# Transconductance Amplifier

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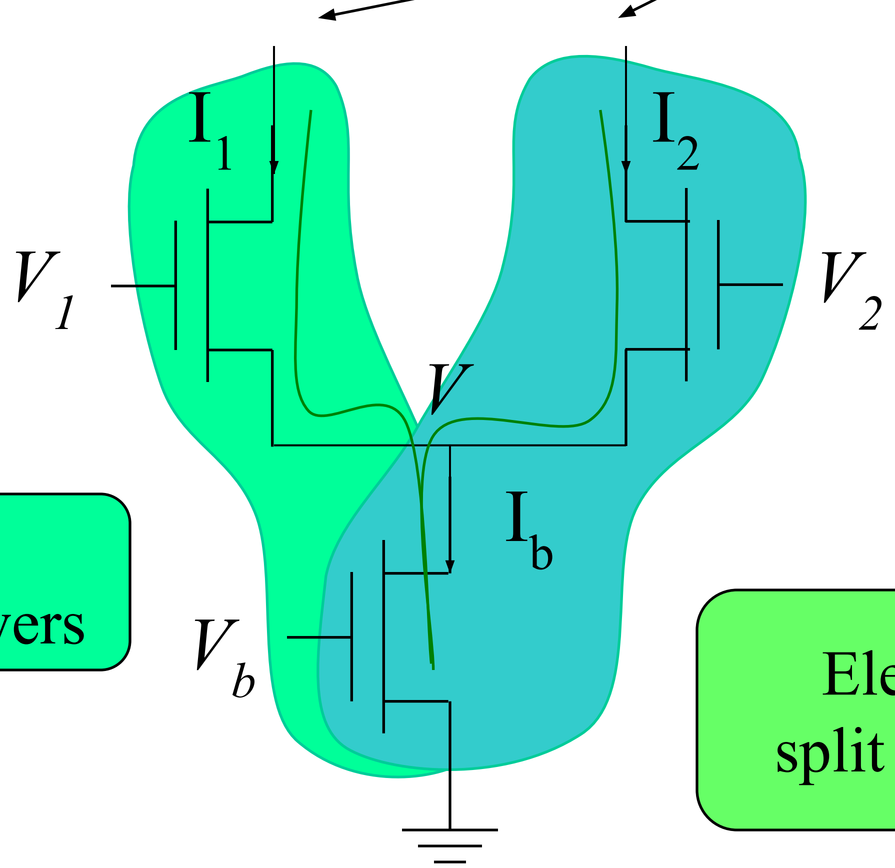
## Outline:

- Differential pair
- Transconductance amplifier (and its $g_m$ and $A$)
- Voltage amplifier
- Wide range transconductance amplifier

# Differential Pair (I)

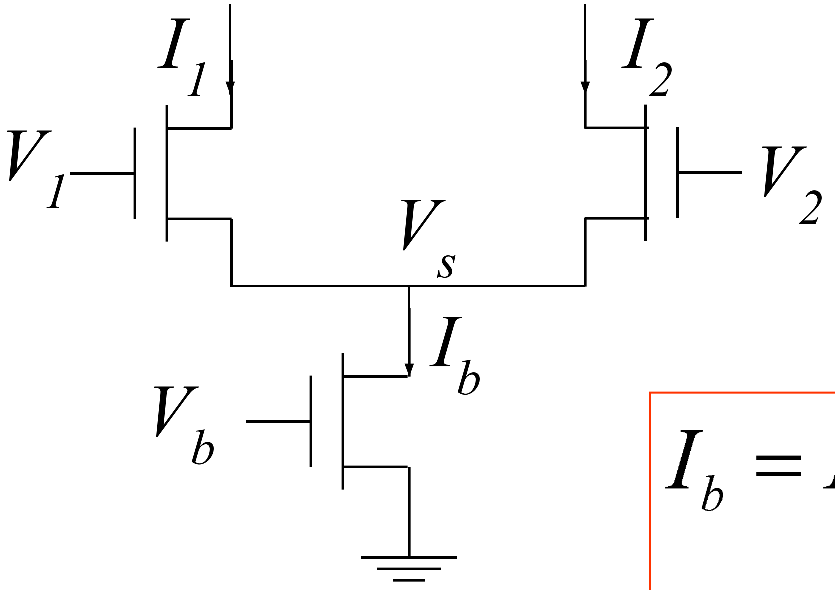
Saturated drains




Two Source-followers

Electrons flow up, split between $I_1$ and $I_2$

# Differential Pair (II)




$$I_1 = I_0 e^{\kappa V_1 - V_s / U_T}$$

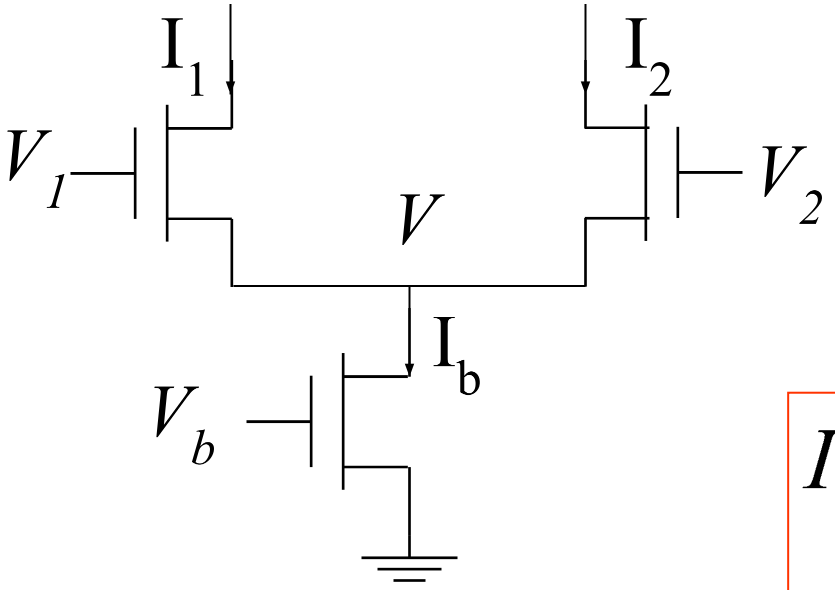
$$I_2 = I_0 e^{\kappa V_2 - V_s / U_T}$$

$$I_b = I_0 e^{\kappa V_b / U_T} = I_1 + I_2$$

$$= I_0 e^{-V_s / U_T} \left( e^{\kappa V_1 / U_T} + e^{\kappa V_2 / U_T} \right)$$

$$\Rightarrow e^{V_s / U_T} = \frac{e^{\kappa V_1 / U_T} + e^{\kappa V_2 / U_T}}{e^{\kappa V_b / U_T}}$$

# Differential Pair (II)

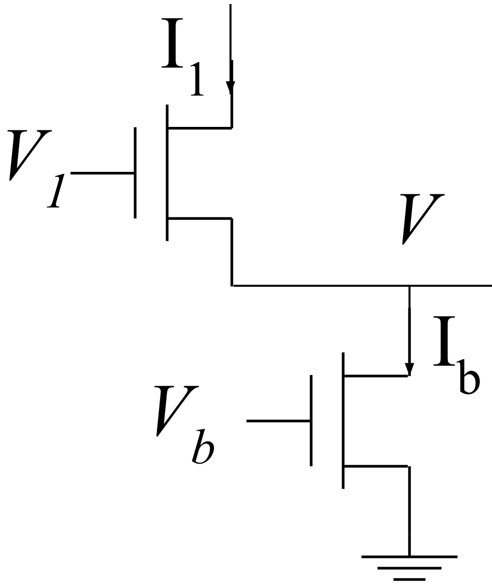



$$\boxed{\begin{aligned} I_1 &= I_0 e^{\kappa V_1 - V} \\ I_2 &= I_0 e^{\kappa V_2 - V} \end{aligned}}$$

$$\boxed{\begin{aligned} I_b = e^{\kappa V_b} &= I_1 + I_2 \\ &= I_0 e^{-V} \left( e^{\kappa V_1} + e^{\kappa V_2} \right) \end{aligned}}$$

$$\boxed{\Rightarrow e^V = \frac{e^{\kappa V_1} + e^{\kappa V_2}}{e^{\kappa V_b}}}$$

# Differential Pair (III)




$$V = \ln\left(e^{\kappa V_1} + e^{\kappa V_2}\right) - \kappa V_b$$

$$\approx \kappa\left(V_1 - V_b\right) \text{ for } V_1 - V_2 \geq 100\text{mV}$$

# Differential Pair (IV)

Output currents:

$$\boxed{I_1 = I_b \frac{e^{\kappa V_1/U_T}}{e^{\kappa V_1/U_T} + e^{\kappa V_2/U_T}} = \frac{I_b}{1 + e^{\kappa (V_2 - V_1)/U_T}}}$$

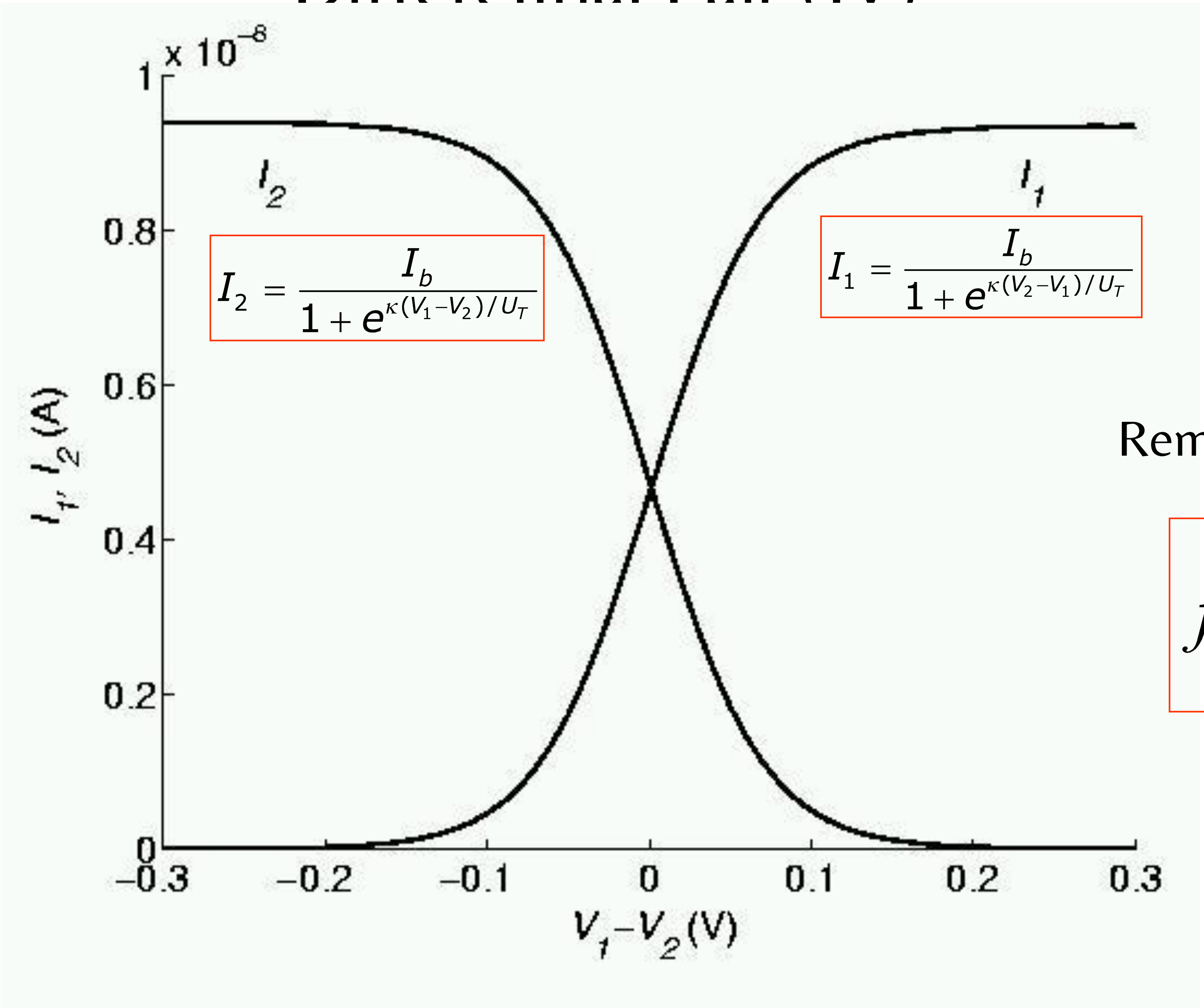
$$\boxed{I_2 = \frac{I_b}{1 + e^{\kappa (V_1 - V_2)/U_T}}}$$

Fermi function

Difference of output currents:

$$\boxed{I_1 - I_2 = I_b \frac{e^{\kappa V_1/U_T} - e^{\kappa V_2/U_T}}{e^{\kappa V_1/U_T} + e^{\kappa V_2/U_T}} = I_b \tanh\left(\kappa \frac{V_1 - V_2}{2U_T}\right)}$$

# Differential Pair (IV)

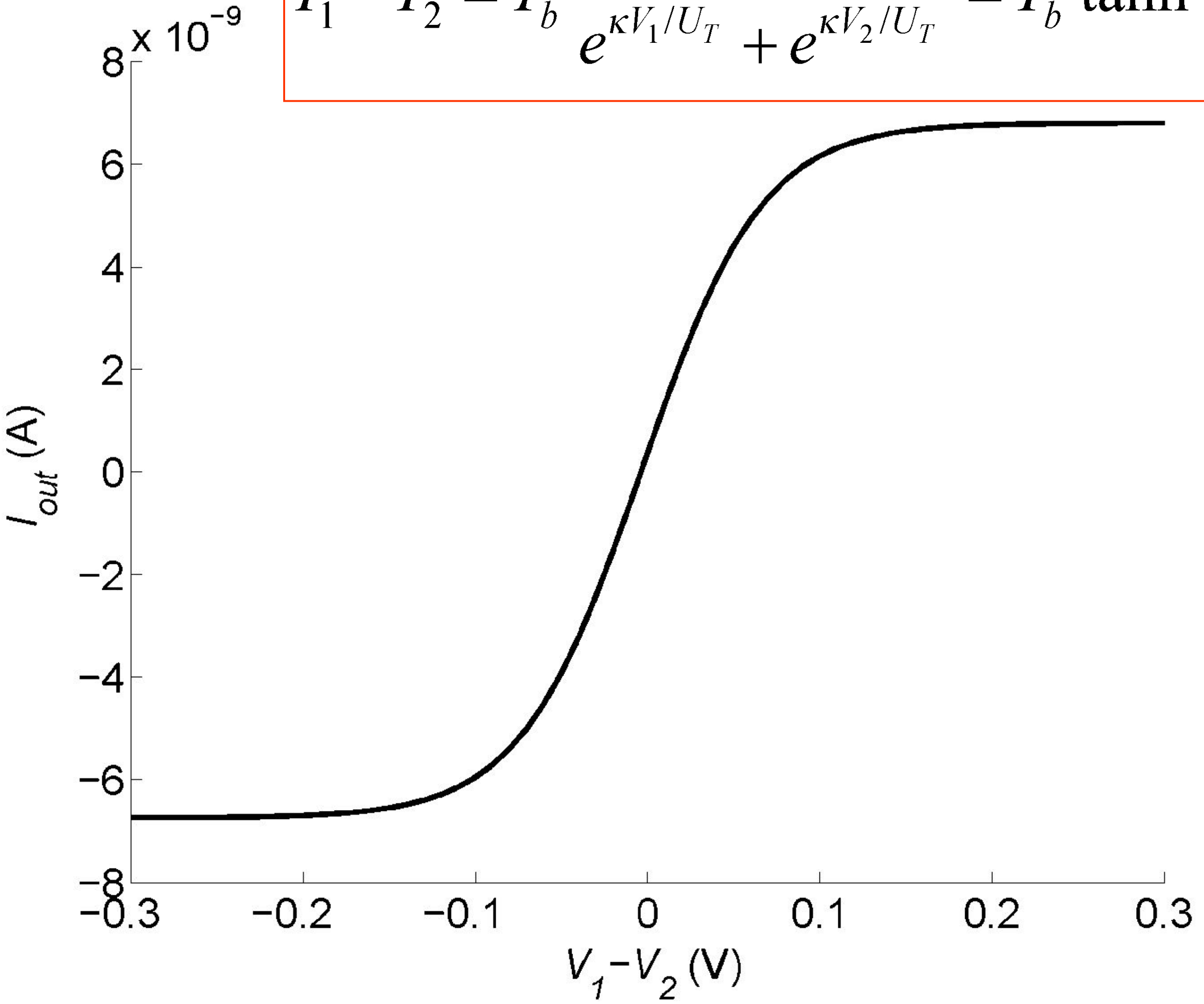



Remember sigmoid is

$$\boxed{f(x) = \frac{1}{1 + e^{-\alpha x}}}$$

# Output Current vs Differential Input

$$I_1 - I_2 = I_b \frac{e^{\kappa V_1/U_T} - e^{\kappa V_2/U_T}}{e^{\kappa V_1/U_T} + e^{\kappa V_2/U_T}} = I_b \tanh\left(\kappa \frac{V_1 - V_2}{2U_T}\right)$$



# Digression: Hyperbolic functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \approx x \text{ for small x}$$

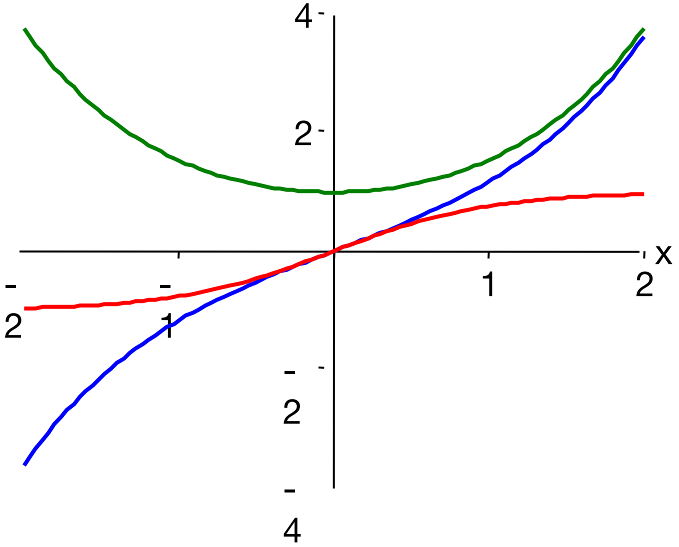
$$\cosh(x) = \frac{e^x + e^{-x}}{2} \approx 1 + \frac{x^2}{2} \text{ for small x}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \approx x \text{ for small x}$$

cosh

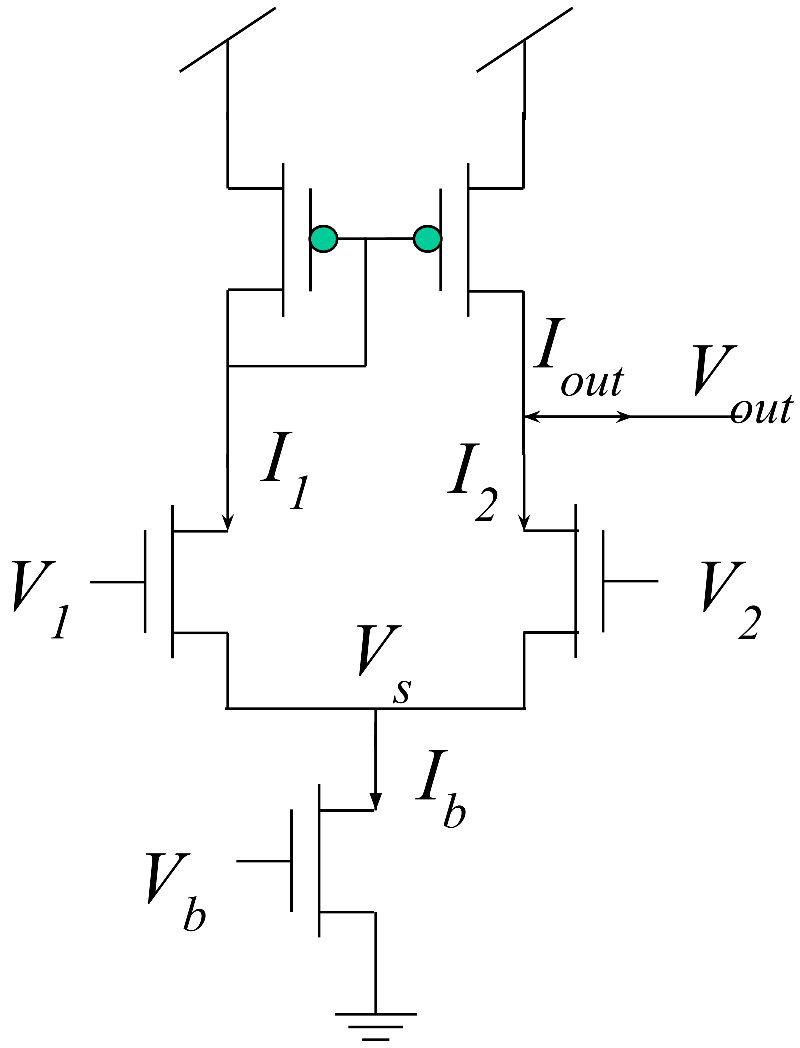
tanh

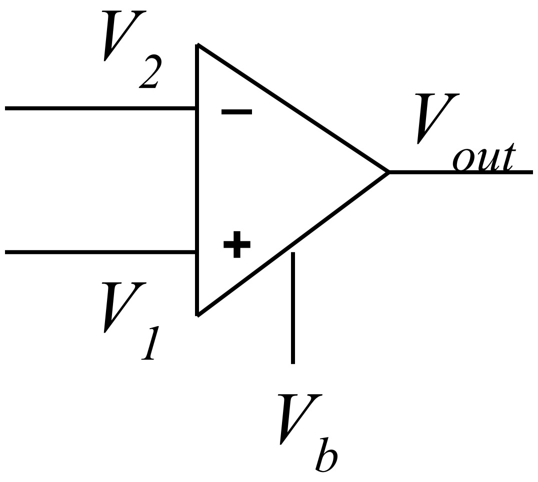
sinh




tanh: *compressive*
sinh: *expansive*

# Transconductance Amplifier




$$
\boxed{\begin{aligned} I_{\text{out}} &= I_1 - I_2 \\ &= I_b \tanh\left(\kappa \frac{V_1 - V_2}{2U_T}\right) \end{aligned}}
$$

# Transconductance Amplifier

In subthreshold:

For small differential voltages ($\left|V_1 - V_2\right|$ e.g. $< 200\text{mV}$),

the tanh(.) relationship is approximately linear:

$$I_{out} = I_b \tanh\left(\kappa \frac{V_1 - V_2}{2U_T}\right)$$

can be reduced to:

$$I_{out} \approx g_m \left(V_1 - V_2\right)$$

where $$g_m \approx \frac{\kappa I_b}{2U_T}$$

# Transconductance Amplifier

Output conductance is defined as :

$$g_d = -\frac{\partial I_{out}}{\partial V_{out}} \approx \frac{I_b}{V_E}$$

# Transconductance in Strong Inversion

In above threshold, the output current of the diff pair is

$$I_1 - I_2 = \frac{\beta}{2}\left(V_1 - V_2\right)\sqrt{\frac{4I_b}{\beta} - \left(V_1 - V_2\right)^2}$$

where $\beta = \mu C_{ox}\dfrac{W}{L}$.

For $\left|V_1 - V_2\right| = \sqrt{2I_b / \beta}$, the transconductance is given by

$$g_m = \sqrt{\beta I_b}$$

# Differential pair in strong inversion (V)

•In **strong inversion**, solve for $I_1$ & $I_2$ this way:

$$\Delta V \equiv V_1 - V_2 = \sqrt{\frac{2I_1}{\beta}} - \sqrt{\frac{2I_2}{\beta}}$$

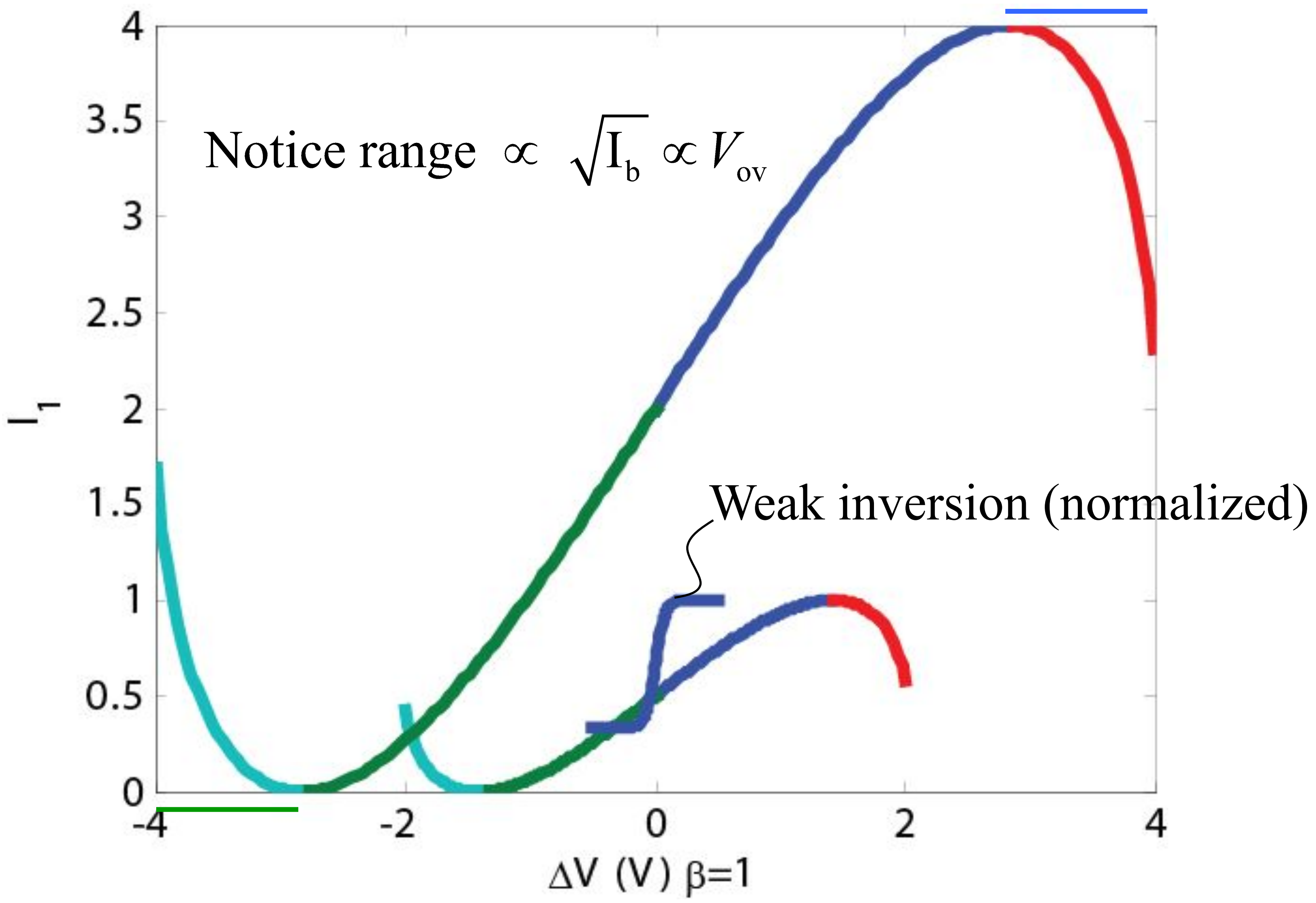
You eventually obtain

$$I_{1,2} = \frac{I_b}{2}\left(1 + - \sqrt{\frac{\beta \Delta V^2}{I_b} - \left(\frac{\beta \Delta V^2 / 2}{I_b}\right)^2}\right)$$

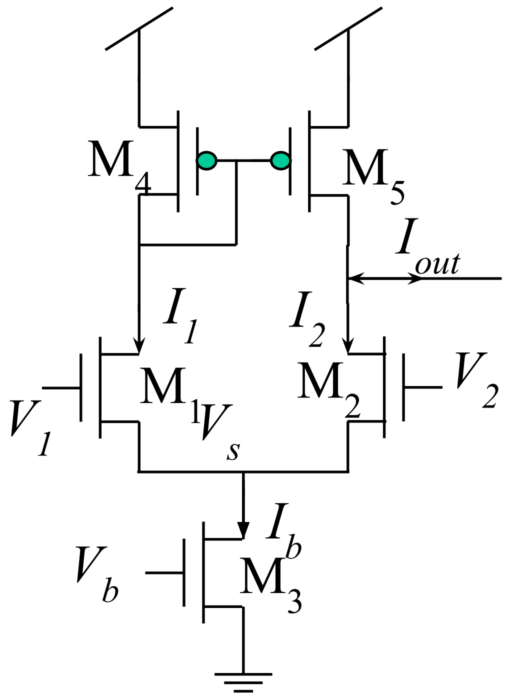
for $\dfrac{\beta \Delta V^2}{2} < I_b$

$$I_1 - I_2 = I_b \sqrt{\frac{\beta \Delta V^2}{I_b} - \left(\frac{\beta \Delta V^2 / 2}{I_b}\right)^2}$$

# Differential pair in weak and strong inversion



# Assumptions for Deriving Tanh Output




To obtain tanh function, we assume $M_1$ to $M_5$ are in saturation:

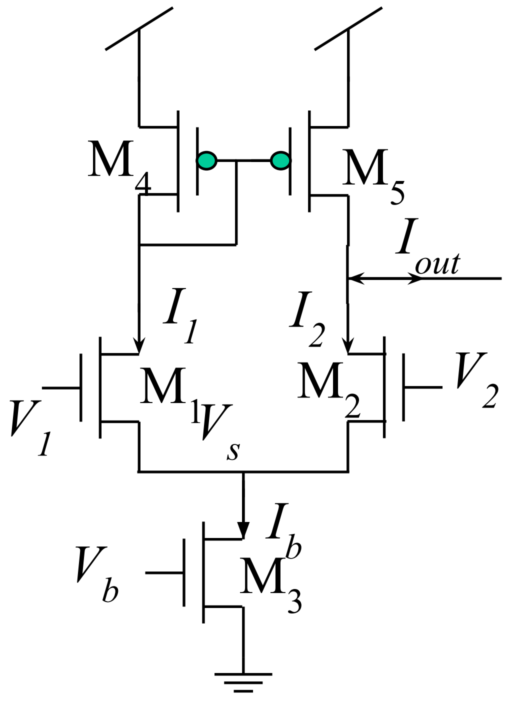
$$I_1 = I_0 e^{\kappa V_1 - V_s / U_T}$$

$$I_2 = I_0 e^{\kappa V_2 - V_s / U_T}$$

$$I_b = I_0 e^{\kappa V_b / U_T}$$

$$I_{out} = I_1 - I_2$$

# Deriving Common Source (I)




Assume bias transistor is not in saturation,

Equation for current through $M_3$ is

$$I_b = I_0 e^{\kappa V_b / U_T}\left(1 - e^{-V_s/U_T}\right)$$

Assuming $M_1$ and $M_2$ are in saturation:

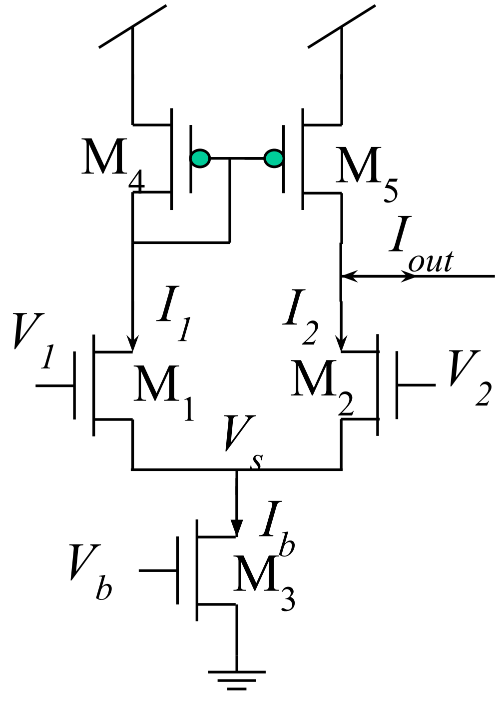
$$I_b = I_1 + I_2$$

$$e^{\kappa V_b / U_T}\left(1 - e^{-V_s/U_T}\right) = e^{(\kappa V_1 - V_s)/U_T} + e^{(\kappa V_2 - V_s)/U_T}$$

Solving for :

$$e^{-V_s/U_T} = \frac{e^{\kappa V_b/U_T}}{e^{\kappa V_b/U_T} + e^{\kappa V_1/U_T} + e^{\kappa V_2/U_T}} \quad (Eq.1)$$

# Deriving Common Source (II)




For the bias transistor to be in saturation,

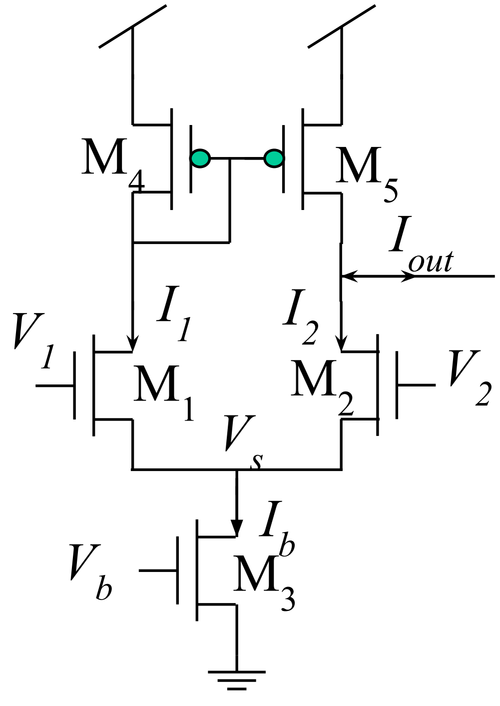
$V_s > 4U_T$ therefore $e^{-V_s/U_T} \ll 1$

$$e^{-V_s/U_T} = \frac{e^{\kappa V_b/U_T}}{e^{\kappa V_b/U_T} + e^{\kappa V_1/U_T} + e^{\kappa V_2/U_T}} \ll 1$$

$$e^{\kappa V_b/U_T} \ll e^{\kappa V_b/U_T} + e^{\kappa V_1/U_T} + e^{\kappa V_2/U_T}$$

$$\frac{e^{\kappa V_1/U_T} + e^{\kappa V_2/U_T}}{e^{\kappa V_b/U_T}} \gg 1 \quad (Eq.2)$$

# Deriving Common Source (III)




From

$$e^{-V_s/U_T} = \frac{e^{\kappa V_b/U_T}}{e^{\kappa V_b/U_T} + e^{\kappa V_1/U_T} + e^{\kappa V_2/U_T}}$$

we can derive:

$$V_s = -\kappa V_b + U_T \ln\left(e^{\kappa V_b/U_T} + e^{\kappa V_1/U_T} + e^{\kappa V_2/U_T}\right)$$

Since $e^{\kappa V_1/U_T} + e^{\kappa V_2/U_T} >> e^{\kappa V_b/U_T}$ and assuming that $\left|V_1 - V_2\right| > 4U_T$ we can simplify the ln(.).

# Deriving Common Source (IV)

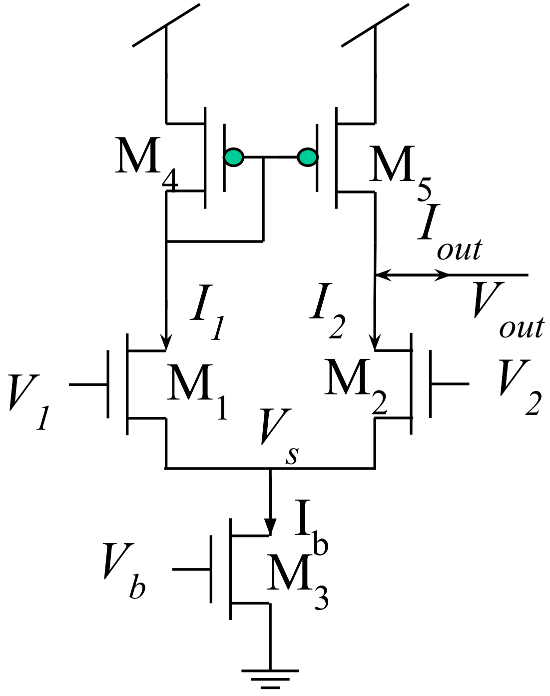
The common node voltage $V_s$ of the transconductance amplifier is:

$$V_s = \kappa\left(\max\left(V_1, V_2\right) - V_b\right)$$

and the saturation condition for $M_3$, $V_s > 4U_T$ is:

$$\max\left(V_1, V_2\right) > V_b + \frac{4U_T}{\kappa}$$

# Output Transistors




The output transistors $M_2$ and $M_5$ should also be in saturation:

To keep $M_5$ in saturation : $V_{dd} - V_{out} > 4U_T$

To keep $M_2$ in saturation : $V_{out} - V_s > 4U_T$

which means

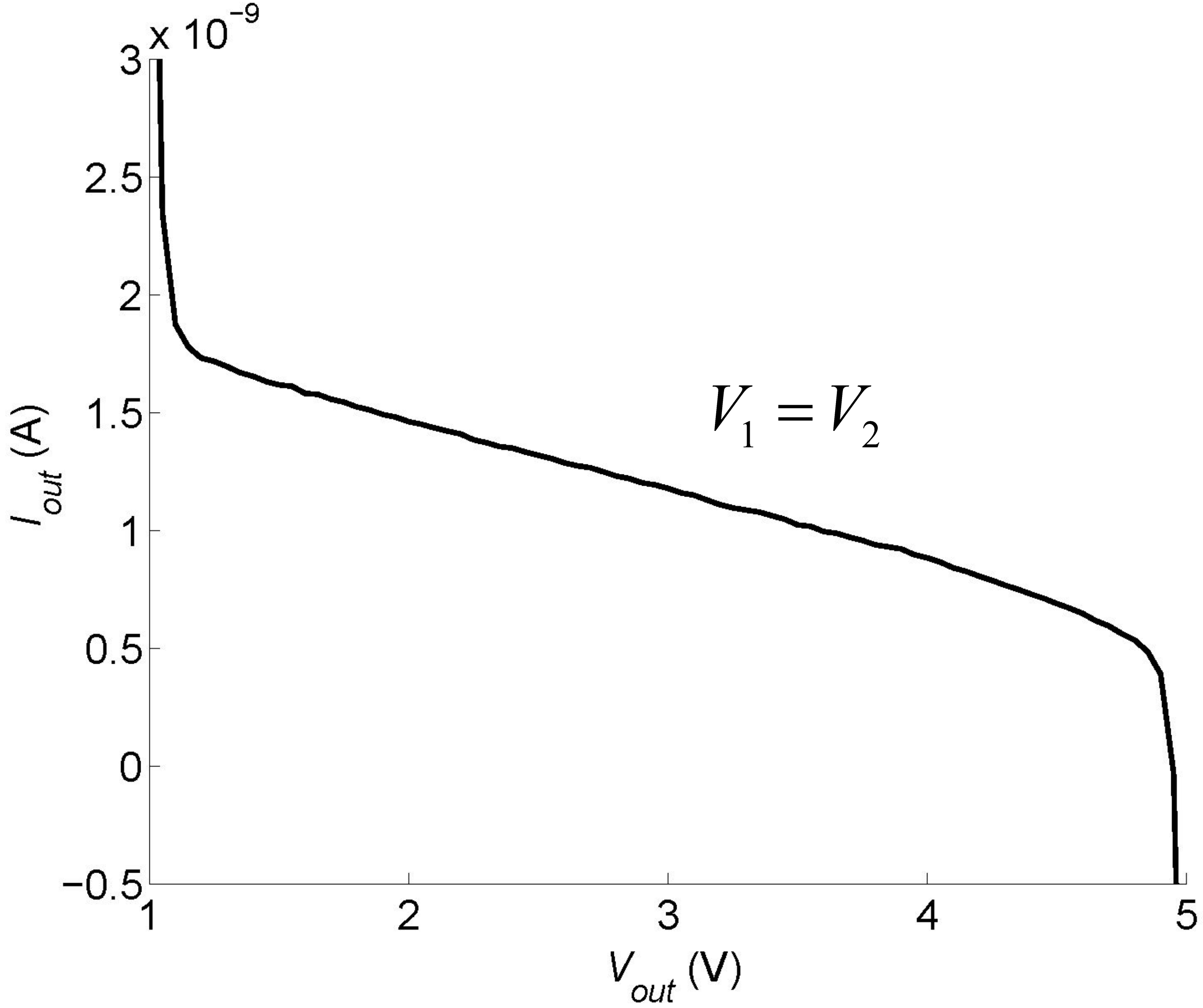
$$V_{out} > \kappa\left(\max\left(V_1, V_2\right) - V_b\right) + 4U_T \quad (Eq.3)$$

Therefore there is a $V_{\min}$ problem, i.e. minimum $V_{out}$ depends on $V_1$ , $V_2$ and $V_b$.

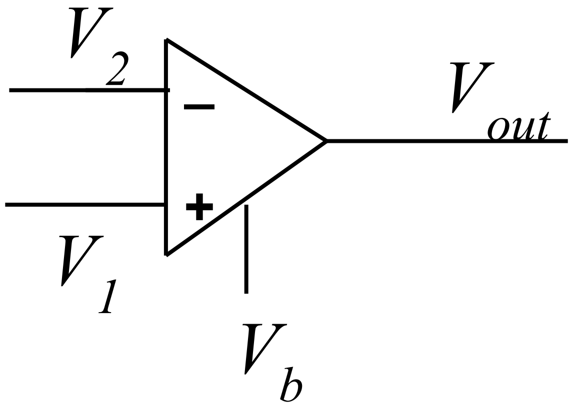
Final condition :

$$\kappa\left(\max\left(V_1, V_2\right) - V_b\right) + 4U_T < V_{out} < V_{dd} - 4U_T$$

# Curves of Iout vs Vout



# Voltage Amplifier




Transconductance amplifier can be used a differential-input voltage amplifier: $V_{out} = A(V_1 - V_2)$

Transfer function:

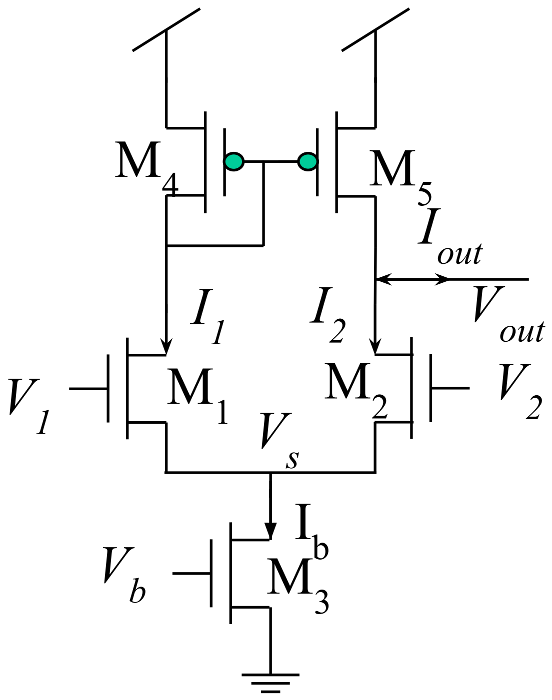
$$A = \frac{dV_{out}}{d(V_1 - V_2)} = \frac{dI_{out}}{d(V_1 - V_2)} \frac{dV_{out}}{dI_{out}} = \frac{g_m}{g_d}$$

$$\text{Subthreshold}: A \approx \frac{\kappa V_E}{2U_T}; \text{Above threshold}: A \approx \sqrt{\frac{\beta}{I_b}} V_E$$

# Voltage Amplifier

- The open-circuit voltage gain $A$ increases with Early voltage, and therefore with the length of the output transistors.
- Typical subthreshold values are between 100 and 1000.
- Because of the large voltage gain and transistor mismatch effects, the amplifier is usually used in a negative-feedback configuration.
- In open-voltage mode, it is used mainly as a comparator. $V_{out}$ is “high” when $V_1 > V_2$ and vice-versa.

# Output Voltage Limits (I)




Compute limits of voltage swing

a) $V_1 > V_2$

For $V_1 > V_2 + 4U_T$ current through $M_2$ is much smaller than $M_1$, $V_{out}$ goes almost to $V_{dd}$ to shut off $M_5$.

$M_5$ goes out of saturation.

b) $V_2 > V_1$

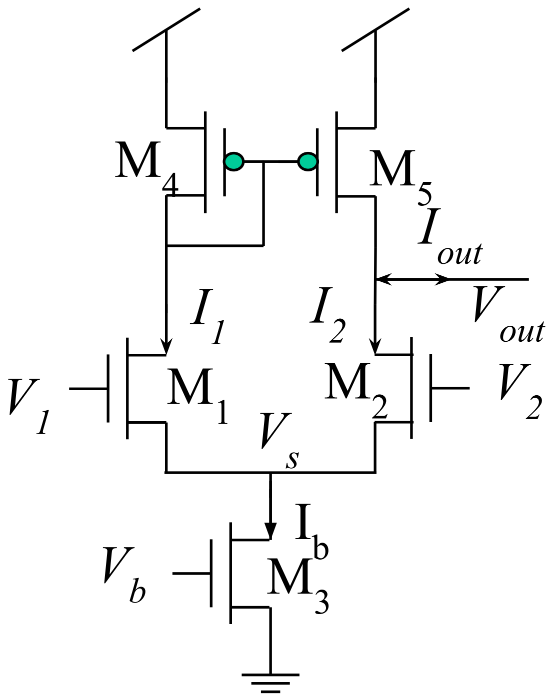
Forward current through $M_2 \gg$ current through $M_1$.

Top current mirror forces $I_1 \approx I_2$.

If $V_2 \gg V_1$, voltage drop across $M_2$ is close to 0,

$V_{out} = V_s$ and $M_2$ goes out of saturation.

# Output Voltage Limits (II)




The output transistors $M_2$ and $M_5$ should also be in saturation:

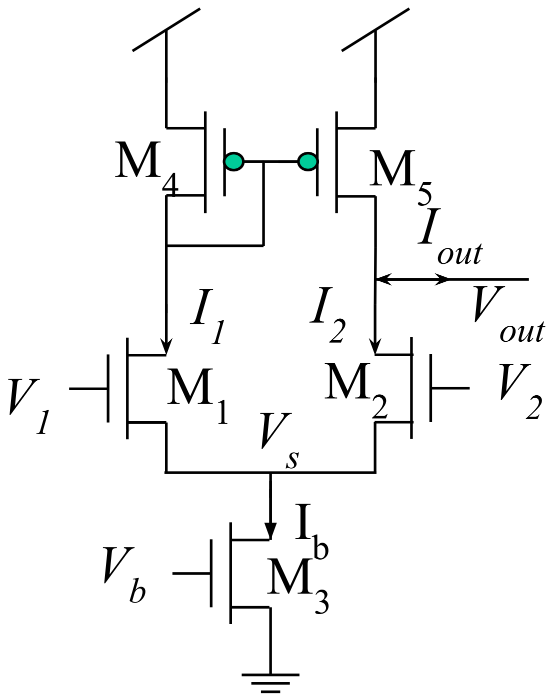
Assume $V_2 > V_1$ therefore $V_{out} \approx V_s$

$$e^{-V_s/U_T} = \frac{e^{\kappa V_b/U_T}}{e^{\kappa V_b/U_T} + e^{\kappa V_1/U_T} + e^{\kappa V_2/U_T}}$$

$$e^{-V_s/U_T} = \frac{e^{\kappa V_b/U_T}}{e^{\kappa V_b/U_T} + 2e^{\kappa V_1/U_T}}$$

$$e^{-V_s/U_T} = \frac{0.5e^{\kappa V_b/U_T}}{0.5e^{\kappa V_b/U_T} + e^{\kappa V_1/U_T}}$$

# Output Voltage Limits (III)




Assume $V_2 > V_1$

a) If $V_1 < V_b - (4+\ln(2))\, U_T / \kappa$

$$V_{out} \approx 0$$

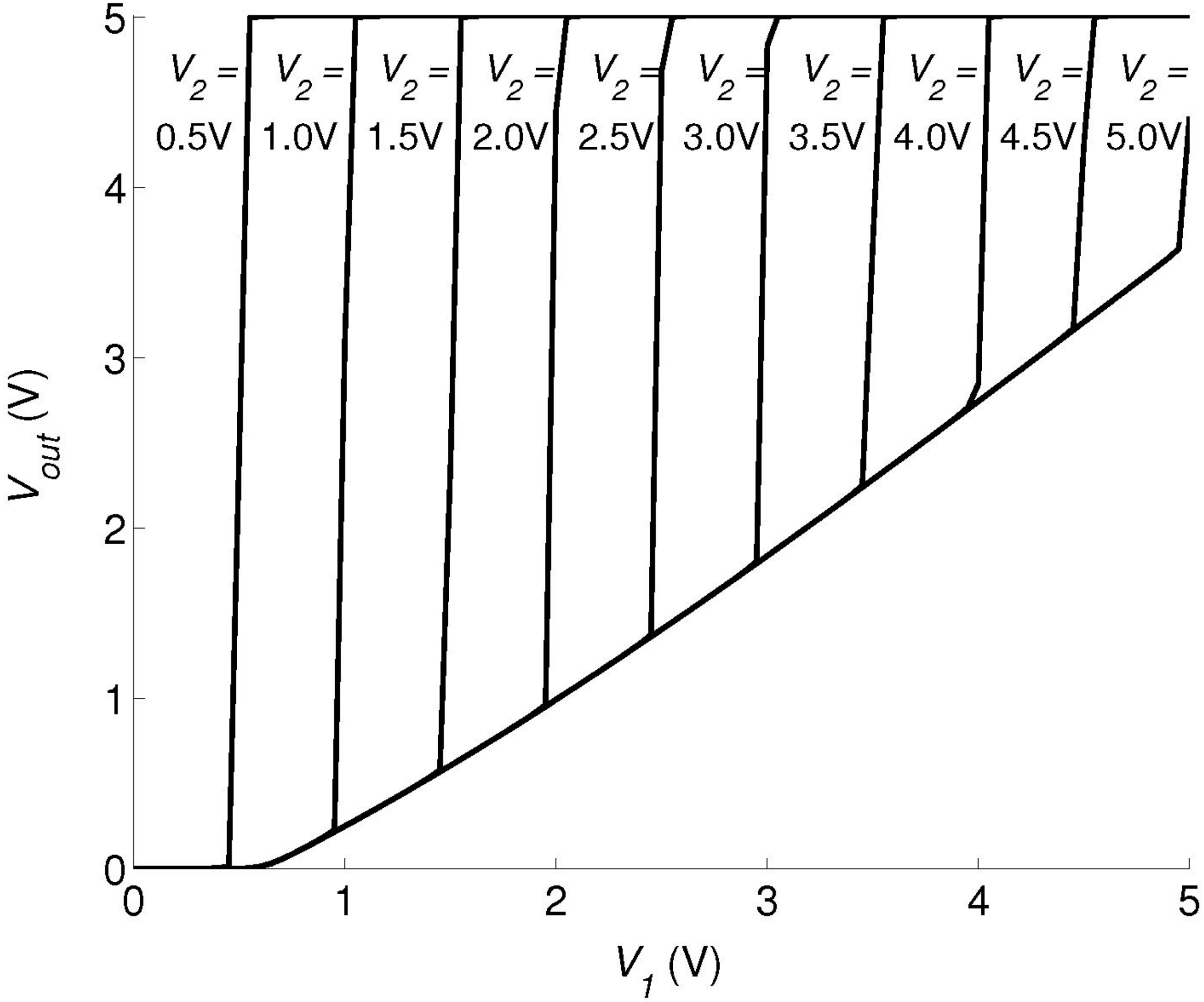
b) If $V_1 > V_b - (4+\ln(2))\, U_T / \kappa$

$$V_{out} \approx \kappa\left(V_1 - V_b\right) + U_T \ln(2)$$

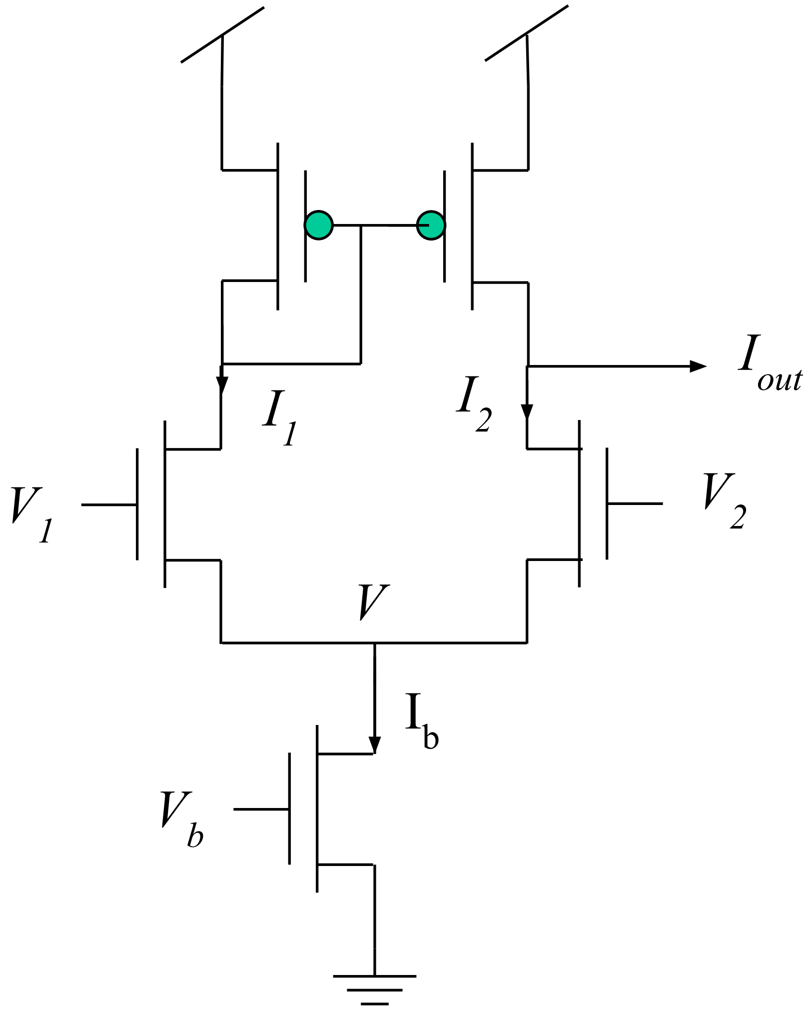
Therefore,

$V_{out}$ increases linearly with $V_1$ with a slope of $\kappa$

# Output Voltage vs Input Voltage



# Transconductance Amplifier

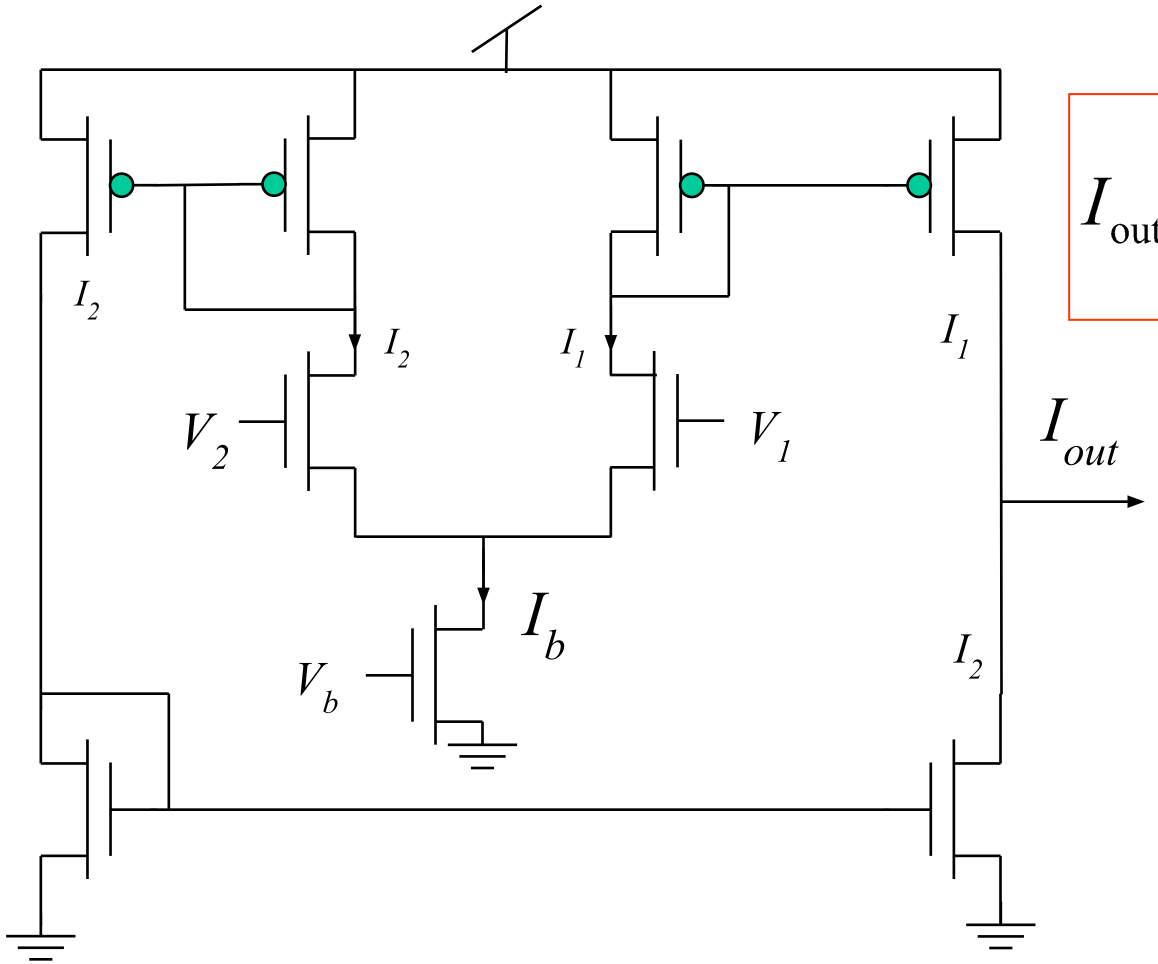



$$I_{\text{out}} = I_1 - I_2$$
$$= I_b \tanh\left(\kappa \frac{V_1 - V_2}{2U_T}\right)$$

Good: Simple, cheap

Bad: output voltage is restricted, voltage gain is limited

# Wide-Output Range Transconductance Amplifier




$$I_{out} = I_b \tanh\left(\kappa \frac{V_1 - V_2}{2U_T}\right)$$

Good: output voltage is unrestricted, gain can be >$10^4$

Bad: larger, more mismatch

# THE END

Next week: Linear systems