Denoising MEMS accelerometer sensors based on L2-norm total variation algorithm

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A method for denoising accelerometer data based on the L2-norm total variation (LTV) algorithm is presented. In order to collect accelerometer data, a wireless accelerometer sensor was developed that is directly connected to a central node. By benefiting from the LTV algorithm, the obtained signals from the accelerometer are denoised. The proposed method is tested by denoising in different accelerometer signals and the results are evaluated by signal-to-noise ratio and power spectral density functions of the signals. The obtained results reveal that noise reduction has been implemented satisfactorily. Hence, the measurement accuracy of accelerometer signals for the proposed method have improved ~4–10% than other the three low-pass filters including Savitzky–Golay, equiripple-pass-band and Butterworth.

Introduction: Accelerometers have recently emerged in different applications especially in healthcare [1] due to the progress in the performance of micro-electromechanical systems (MEMS), so use of MEMS-based accelerometers are increasing in the healthcare arena. Due to the sensor's physical features, which include circuit devices and complex environmental factors, accelerometer data accompanied by noise during signal acquisition. Accordingly, noise reduction is essential in MEMS accelerometer sensors [2].

Digital filters such as the Butterworth (BW) and low-pass were initially used [3] for denoising MEMS-based accelerometer sensors. In spite the simple architecture of these filters, it is difficult to evaluate the boundary between noise and signal. In another work, denoising was performed to calculate a characteristic function for correcting the amplitude and phase of the accelerometer signal by employing a custom-designed discrete-time filter [4]. However, this denoising is associated to special accelerometers, and accordingly, one can say the mentioned method is subjective. Hence, the Kalman filter was employed to estimate accelerometer data in the presence of noise [5]. Considering the accurate determination of statistical properties of noise measurement is difficult in actual conditions, consequently, the Kalman filter might not deliver desired performance. To this end, the adaptive Kalman filter was improved, and the innovation sequence and optimisation approach were applied [6]. One of the disadvantages of the adaptive Kalman filter is its complexity.

In this Letter, a noise reduction method for accelerometer data is presented based on the L2-norm total variation (LTV) algorithm. To that end, first a hardware is developed with features including low profile and cost effective while being comfortably wearable and non-intrusive. Then, obtained data from the developed sensor is denoised using the LTV algorithm. In comparison to the equiripple-pass-band (EP), BW and Savitzky–Golay (SG) low-pass filters, the proposed method exhibits higher signal-to-noise ratio (SNR) and lower power spectral density (PSD).

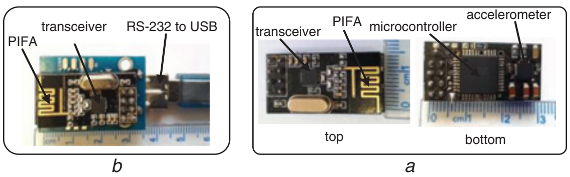

Fig. 1 *Schematic depicting developed measurement system*

a Wireless accelerometer
b Central node

Developed measurement system: To measure acceleration, a measurement system (Fig. 1) is developed including a wireless accelerometer sensor as well as a central node, which is connected to a personal computer to record the measurements at sampling rate of 50 Hz. Similar to a typical wireless sensor [7], the essential components of each wireless accelerometer are a MEMS-based accelerometer, a microcontroller, a transceiver operating at the Industrial, Scientific and Medical (ISM) frequency of 2.45 GHz and a built-in meander planar inverted-f antenna as shown in Fig. 1*a*. Furthermore, the central node consists of a transceiver and an RS-232 to USB port converter (Fig. 1*b*).

Proposed method and results: Total variation regularisation is known in signal processing and is most often used for noise reduction in digital image processing. Original signal (without noise) can be obtained by decreasing the total variation of the digital signal [8]. In order to increase SNR of the MEMS-based accelerometer data, a method is proposed based on a function which is called LTV algorithm. This function is able to eliminate the noise included in the accelerometer signal. This function attempts to minimise the following equation:

$$\min_{x,u} \frac{\mu}{2}\left\|f-g\right\|^2 + \left\|Df\right\| \tag{1}$$

where μ and D are regularisation parameter and first-order forward finite difference operator, respectively. f and g are accelerometer data before and after denoising. In order to solve (1), (2) can be obtained by using the intermediate variable $u = Df$. In other words, the unconstrained problem (1) is converted to a constraint problem (2)

$$\begin{gathered}\min_{x,u} \frac{\mu}{2}\left\|f-g\right\|^2 + \|u\| \\ \text{Subject to } u = Df\end{gathered} \tag{2}$$

By using augmented Lagrangian method [9]

$$l(f,u,y) = \frac{\mu}{2}\left\|f-g\right\|^2 + \|u\| - y^{\mathrm{T}}(u-Df) + \frac{p_{\mathrm{r}}}{2}\left\|u-Df\right\|^2 \tag{3}$$

where p_{r} is the regularisation parameter associated with the quadric penalty term $\left\|u-Df\right\|^2$, y is Lagrange multiplier associated with the constraint $u = Df$. By solving the resulting equation using the alternating direction method [10], it can be obtained as follows:

$$f_{k+1} = \arg\min \frac{\mu}{2}\left\|f-g\right\|^2 - y^{\mathrm{T}}(u_k-Df) + \frac{p_{\mathrm{r}}}{2}\left\|u_k-Df\right\|^2 \tag{4}$$

$$u_{k+1} = \|u\| - y^{\mathrm{T}}(u-Df_{k+1}) + \frac{p_{\mathrm{r}}}{2}\left\|u-Df_{k+1}\right\|^2 \tag{5}$$

$$y_{k+1} = y_k - p_{\mathrm{r}}(u_{k+1}-Df_{k+1}) \tag{6}$$

By dropping indexes k and using normal equation, the solution of f is as follows:

$$(\mu + p_{\mathrm{r}}D^{\mathrm{T}}D)f = \mu g + p_{\mathrm{r}}D^{\mathrm{T}}u - D^{\mathrm{T}}y \tag{7}$$

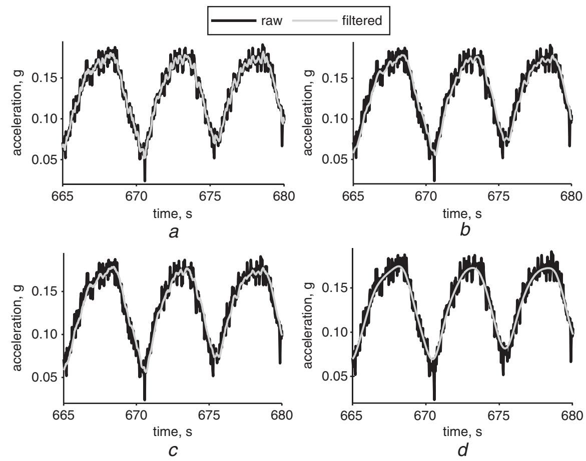

Fig. 2 *Accelerometer output of developed sensor for lung movement before (raw) and after (filtered) passing through filters*

a SG
b BW
c EP
d Presented method (LTV) with $p_{\mathrm{r}} = 0.4$ and $\mu = 0.5$

In order to achieve solution of f, the discrete Fourier transform of (7) is obtained

$$f = F^{-1}\left[\frac{F(\mu g + p_{\mathrm{r}}D^{\mathrm{T}}u - D^{\mathrm{T}}y)}{\mu + p_{\mathrm{r}}|F[D]|^2}\right] \tag{8}$$

where D^{T} and D are inverse and forward finite difference operators.

The parameter u should be obtained by

$$u = \max\left\{|V| - \frac{1}{p_r}, 0\right\} \cdot \mathrm{sign}(V) \tag{9}$$

$$V_x = Df + \frac{1}{p_r} y \tag{10}$$

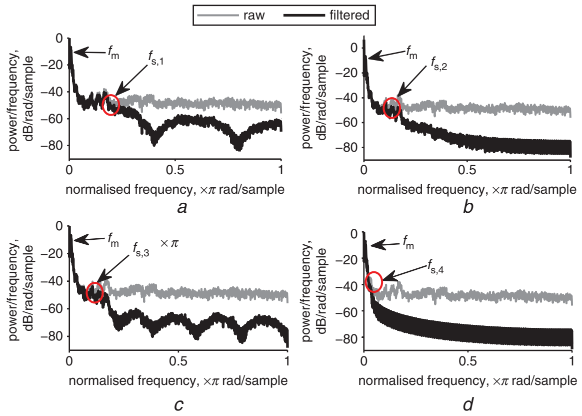

Fig. 3 *PSD of signal in Fig. 2 before (raw) and after (filtered) passing on filters*

a SG
b BW
c EP
d Presented method (LTV). f_m is frequency of lung movement. f_s is frequency where PSD of filtering data is separated from PSD of raw accelerometer data

Table 1: Developed method comparison to other methods

Method	Small signal			Large signal		
	SNR	STD	Mean	SNR	STD	Mean
–	5.43	0.0486	0.1226	15.55	0.5445	–0.6967
EP	5.93	0.0480	0.1225	17.64	0.5368	–0.6942
SG	5.87	0.0481	0.1226	17.61	0.5382	–0.6966
BW	5.92	0.0481	0.1226	17.14	0.5419	–0.6953
LTV	6.08	0.0456	0.1226	18.73	0.5252	–0.6967

Posteriori y obtained from (8)–(10), and priori y from (6). This iteration is repeated for denoising any accelerometer data. In order to investigate the presented method, two scenarios are performed. In one scenario, the wireless accelerometer sensor is fixed on a volunteer's chest and his lung movement data is collected for a duration of about 14 min. Another scenario is done by situating the wireless accelerometer sensor on the volunteer's wrist to record wrist movement in jumping jack exercise. Then, the obtained raw data are passed on to the presented method and other three filters including SG, EP and BW that their parameters are separately set to obtain highest SNR. The BW filter is a second-order low-pass digital filter as [11] with the sampling rate of 50 Hz and a cutoff frequency of 0.5 Hz, which corresponds to a normalised value of 0.01. The SG filter is a digital low-pass filter for the purpose of smoothing the data [12]. The SG filter is a generalised moving average with filter coefficients determined by an unweighted linear least-squares regression and a polynomial model of second degree. Moreover, EP is a fourth-order low-pass filter that its parameter as [13] consists of the amount of ripple allowed in the pass band (0.1 dB), attenuation in the stop band (1 dB), frequency at the start of the pass band (0.01π radians/sample) and end of the stop band (0.08π radians/sample). Fig. 2 shows the obtained results of all the four filters used for denoising the MEMS-based accelerometer data for volunteer's lung movement while showing that the presented method presents the highest accuracy and lowest noise among other methods. For further investigation, the SNR as well as standard deviation (STD) and mean of the signal of the presented method (LTV), EP, SG and BW filters beside raw accelerometer data were obtained for lung movement (small signal) and jumping jack exercise (large signal) and are shown in Table 1. These results proved that the SNR of the presented method has improved 12–20% compared with the unfiltered data for lung movement and jumping jack exercise. These values for EP, SG and BW filters are 9–13, 8–13 and 9–10%, respectively. Also, the STD of the presented method, EP, SG and BW filters has decreased ~6, 1, 1, 1% than the unfiltered data in turn. Moreover, the obtained PSDs (Fig. 3) for the raw accelerometer signal (grey) and four mentioned methods (black) were determined for lung movement. The red circle in Fig. 3 shows a frequency where PSD of filtering and raw accelerometer data are separated. As it can be seen, the mentioned frequency for the presented method ($f_{s,4}$) is smaller than the frequency of EP ($f_{s,3}$), BW ($f_{s,2}$) and SG ($f_{s,1}$) filters, respectively. Also, the PSD level of the presented method is lower than other filters in $f > f_{s,4}$. These results are evidences for high denoising capability in the presented method. Correspondingly, the effect of regularisation parameter (p_r) on SNR was investigated for the presented method. Obtained results illustrated that SNR changes in different p_r and optimum value occurred in $p_r = 0.4$ and $p_r = 0.025$ for lung movement and jumping jack exercise, respectively. Likewise, μ parameter was selected 0.5 in the LTV for the two movements.

Conclusions: In this Letter, a new method for denoising MEMS accelerometers based on the LTV algorithm was presented. By using a developed wireless accelerometer sensor, acceleration data of lung movement and also jumping jack exercise for several volunteers were collected. Obtained results showed that the SNR of the presented method is ~3–10, 4–7 and 3–7% more than the BW, SG and EP filters, respectively. Also, the STD of the presented method was obtained 3–5% lower than the EP, SG and BW filters. Correspondingly, the PSD of the signals in the presented method was lower than the other mentioned filters in $f > f_{s,4}$. Due to the satisfying results of the proposed method with respect to other filters, it can be used in a variety of healthcare applications.

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One or more of the Figures in this Letter are available in colour online.

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