

Solutions to Walter Rudin's  
*Real and Complex Analysis*

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ABSTRACT. This document contains solutions to the exercises of Walter Rudin's *Real and Complex Analysis*.

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