# Solutions to K. Ireland and M. Rosen's
# *A Classical Introduction to Modern Number Theory*

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ABSTRACT. This document contains solutions to the exercises of Kenneth Ireland and Michael Rosen's *A Classical Introduction to Modern Number Theory*.

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