

Solutions to Serge Lang's
Algebra

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ABSTRACT. This document contains solutions to the exercises of Serge Lang's *Algebra*.

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