Solutions to William Fulton's
Algebraic Curves: An Introduction to Algebraic Geometry

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ABSTRACT. This document contains solutions to the problems of William Fulton's *Algebraic Curves: An Introduction to Algebraic Geometry*.

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