

Solutions to C. Voisin's
Hodge Theory and Complex Algebraic Geometry I

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ABSTRACT. This document contains solutions to the exercises of Claire Voisin's *Hodge Theory and Complex Algebraic Geometry I*.

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Part I

Preliminaries

CHAPTER 1

Holomorphic Functions of Many Variables

1.1.

1.2.

1.3.

CHAPTER 2

Complex Manifolds

2.1.

2.2.

2.3.

CHAPTER 3

Kähler Metrics

3.1.

3.2.

CHAPTER 4

Sheaves and Cohomology

4.1.

4.2.

Part II

The Hodge Decomposition

CHAPTER 5

Harmonic Forms and Cohomology

5.1.

5.2.

CHAPTER 6

The Case of Kähler Manifolds

6.1.

6.2.

CHAPTER 7

Hodge Structures and Polarisation

7.1.

7.2.

CHAPTER 8

Holomorphic de Rham Complexes and Spectral Sequences

8.1.

8.2.

Part III

Variations of Hodge Structures

CHAPTER 10

Variations of Hodge Structure

10.1.

10.2.

Part IV

Cycles and Cycle Classes

CHAPTER 11

Hodge Classes

11.1.

11.2.

11.3.

CHAPTER 12

Deligne-Beilinson Cohomology and the Abel-Jacobi Map

12.1.

12.2.