

Base Case Left Hand Side	Base Case Right Hand Side
<p>L.H.S let x be '()' =&gt; empty list  Apply-Compare law: <b>Base Case</b></p>	<p>Let x be '()' =&gt; <b>Base Case</b></p>
<p><math>((o\ f\ g)\ x) == (f\ (g\ x))</math>  <math>((((curry\ map)\ f)\ (((curry\ map)\ g)\ '())))</math></p>	<p><math>((((curry\ map)\ (o\ f\ g))\ '()))</math>  <u>Apply curried law</u>  <math>(map\ (o\ f\ g)\ '())</math></p>
<p>Apply-curried law:  <math>((((curry\ f)\ x)\ y) == (f\ x\ y))</math></p>	<p><u>Apply definition of map</u>  <u>If #t law</u></p>
<p><math>(map\ f\ (((curry\ map)\ g)\ '()))</math></p>	<p><math>(if\ (null?\ '())\ '() =&gt; \#t</math></p>
<p><math>(map\ f\ (map\ g\ '()))</math></p>	<p><u>if #f non-empty law</u></p>
<p><u>Apply map-empty law in the inner map</u>  <math>(map\ g\ '())</math> is '()</p>	<p><math>(cons\ (f\ (car\ '())\ (map\ f\ (cdr\ '())))) =&gt; \#f</math></p>
<p><u>Map-empty law in outer map</u>  <math>(map\ f\ '())</math></p>	<p>The first case is true hence:  <math>(map\ (o\ f\ g)\ '()) =&gt; '()</math></p>
<p><u>Apply definition of map</u>  <u>If #t =&gt; empty law</u></p>	<p>So both sides evaluate to '()' at the base case so they are equal at their base cases.</p>
<p><math>(if\ (null?\ '())\ '() =&gt; \#t</math></p>	
<p><u>if #f =&gt; Non-empty law</u></p>	
<p><math>(Map\ f\ '())</math> will return '()</p>	
Inductive Case	Inductive Case
<p>Assume <math>x = (cons\ z\ zs)</math></p>	<p>Assume <math>x = (cons\ z\ zs)</math></p>
<p><u>Compose law</u>  <math>((((curry\ map)\ f)\ (((curry\ map)\ g)\ x)))</math></p>	
<p><u>Curried law</u>  <math>(map\ f\ (map\ g\ x))</math></p>	<p><u>= {curried-law}</u>  <math>(map\ (o\ f\ g)\ (cons\ z\ zs))</math></p>
<p><u>X is not an empty</u>  <math>(map\ f\ (map\ g\ (cons\ z\ zs)))</math></p>	<p><u>= {Apply definition of map}</u></p>

<p><u>substitute actual parameter into inner map with definition of map</u></p> <pre>(map f   (if (null? (cons z zs)) '()       (cons (g (car (cons z zs)))             (map g (cdr (cons z zs))))))</pre> <p><u>={null? - cons law}</u></p> <pre>(map f   (if #f       '()       (cons (g (car (cons z zs)))             (map g (cdr (cons z zs))))))</pre> <p><u>={if #f law}</u></p> <pre>(map f   (cons (g (car (cons z zs)))         (map g (cdr (cons z zs)))))</pre> <p><u>={car-cons law}</u></p> <pre>(map f   (cons (g z) (map g (cdr (cons z zs)))))</pre> <p><u>={cdr-cons law}</u></p> <pre>(map f   (cons (g z) (map g zs)))</pre> <p><u>=Apply Map function Definition</u></p> <pre>(if (null? (cons (g z) (map g zs))) '()     (cons (f (car (cons (g z) (map g zs)))           (map f (cdr (cons (g z) (map g zs))))))</pre> <p><u>={null?-cons law}</u></p> <pre>(if #f     (cons (f (car (cons (g z) (map g zs)))           (map f (cdr (cons (g z) (map g zs))))))     (cons (f (car (cons (g z) (map g zs))))))</pre> <p><u>={if -#f law}</u></p> <pre>(cons (f (car (cons (g z) (map g zs))))       (map f (cdr (cons (g z) (map g zs)))))</pre>	<pre>(if (null? (cons z zs)) '()     (cons ((o f g) (car (cons z zs)))           (map (o f g) (cdr (cons z zs)))))</pre> <p><u>=null? -cons law</u></p> <pre>(if #f '()     (cons ((o f g) (car (cons z zs)))           (map (o f g) (cdr (cons z zs)))))</pre> <p><u>={if -#f law}</u></p> <pre>(cons ((o f g) (car (cons z zs)))       (map (o f g) (cdr (cons z zs)))))</pre> <p><u>={car-cons law}</u></p> <pre>(cons ((o f g) z) (map (o f g) (cdr (cons z zs)))))</pre> <p><u>={cdr-cons law}</u></p> <pre>(cons ((o f g) z) (map (o f g) zs)))</pre> <p><u>={compose law}</u></p> <pre>(cons (f (g z)) (map (o f g) zs)))</pre> <p><u>={curried law}</u></p> <pre>(cons (f (g z)) (((curry map) (o f g)) zs))</pre> <p><b>Using the inductive hypothesis</b></p> <pre>(cons (f (g z)) ((o ((curry map) f) (curry map) g) zs)))</pre> <p><u>={compose law}</u></p> <pre>(cons (f (g z)) (((curry map) f) (((curry map) g) zs))))</pre> <p><u>={curried law}</u></p> <p><b>(cons (f (gz))(map f g zs)))</b></p> <p><b>LHS = RHS</b></p>
--	--

<pre> (map f (cdr (cons (g z) (map g zs)))  =<u>{car-cons law}</u> (cons (f (g z))(map f (cdr (cons (g z)(map g zs))) =<u>{cdr-cons law}</u>  <b>(cons (f (g z))(map f (map g zs)))</b> </pre>	
--	--

### Starting equation

$$((o((\text{curry map}) f) ((\text{curry map}) g))x) == (((\text{curry map}) (o f g))x)$$