

Solutions to Gregory Lawler's
Introduction to Stochastic Processes

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ABSTRACT. This document contains solutions to the exercises of Gregory Lawler's *Introduction to Stochastic Processes*.

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CHAPTER 0

Preliminaries

Exercise 0.1.

Exercise 0.2.

Exercise 0.3.

Exercise 0.4.

Exercise 0.5.

Exercise 0.6. (a)
(b)

CHAPTER 1

Finite Markov Chains

Exercise 1.1.

Exercise 1.2.

Exercise 1.3.

Exercise 1.4.

Exercise 1.5.

Exercise 1.6.

Exercise 1.7.

Exercise 1.8. (a)

- (b)
- (c)
- (d)
- (e)

Exercise 1.9. (a)

- (b)
- (c)
- (d)
- (e)

Exercise 1.10. (a)

- (b)
- (c)
- (d)
- (e)

Exercise 1.11.

Exercise 1.12. (a)

- (b)
- (c)

Exercise 1.13. (a)

- (b)

Exercise 1.14. (a)

- (b)
- (c)

- (d)
- (e)

Exercise 1.15. (a)

- (b)
- (c)

Exercise 1.16.

Exercise 1.17. (a)

- (b)
- (c)

Exercise 1.18.

Exercise 1.19.

Exercise 1.20. (a)

- (b)
- (c)
- (d)
- (e)
- (f)
- (g)
- (h)
- (i)
- (j)

Exercise 1.21. (a)

- (b)

CHAPTER 2

Countable Markov Chains

Exercise 2.1.

Exercise 2.2.

Exercise 2.3.

Exercise 2.4.

Exercise 2.5. (a)

(b)

(c)

(d)

Exercise 2.6. (a)

(b)

(c)

(d)

Exercise 2.7. (a)

(b)

(c)

Exercise 2.8. (a)

(b)

(c)

(d)

Exercise 2.9. (a)

(b)

Exercise 2.10.

Exercise 2.11.

Exercise 2.12.

Exercise 2.13. (a)

(b)

Exercise 2.14. (a)

(b)

(c)

Exercise 2.15. (a)

- (b)
- (c)
- (d)

Exercise 2.16. (a)

- (b)
- (c)

Exercise 2.17. (a)

- (b)
- (c)
- (d)

Exercise 2.18. (a)

- (b)
- (c)

CHAPTER 3

Continuous-Time Markov Chains

Exercise 3.1. (a)

(b)

(c)

(d)

(e)

Exercise 3.2. (a)

(b)

(c)

(d)

Exercise 3.3. (a)

(b)

(c)

Exercise 3.4. (a)

(b)

Exercise 3.5.

Exercise 3.6.

Exercise 3.7.

Exercise 3.8. (a)

(b)

(c)

Exercise 3.9.

Exercise 3.10.

Exercise 3.11.

Exercise 3.12.

Exercise 3.13.

Exercise 3.14.

Exercise 3.15.

Exercise 3.16.

CHAPTER 4

Optimal Stopping

Exercise 4.1.

Exercise 4.2.

Exercise 4.3. (a)

(b)

(c)

Exercise 4.4. (a)

(b)

(c)

Exercise 4.5.

Exercise 4.6. (a)

(b)

Exercise 4.7.

Exercise 4.8.

Exercise 4.9. (a)

(b)

Exercise 4.10. (a)

(b)

CHAPTER 5

Martingales

Exercise 5.1.

Exercise 5.2.

Exercise 5.3. (a)

(b)

(c)

(d)

(e)

(f)

Exercise 5.4.

Exercise 5.5.

Exercise 5.6. (a)

(b)

Exercise 5.7. (a)

(b)

Exercise 5.8. (a)

(b)

(c)

Exercise 5.9. (a)

(b)

Exercise 5.10. (a)

(b)

(c)

Exercise 5.11. (a)

(b)

(c)

Exercise 5.12.

Exercise 5.13.

Exercise 5.14.

Exercise 5.15.

Exercise 5.16. (a)

- (b)
- (c)
- (d)

Exercise 5.17. (a)

- (b)
- (c)

CHAPTER 6

Renewal Processes

Exercise 6.1. (a)

(b)

(c)

(d)

Exercise 6.2.

Exercise 6.3.

Exercise 6.4.

Exercise 6.5. (a)

(b)

(c)

Exercise 6.6. (a)

(b)

Exercise 6.7. (a)

(b)

(c)

(d)

Exercise 6.8. (a)

(b)

(c)

Exercise 6.9.

Exercise 6.10.

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Reversible Markov Chains

Exercise 7.1.

Exercise 7.2. (a)
(b)

Exercise 7.3.

Exercise 7.4. (a)
(b)
(c)
(d)

Exercise 7.5.

Exercise 7.6.

Exercise 7.7.

Exercise 7.8. (a)
(b)
(c)

Exercise 7.9.

Exercise 7.10. (a)
(b)

CHAPTER 8

Brownian Motion

Exercise 8.1.

Exercise 8.2.

Exercise 8.3.

Exercise 8.4. (a)

(b)

(c)

(d)

(e)

(f)

Exercise 8.5.

Exercise 8.6. (a)

(b)

(c)

Exercise 8.7.

Exercise 8.8.

Exercise 8.9.

Exercise 8.10. (a)

(b)

Exercise 8.11.

Exercise 8.12. (a)

(b)

Exercise 8.13. (a)

(b)

Exercise 8.14.

Exercise 8.15.

Exercise 8.16. (a)

(b)

(c)

Exercise 8.17. (a) (a)

- (b)
 - (c)
 - (d)
- (b)

CHAPTER 9

Stochastic Integration

Exercise 9.1. (a)
(b)

Exercise 9.2.

Exercise 9.3. (a)
(b)
(c)
(d)
(e)

Exercise 9.4.

Exercise 9.5. (a)
(b)
(c)
(d)

Exercise 9.6. (a)
(b)
(c)
(d)

Exercise 9.7.

Exercise 9.8. (a)
(b)

Exercise 9.9.