Solutions to Daniel Huybrechts'
Complex Geometry: An Introduction

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ABSTRACT. This document contains solutions to the exercises of Daniel Huybrechts' *Complex Geometry: An Introduction*.

Contents

CHAPTER 1

Local Theory

1.1. Holomorphic Functions of Several Variables

Exercise 1.1.1.

Exercise 1.1.2.

Exercise 1.1.3.

Exercise 1.1.4.

Exercise 1.1.5.

Exercise 1.1.6.

Exercise 1.1.7.

Exercise 1.1.8.

Exercise 1.1.9.

Exercise 1.1.10.

Exercise 1.1.11.

Exercise 1.1.12.

Exercise 1.1.13.

Exercise 1.1.14.

Exercise 1.1.15.

Exercise 1.1.16.

Exercise 1.1.17.

Exercise 1.1.18.

Exercise 1.1.19.

Exercise 1.1.20.

1.2. Complex and Hermitian Structures

Exercise 1.2.1.

Exercise 1.2.2.

Exercise 1.2.3.

Exercise 1.2.4.

Exercise 1.2.5.

Exercise 1.2.6.

Exercise 1.2.7.

Exercise 1.2.8.

1.3. Differential Forms

Exercise 1.3.1.

Exercise 1.3.2.

Exercise 1.3.3.

Exercise 1.3.4.

Exercise 1.3.5.

Exercise 1.3.6.

Exercise 1.3.7.

Complex Manifolds

2.1. Complex Manifolds: Definition and Examples

Exercise 2.1.1.

Exercise 2.1.2.

Exercise 2.1.3.

Exercise 2.1.4.

Exercise 2.1.5.

Exercise 2.1.6.

Exercise 2.1.7.

Exercise 2.1.8.

Exercise 2.1.9.

Exercise 2.1.10.

2.2. Holomorphic Vector Bundles

Exercise 2.2.1.

Exercise 2.2.2.

Exercise 2.2.3.

Exercise 2.2.4.

Exercise 2.2.5.

Exercise 2.2.6.

Exercise 2.2.7.

Exercise 2.2.8.

Exercise 2.2.9.

Exercise 2.2.10.

Exercise 2.2.11.

Exercise 2.2.12.

Exercise 2.2.13.

2.3. Divisors and Line Bundles

Exercise 2.3.1.

Exercise 2.3.2.

Exercise 2.3.3.

Exercise 2.3.4.

Exercise 2.3.5.

Exercise 2.3.6.

Exercise 2.3.7.

Exercise 2.3.8.

Exercise 2.3.9.

2.4. The Projective Space

Exercise 2.4.1.

Exercise 2.4.2.

Exercise 2.4.3.

Exercise 2.4.4.

Exercise 2.4.5.

Exercise 2.4.6.

Exercise 2.4.7.

Exercise 2.4.8.

Exercise 2.4.9.

2.5. Blow-ups

Exercise 2.5.1.

Exercise 2.5.2.

Exercise 2.5.3.

Exercise 2.5.4.

2.6. Differential Calculus on Complex Manifolds

Exercise 2.6.1.

Exercise 2.6.2.

Exercise 2.6.3.

Exercise 2.6.4.

Exercise 2.6.5.

Exercise 2.6.6.

Exercise 2.6.7.

Exercise 2.6.8.

Exercise 2.6.9.

CHAPTER 3

Kähler Manifolds

3.1. Kähler Identities

Exercise 3.1.1.

Exercise 3.1.2.

Exercise 3.1.3.

Exercise 3.1.4.

Exercise 3.1.5.

Exercise 3.1.6.

Exercise 3.1.7.

Exercise 3.1.8.

Exercise 3.1.9.

Exercise 3.1.10.

3.2. Hodge Theory on Kähler Manifolds

Exercise 3.2.1.

Exercise 3.2.2.

Exercise 3.2.3.

Exercise 3.2.4.

Exercise 3.2.5.

Exercise 3.2.6.

Exercise 3.2.7.

Exercise 3.2.8.

Exercise 3.2.9.

Exercise 3.2.10.

Exercise 3.2.11.

Exercise 3.2.12.

Exercise 3.2.13.

3.3. Lefschetz Theorems

A. Formality of Compact Kähler Manifolds

Exercise A.1.

Exercise A.2.

Exercise A.3.

Exercise A.4.

Vector Bundles

4.1. Hermitian Vector Bundles and Serre Duality

Exercise 4.1.1.

Exercise 4.1.2.

Exercise 4.1.3.

Exercise 4.1.4.

4.2. Connections

Exercise 4.2.1.

Exercise 4.2.2.

Exercise 4.2.3.

Exercise 4.2.4.

Exercise 4.2.5.

Exercise 4.2.6.

Exercise 4.2.7.

4.3. Curvature

Exercise 4.3.1.

Exercise 4.3.2.

Exercise 4.3.3.

Exercise 4.3.4.

Exercise 4.3.5.

Exercise 4.3.6.

Exercise 4.3.7.

Exercise 4.3.8.

4.4. Chern Classes

Exercise 4.4.1.

Exercise 4.4.2.

Exercise 4.4.3.

Exercise 4.4.4.

Exercise 4.4.5.

Exercise 4.4.6.

Exercise 4.4.7.

Exercise 4.4.8.

Exercise 4.4.9.

Exercise 4.4.10.

A. Levi-Civita Connection and Holomony on Complex Manifolds

B. Hermite-Einstein and Kähler Einstein Metrics

CHAPTER 5

Applications of Cohomology

CHAPTER 6

Deformations of Complex Structures