

Solutions to John Milnor and James Stasheff's
Characteristic Classes

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ABSTRACT. This document contains solutions to the problems of John Milnor and James Stasheff's *Characteristic Classes*.

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Singular Homology and Cohomology

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