   -0.040032
User 7   -0.303697
User 8   -0.762493
User 9    0.023702
dtype: float64
```

Now, let's find the 3 most similar users in the remaining ones. We can do that by sorting the similarities series (from smaller to larger) and keeping the last three entries. We can do this using the `sort_values()` function of pandas.

> See the documentation here: https://pandas.pydata.org/pandas-docs/stable/reference/api/pandas.DataFrame.sort_values.html

```
topN = 3

similarity.sort_values(ascending = False, inplace = True)
idx = similarity.index[:topN] # get the indices of the top N most similar users
idx
```

```
Index(['User 3', 'User 2', 'User 9'], dtype='object')
```

Now that we have the most similar users, we can use their ratings to derive ratings for the target user. To combine the ratings of the most similar users we will use the formula:

$ pred(a, i) = \hat{r_a} + \frac{\sum_{b \in N} sim(a, b) (r_{b,i} - \hat{r_b})}{\sum_{b \in N} sim(a,b)} $$ pred(a, i) = \hat{r_a} + \frac{\sum_{b \in N} sim(a, b) (r_{b,i} - \hat{r_b})}{\sum_{b \in N} sim(a,b)} $

where a and i are the user and item for which we want to predict a rating, and N is the set of similar users we have identified.

For example, if we wanted to derive the rating for the Movie 2 (`target["Movie 2"]`), we would combine ratings in the following way:

```
m = "Movie 2" #The movie we are interested in predicting a rating for

# initialise two variables in which we will hold the sums of the nominator and
sum_nom = sum_denom = 0

# Loop through the list of similar users
for u in idx:
    user_ratings = ratings.loc[u] # the row of ratings corresponding to user u
    print("\nuser:", u, "Similarity: ", similarity[u])

    # Only use the users that have rated the movie we are interested in
    if ~(user_ratings.isna()[m]):
        print("Has seen the movie:", m)
        print("Gave it a rating of: ", user_ratings[m])
        mu = user_ratings.mean() # Calculate the mean rating of the user, this
        print("Mean: ", mu)
        sum_nom = sum_nom + similarity[u]*(user_ratings[m] - mu) # Nominator
        sum_denom = sum_denom + similarity[u] # Denominator
    else:
        print("Has not seen the movie. Ignore.")

if sum_denom != 0: # Check if at least one of the similar users had rated the m
    target[m] = target.mean() + sum_nom / sum_denom

print("\nThe predicted ratings of the target user are: \n", target)
```

```
user: User 3 Similarity:  0.6454972243679027
Has seen the movie: Movie 2
Gave it a rating of:  3.0
Mean:  2.8181818181818183

user: User 2 Similarity:  0.06299407883487121
Has seen the movie: Movie 2
Gave it a rating of:  1.0
Mean:  2.3333333333333335

user: User 9 Similarity:  0.02370227315699887
Has seen the movie: Movie 2
Gave it a rating of:  1.0
Mean:  2.8461538461538463

The predicted ratings of the target user are:
 Movie 0     1.000000
Movie 1     1.107035
Movie 2     2.577300
Movie 3     2.000000
Movie 4     4.000000
Movie 5     0.883737
Movie 6     1.000000
Movie 7     4.000000
Movie 8     3.436815
Movie 9     1.369908
Movie 10    4.348525
Movie 11    2.796355
Movie 12    2.086963
Movie 13    3.000000
Movie 14    5.000000
Name: 0, dtype: float64
```

Do the same for all the movies that the target user has not rated yet.

```
# YOUR CODE HERE
for m in target[target.isna()].index:
    sum_nom = sum_denom = 0

    for u in idx:
        user_ratings = ratings.loc[u]

        if not user_ratings.isna()[m]:
            mu = user_ratings.mean()
            sum_nom += similarity[u] * (user_ratings[m] - mu)
            sum_denom += similarity[u]

    if sum_denom != 0:
        target[m] = target.mean() + sum_nom / sum_denom

print("\nThe predicted ratings of the target user are: \n", target)
```

```
The predicted ratings of the target user are: 
 Movie 0     1.000000
Movie 1     1.107035
Movie 2     2.577300
Movie 3     2.000000
Movie 4     4.000000
Movie 5     0.883737
Movie 6     1.000000
Movie 7     4.000000
Movie 8     3.436815
Movie 9     1.369908
Movie 10    4.348525
Movie 11    2.796355
Movie 12    2.086963
Movie 13    3.000000
Movie 14    5.000000
Name: 0, dtype: float64
```