



One-mode and two-mode networks

Network Science '21: Session 1.3 (self-study content)

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**Blockchain & Distributed Ledger Technologies
UZH Blockchain Center**



Outlook



Contents

1 Outlook

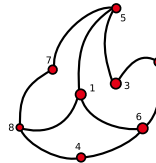


One-mode and two-mode networks

Unipartite networks

Unipartite networks (one mode)

- + All nodes are of the same nature;
- + E.g.: Social networks, Internet, WWW, Firms



$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

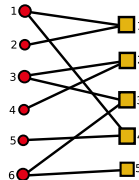
Bipartite networks

Bipartite networks (two modes)

- + Nodes are of two **well-differentiated** nature
- + Node of one type **can only be connected** to a node of another type;
- + e.g.:
 - Recommender systems (product/user)
 - Goods (buyer/product; buyer/seller; manufacturer/contractor)

Bipartite graph can also be:

- + **Unweighted:** $a_{ij} \in \{0, 1\}$, A is rectangular
- + **Weighted:** $a_{ij} \in \mathbb{R}$, A is rectangular;



$$A = \begin{matrix} & \text{red circle} & \\ \text{yellow square} & \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

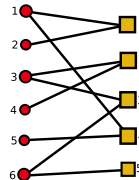
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$$A = \begin{matrix} & \text{Red Node} & \text{Yellow Node} & & & & \\ & & & & & & \\ \text{Yellow Node} & \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

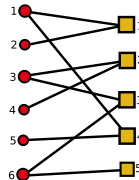
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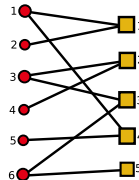
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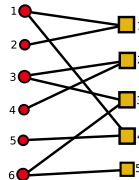
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Bipartite networks: example

A supermarket chain wants to know which products are frequently bought together.

They have the following data:

Receipt ID	Date	Product	Category	Shop ID
ID1	19.02.2014	Milch Naturaplan 1.5L	Milk	SID1
ID1	19.02.2014	Gruyère Reibkäse	Cheese	SID1
ID1	19.02.2014	Naturaplan Bio Butter	Butter	SID1
ID1	19.02.2014	Rispen Tomaten	Tomatoes	SID1
ID2	20.02.2014	De Cecco Olivenöl	Oil	SID1
ID2	20.02.2014	Vorzugsbutter	Butter	SID1
ID2	20.02.2014	Persil Gel Universal	Washing powder	SID1
ID2	20.02.2014	Naturaplan Bio Tomaten	Tomatoes	SID1
ID3	22.02.2014	Rispen Tomaten	Tomatoes	SID1
ID3	22.02.2014	Nero d'Avola Sicilia IGT	Wine	SID1

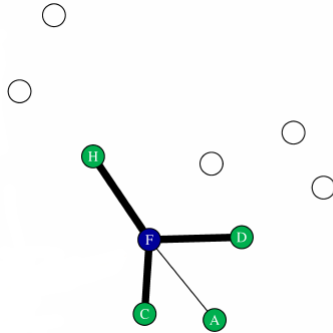


Bipartite network: Nodes

Node Index	Subject	Type
A	Milk	Product
B	Oil	Product
C	Cheese	Product
D	Butter	Product
E	Washing powder	Product
F	ID1	Receipt ID
G	ID2	Receipt ID
H	Tomatoes	Product
I	ID3	Receipt ID
J	Wine	Product

Bipartite network: Edges

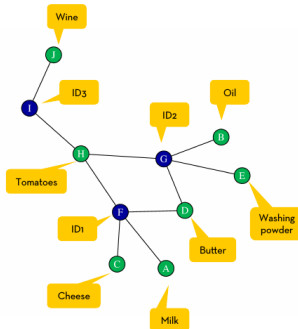
Receipt ID	Product
ID1	Milk
ID1	Cheese
ID1	Butter
ID1	Tomatoes
ID2	Oil
ID2	Butter
ID2	Washing powder
ID2	Tomatoes
ID3	Tomatoes
ID3	Wine



Bipartite networks: adjacency matrix

- + **Blue** nodes - receipts; **Green** nodes - products
- + Edges exist only between nodes of **different types**.
- + Adjacency matrix for **bipartite** networks: block-matrix;

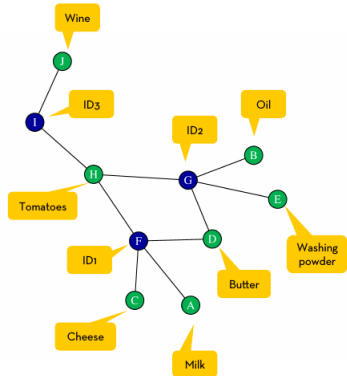
	A	B	C	D	E	H	J	F	G	I
A	0	0	0	0	0	0	0	1	0	0
B	0	0	0	0	0	0	0	0	1	0
C	0	0	0	0	0	0	0	1	0	0
D	0	0	0	0	0	0	0	1	1	0
E	0	0	0	0	0	0	0	0	1	0
H	0	0	0	0	0	0	0	1	1	1
J	0	0	0	0	0	0	0	0	0	1
F	1	0	1	1	0	1	0	0	0	0
G	0	1	0	1	1	1	0	0	0	0
I	0	0	0	0	0	1	1	0	0	0



Bipartite networks: edge list

- + **Blue** nodes - receipts; **Green** nodes - products
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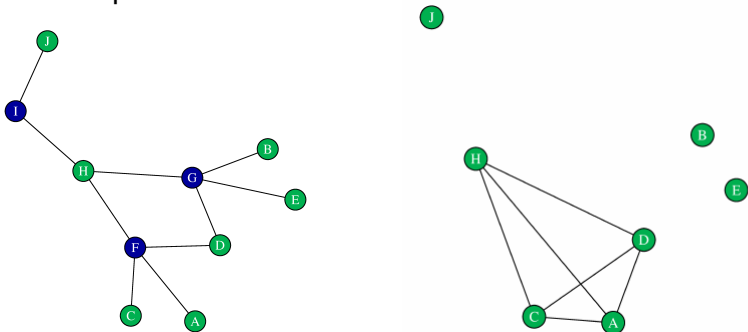
Source (Products)	Target (Receipts)
A	F
B	G
C	F
D	F
D	G
E	G
H	G
H	F
H	I
J	I



One mode projection

Link all products that were bought together on the same receipt

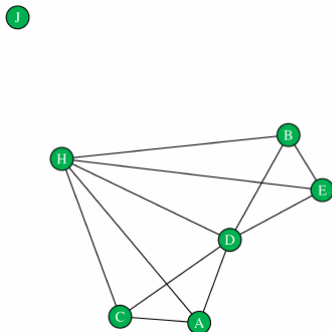
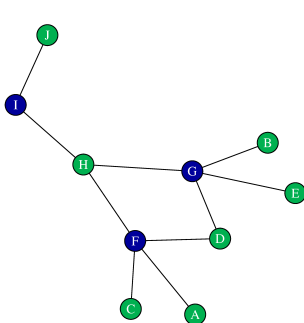
Consider receipt **F** first



One mode projection

Link all products that were bought together on the same receipt

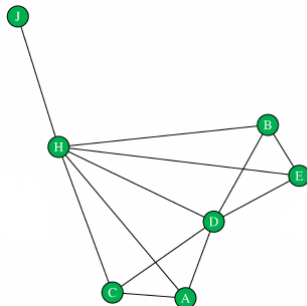
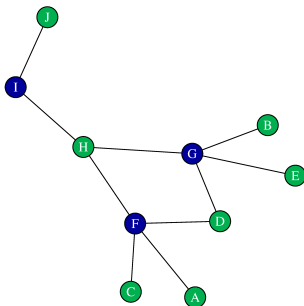
Now consider receipt **G**



One mode projection

Link all products that were bought together on the same receipt

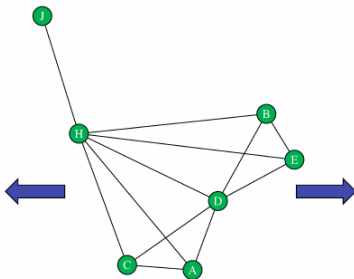
Finally, consider receipt I



One mode projection

Resulting graph is **unipartite**, undirected, unweighted

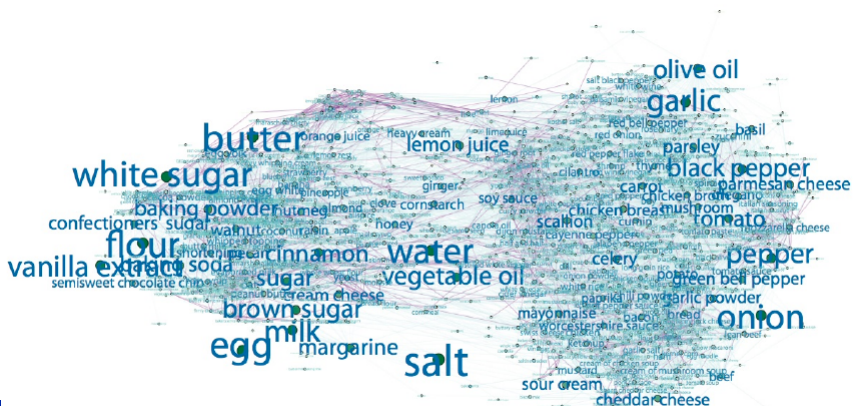
	A	B	C	D	E	H	J
A	0	0	1	1	0	1	0
B	0	0	0	1	1	1	0
C	1	0	0	1	0	1	0
D	1	1	1	0	1	1	0
E	0	1	0	1	0	1	0
H	1	1	1	1	1	0	1
J	0	0	0	0	0	1	0



Source Products	Target Products
A	C
A	D
A	H
B	D
B	E
B	H
C	D
C	H
D	E
D	H
E	H
H	J

Network of ingredients

Network of ingredients that occur together more than by chance:





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