Financial Networks

Network Science - Lecture 9

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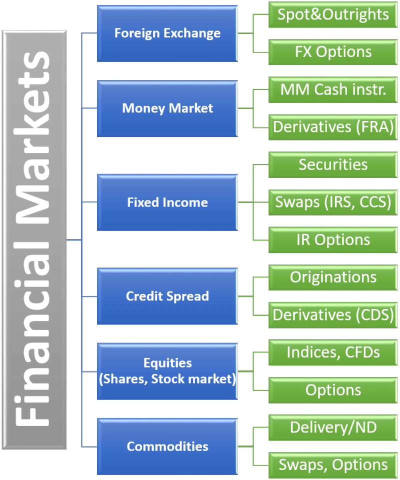
Lecture Objectives

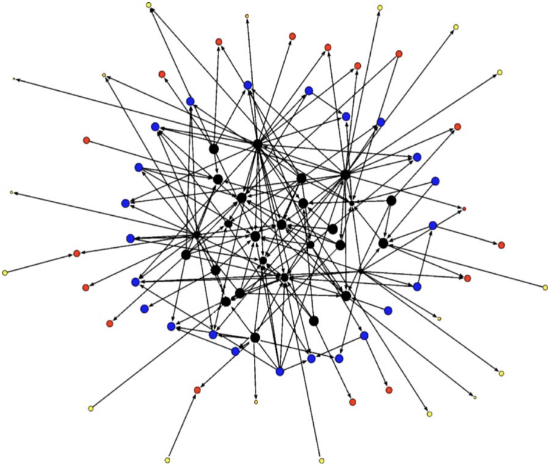
1. Get to know different network representations of financial data
2. Learn how networks help regulators controlling markets...
3. ...how they help traders improving their strategies...
4. ...and platforms profiling their clients

...more like this



Networks of institutions

The interbank market



Nodes are Italian banks
Links are overnight loans
Direction from cred. to deb.

Small world network
Core-Periphery structure

[De Masi et al., *Phys. Rev. E 74*, *doi.org/10.1103/PhysRevE.74.066112* (2006)]

How does this network form?

How do banks select a credit partner?

Fitness model: call V_i the total volume lent to/by bank i and assume this determines linkage

$$p_{ij} = \frac{v_i + v_j}{\sum_{i<j}(v_i + v_j)}$$

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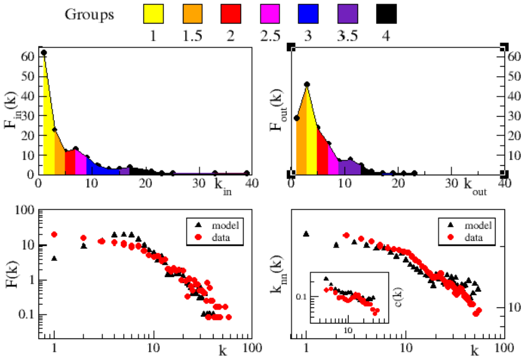


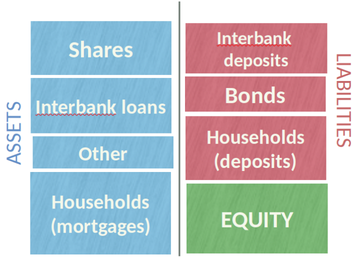
Can we do more?

The fitness model is rather simplistic: the links lose meaning and are unweighted!

We can be more specific

[Bardoscia et al., *PLoS ONE 10, doi.org/10.1371/journal.pone.0130406* (2015)]

“normal” balance sheet



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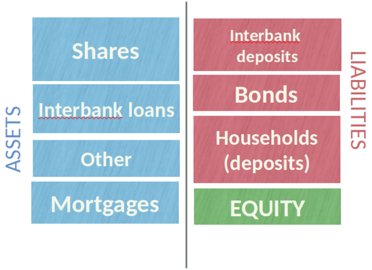
shock on mortgages



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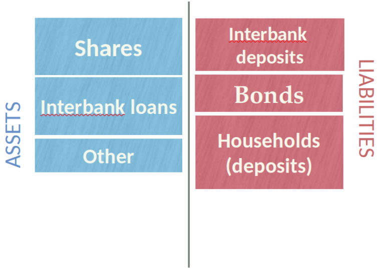
large shock → bankruptcy



DebtRank

Let's call

- A_{ij} are assets on i's balance sheet and liabilities on j's
- $A_i = \sum_j A_{ij}$ are i's total assets, $L_i = \sum_j A_{ji}$ are i's liabilities
- E_i is i's equity $E_i = A_i - L_i$
- $\Lambda_{ij} = A_{ij}/E_i$ is i's exposure to j
- $h_i(t) = [E_i(0) - E_i(t)]/E_i(0)$ is i's relative equity loss at time t

Let's say that A_{ij} devaluates proportionally to an equity loss of j, i.e.

$$\frac{A_{ij}(t+1)}{A_{ij}(t)} = \frac{E_j(t)}{E_j(t-1)}$$

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Then

$$h_i(t+1) = \min\left\{1, h_i(t) + \sum_j \Lambda_{ij}[h_j(t) - h_j(t-1)]\right\}$$

This can be simulated from real balance sheet data, and has been used by the ECB in stress testing

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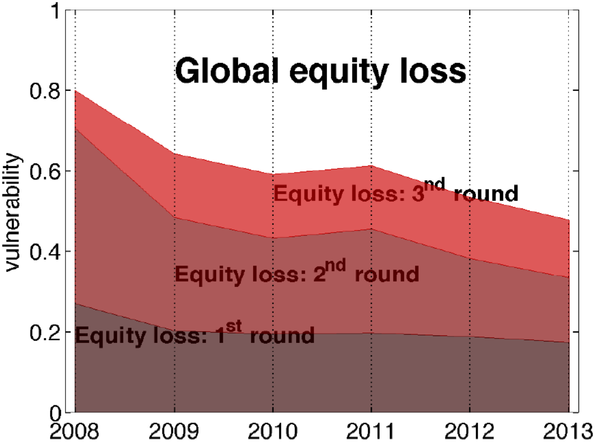
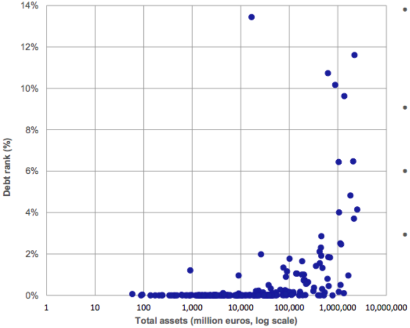
DebtRank application



DebtRank application

WS3: Indicator of marginal bank contagion risk

➢ Effect of bank failure on euro interbank network (example Dec. 08)



- Transmission not only through defaults but also proportional to Furfine exposures, relative losses and relative capitalisation of banks
- Contagion risk larger than found in traditional default simulations
- Largest banks have systemic effect (non-linear) but wide dispersion
- Helps, inter alia, to understand the systemic importance of individual banks and how it evolves over time

Simulation of the overall loss of equity (in % of total) among all banks active in TARGET2 caused by individual bank failures ("debt rank" methodology based on a further development of Battiston et al. (2012)) and bank size. Source: di Iasio, Rainone, Rocco and Vacirca (2013).

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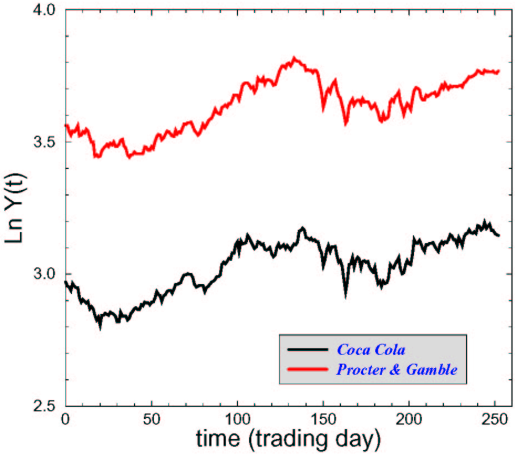
Networks of assets

Correlations among stocks



Pearson's cross-correlations

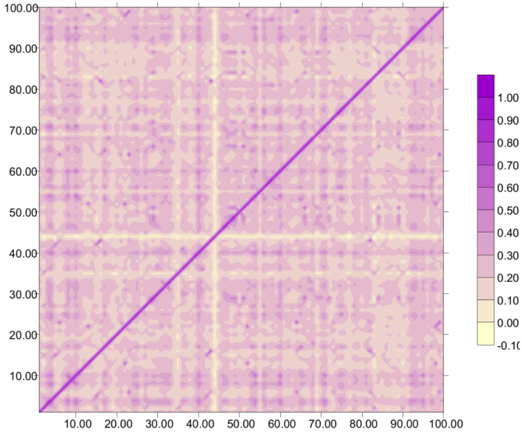
One of the simplest measures of correlation among prices is Pearson's r

$$r_{ij} = \frac{\langle S_i S_j \rangle - \langle S_i \rangle \langle S_j \rangle}{\sqrt{\left\langle S_i^2 - \langle S_i \rangle^2 \right\rangle \left\langle S_j^2 - \langle S_j \rangle^2 \right\rangle}}$$

$$S_i(t) = \log P(t) - \log P(t-1)$$

$S_i(t)$ is called **log-returns** of asset i: it is a way of measuring relative variation of the prices, which returns a well behaved time series

The Correlation Matrix



R_{ij} is the correlation between stock i and j (here 100 stocks from S&P100 index, 1995-98)

Statistical testing

By now you should know that statistics have no meaning if they don't have statistical significance

Null hypothesis: stocks are independent identically distributed

What do correlations look like in this case?

Random correlation matrices

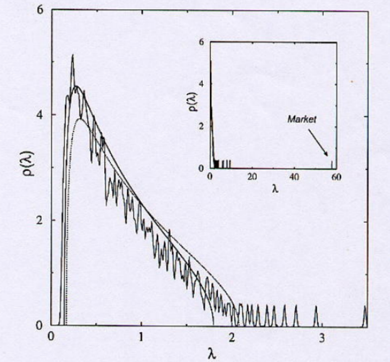
Consider a random data matrix $X \in \mathbb{R}^{t\times N}$ where $X_{ij} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$ for all i, j.
The covariance matrix is $Y_t = \frac{1}{t} X X^T$, with eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$ that are themselves random variables
Let $N, t \to \infty$, keeping $Q = T/N \geq 1$ **fixed**. Then the distribution of the eigenvalues follows

$$\rho(\lambda) = \frac{Q}{2\pi\sigma^2} \frac{\sqrt{(\lambda_{max} - \lambda)(\lambda - \lambda_{min})}}{\lambda}$$

where $\lambda_{min,max} = \sigma^2 \left(1 + \frac{1}{Q} \pm 2\sqrt{\frac{1}{Q}}\right)$

This is called the Marčenko-Pastur law

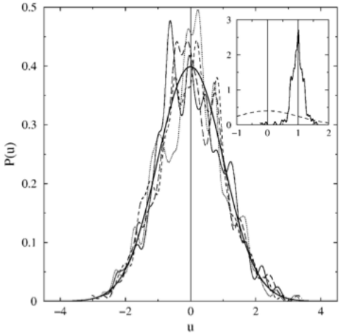
The Marčenko-Pastur law of stocks



The null distribution (MP) only explains a fraction of the eigenvalues of real correlation matrices, and typically the largest eigenvalue λ_1 is a lot larger than expected

λ_1 is often called the **market mode**

Eigenvectors



The eigenvectors related to the *predictable* eigenvalues are well predicted by the theory, whereas the one of the market mode is completely different

Different eigenvectors have been linked to business sectors

Filtering correlations

The correlation matrix can be *filtered* by the MP law:

- Fit the MP to the empirical eigenvalues
- Determine the theoretical λ_{max}: any $\lambda < \lambda_{max}$ is classified as *noise*, others are *signal*
- Replace noise λs with a constant, then renormalize so that $\sum_{i=1}^{N} \lambda_i = N$
- Undo diagonalisation to get the **denoised correlation matrix**

$$\overline{Y} = V \Lambda V^{-1}$$

where V is the eigenvectors matrix and Λ is the diagonal matrix of eigenvalues

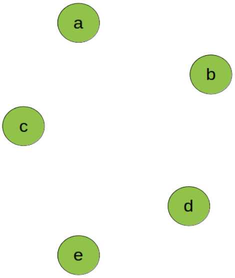
Minimal Spanning Trees

Another way of filtering correlations is using them to define some *metric distance* as

$$d_{ij} = \sqrt{2(1 - r_{ij})}$$

and based on it do a **hierarchical clustering**

	a	b	c	d	e
a	0	0.85	0.94	0.89	0.96
b		0	0.88	0.78	0.93
c			0	0.95	1.01
d				0	0.97
e					0



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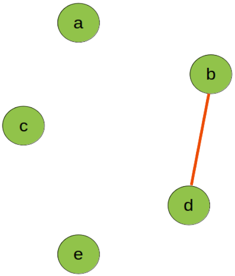

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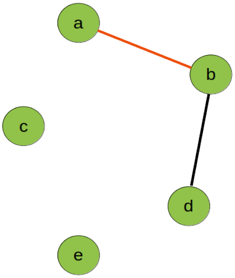

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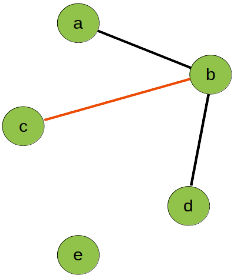

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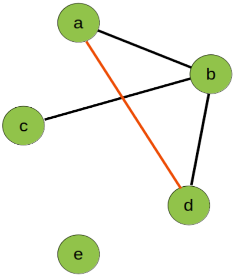

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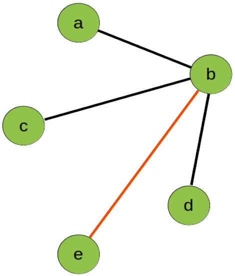

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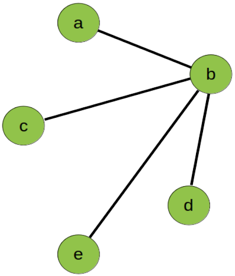
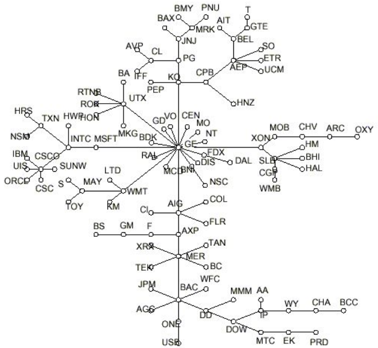
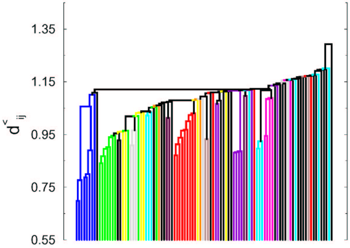


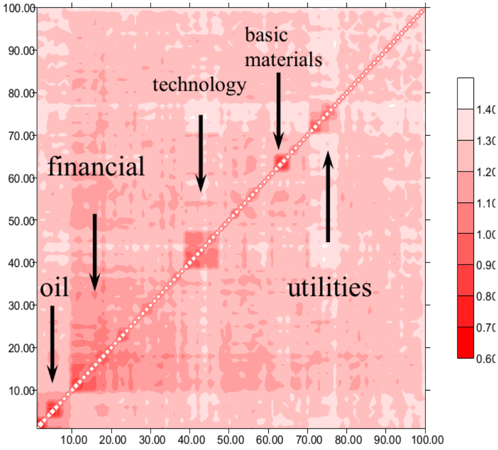
MST of stocks





The MST is also visualizable as a **hierarchical tree** (dendrogram)

MST of stocks

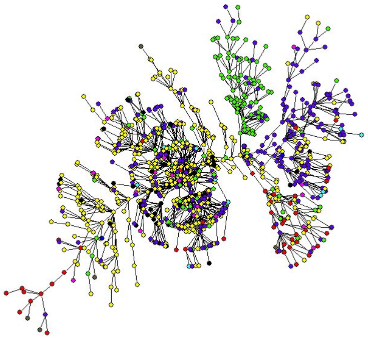


Sorting the rows/columns according to the hierarchical tree highlights the structure of correlations!

MSTs in synthetic models

One can compare the MST between measurements and simple models

1071 NYSE stocks 1987-1998



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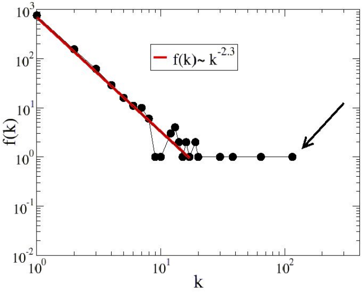

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degree distribution

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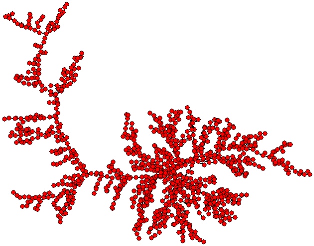

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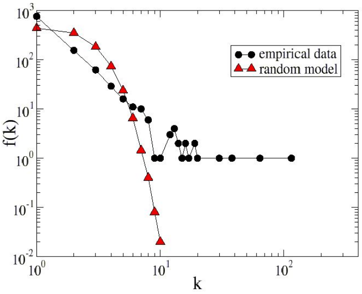

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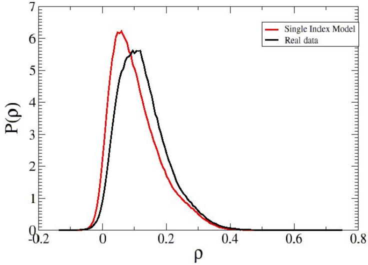

One-factor model

$$R_i(t) = \alpha_i + \beta_i R_M(t) + \epsilon_i(t)$$

explains 85% of correlations

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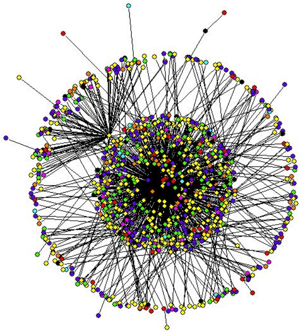

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BAD MST

MSTs in synthetic models

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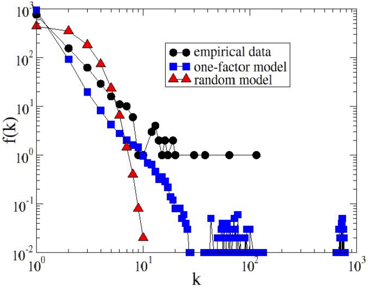


One-factor model

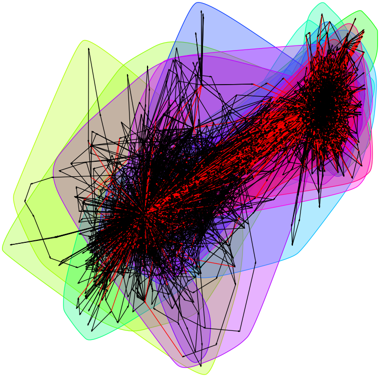
$$R_i(t) = \alpha_i + \beta_i R_M(t) + \epsilon_i(t)$$

explains 85% of correlations
degree distribution is not reproduced

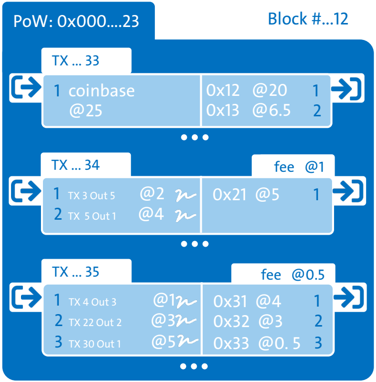
Networks of transactions

Cryptocurrencies

Cryptocurrencies



UTXO

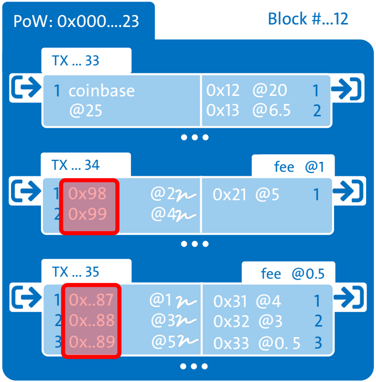


Each transaction always contains both:

a set of outputs the recipients of the transaction and amount of assets transferred

a set of inputs pointers to outputs of previous transactions that are being spent in this transaction

Multiple Inputs

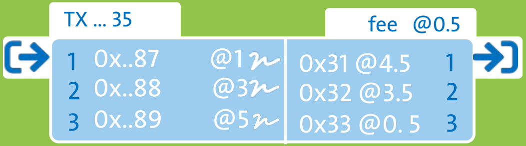


Input addresses

All input addresses in the same transaction must belong to the same user (because she has access to all private keys)

- Very reliable (until ~2017)
- Large level of aggregation

Change output



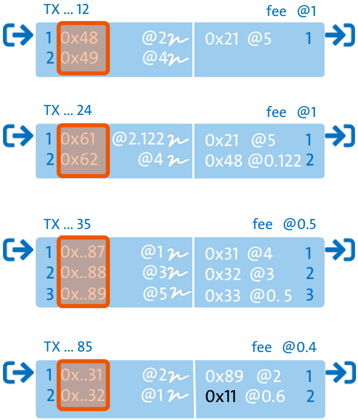
It is not always possible to match the sum of all inputs with the output. There is always an output that serves as change; i.e., it is sent to the transaction creator

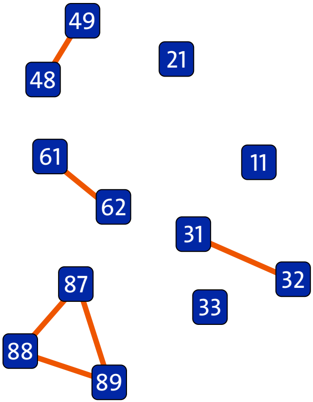
The system attempts to minimise number of inputs (transaction size has an impact on cost)

It also attempts to minimise the change

How to reconstruct identities?

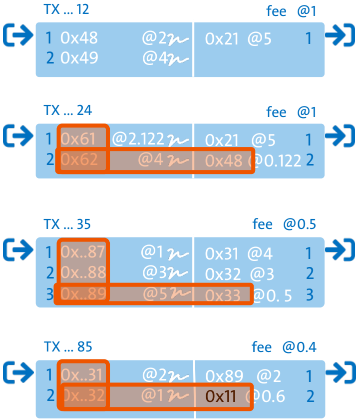
1. Multiple input

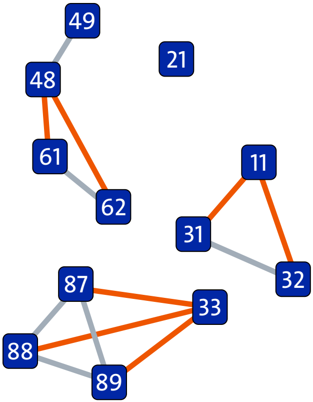




How to reconstruct identities?

2. Change addresses and others





How to reconstruct identities?

3. Cluster them

TX … 12 fee @1

Inputs		Outputs	
1 0x48	@2	0x21 @5	1
2 0x49	@4		

TX … 24 fee @1

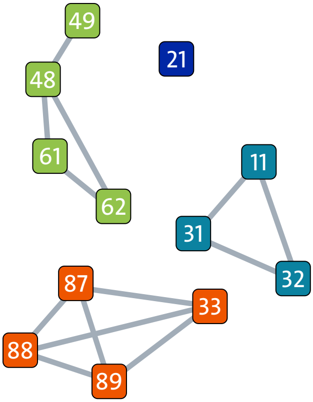
Inputs		Outputs	
1 0x61	@2.122	0x21 @5	1
2 0x62	@4	0x48 @0.122	2

TX … 35 fee @0.5

Inputs		Outputs	
1 0x..87	@1	0x31 @4	1
2 0x..88	@3	0x32 @3	2
3 0x..89	@5	0x33 @0. 5	3

TX … 85 fee @0.4

Inputs		Outputs	
1 0x..31	@2	0x89 @2	1
2 0x..32	@1	0x11 @0.6	2



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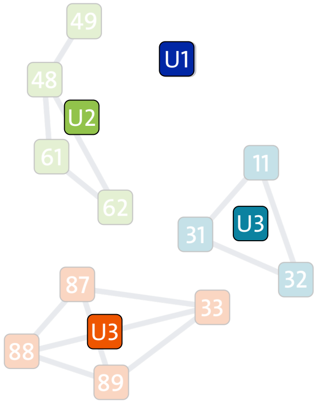

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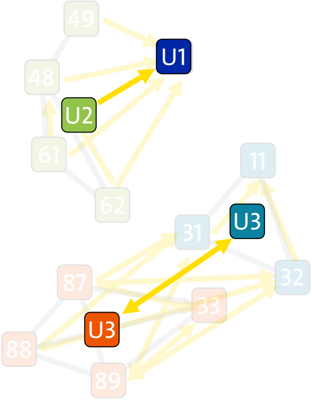
Inputs	Outputs
1 0x61 @2.122	0x21 @5 1
2 0x62 @4	0x48 @0.122 2

TX ... 35 fee @0.5

Inputs	Outputs
1 0x..87 @1	0x31 @4 1
2 0x..88 @3	0x32 @3 2
3 0x..89 @5	0x33 @0. 5 3

TX ... 85 fee @0.4

Inputs	Outputs
1 0x..31 @2	0x89 @2 1
2 0x..32 @1	0x11 @0.6 2



How to reconstruct identities?

3. Cluster them

TX ... 12 — fee @1

1 0x48 @2 2 0x49 @4	0x21 @5 1

TX ... 24 — fee @1

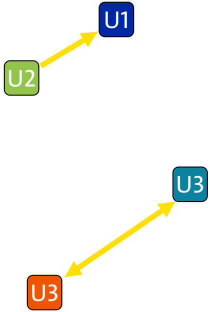
1 0x61 @2.122 2 0x62 @4	0x21 @5 1 0x48 @0.122 2

TX ... 35 — fee @0.5

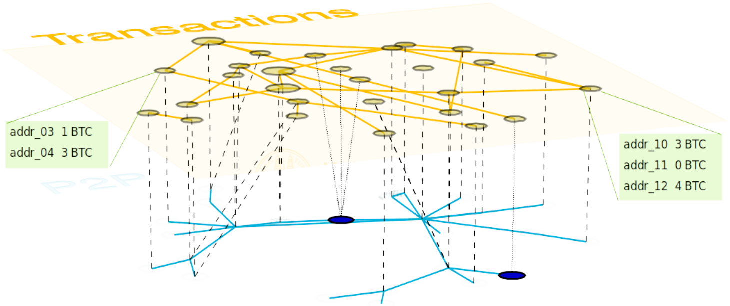
1 0x..87 @1 2 0x..88 @3 3 0x..89 @5	0x31 @4 1 0x32 @3 2 0x33 @0. 5 3

TX ... 85 — fee @0.4

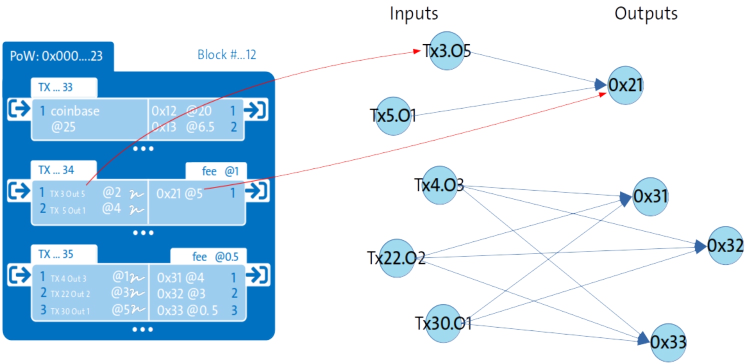
1 0x..31 @2 2 0x..32 @1	0x89 @2 1 0x11 @0.6 2



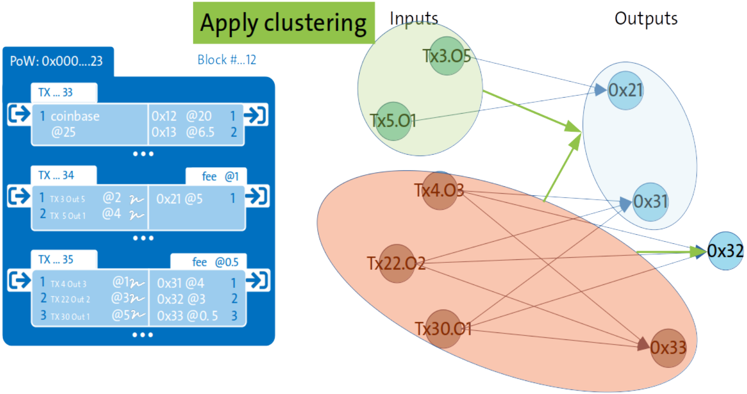
How to define a tx network



How to define a tx network

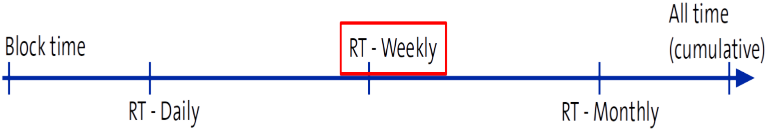


How to define a tx network



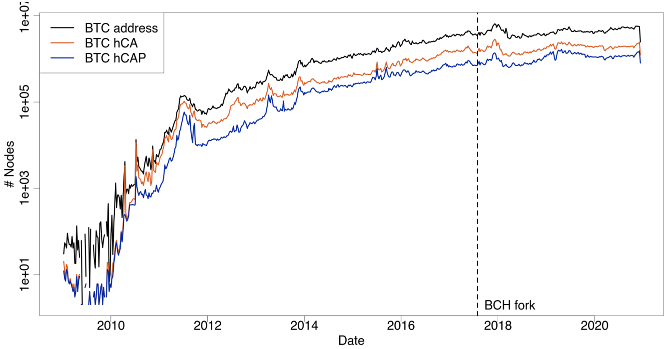
Time aggregation

The time aggregation is largely arbitrary



The larger the time aggregation, the least "causal" the network is. Need to make choices depending on research questions.

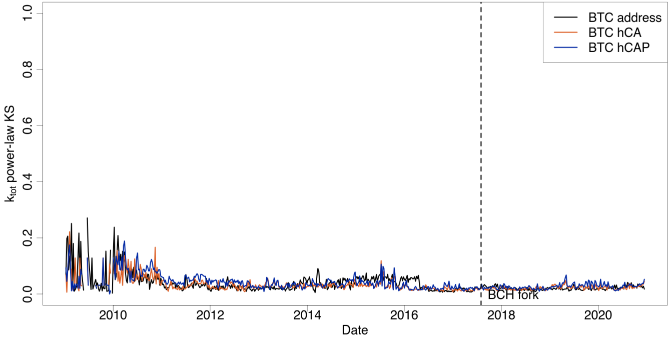
Transaction Networks: basics



Transaction Networks: basics

Transaction Networks: degree distribution

Degree distributions are strongly skewed and close to power-law behaviour



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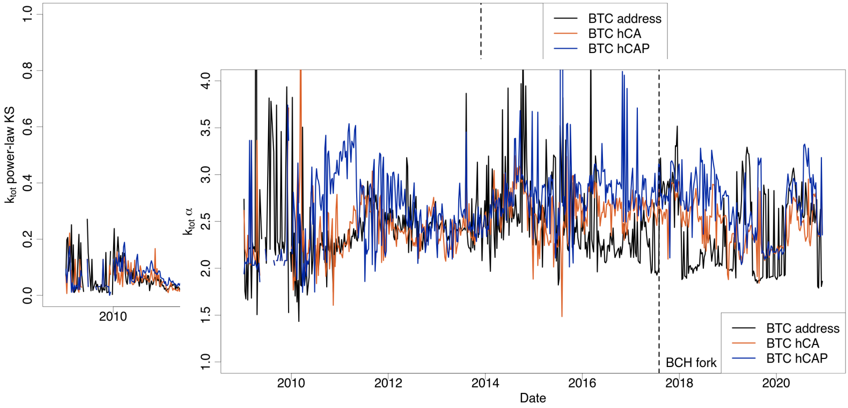
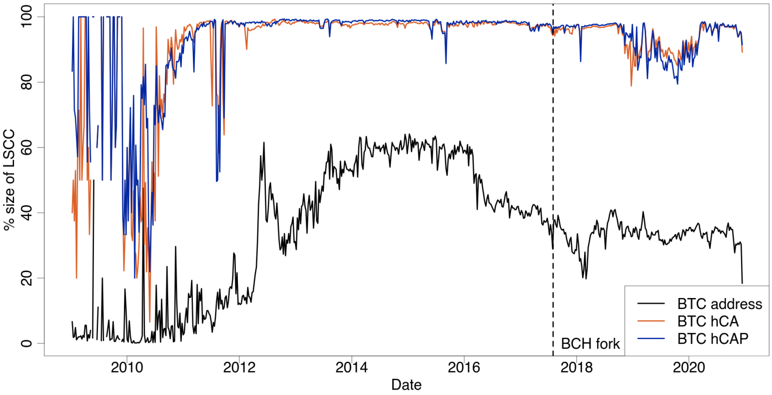


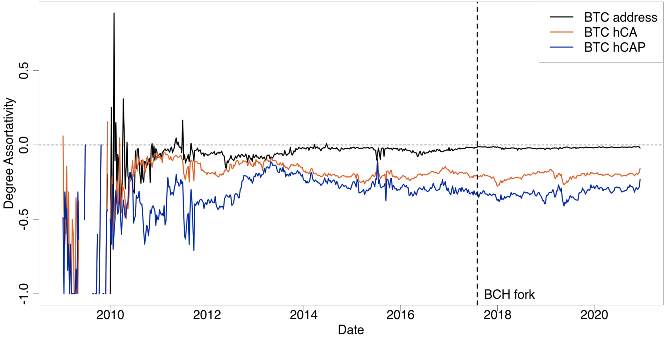
Transaction Networks: connectedness

Clustering makes networks cohesive and strongly connected



Transaction Networks: signs of centralisation

Importantly, clustered networks are degree-disassortative



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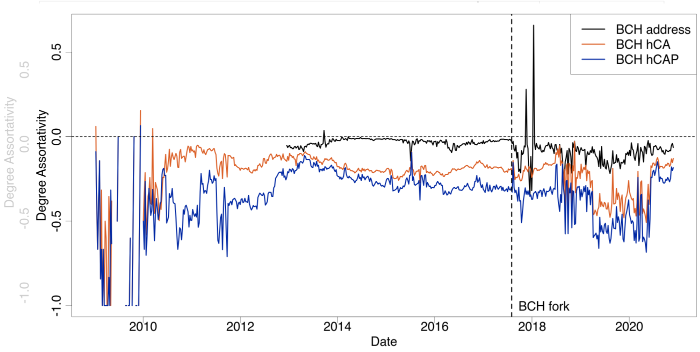

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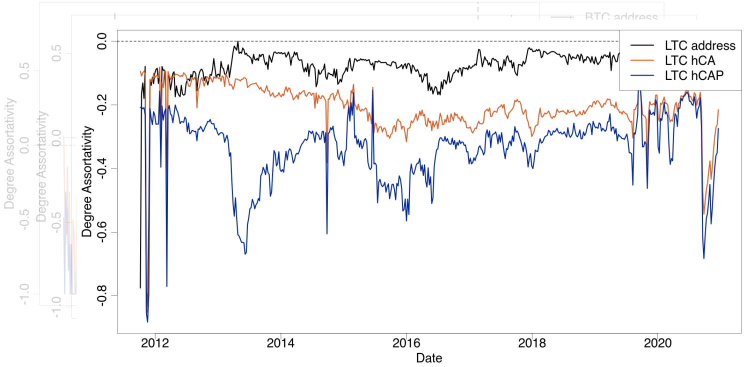

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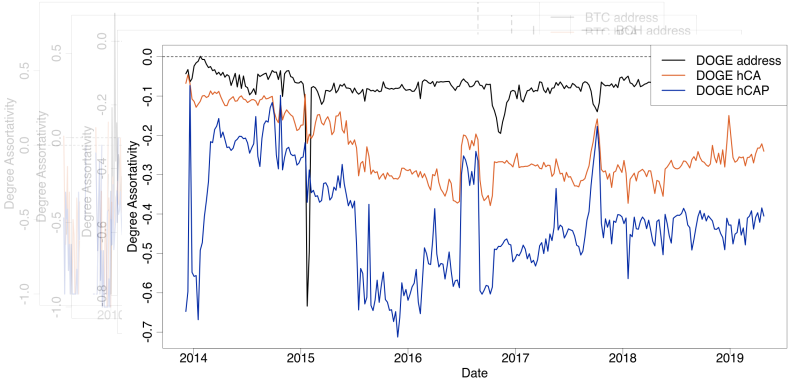

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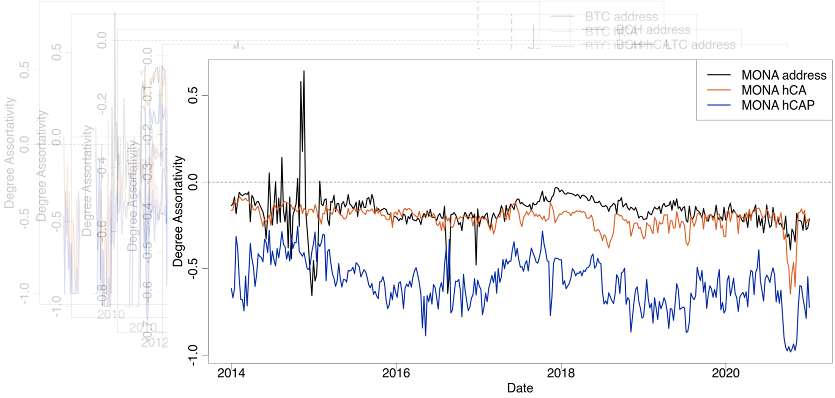
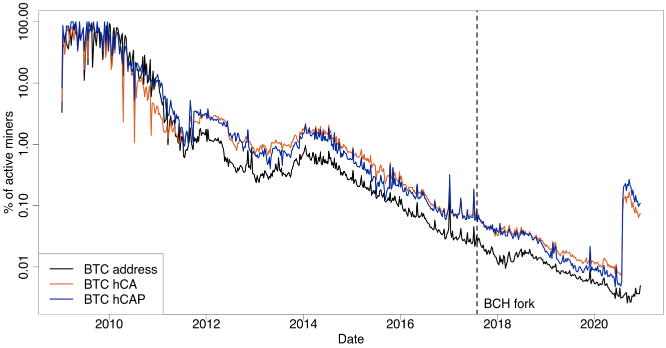


Transaction Networks: Miner Activity

A sign of mining power centralisation



Transaction Networks: Miner Activity

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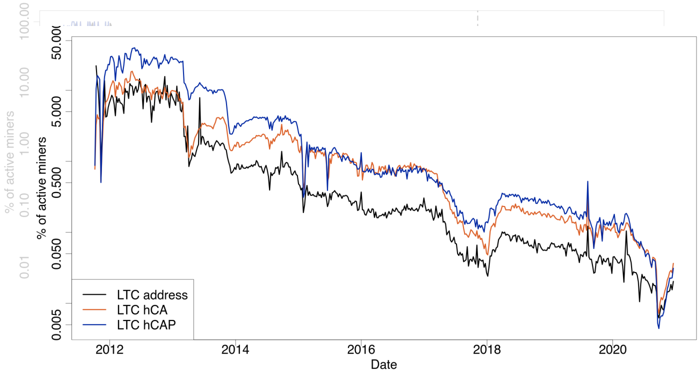

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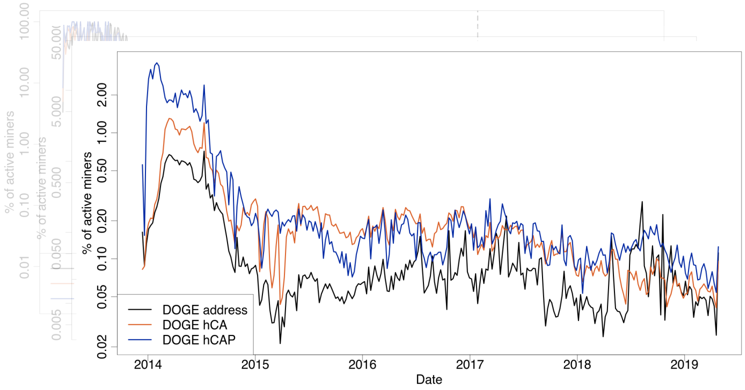

Transaction Networks: Miner Activity

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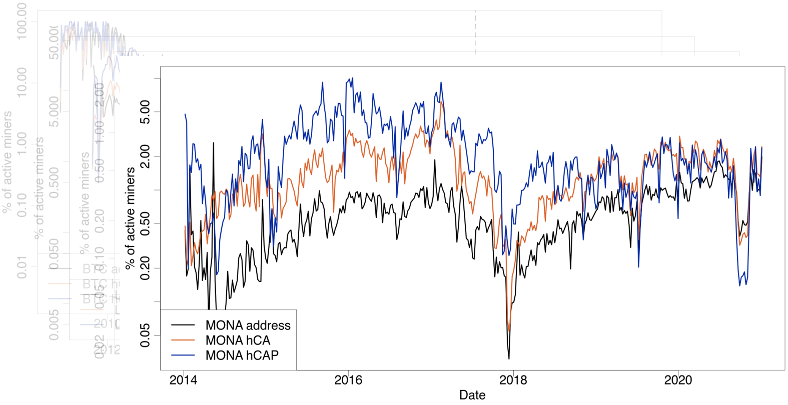

Transaction Networks: Miner Activity

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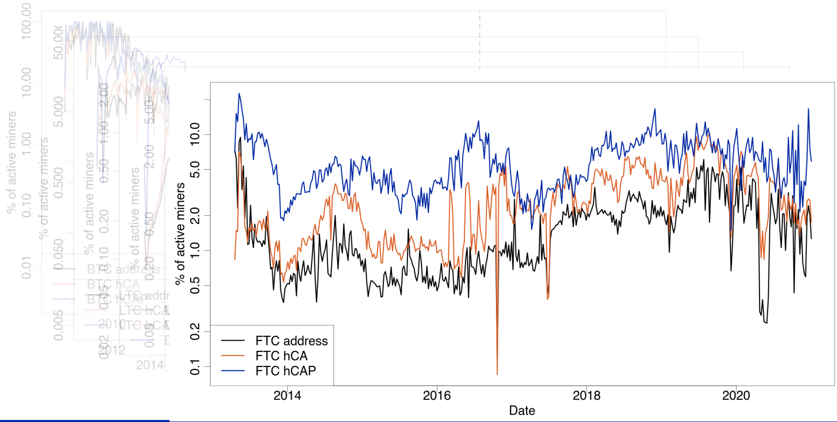


Some conclusions

- Cryptocurrencies are thought for anonymity, but this is not uncrackable;
- Transaction networks help sorting the information, highlighting centralisation arising naturally.

Decentralised technology $\neq$ decentralised economy

Networks of traders

Centralised markets

Several asset classes are traded on **exchange markets**

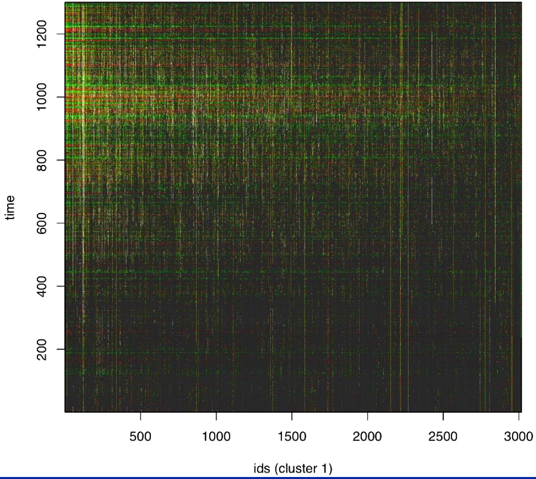
Nasdaq

Trading

Trading is the process of buying/selling assets on the market and different investor classes adopt different **trading strategies**

Can we find clusters of investors that behave similarly?

Traders



~ 3000 traders
~ 1300 days
sells
buys
buys&sells
trader inactive

[Tumminello et al., *New J. Phys. 14, doi:10.1088/1367-2630/14/1/013041* (2012)]

Statistically validated networks

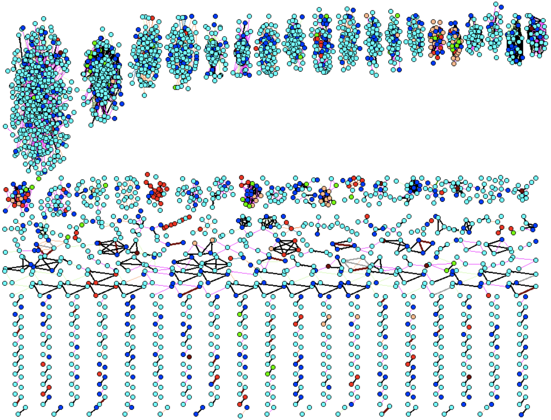
N days, I investors each having state $i \in \{S, B, BS, 0\}$ every day
For each pair of investors a, b, count the number of co-occurrences of combinations of states X_{i_a, i_b}

Investor i_a \ Investor i_b	B	S	BS
B	$X_{B.B}$	$X_{B.S}$	$X_{B.BS}$
S	$X_{S.B}$	$X_{S.S}$	$X_{S.BS}$
BS	$X_{BS.B}$	$X_{BS.S}$	$X_{BS.BS}$

*X_{i_a, i_b} follows the **hypergeometric** distribution!*

Investor networks

Using SVNs + Bonferroni correction



Investor networks

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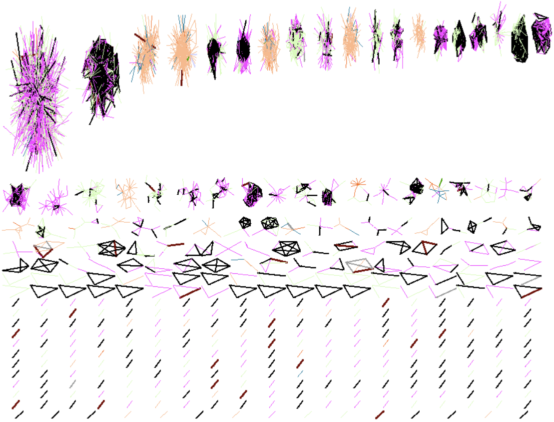

B.B
S.S
B.B & S.S
BS.BS
B.BS

References I

- M. Bardoscia et al., *The physics of financial networks*, Nature Reviews Physics, 2021.
- G. Bonanno et al., *Topology of correlation-based minimal spanning trees in real and model markets*, Physical Review E 68.4 2003.
- M. Tumminello et al., *Identification of clusters of investors from their real trading activity in a financial market*, New Journal of Physics 14.1, 2012.