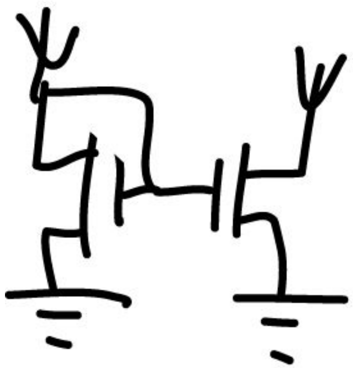
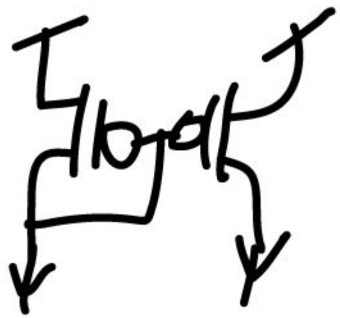
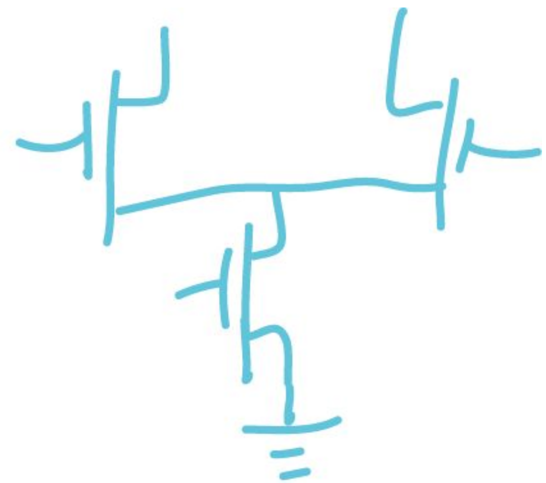
Static Circuits

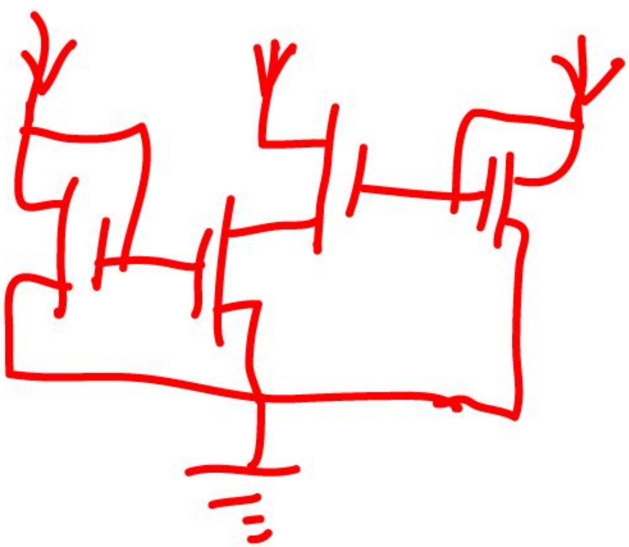
- Results from Lab3 (Strong inversion+Early)
- Intrinsic gain of a transistor (take 2)
- Diode-connected transistor
- Current mirrors & Scaling (tilted) current mirrors
- Differential pair
- Current correlator
- Bump-Antibump circuit
- Transconductance amplifier (and its g_m and A)
- Wide range transamp

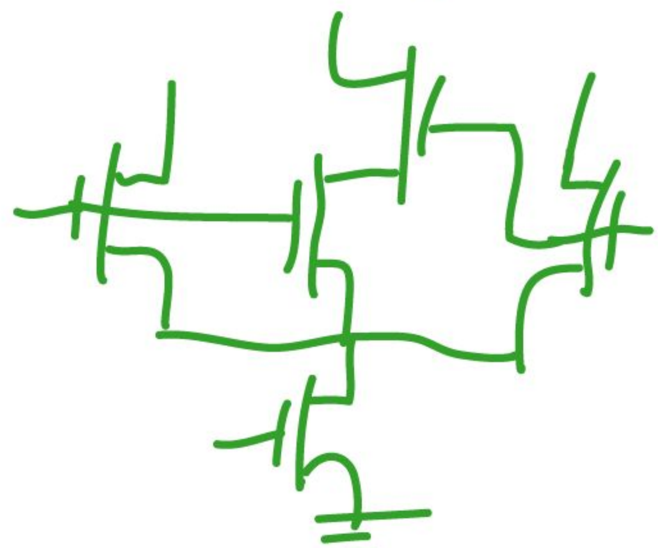
Current mirror, diff-pair, current correlator, bump-antibump circuit



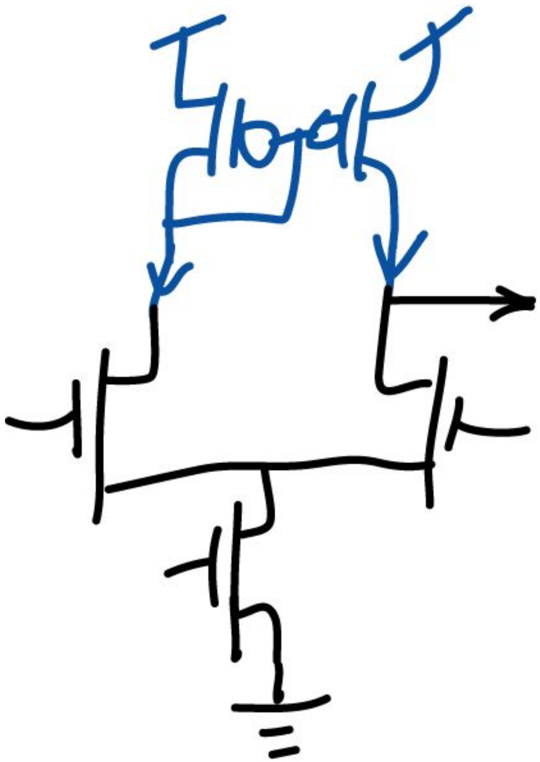


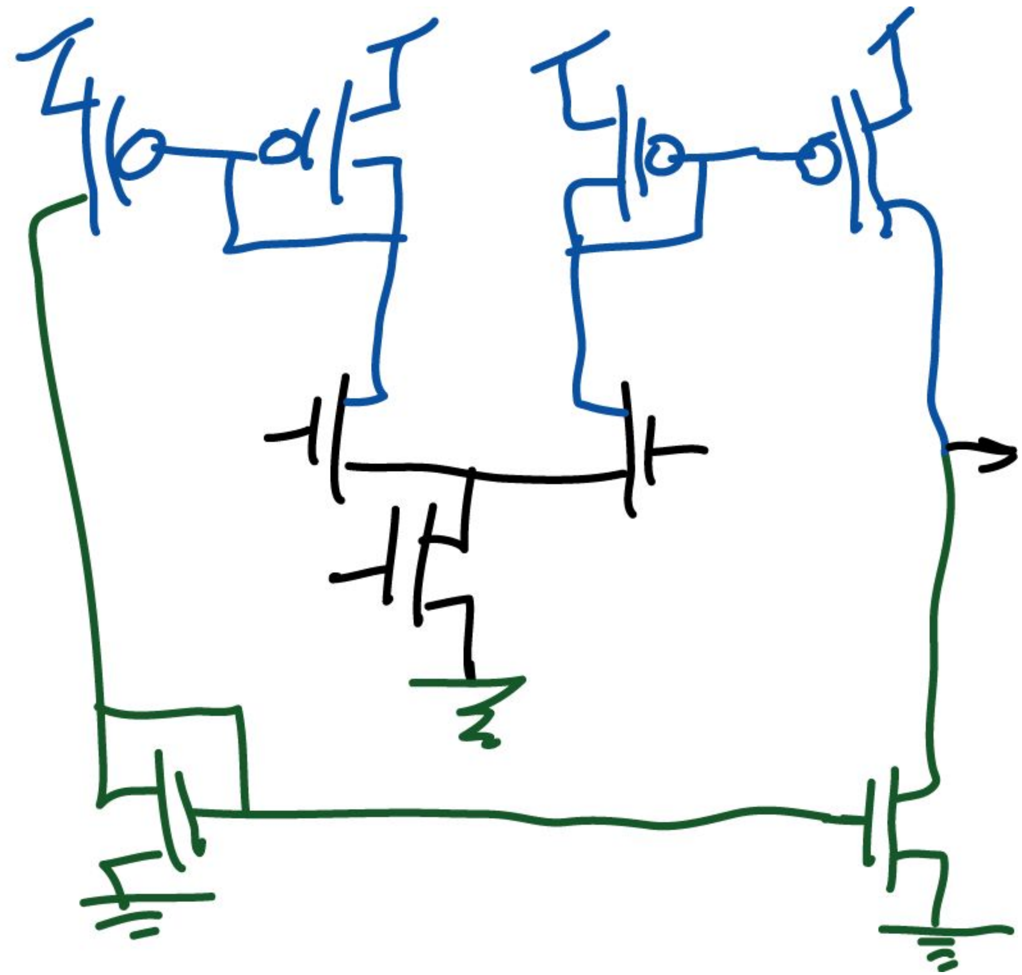






Transconductance amplifier and wide range transconductance amplifier

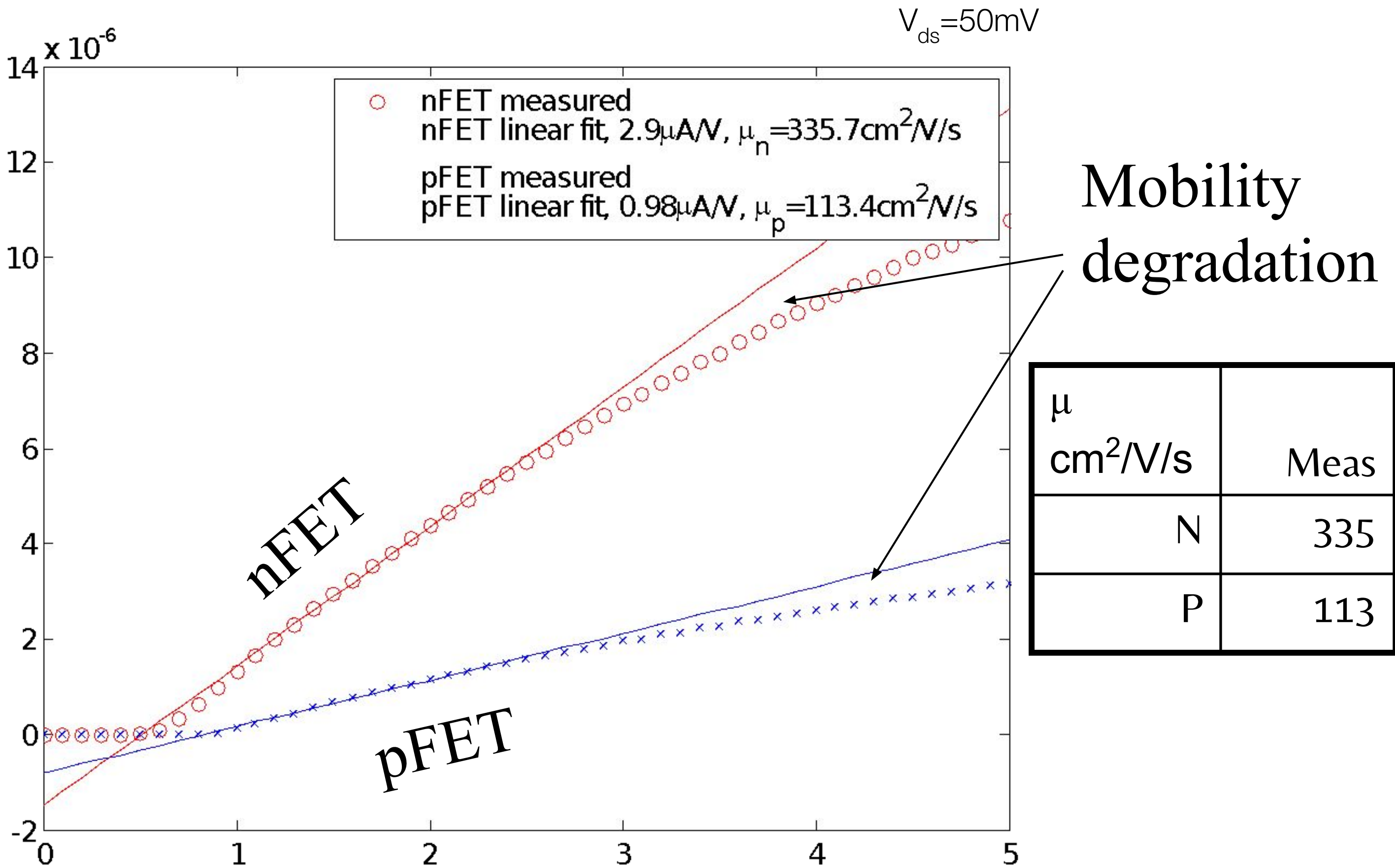




Results from lab 3
Above-threshold transistor characteristics

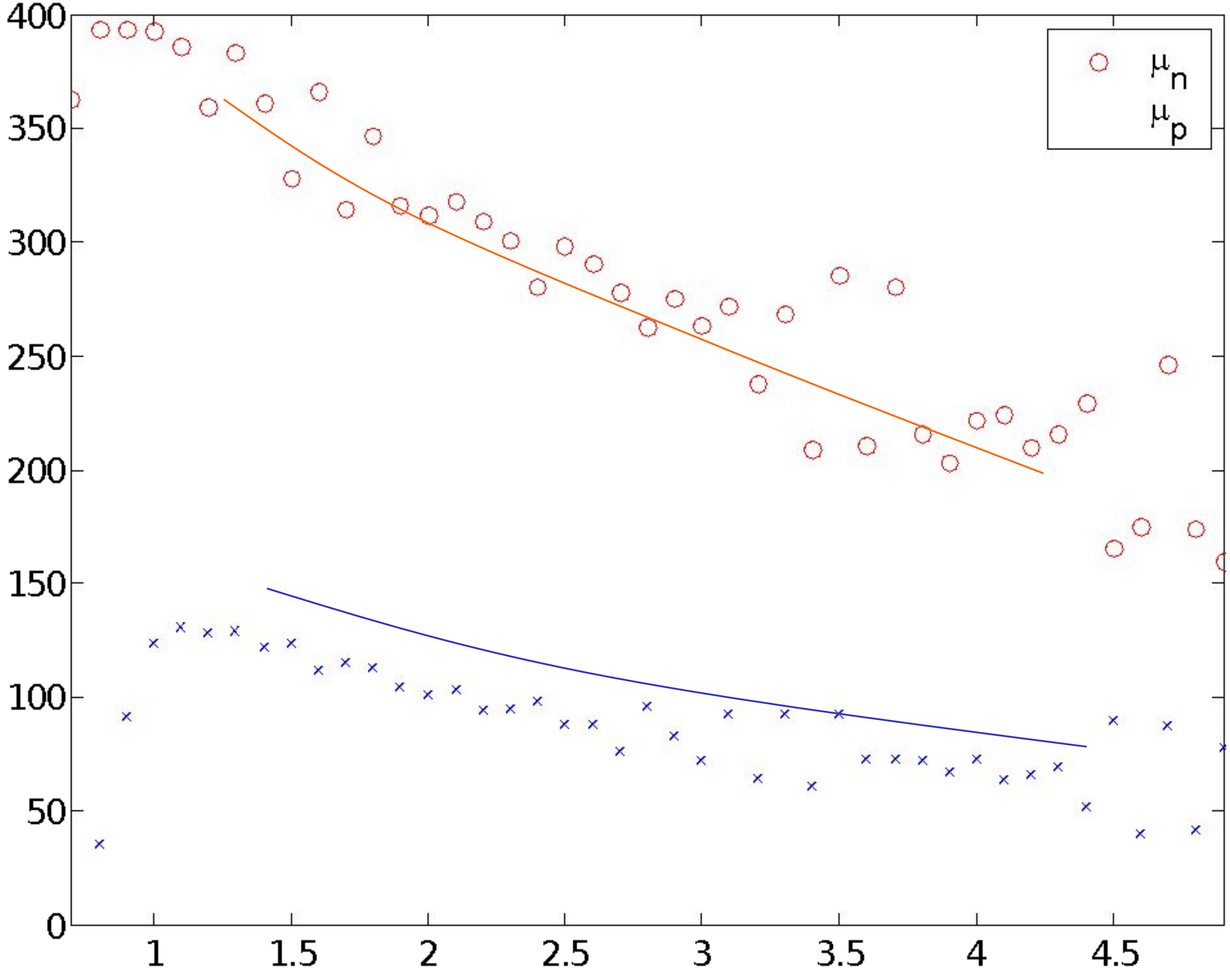
How does I_{dlin} scale with V_g-V_T?

$$\boxed{I_{ds} = \mu C_{\text{ox}} \frac{W}{L} (V_{gs} - V_{\text{T}}) V_{ds}}$$



µ cm²/V/s	Meas	Bulk
N	335	1450
P	113	500

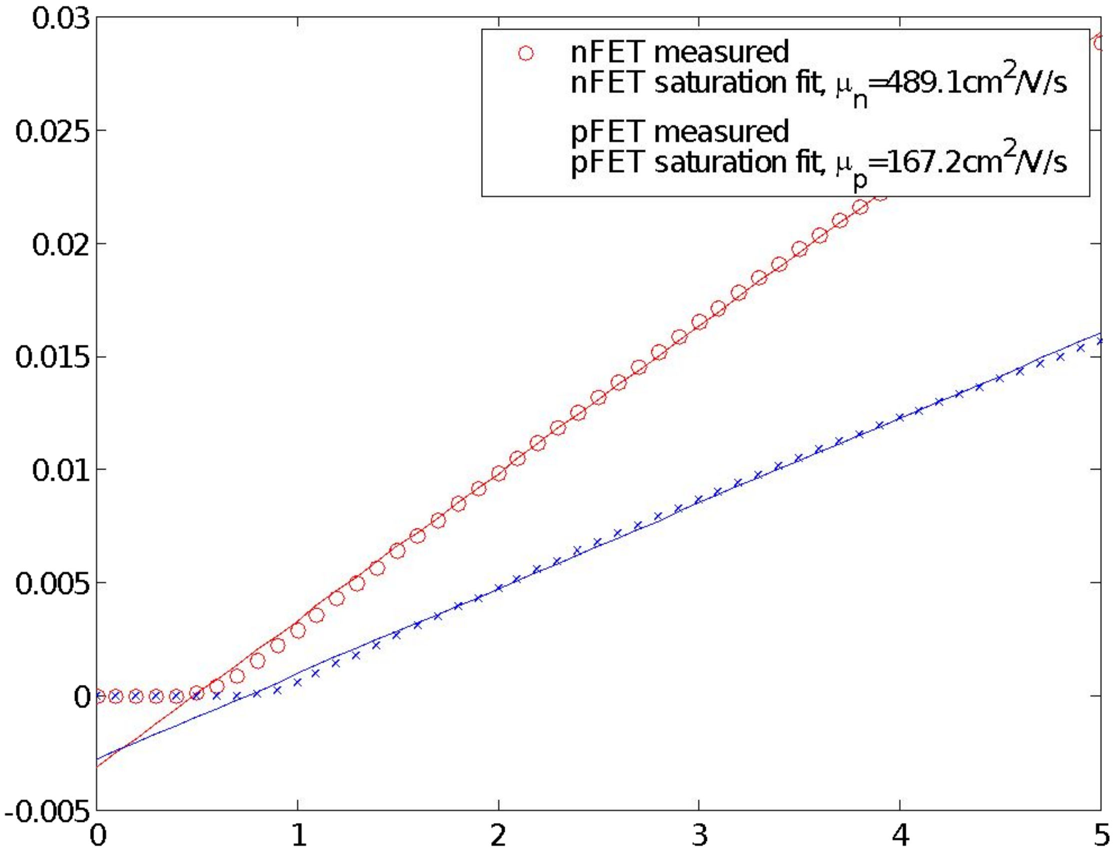
What is μ as function of V_g?



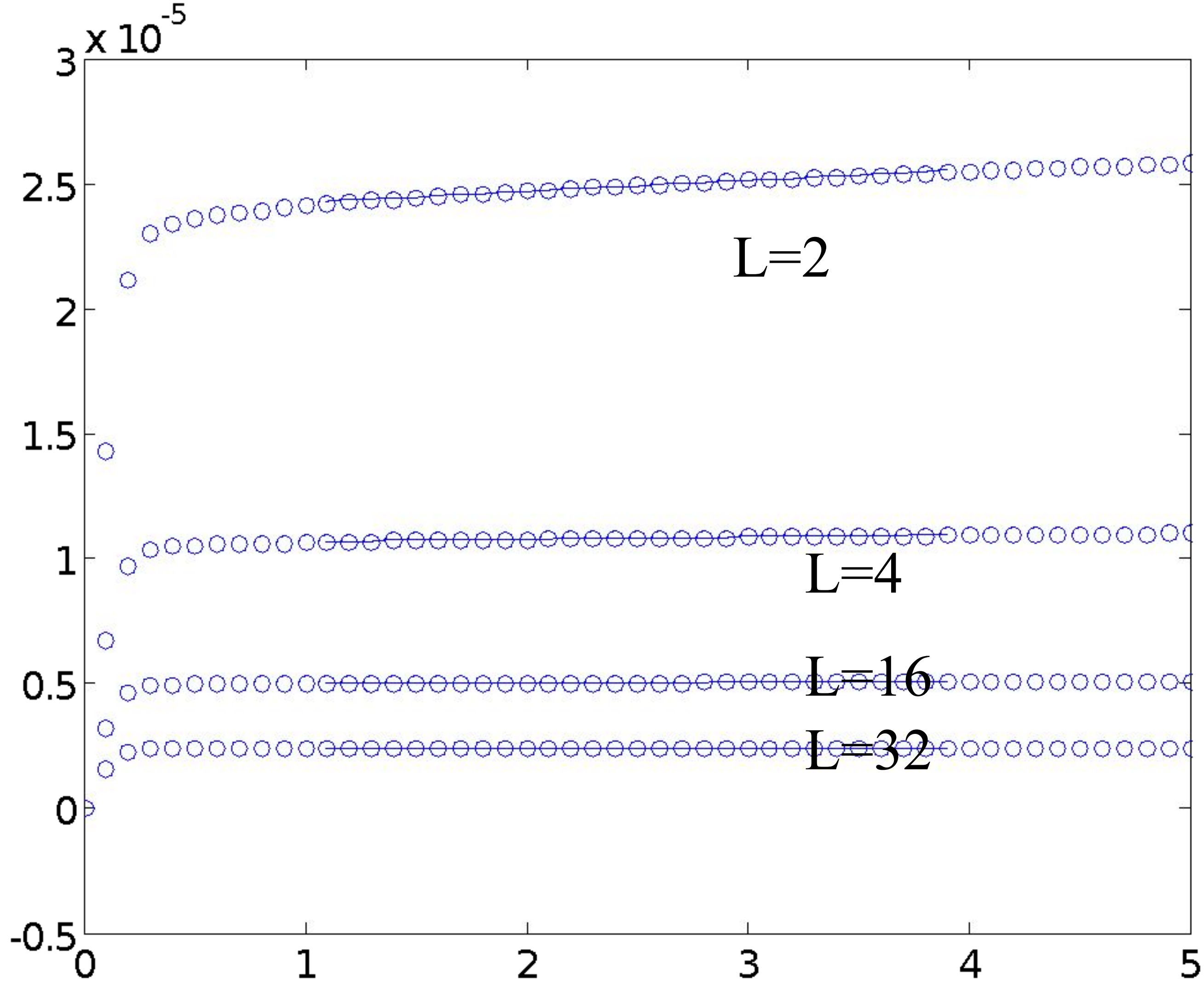
How does $\sqrt{I_{\mathrm{dsat}}}$ scale with V_g-V_T?

Saturation current

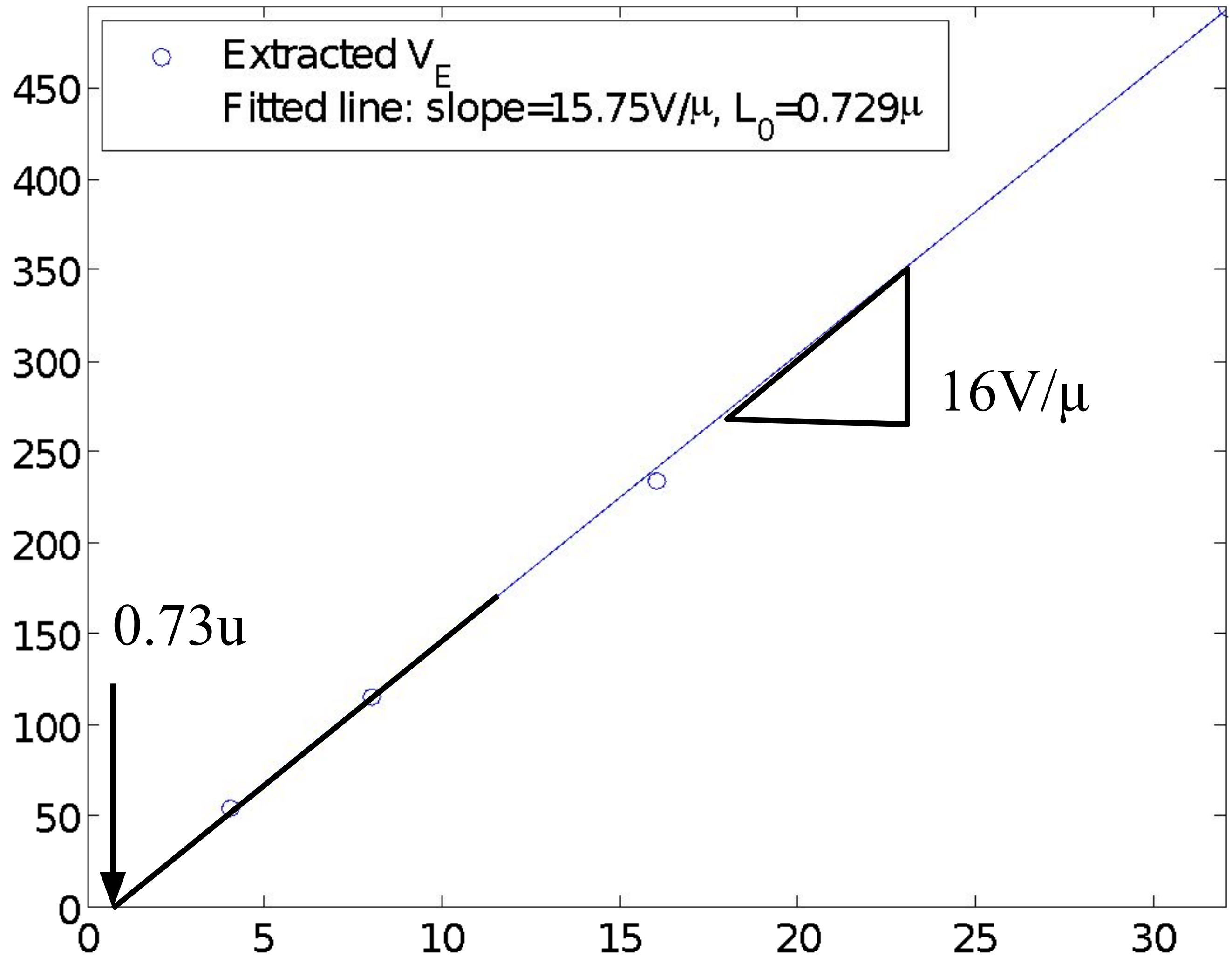
$$\boxed{\sqrt{I_{ds,sat}} = \sqrt{\frac{\mu C_{\mathrm{ox}}}{2}\frac{W}{L}}(V_{gs} - V_{\mathrm{T}})}$$



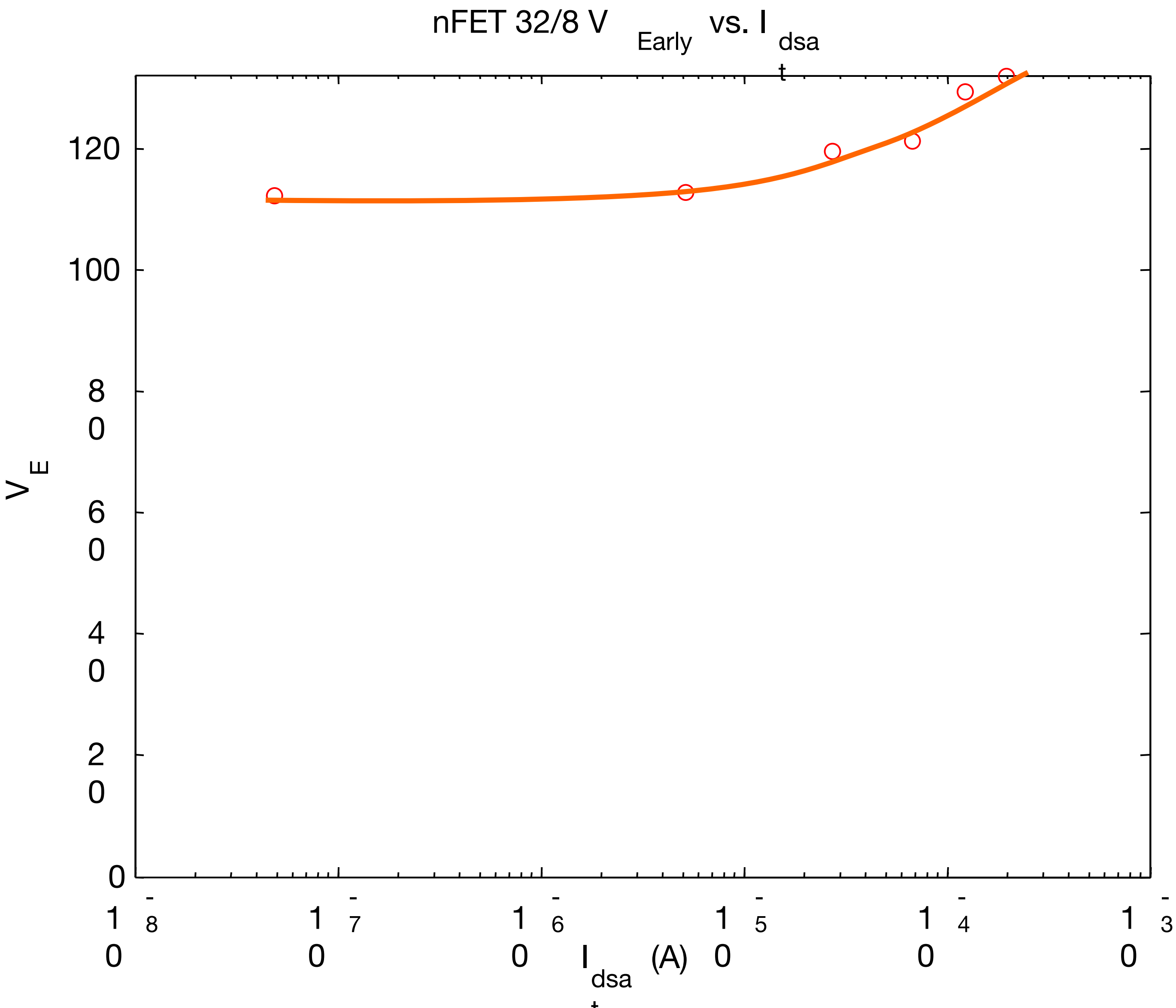
How does I_{ds} scale with L?



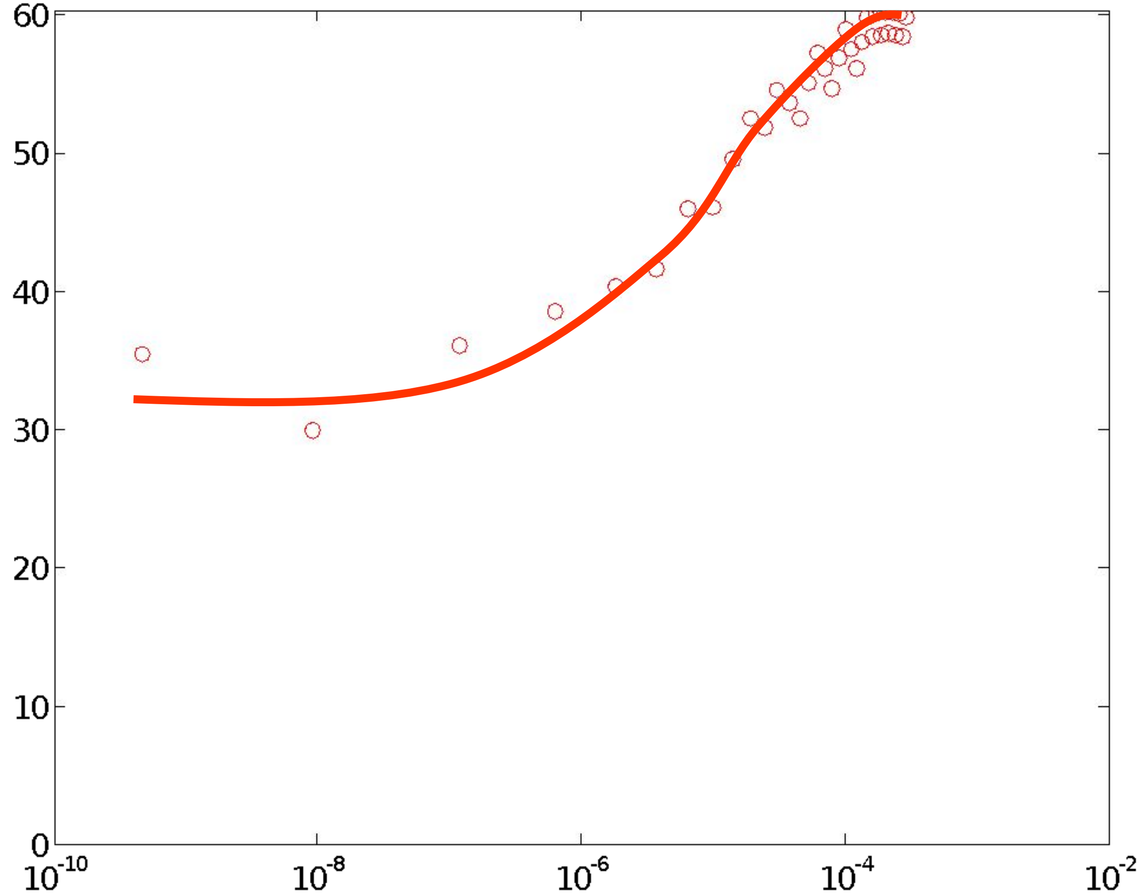
How does V_E change with L?



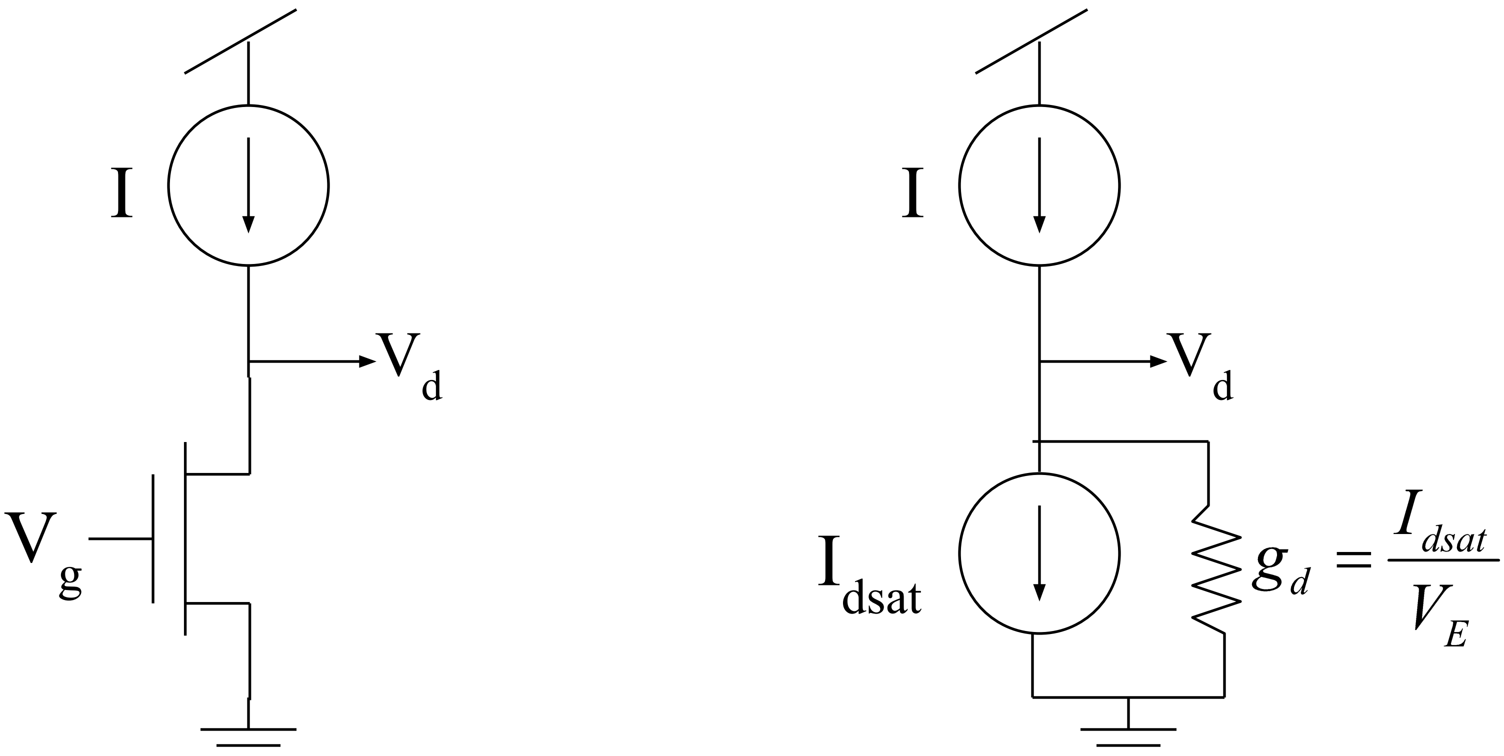
Is V_E constant with I_{dsat}?



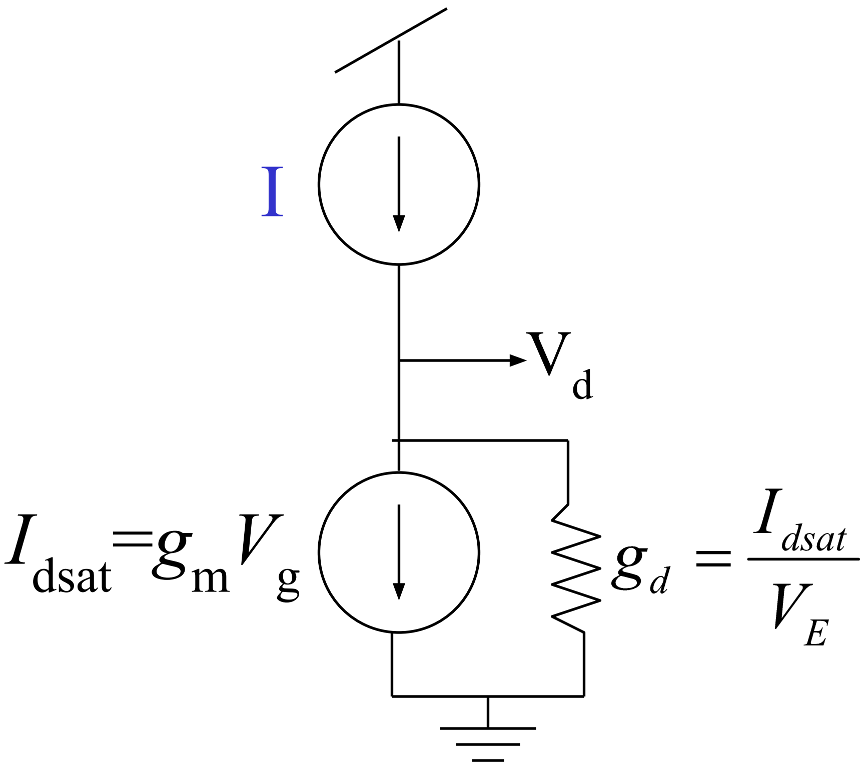
pFET V_E vs. I_{dsat}

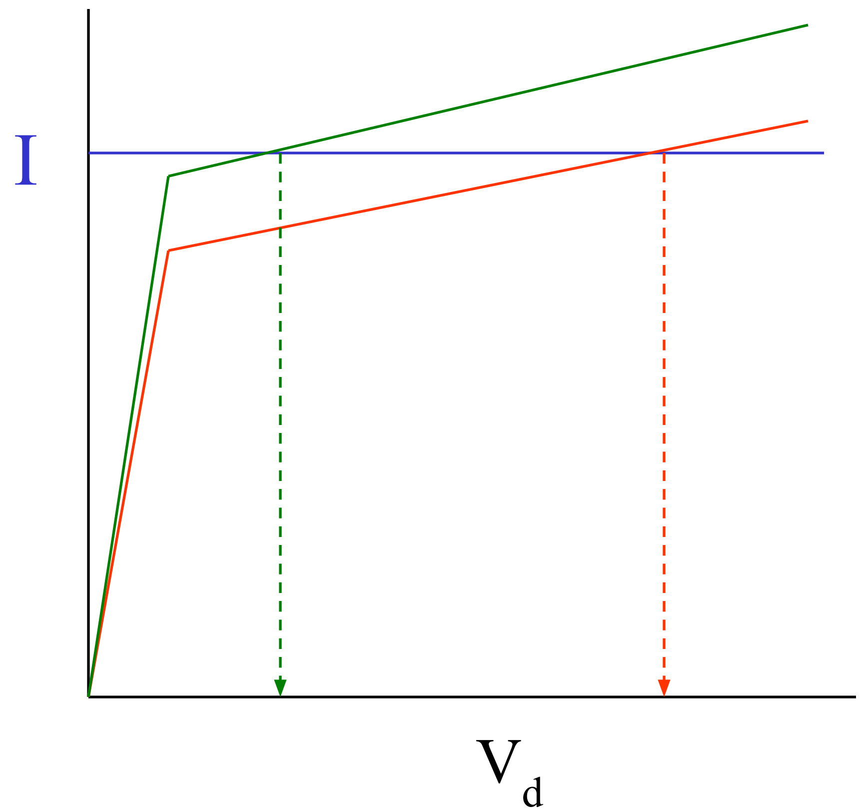


Solution: Intrinsic transistor voltage gain

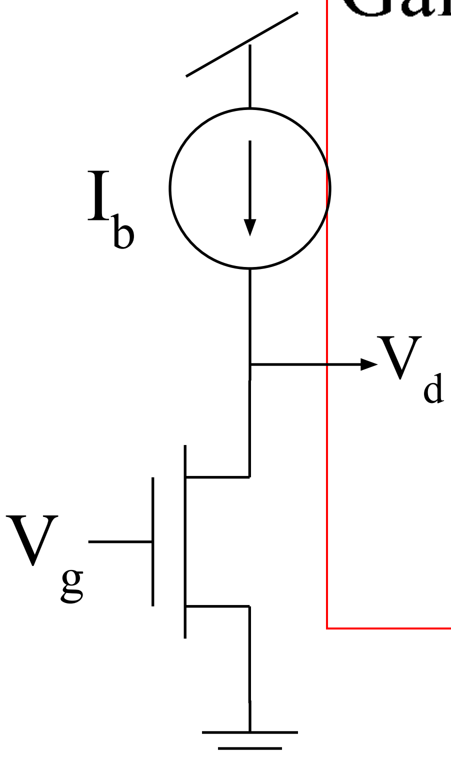


Intrinsic transistor voltage gain





Intrinsic transistor voltage gain



$$\text{Gain } -A = \frac{\partial V_d}{\partial V_g}$$

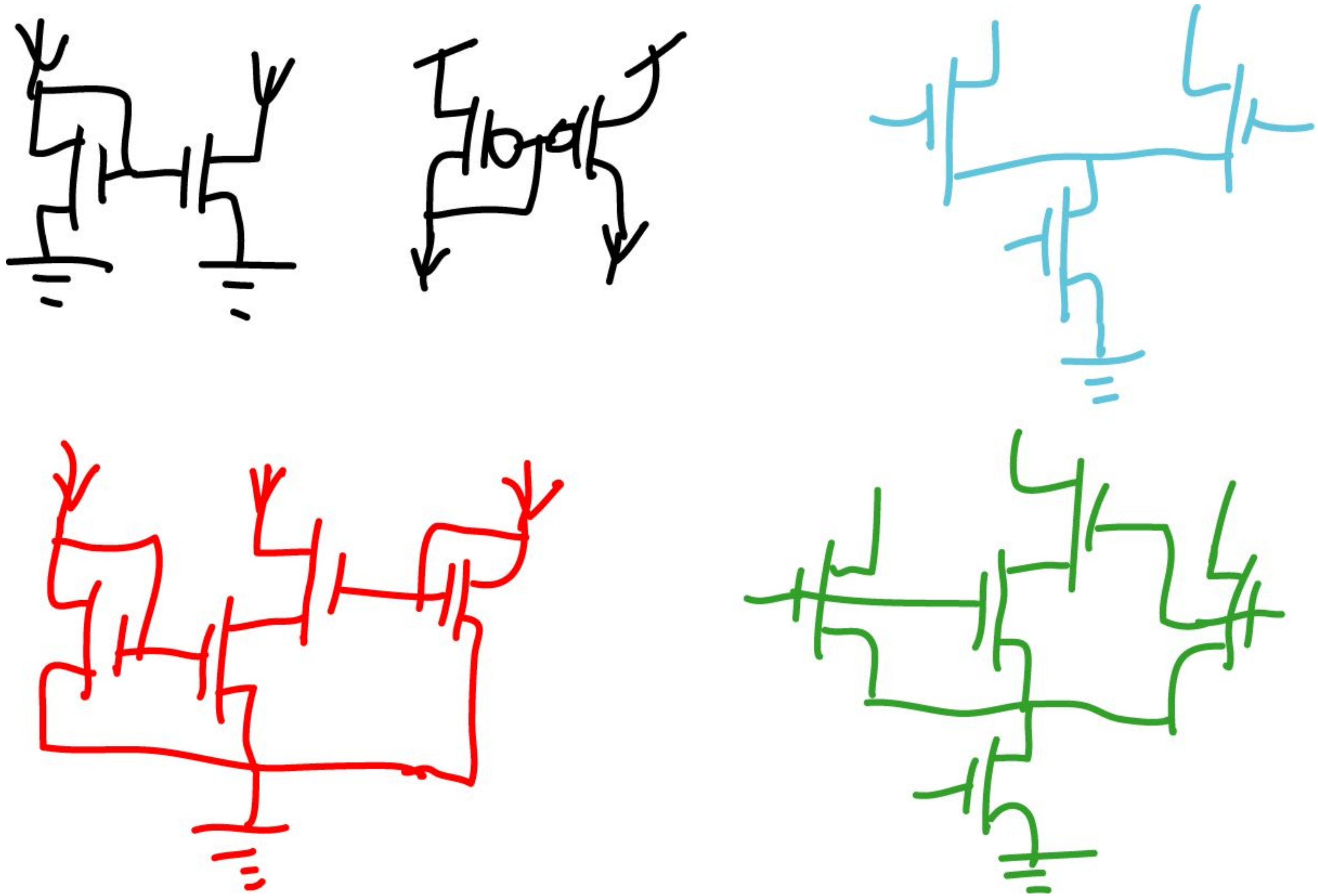
$$= \frac{\partial I}{\partial V_g}\frac{\partial V_d}{\partial I}$$

$$= \frac{g_m}{g_{ds}} = g_m r_o = \frac{\kappa V_E}{U_T} \quad \text{(subthreshold)}$$

Typical value: $A = \dfrac{.75 * 100\text{V}}{(1/40)\text{V}} = 3000$

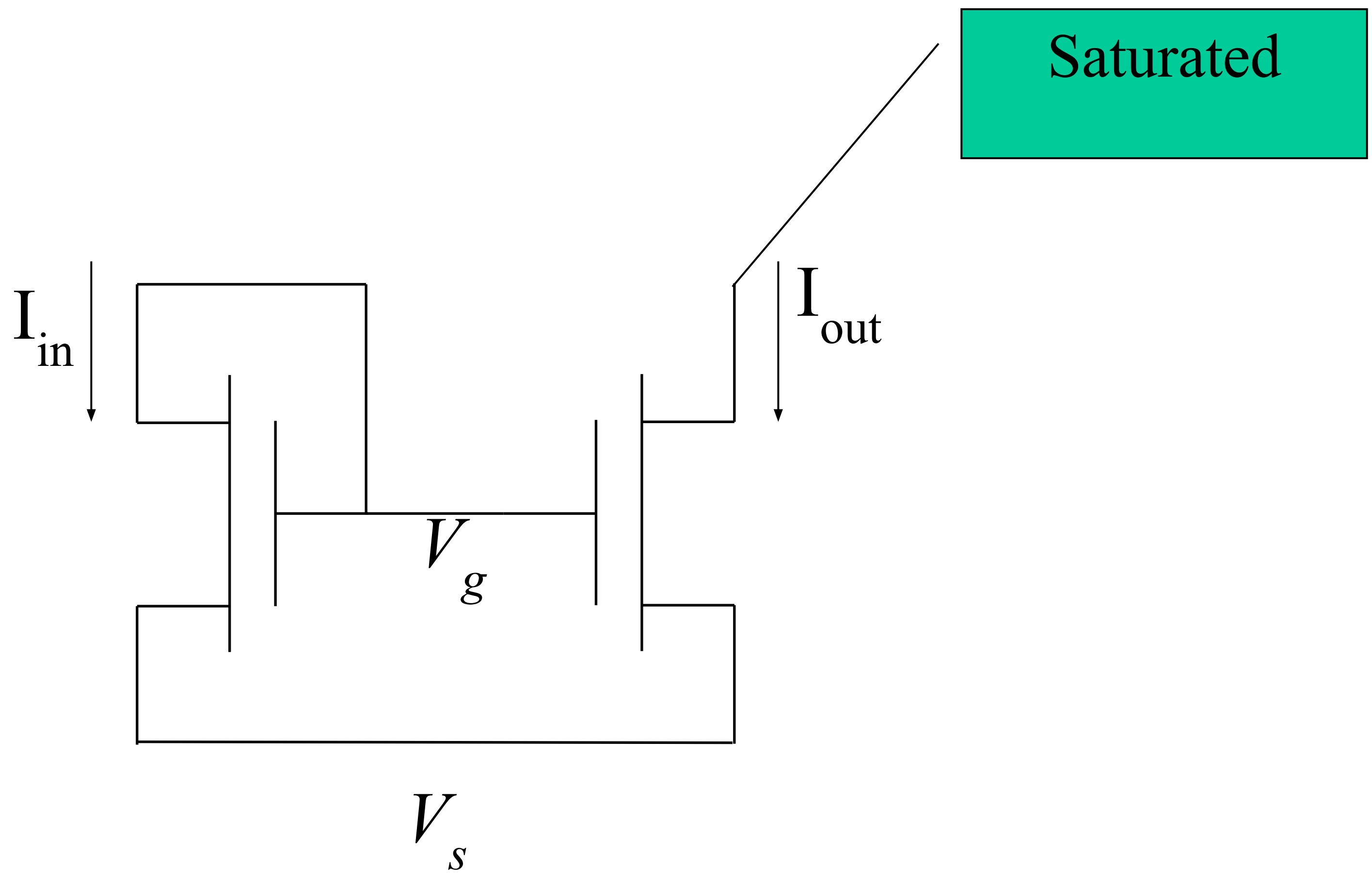
The same equation $A = g_m/g_{out}$ applies to any amplifier, e.g. transamp. g_{out} refers to the total output conductance.

Current mirror, diff-pair, current correlator, bump-antibump circuit

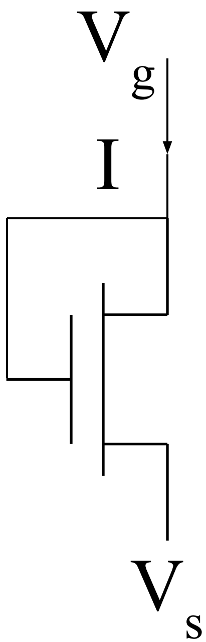


Current Mirror

The output current is a copy of the input current.
It is *mirrored* because it is sunk, not sourced.



Diode-connected transistors



$$I = e^{\kappa V_g - V_s}\left(1 - e^{-(V_d - V_s)}\right)$$

$$V_g = V_d$$

$$= e^{\kappa V_g - V_s}\left(1 - e^{-(V_g - V_s)}\right)$$

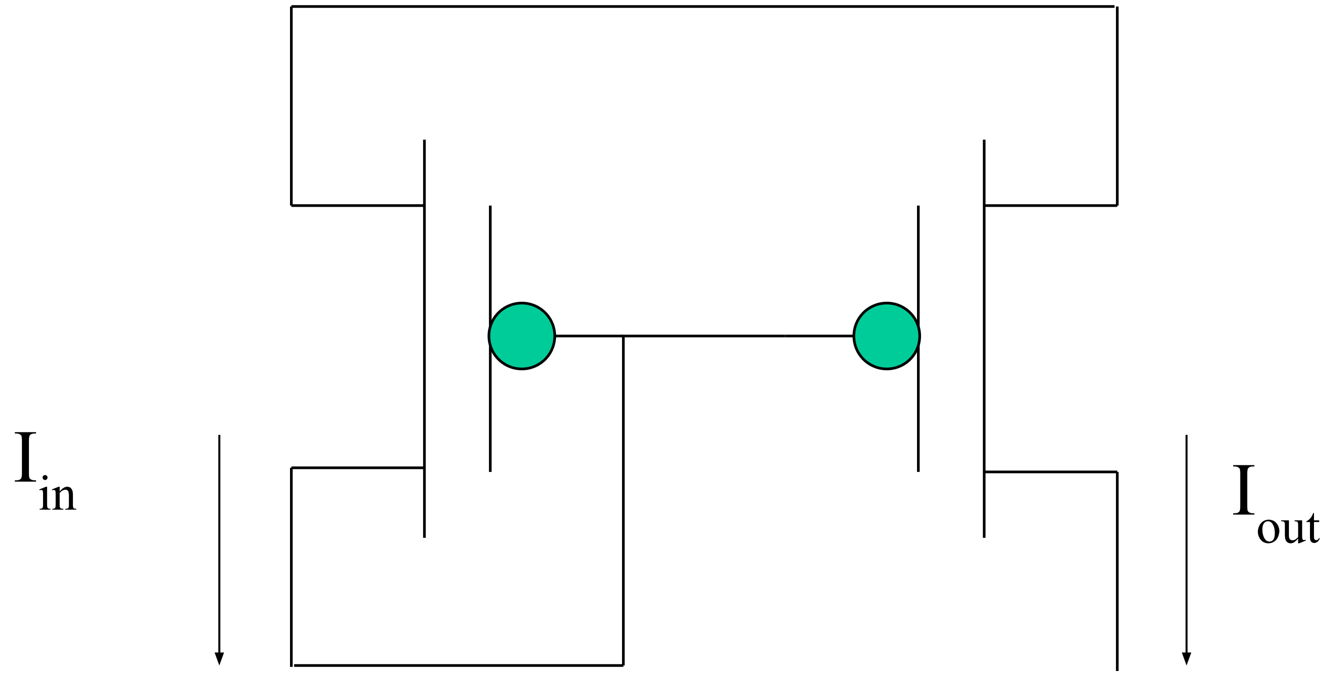
$$= e^{\kappa V_g - V_s} - e^{-(1-\kappa)V_g}$$

$$e^{-(1-\kappa)V_g} \to 0$$

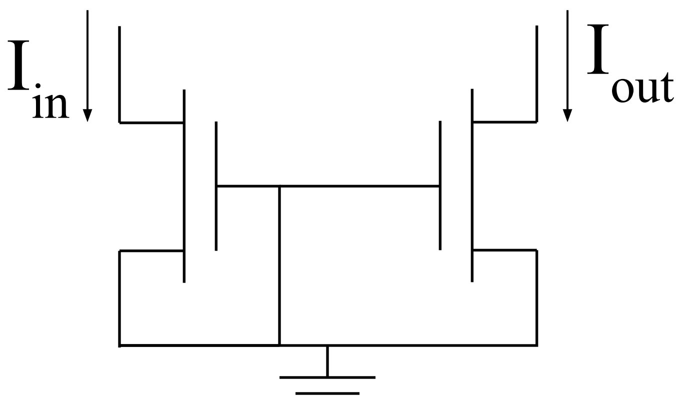
whenever a current more than a few I_0 flows

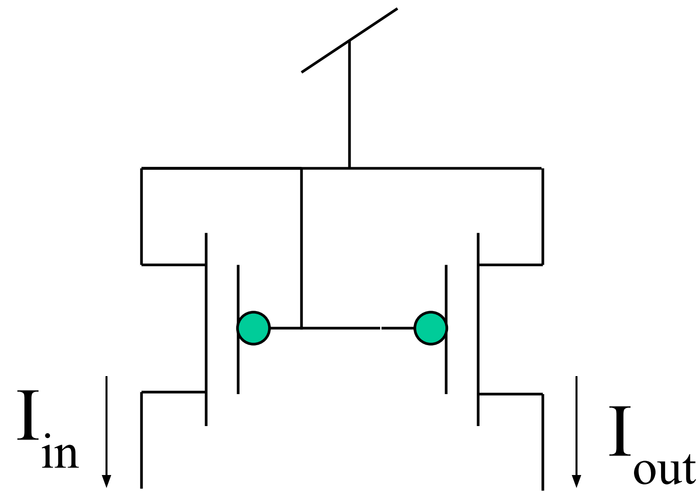
$$\boxed{I = e^{\kappa V_g - V_s}}$$

pFET mirror



How about these configurations?



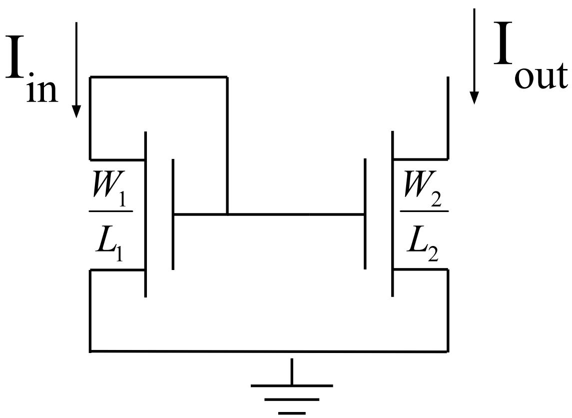


$$V_{gs}\!=\!0\text{, so } I_{out}\!=\!I_0$$

Current mirror with gain (tilted mirror) (I)

How do you make an output current that is M times the input current?

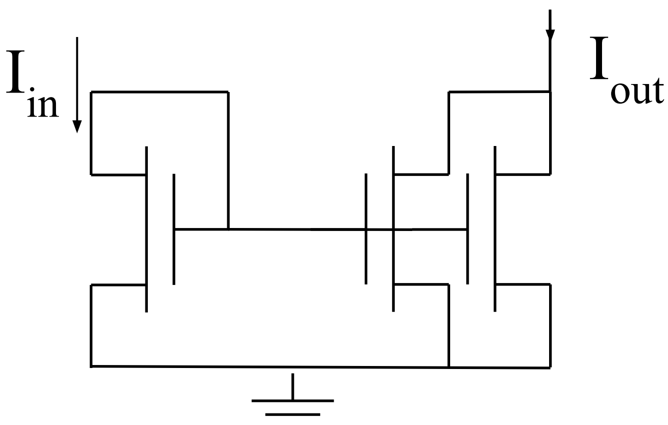
METHOD 1: USE TRANSISTOR GEOMETRY



$$\text{Gain } M = \frac{W_2/L_2}{W_1/L_1}$$

Not very accurate when M!=1

METHOD 2: USE MULTIPLE TRANSISTORS



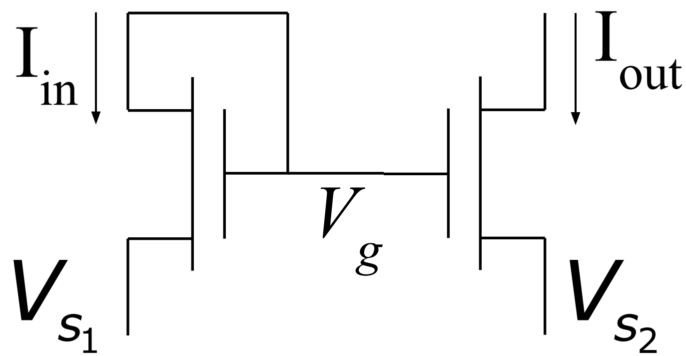
$$\text{e.g. Gain } M = 2$$

More accurate

$$\text{Precision} \propto \frac{1}{\sqrt{WL}}$$

Current mirror with gain (tilted mirror) (II)

METHOD 3: USE A DIFFERENT V_s

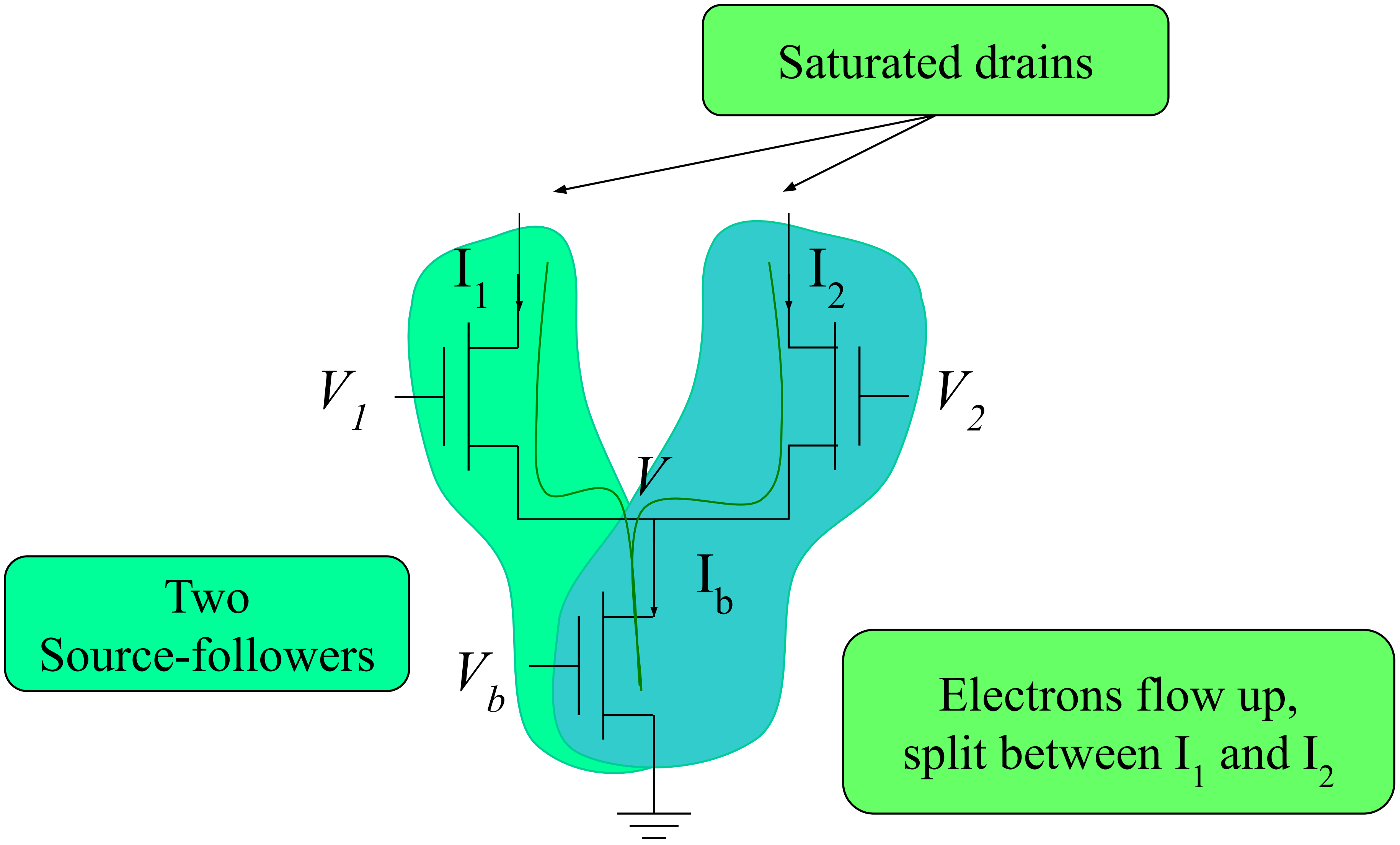


$$\boxed{\begin{aligned} I_{in} &= e^{\kappa V_g - V_{s1}} \\ I_{out} &= e^{\kappa V_g - V_{s2}} \end{aligned}}$$

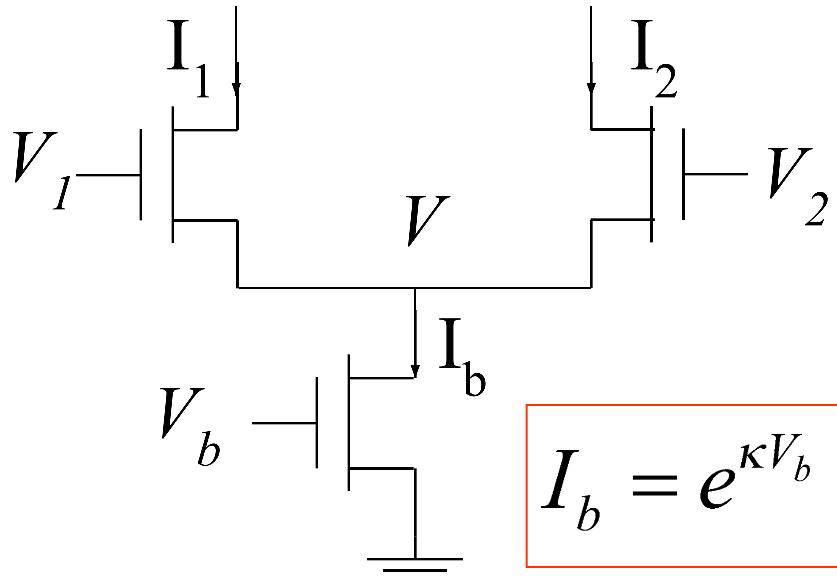
$$[V] = U_T$$

$$\boxed{\text{Gain } M = \frac{I_2}{I_1} = e^{V_{s1} - V_{s2}}}$$

Differential Pair (I)



Differential Pair (II)



$$I_1 = I_0 e^{\kappa V_1 - V}$$

$$I_2 = I_0 e^{\kappa V_2 - V}$$

$$I_b = e^{\kappa V_b} = I_1 + I_2 = I_0 e^{-V}\left(e^{\kappa V_1} + e^{\kappa V_2}\right)$$

$$\Rightarrow e^{V} = \frac{e^{\kappa V_1} + e^{\kappa V_2}}{e^{\kappa V_b}}$$

$$V = \ln\left(e^{\kappa V_1} + e^{\kappa V_2}\right) - \kappa V_b$$

$$\approx \kappa\left(V_1 - V_b\right) \text{ for } V_1 - V_2 \geq 100\text{mV}$$

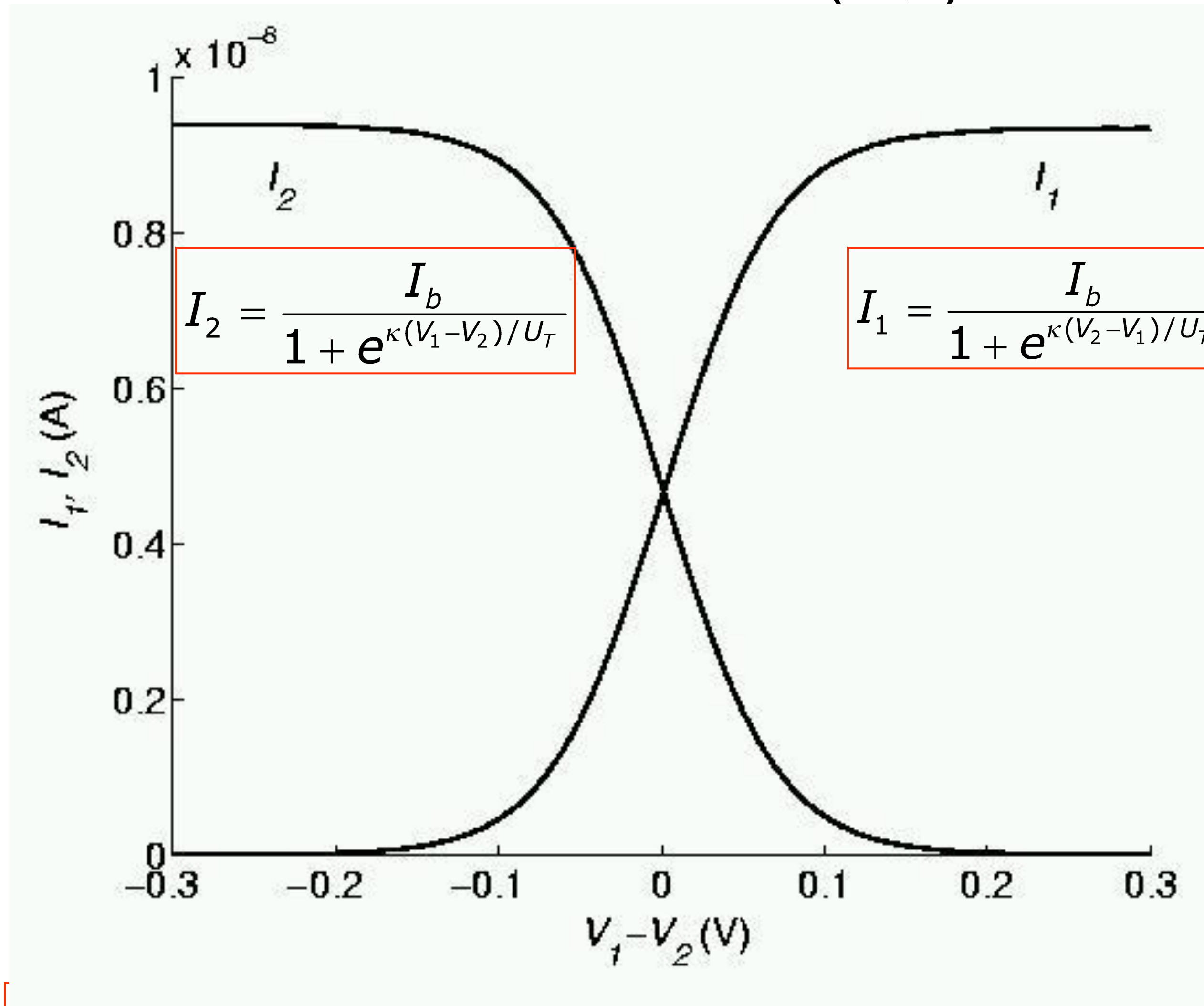
Differential Pair (III)

$$\boxed{\begin{aligned} I_1 &= \frac{I_b e^{\kappa V_1}}{e^{\kappa V_1} + e^{\kappa V_2}} \\ &= \frac{I_b}{1 + e^{\kappa (V_2 - V_1)}} \end{aligned}}$$

$$\boxed{I_2 = \frac{I_b}{1 + e^{\kappa (V_1 - V_2)}}}$$

FERMI
FUNCTIONS

Differential Pair (IV)



$$I_2 = \frac{I_b}{1 + e^{\kappa (V_1 - V_2)/U_T}}$$

$$I_1 = \frac{I_b}{1 + e^{\kappa (V_2 - V_1)/U_T}}$$

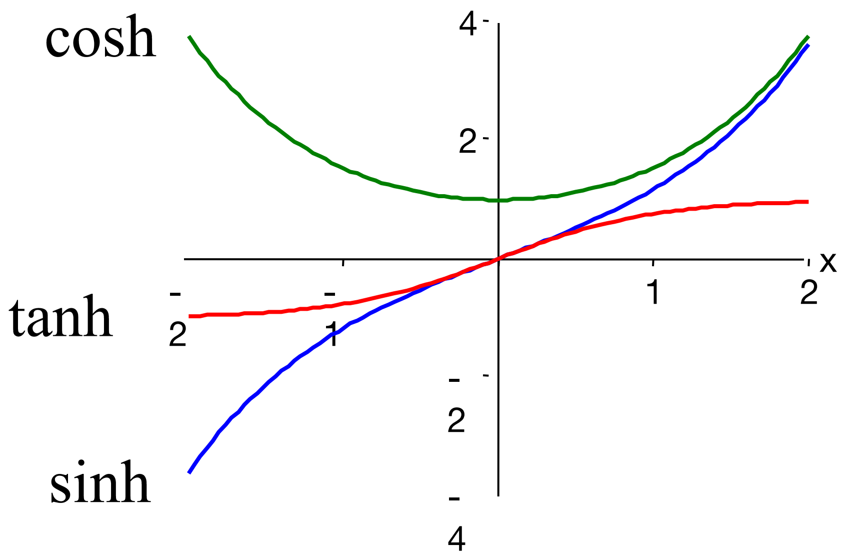
$$I_1 - I_2 = I_b \frac{e^{\kappa V_1} - e^{\kappa V_2}}{e^{\kappa V_1} + e^{\kappa V_2}} = I_b \tanh\left(\kappa \frac{V_1 - V_2}{2}\right)$$

Digression: Hyperbolic functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \approx x \text{ for small x}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \approx 1 + \frac{x^2}{2} \text{ for small x}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \approx x \text{ for small x}$$



tanh: *compressive*
sinh: *expansive*

Differential pair in strong inversion (V)

•In **strong inversion**, solve for I_1 & I_2 this way:

$$\Delta V \equiv V_1 - V_2 = \sqrt{\frac{2I_1}{\beta}} - \sqrt{\frac{2I_2}{\beta}}$$

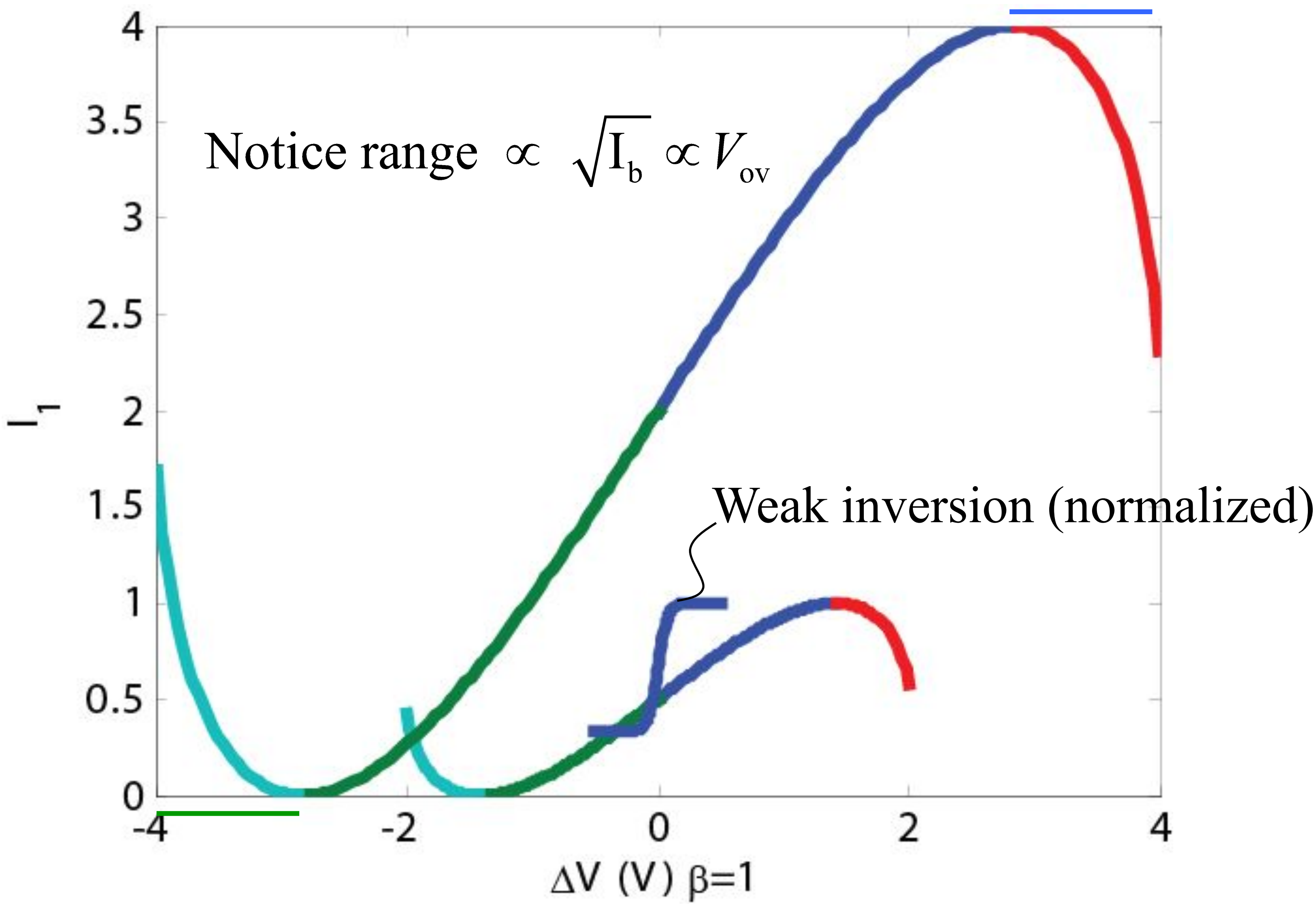
You eventually obtain

$$I_{1,2} = \frac{I_b}{2}\left(1 \pm \sqrt{\frac{\beta \Delta V^2}{I_b} - \left(\frac{\beta \Delta V^2/2}{I_b}\right)^2}\right)$$

for $\dfrac{\beta \Delta V^2}{2} < I_b$

$$I_1 - I_2 = I_b \sqrt{\frac{\beta \Delta V^2}{I_b} - \left(\frac{\beta \Delta V^2/2}{I_b}\right)^2}$$

Differential pair in weak and strong inversion



Differential pair transconductance (VII)

Transconductance of differential tail current

$$g_m = \frac{d\left(I_1 - I_2\right)}{d\left(V_1 - V_2\right)}$$

Weak inversion

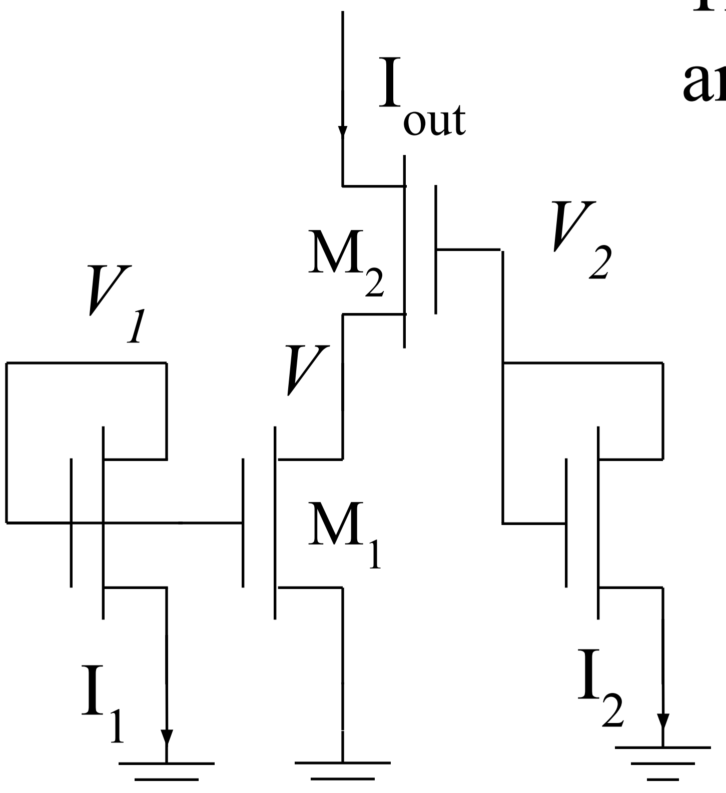
$$g_m = \frac{\kappa I_b}{2U_T}$$

Strong inversion

$$g_m = \sqrt{\beta I_b}$$

$$= \sqrt{2}\,\frac{I_b}{V_b - V_T}$$

Current Correlator



Transistor M_1 is in ohmic region and transistor M_2 is in saturation.

$$I_{out} = I_0 e^{\kappa V_1}\left(1 - e^{V}\right) \quad (M_1)$$

$$I_{out} = I_0 e^{\kappa V_2 - V} \quad (M_2)$$

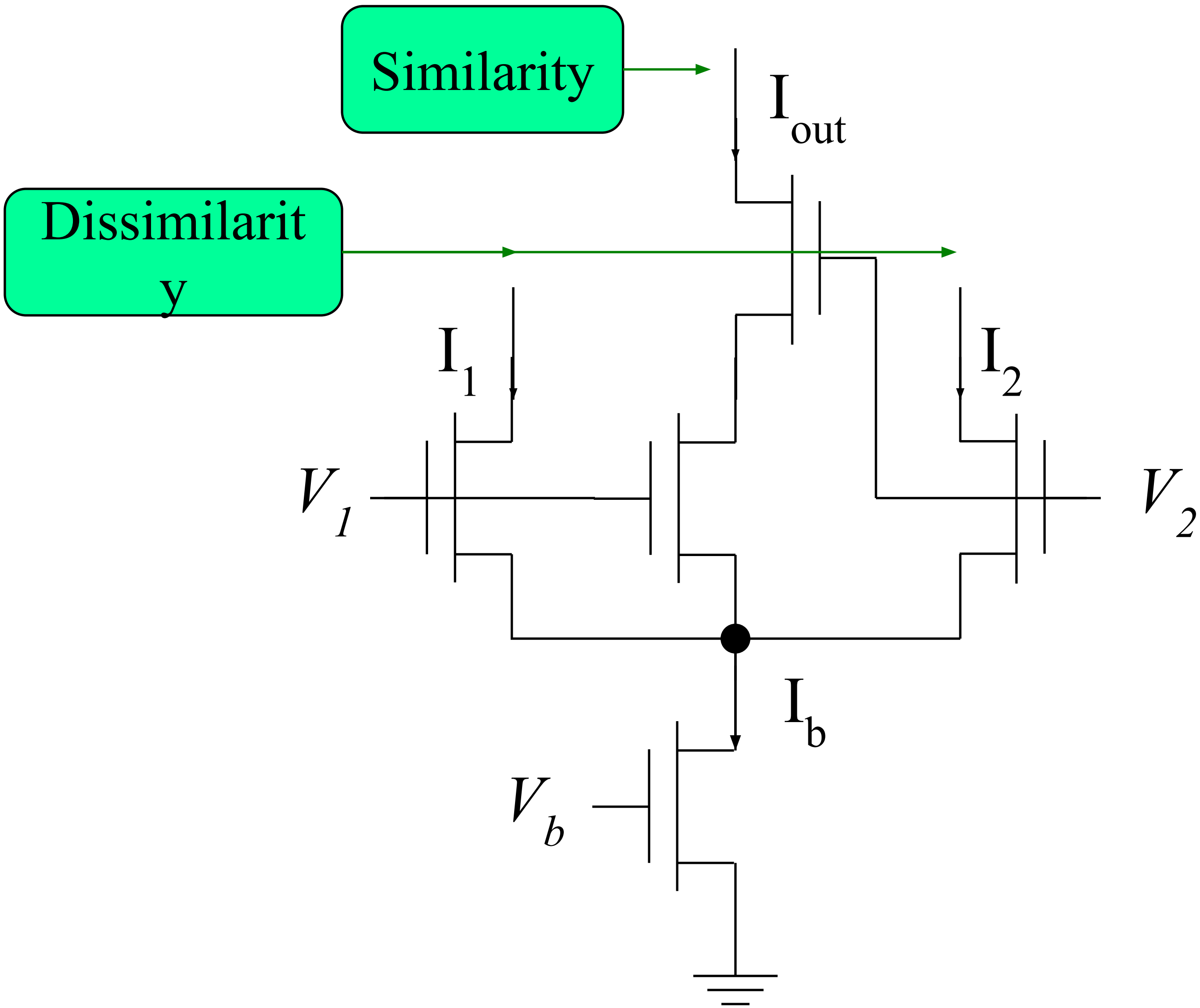
$$e^{V} = \frac{I_0 e^{\kappa V_2}}{I_{out}}$$

$$I_1 = I_0 e^{\kappa V_1} \qquad I_2 = I_0 e^{\kappa V_2}$$

$$\boxed{I_{out} = \frac{I_1 I_2}{I_1 + I_2}}$$

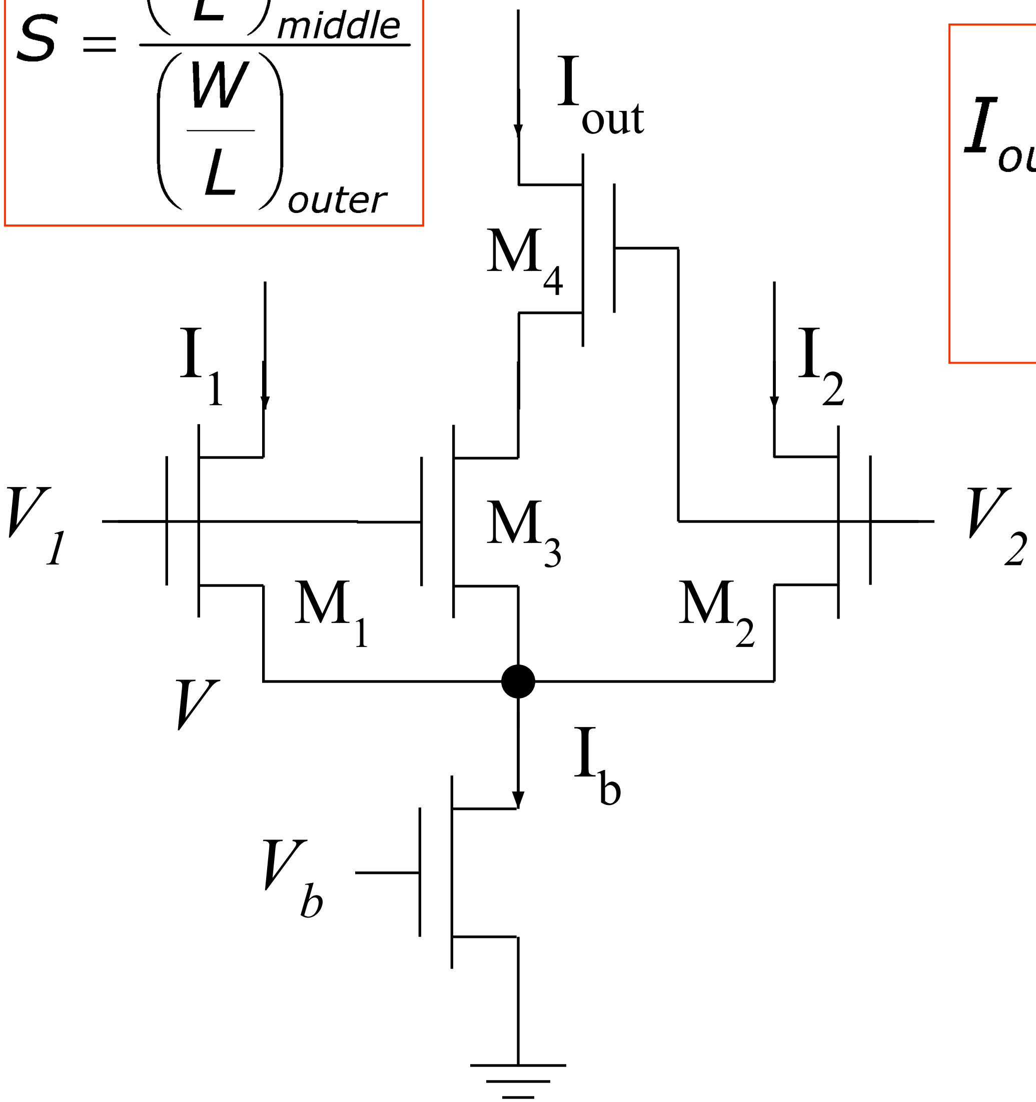
Self-normalized product

Bump-Antibump circuit



Bump Circuit

$$S = \frac{\left(\frac{W}{L}\right)_{middle}}{\left(\frac{W}{L}\right)_{outer}}$$

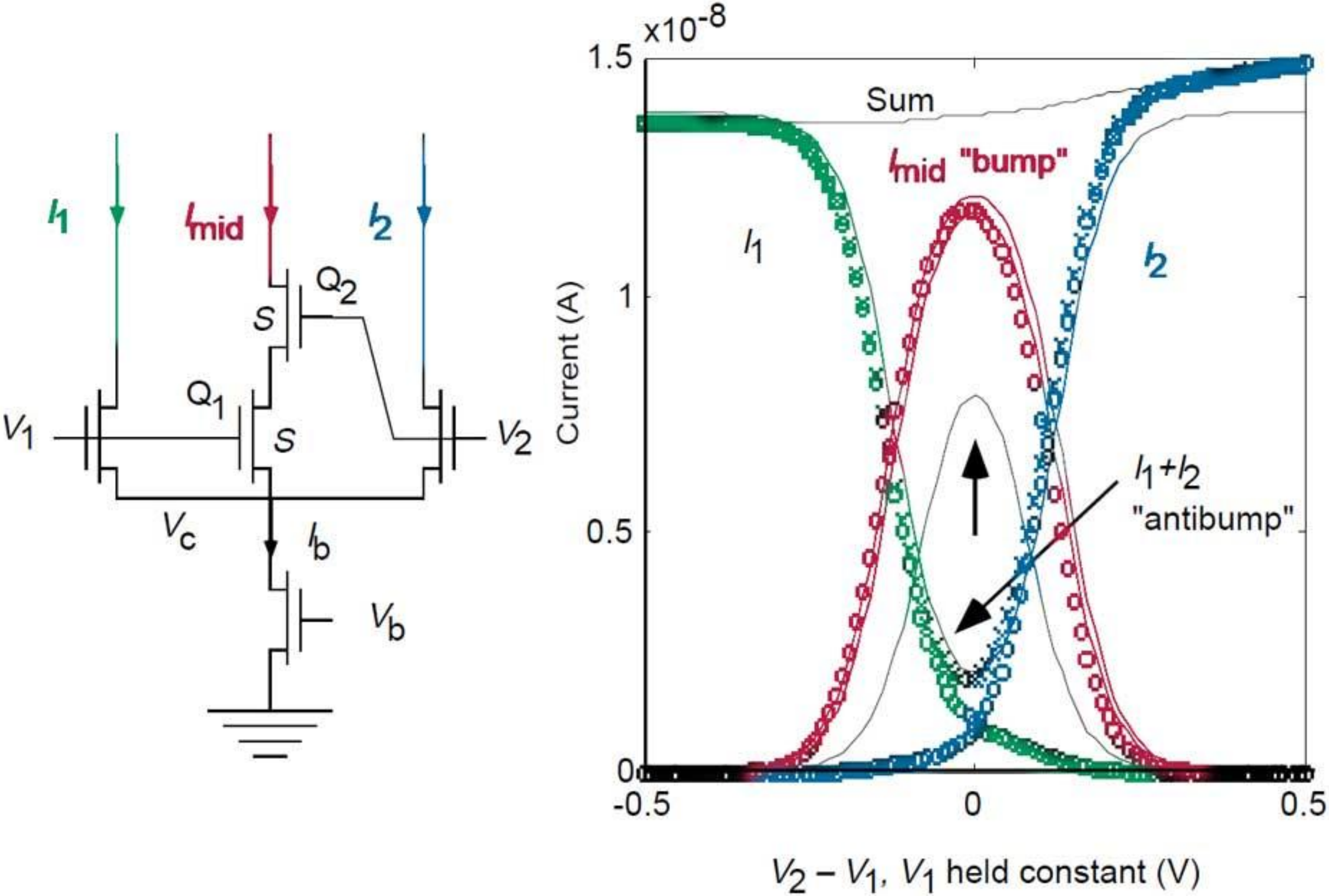


$$I_{out} = \frac{I_b}{1 + \frac{4}{S}\cosh^2\left(\frac{\kappa \Delta V}{2U_T}\right)}$$

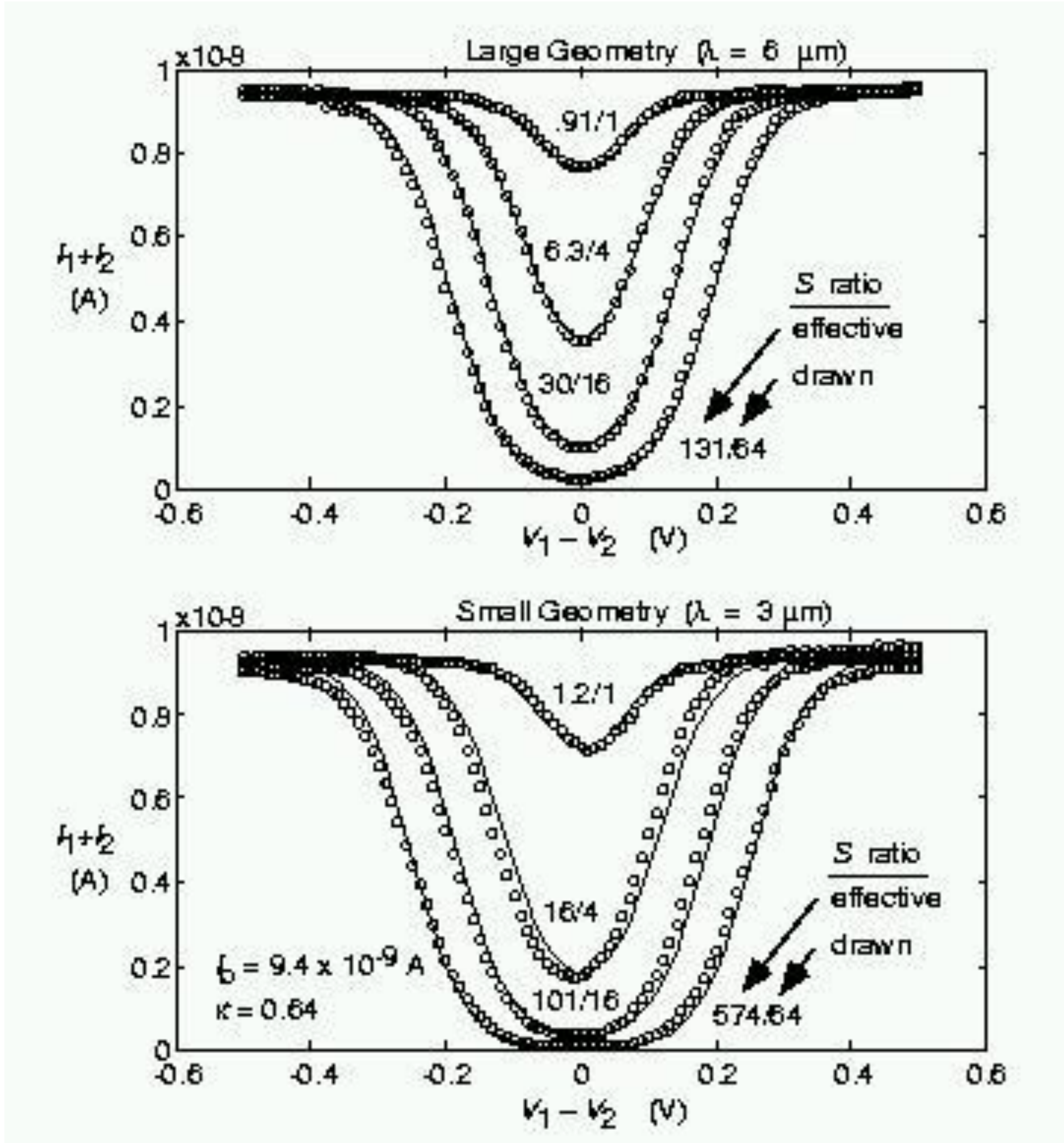
$$I_1 + I_2 = I_b - I_{out}$$
$$= \frac{I_b}{\frac{S}{4}\cosh^{-2}\left(\frac{\kappa \Delta V}{2U_T}\right) + 1}$$

$$(I_1 + I_2)_{min} = \frac{I_b}{\frac{S}{4} + 1}$$

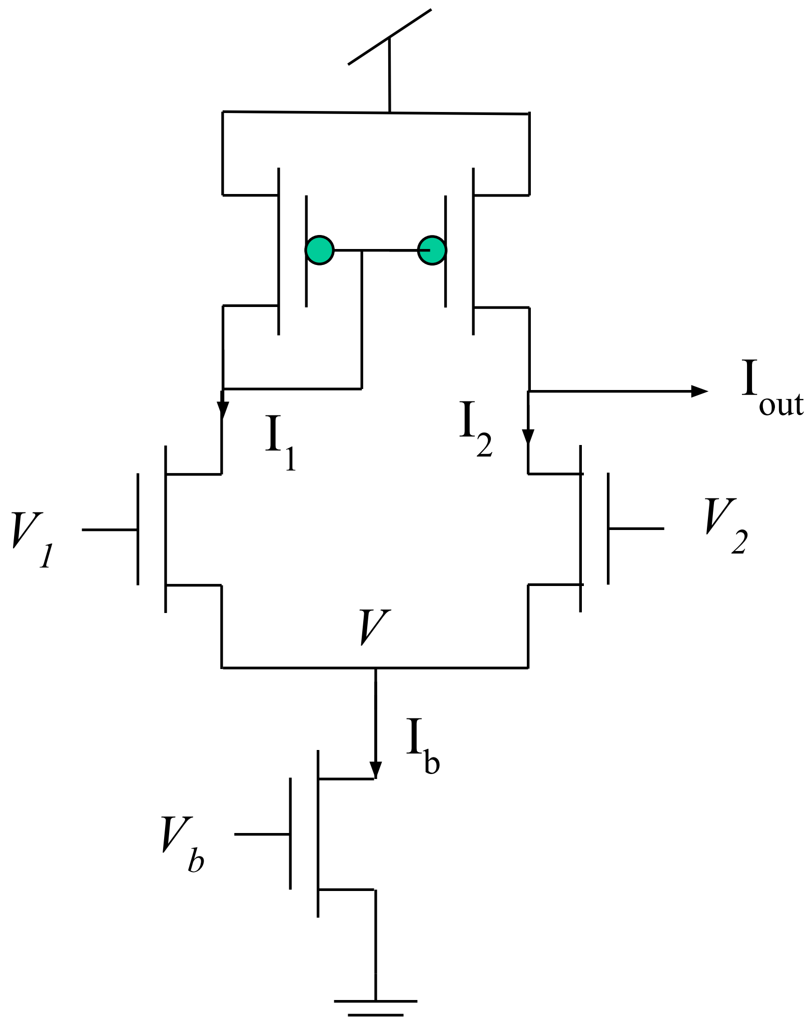
Bump Circuit



Anti-Bump Circuit



Transconductance amplifier

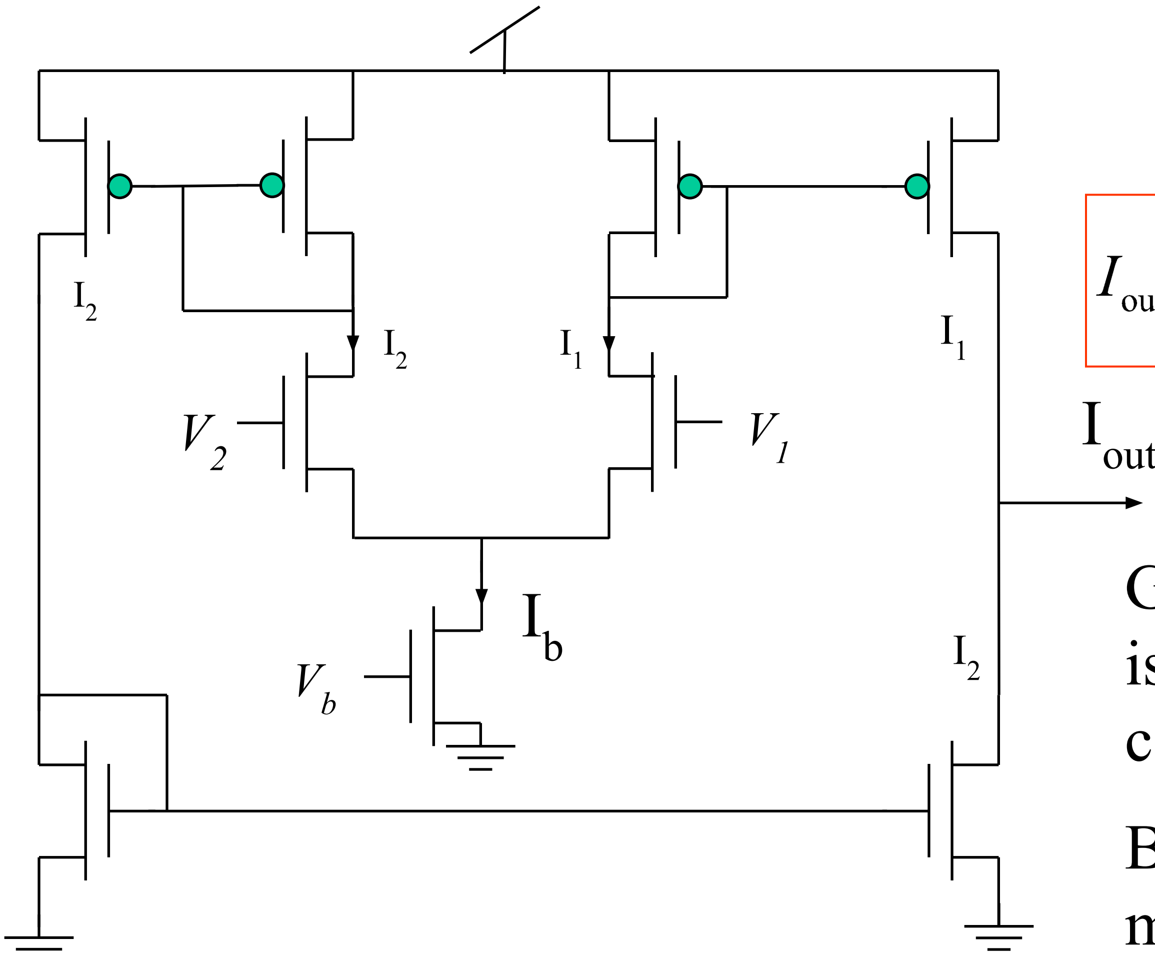


$$I_{out} = I_1 - I_2$$
$$= I_b \tanh\left(\kappa \frac{V_1 - V_2}{2}\right)$$

Good: Simple, cheap

Bad: output voltage is restricted, voltage gain is limited

Wide-range transconductance amplifier



$$I_{out} = I_b \tanh\left(\kappa \frac{V_1 - V_2}{2}\right)$$

Good: output voltage is unrestricted, gain can be $>10^4$

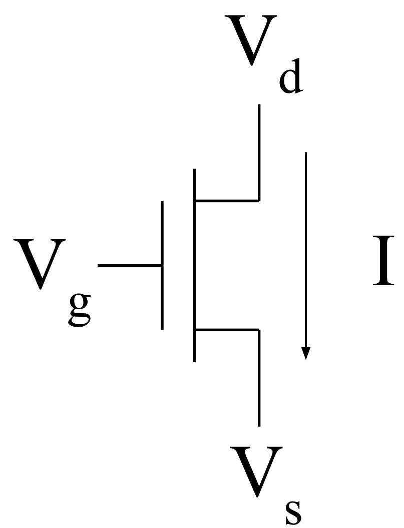
Bad: larger, more mismatch

nFET Conductances

$$\boxed{I = I_0 e^{\kappa V_g / U_T} \left(e^{-V_s / U_T} - e^{-V_d / U_T} \right)}$$

<u>Gate Transconductance</u>

$$\boxed{g_m = \frac{\partial I}{\partial V_g} = \frac{\kappa I}{U_T}}$$



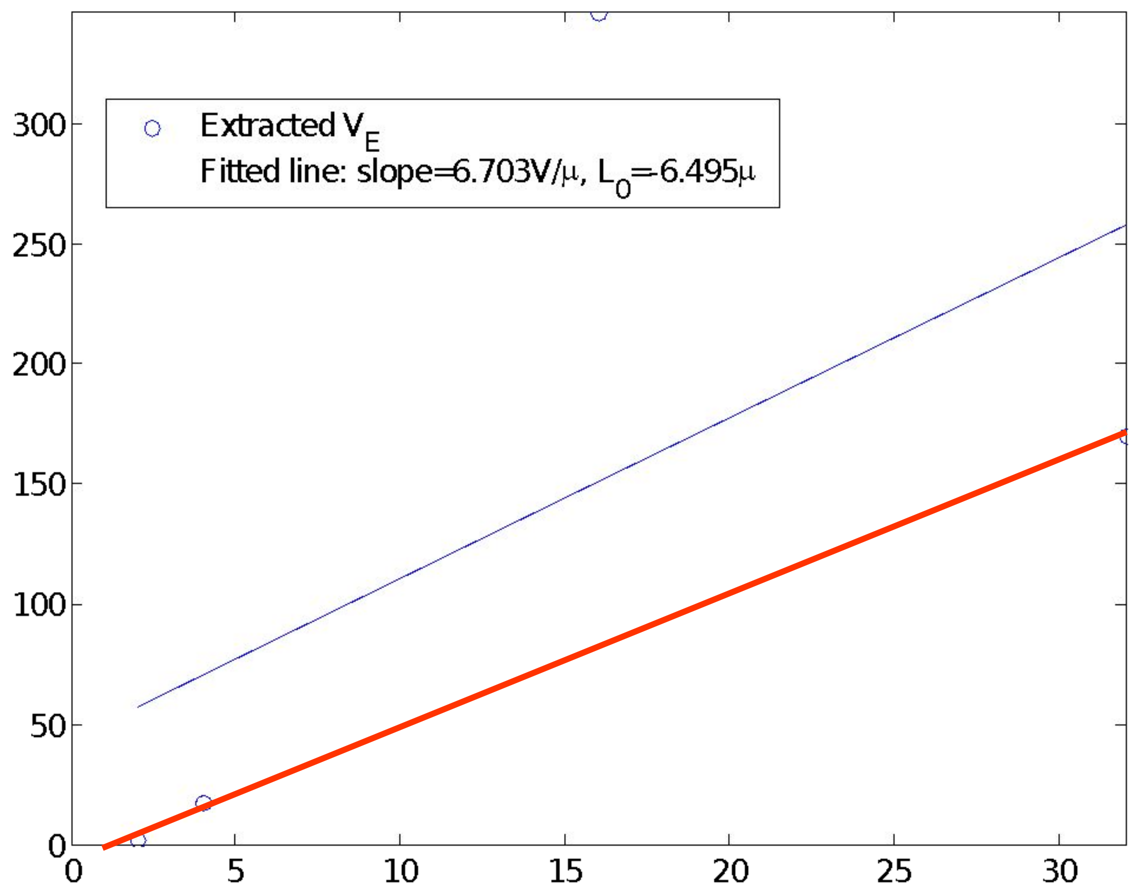
<u>Drain Conductance</u>

$$\boxed{g_d = \frac{\partial I}{\partial V_d} = \frac{I}{V_E}; \quad V_E = \text{Early Voltage}}$$

THE END

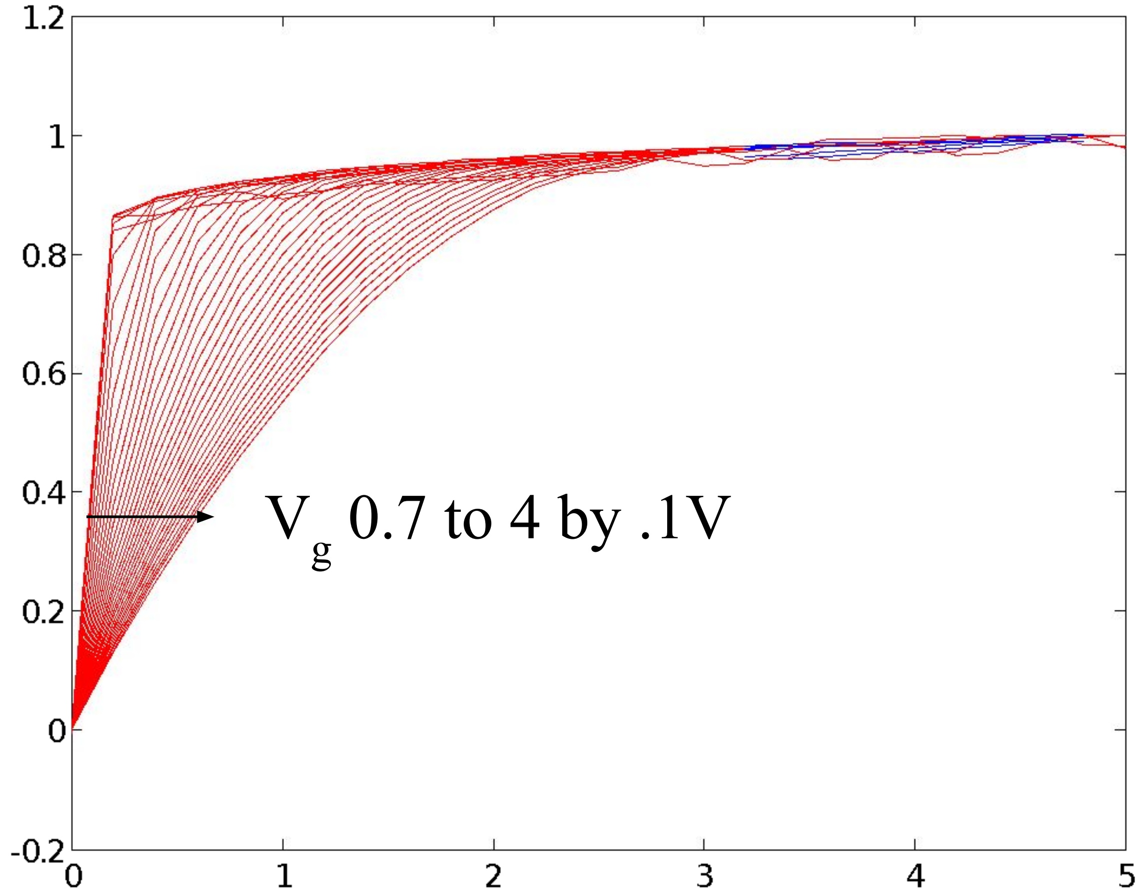
Next week: Linear systems, follower-integrator, follower-differentiator

pFET V_E vs L

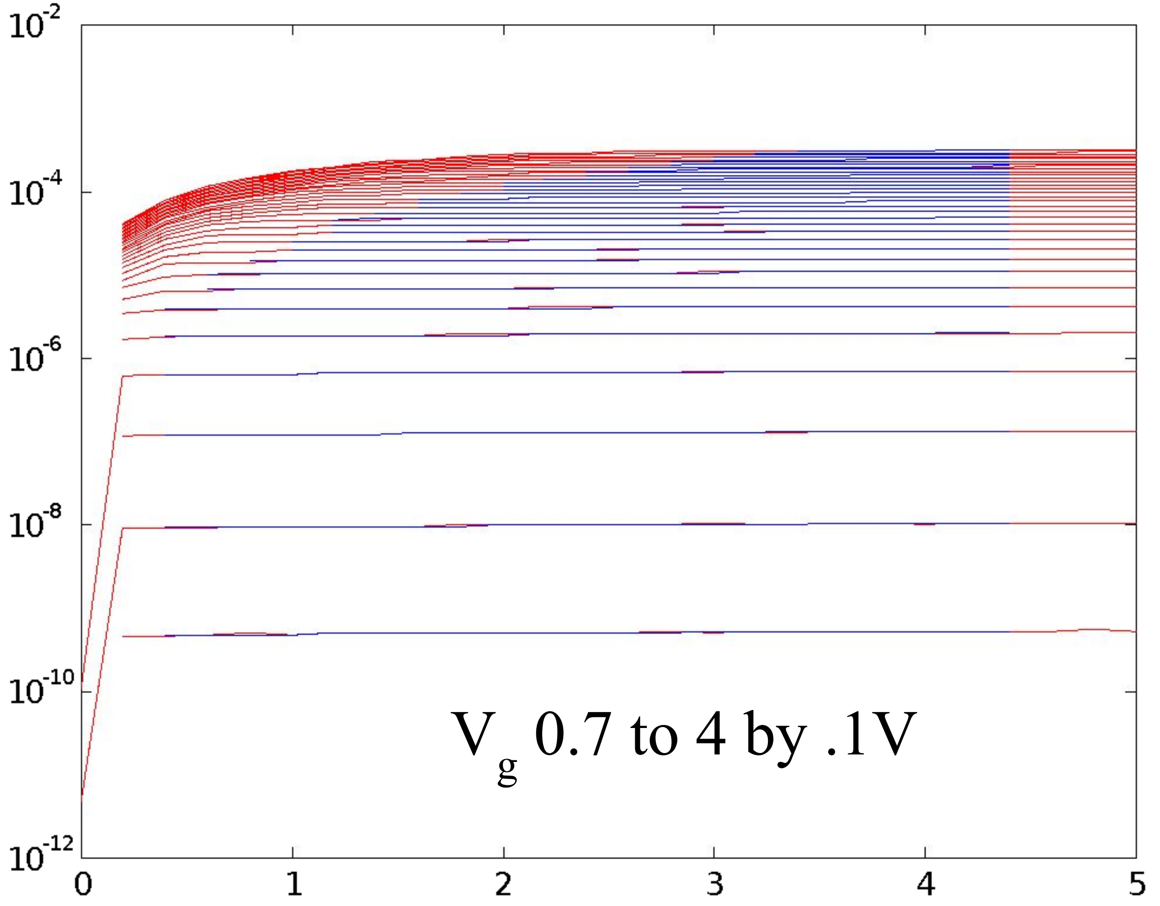


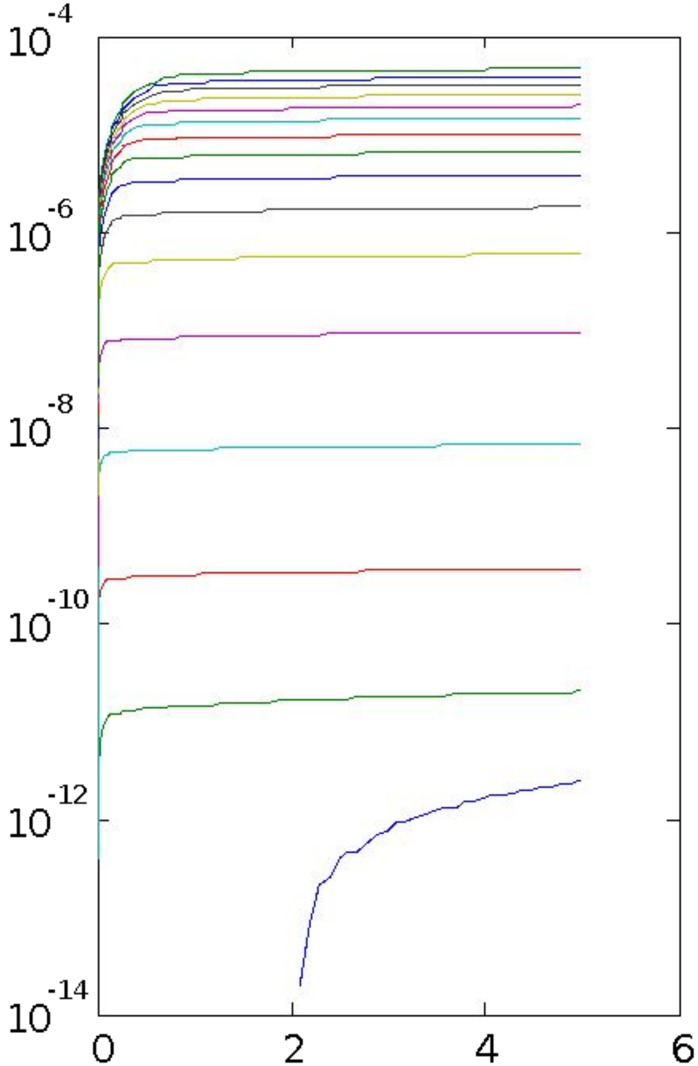
Extracted V_E
Fitted line: slope=6.703V/µ, L_0=-6.495µ
300
250
200
150
100
50
0
0
5
10
15
20
25
30

pFET normalized I_{ds} vs V_{ds}

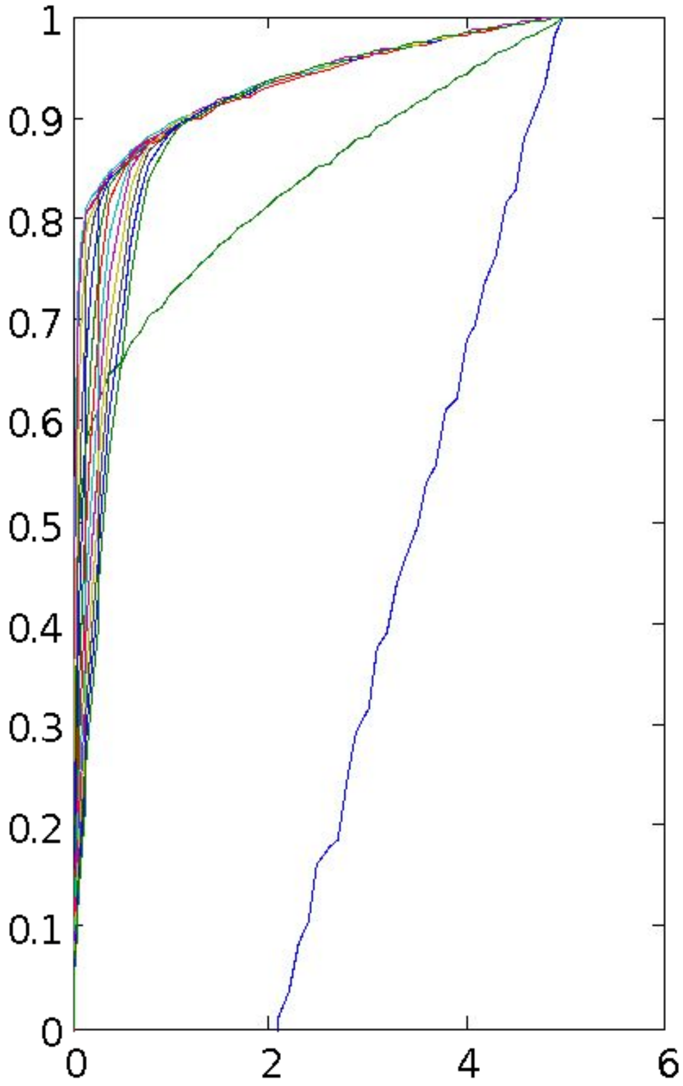


pFET I_{ds} vs V_{ds}

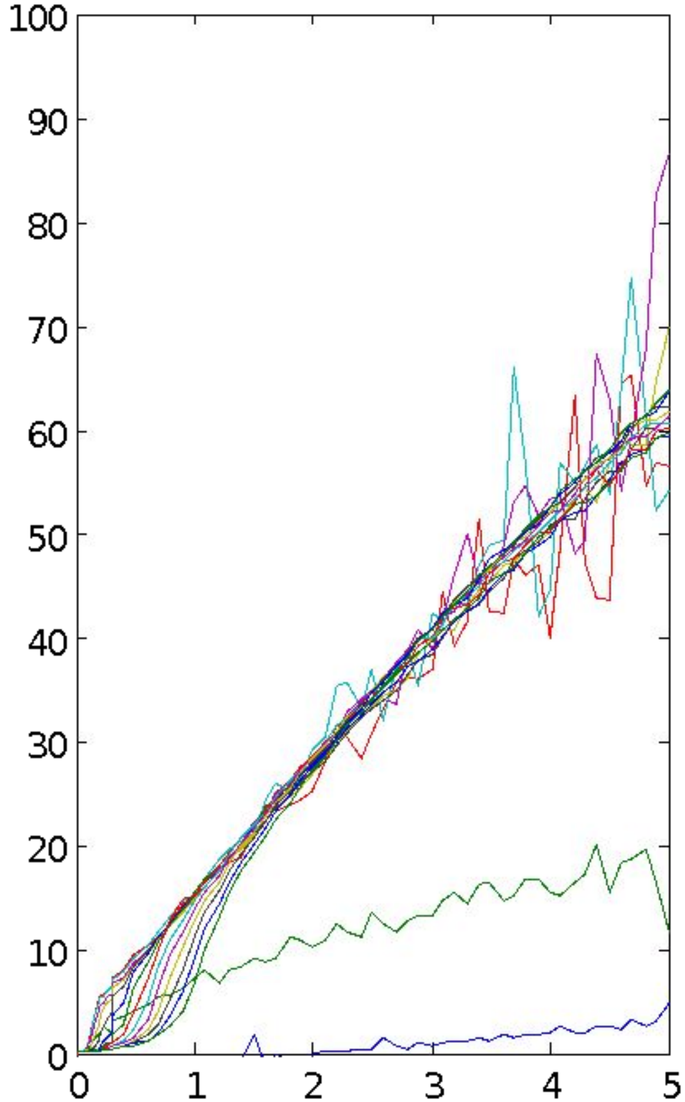




10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
0
2
4
6



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6



100
90
80
70
60
50
40
30
20
10
0
0
1
2
3
4
5

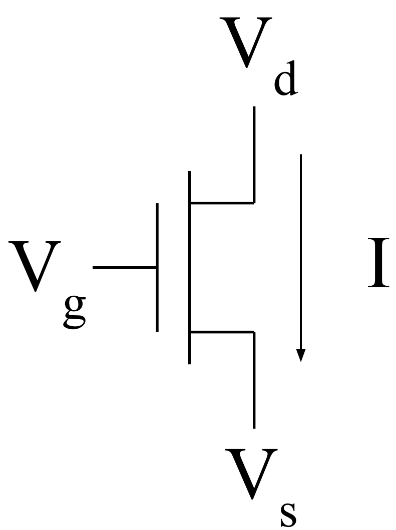
Current source

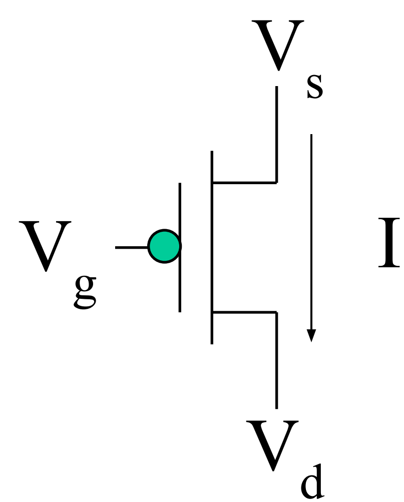
Subthreshold nFET Equation

$$I = I_0 e^{(\kappa V_g - V_s)/U_T} \quad for \ |V_d - V_s| \geq 100mV$$

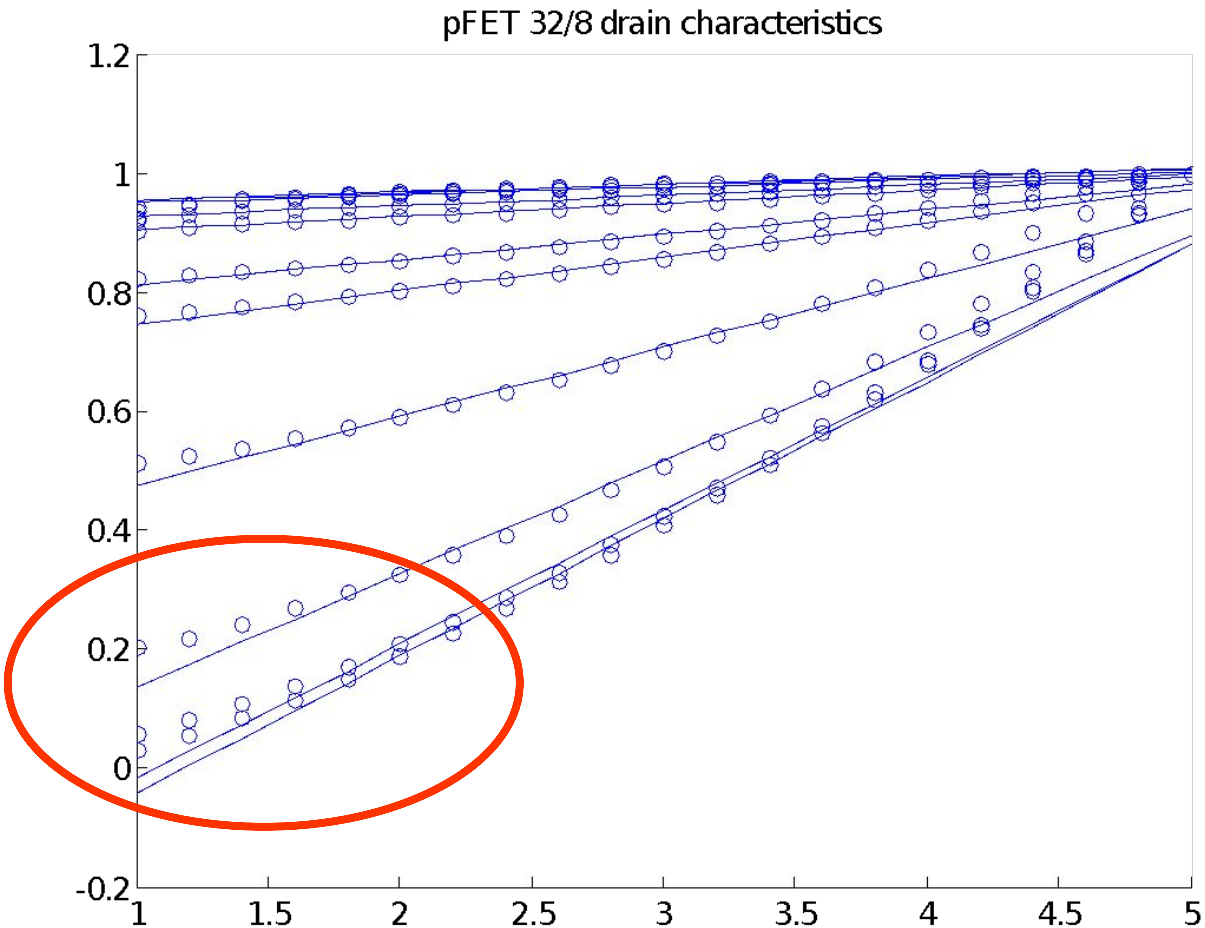
Above Threshold nFET Equation

$$I = \frac{\beta}{2\kappa}\left[\left(\kappa(V_g - V_{T0}) - V_s\right)^2\right] \quad for \ V_d \geq V_g - V_T$$

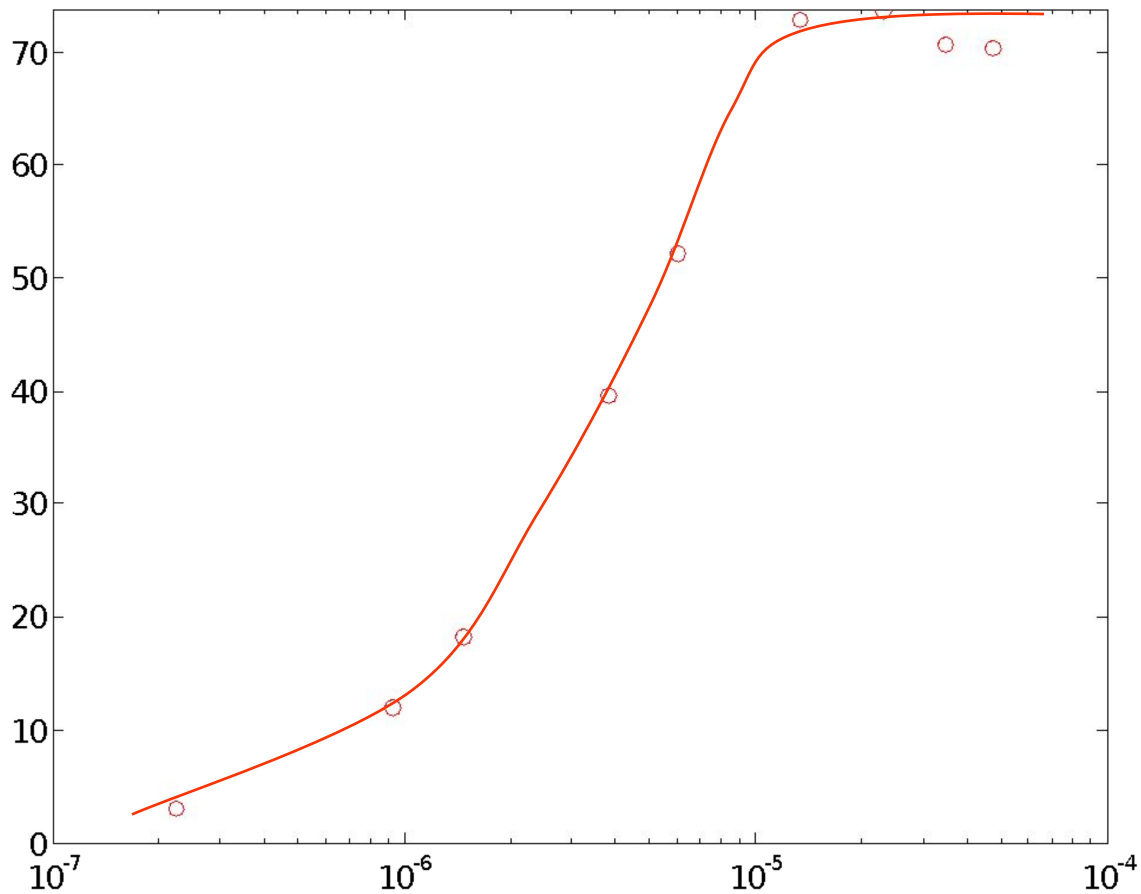




pFET normalized I_{ds} vs V_{ds}



pFET V_E vs I_{dsat}



Transistor Equations

Subthreshold nFET Equation

$$\boxed{I = I_0 e^{\kappa V_g / U_T} \left(e^{-V_s / U_T} - e^{-V_d / U_T} \right)}$$

Above Threshold nFET Equation

$$\boxed{I = \frac{\beta}{2\kappa}\left[\left(\kappa (V_g - V_{T0}) - V_s \right)^2 - \left(\kappa (V_g - V_{T0}) - V_d \right)^2 \right]}$$

Linear Resistor

Subthreshold nFET Equation

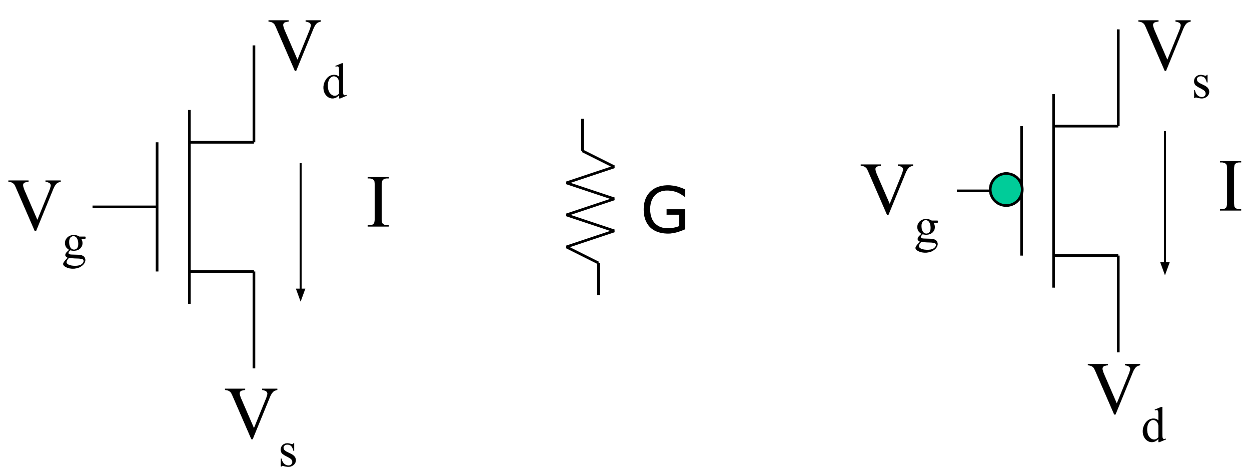
$$I \approx I_0 e^{(\kappa V_g - (V_d + V_s)/2)/U_T} \frac{V_d - V_s}{U_T}$$

$$G = \frac{U_T}{I_0} e^{((V_d + V_s)/2 - \kappa V_g)/U_T}$$

Ohmic region

Above Threshold nFET Equation

$$I = \beta(V_g - V_T)(V_d - V_s) = G(V_d - V_s); G = \beta(V_g - V_T)$$



Diode-connected transistor

<u>Nonlinear Voltage-Current/Current-Voltage Converter</u>

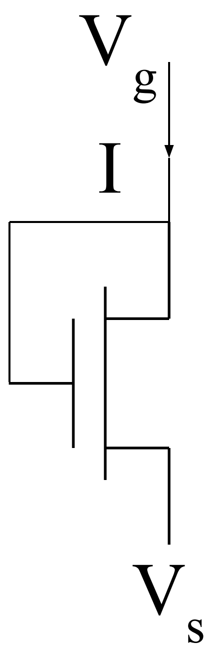
In a diode configuration: $V_d = V_g$

The transistor is then a 2 – terminal device.

Transistor operates in saturation and uses negative feedback

Subthreshold: Logarithmic I-V Converter

Above Threshold: Square root I-V Converter

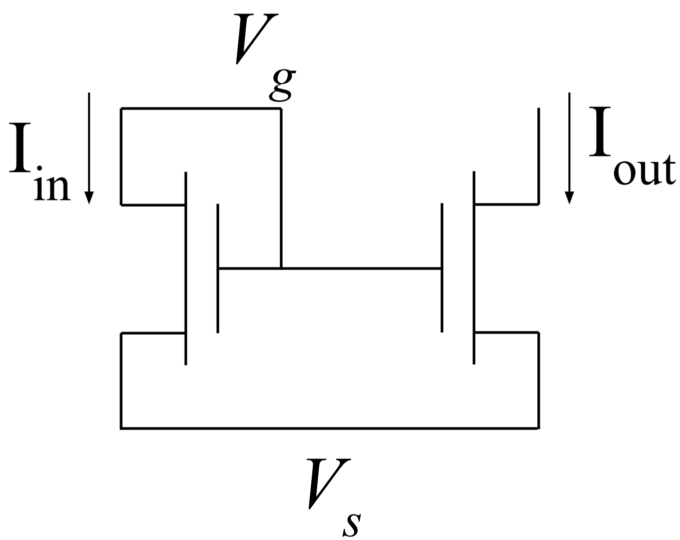


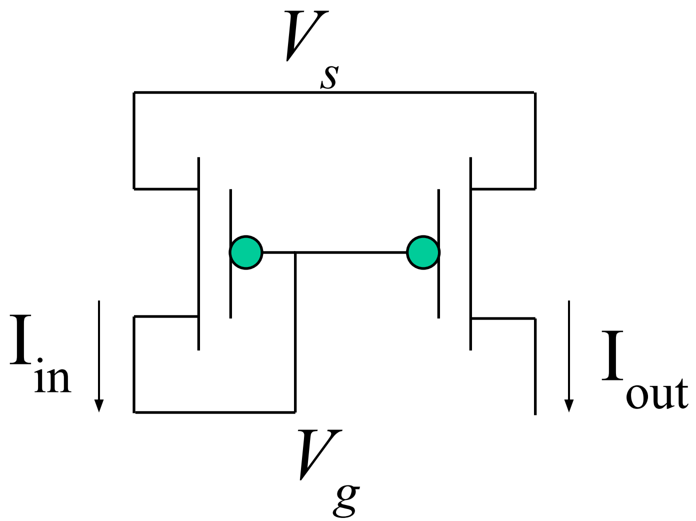
$$\text{Output:} V_s = \kappa V_g - U_T \log\left(\frac{I}{I_0}\right)$$

$$\text{Output:} V_g = \kappa^{-1}\left(V_s + U_T \log\left(\frac{I}{I_0}\right)\right)$$

Current Mirror

The output current is a copy of the input current.

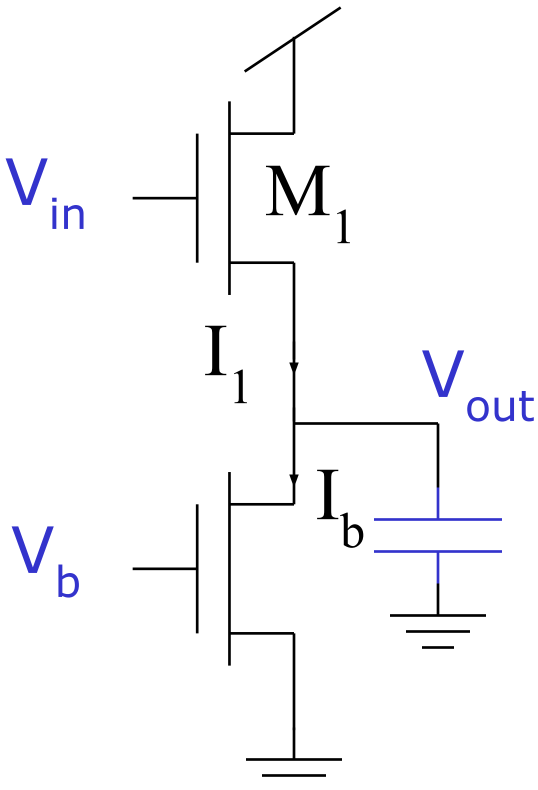




$$I_{out} \approx I_{in} \quad \text{if output drain is in saturation}$$

$$= e^{\kappa V_g - V_s}$$

Source Follower

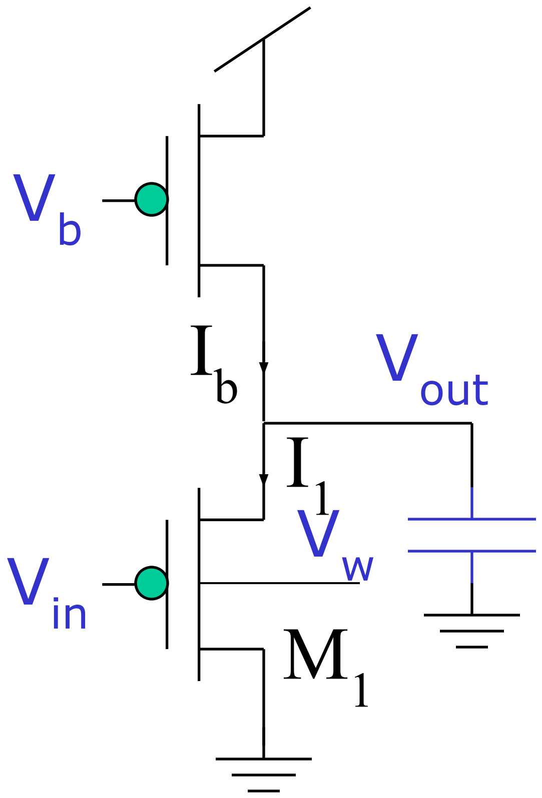


<u>Subthreshold nFET Equation</u>

$$\boxed{I_1 = I_0 e^{(\kappa V_{in} - V_{out})/U_T} = I_b}$$

$$\boxed{\begin{aligned} V_{out} &= \kappa V_{in} - U_T \log\left(\frac{I_b}{I_0}\right) \\ &= \kappa V_{in} - \kappa_b V_b \end{aligned}}$$

Source Follower

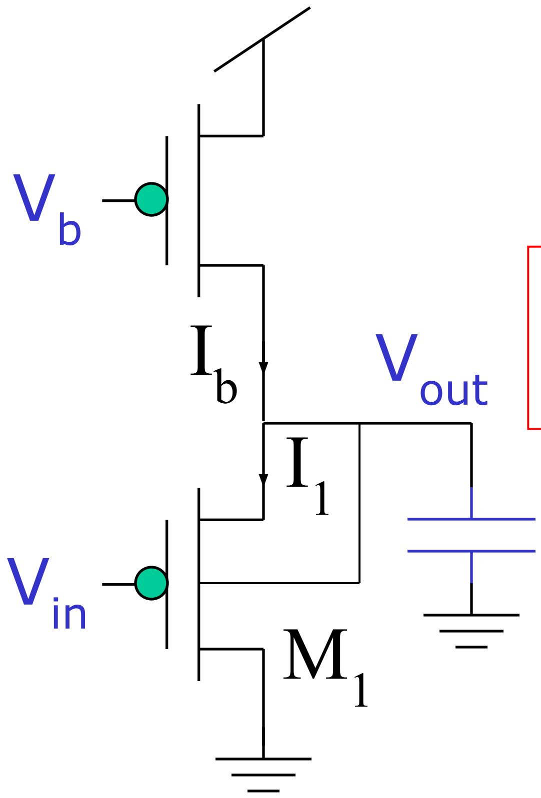


<u>Subthreshold pFET Equation</u>

$$I_1 = I_0 e^{(\kappa_p (V_w - V_{in}) - (V_w - V_{out}))/U_T}$$
$$= I_b$$

$$V_{out} = (1 - \kappa_p) V_w + \kappa_p V_{in} + U_T \log\left(\frac{I_b}{I_0}\right)$$

Source Follower

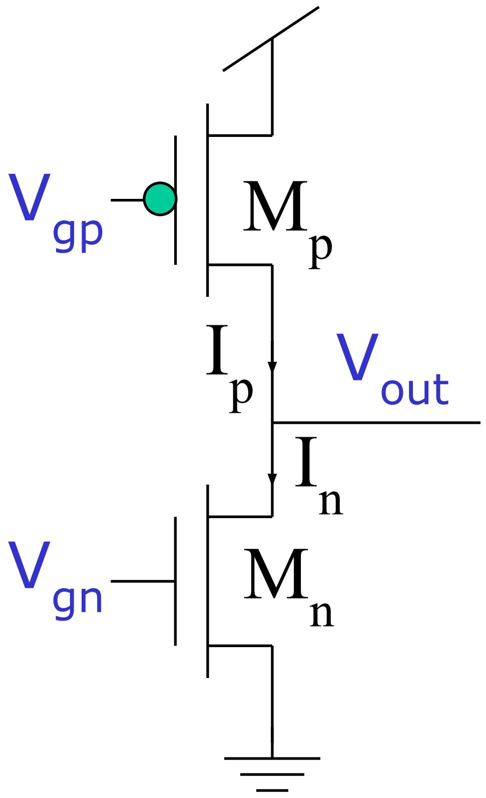


Subthreshold pFET Equation

$$I_1 = I_0 e^{(\kappa_p (V_{out} - V_{in}) - (V_{out} - V_{out}))/U_T}$$

$$V_{out} = V_{in} + \frac{U_T}{\kappa_p} \log\left(\frac{I_b}{I_0}\right) = V_{in} + (V_{dd} - V_b)$$

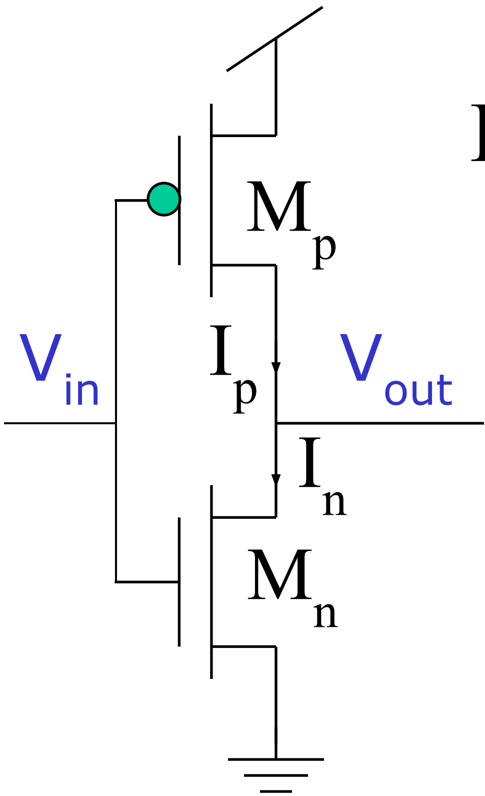
Inverting Amplifier



$$A_p = \frac{\partial V_{out}}{\partial V_{gp}} = -\frac{\kappa_p}{U_T}\frac{V_{nE}V_{pE}}{V_{nE}+V_{pE}}$$

$$A_n = \frac{\partial V_{out}}{\partial V_{gn}} = -\frac{\kappa_n}{U_T}\frac{V_{nE}V_{pE}}{V_{nE}+V_{pE}}$$

Inverter



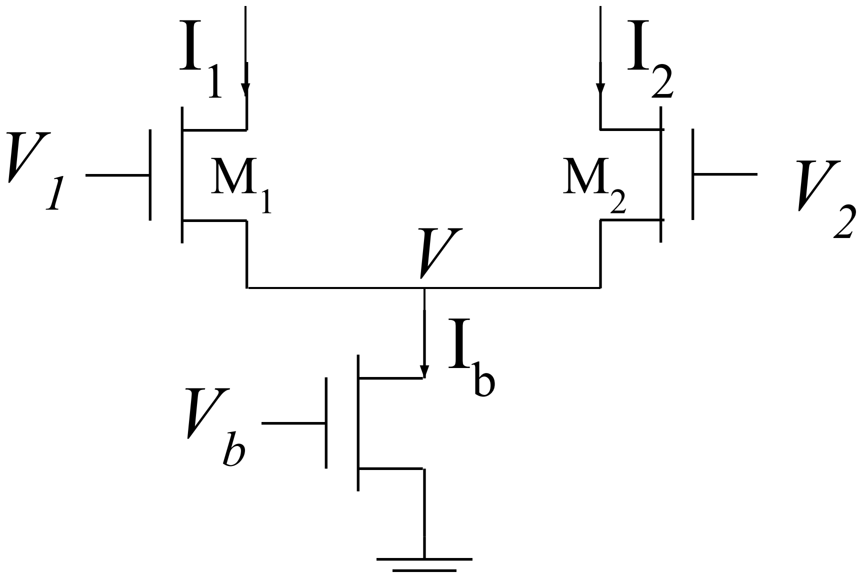
Basic building block of CMOS Logic

$$\boxed{A_p = \frac{\partial V_{out}}{\partial V_{in}} = -\frac{\kappa_p}{U_T}\frac{V_{nE}V_{pE}}{V_{nE}+V_{pE}}}$$

$$\boxed{A_n = \frac{\partial V_{out}}{\partial V_{in}} = -\frac{\kappa_n}{U_T}\frac{V_{nE}V_{pE}}{V_{nE}+V_{pE}}}$$

$$\boxed{A = A_p + A_n}$$

Differential Pair (III)

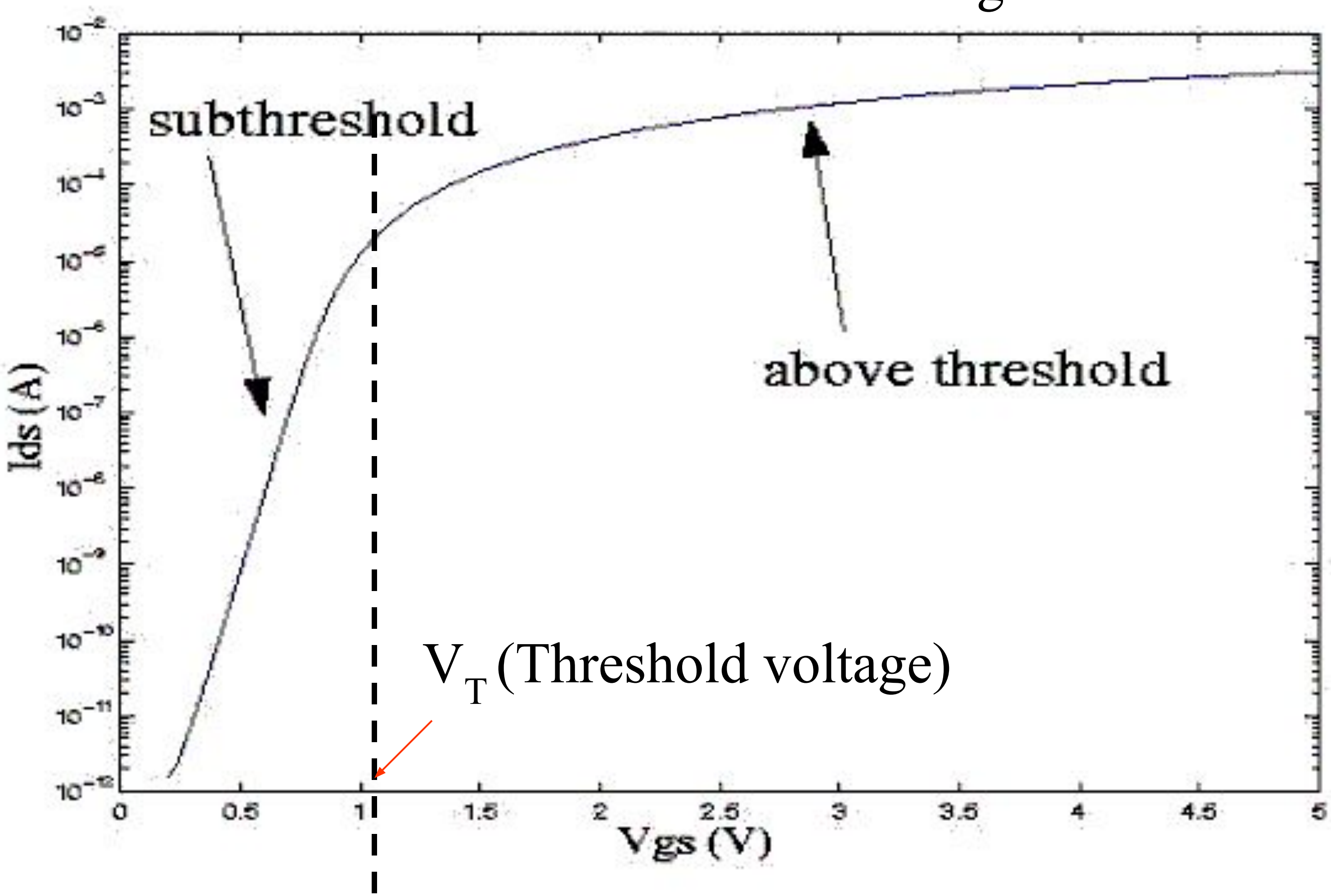


$$\boxed{\begin{aligned} I_1 &= I_0 e^{(\kappa V_1 - V)/U_T} \\ I_2 &= I_0 e^{(\kappa V_2 - V)/U_T} \end{aligned}}$$

$$\boxed{I_1 + I_2 = I_0 e^{-V/U_T}\left(e^{\kappa V_1/U_T} + e^{\kappa V_2/U_T}\right) = I_b}$$

$$\boxed{\Rightarrow e^{-V/U_T} = \frac{I_b}{I_0(e^{\kappa V_1/U_T} + e^{\kappa V_2/U_T})}}$$

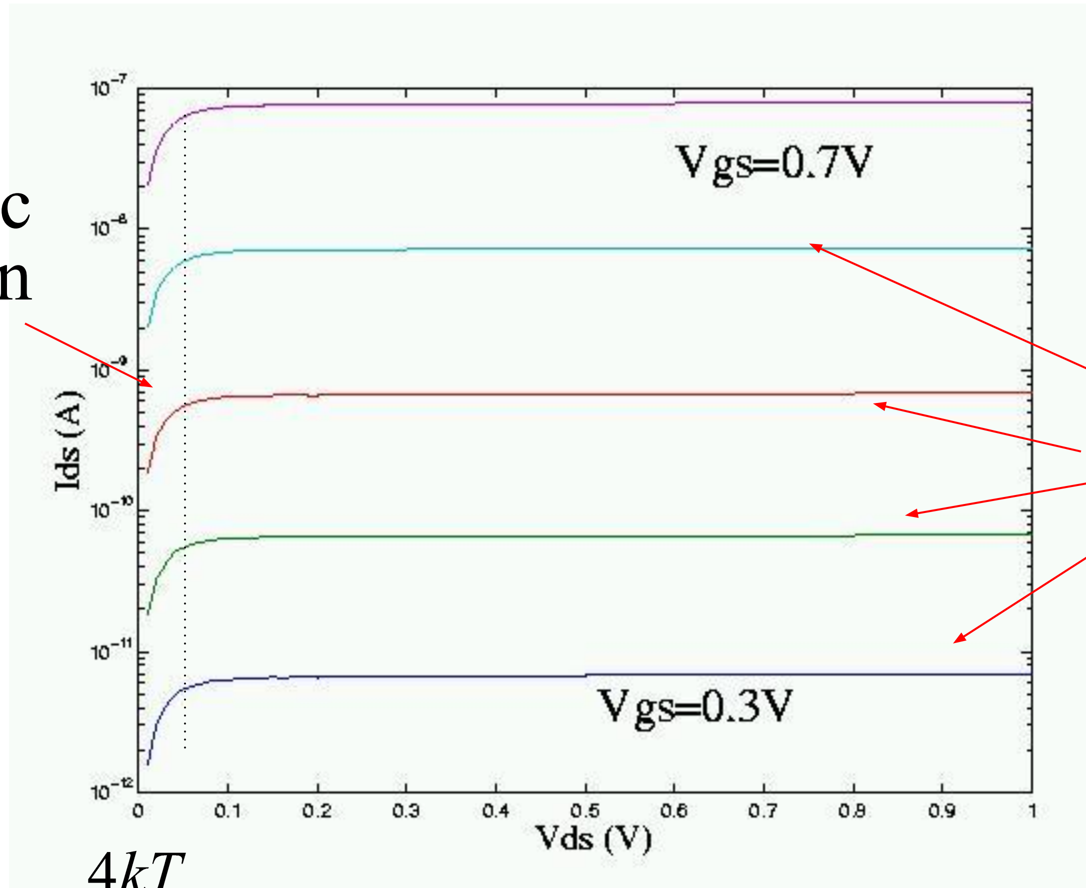
nFet curve: I vs V_{gs}



Threshold voltage is the voltage where the measured I is half of the I computed from the exponential equation.

nFET curve: I vs V_{ds}

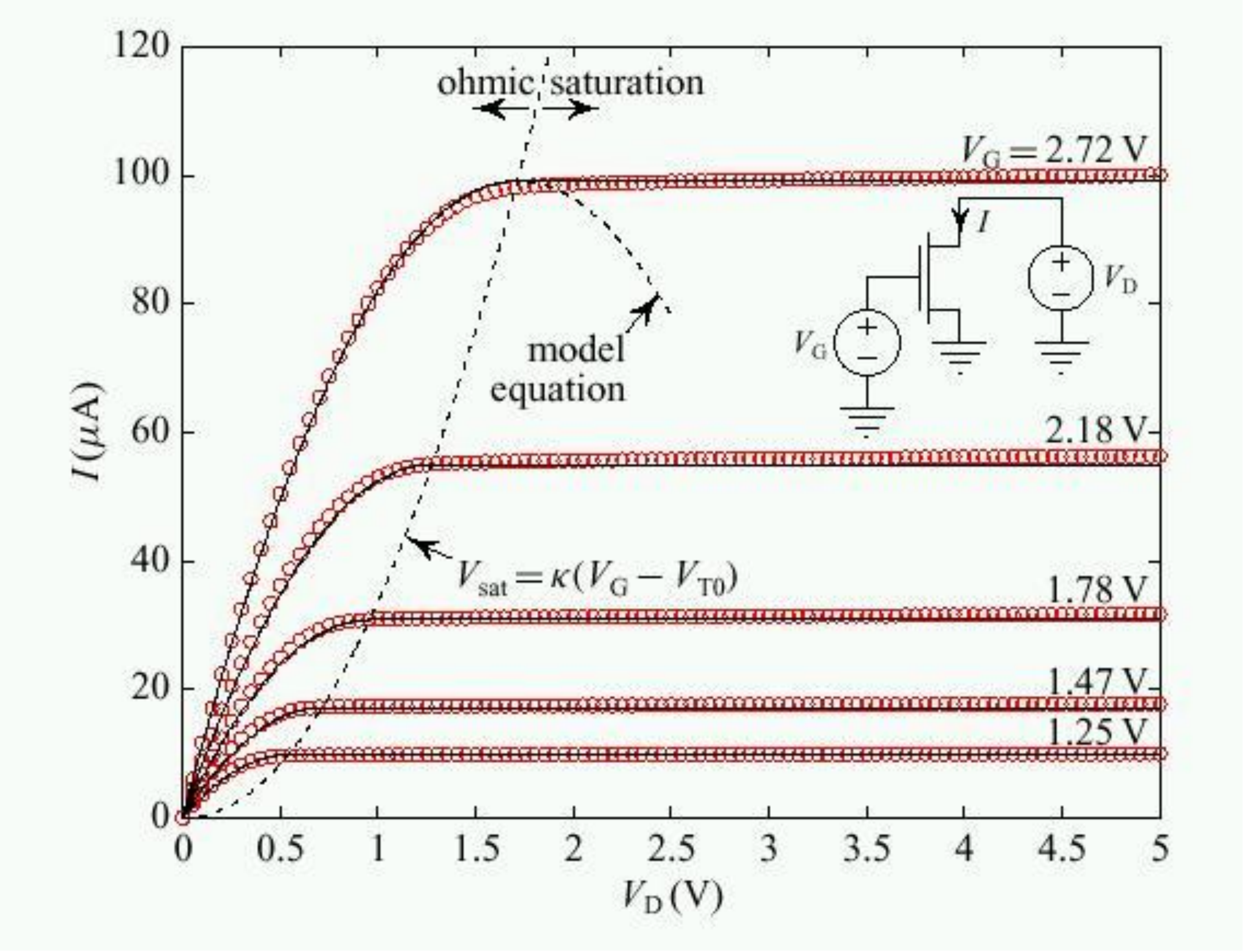
Ohmic region



Saturation region

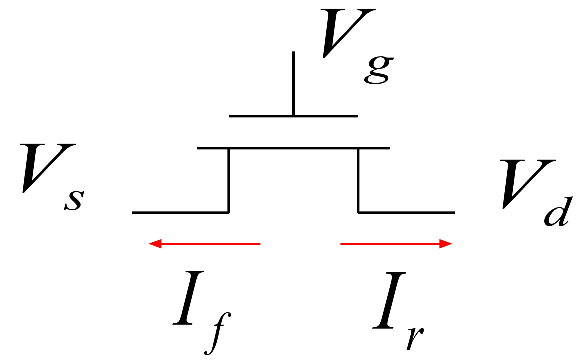
$$\frac{4kT}{q} \approx 100mV$$

Above Threshold nFET curve: I vs V_{ds}



Body Effect

What is body effect?



In subthreshold, for a constant I, a **ΔV** change in the source voltage means that the gate voltage has to increase by **κ ΔV** and not just ΔV.

In above threshold, this effect is often taken to mean that the threshold voltage of the transistor **increases** with the source voltage.