# Solutions to Michael Atiyah and Ian MacDonald's $Introduction\ to\ Commutative\ Algebra$

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 $\label{lem:ABSTRACT.} Abstract. This document contains solutions to the exercises of Michael Atiyah and Ian MacDonald's \\ Introduction to Commutative Algebra.$ 

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## Rings and Ideals

- Exercise 1.1.
- Exercise 1.2.
- Exercise 1.3.
- Exercise 1.4.
- Exercise 1.5.
- Exercise 1.6.
- Exercise 1.7.
- Exercise 1.8.
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- Exercise 1.26.
- Exercise 1.27.
- Exercise 1.28.

## Modules

- Exercise 2.1.
- Exercise 2.2.
- Exercise 2.3.
- Exercise 2.4.
- Exercise 2.5.
- Exercise 2.6.
- Exercise 2.7.
- Exercise 2.8.
- Exercise 2.9.
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- Exercise 2.26.
- Exercise 2.27.
- Exercise 2.28.

## Rings and Modules of Fractions

- Exercise 3.1.
- Exercise 3.2.
- Exercise 3.3.
- Exercise 3.4.
- Exercise 3.5.
- Exercise 3.6.
- Exercise 3.7.
- Exercise 3.8.
- Exercise 3.9.
- Exercise 3.10.
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- Exercise 3.26.
- Exercise 3.27.
- Exercise 3.28.
- Exercise 3.29.
- Exercise 3.30.

# **Primary Decomposition**

- Exercise 4.1.
- Exercise 4.2.
- Exercise 4.3.
- Exercise 4.4.
- Exercise 4.5.
- Exercise 4.6.
- Exercise 4.7.
- Exercise 4.8.
- Exercise 4.9.
- Exercise 4.10.
- Exercise 4.11.
- Exercise 4.12.
- Exercise 4.13.
- Exercise 4.14.
- Exercise 4.15.
- Exercise 4.16.
- Exercise 4.17.
- Exercise 4.18.
- Exercise 4.19.
- Exercise 4.20.
- Exercise 4.21.
- Exercise 4.22.
- Exercise 4.23.

## Integral Dependence and Valuations

- Exercise 5.1.
- Exercise 5.2.
- Exercise 5.3.
- Exercise 5.4.
- Exercise 5.5.
- Exercise 5.6.
- Exercise 5.7.
- Exercise 5.8.
- Exercise 5.9.
- Exercise 5.10.
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- Exercise 5.26.
- Exercise 5.27.
- Exercise 5.28.
- Exercise 5.29.
- Exercise 5.30.
- Exercise 5.31.
- Exercise 5.32.
- Exercise 5.33.
- Exercise 5.34.
- Exercise 5.35.

## **Chain Conditions**

- Exercise 6.1.
- Exercise 6.2.
- Exercise 6.3.
- Exercise 6.4.
- Exercise 6.5.
- Exercise 6.6.
- Exercise 6.7.
- Exercise 6.8.
- Exercise 6.9.
- Exercise 6.10.
- Exercise 6.11.
- Exercise 6.12.

## Noetherian Rings

- Exercise 7.1.
- Exercise 7.2.
- Exercise 7.3.
- Exercise 7.4.
- Exercise 7.5.
- Exercise 7.6.
- Exercise 7.7.
- Exercise 7.8.
- Exercise 7.9.
- Exercise 7.10.
- Exercise 7.11.
- Exercise 7.12.
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- Exercise 7.21.
- Exercise 7.22.
- Exercise 7.23.
- Exercise 7.24.
- Exercise 7.25.
- Exercise 7.26.
- Exercise 7.27.

# Artin Rings

- Exercise 8.1.
- Exercise 8.2.
- Exercise 8.3.
- Exercise 8.4.
- Exercise 8.5.
- Exercise 8.6.

# Discrete Valuation Rings and Dedekind Domains

- Exercise 9.1.
- Exercise 9.2.
- Exercise 9.3.
- Exercise 9.4.
- Exercise 9.5.
- Exercise 9.6.
- Exercise 9.7.
- Exercise 9.8.
- Exercise 9.9.

## $CHAPTER \ 10$

# Completions

- Exercise 10.1.
- Exercise 10.2.
- Exercise 10.3.
- Exercise 10.4.
- Exercise 10.5.
- Exercise 10.6.
- Exercise 10.7.
- Exercise 10.8.
- Exercise 10.9.
- Exercise 10.10.
- Exercise 10.11.
- Exercise 10.12.

# Dimension Theory

- Exercise 11.1.
- Exercise 11.2.
- Exercise 11.3.
- Exercise 11.4.
- Exercise 11.5.
- Exercise 11.6.
- Exercise 11.7.