

Solutions to Joseph Silverman's  
*The Arithmetic of Elliptic Curves*

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ABSTRACT. This document contains solutions to the exercises of Joseph Silverman's *The Arithmetic of Elliptic Curves*.

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## CHAPTER I

# Algebraic Varieties

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# Algebraic Curves

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# Elliptic Curves over Finite Fields

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