Solutions to Glen Bredon's
Topology and Geometry

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ABSTRACT. This document contains solutions to the problems of Glen Bredon's *Topology and Geometry*.

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