Solutions to Joe Harris' *Algebraic Geometry: A First Course*

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Abstract. This document contains solutions to the exercises of Joe Harris' *Algebraic Geometry: A First Course*.

Contents

Part I

Examples of Varieties and Maps

LECTURE 1

Affine and Projective Varieties

Exercise 1.3.

Exercise 1.5.

Exercise 1.6.

Exercise 1.11.

Exercise 1.12. s

Exercise 1.13.

Exercise 1.15.

Exercise 1.19.

Exercise 1.21.

Exercise 1.23.

Exercise 1.24.

Exercise 1.25.

Exercise 1.27.

Exercise 1.28.

Exercise 1.29.

LECTURE 2

Regular Functions and Maps

Exercise 2.2.

Exercise 2.3.

Exercise 2.5.

Exercise 2.8.

Exercise 2.9.

Exercise 2.10.

Exercise 2.12.

Exercise 2.13.

Exercise 2.14.

Exercise 2.15.

Exercise 2.17.

Exercise 2.18.

Exercise 2.19.

Exercise 2.20.

Exercise 2.22.

Exercise 2.25.

Exercise 2.26.

Exercise 2.28.

Exercise 2.29.

LECTURE 3

Cones, Projections, and More About Products

Exercise 3.2.

Exercise 3.7.

Exercise 3.8.

Exercise 3.9.

LECTURE 4

Families and Parameter Spaces

Exercise 4.2.

Exercise 4.3.

Exercise 4.4.

Exercise 4.7.

Exercise 4.10.

Exercise 4.11.

Exercise 4.12.

Exercise 4.13.

Exercise 4.14.

Exercise 4.16.

Exercise 4.17.

LECTURE 5

Ideals of Varieties, Irreducible Decomposition, and the Nullstellensatz

Exercise 5.2.

Exercise 5.3.

Exercise 5.4.

Exercise 5.5.

Exercise 5.9.

Exercise 5.10.

Exercise 5.13.

Exercise 5.14.

LECTURE 6

Grassmannians and Related Varieties

Exercise 6.2.

Exercise 6.4.

Exercise 6.5.

Exercise 6.6.

Exercise 6.7.

Exercise 6.8.

Exercise 6.9.

Exercise 6.15.

Exercise 6.16.

Exercise 6.20.

LECTURE 7

Rational Functions and Rational Maps

Exercise 7.1.

Exercise 7.5.

Exercise 7.6.

Exercise 7.7.

Exercise 7.8.

Exercise 7.9.

Exercise 7.10.

Exercise 7.12.

Exercise 7.13.

Exercise 7.14.

Exercise 7.19.

Exercise 7.20.

Exercise 7.23.

Exercise 7.25.

Exercise 7.26.

Exercise 7.27.

Exercise 7.28.

Exercise 7.29.

LECTURE 8

More Examples

Exercise 8.2.

Exercise 8.4.

Exercise 8.6.

Exercise 8.7.

Exercise 8.8.

Exercise 8.10.

Exercise 8.11.

Exercise 8.12.

Exercise 8.13.

Exercise 8.15.

Exercise 8.16.

Exercise 8.18.

Exercise 8.19.

Exercise 8.21.

Exercise 8.22.

Exercise 8.23.

Exercise 8.24.

Exercise 8.25.

Exercise 8.27.

Exercise 8.28.

Exercise 8.31.

Exercise 8.32.

Exercise 8.33.

Exercise 8.37.

Exercise 8.38.

LECTURE 9

Determinantal Varieties

Exercise 9.5.

Exercise 9.9.

Exercise 9.11.

Exercise 9.13.

Exercise 9.14.

Exercise 9.16.

Exercise 9.17.

Exercise 9.18.

Exercise 9.19.

Exercise 9.20.

Exercise 9.23.

Exercise 9.24.

LECTURE 10

Algebraic Groups

Exercise 10.5.

Exercise 10.6.

Exercise 10.7.

Exercise 10.10.

Exercise 10.11.

Exercise 10.13.

Exercise 10.14.

Exercise 10.20.

Exercise 10.22.

Exercise 10.24.

Exercise 10.25.

Exercise 10.26.

Exercise 10.28.

Exercise 10.29.

Exercise 10.30.

Exercise 10.31.

Part II

Attributes of Varieties

LECTURE 11

Definitions of Dimension and Elementary Examples

Exercise 11.6.

Exercise 11.9.

Exercise 11.10.

Exercise 11.11.

Exercise 11.15.

Exercise 11.16.

Exercise 11.19.

Exercise 11.20.

Exercise 11.21.

Exercise 11.23.

Exercise 11.25.

Exercise 11.26.

Exercise 11.27.

Exercise 11.28.

Exercise 11.29.

Exercise 11.31.

Exercise 11.33.

Exercise 11.39.

Exercise 11.41.

Exercise 11.43.

Exercise 11.44.

Exercise 11.45.

LECTURE 12

More Dimension Computations

Exercise 12.3.

Exercise 12.4.

Exercise 12.6.

Exercise 12.7.

Exercise 12.10.

Exercise 12.11.

Exercise 12.13.

Exercise 12.16.

Exercise 12.17.

Exercise 12.19.

Exercise 12.20.

Exercise 12.21.

Exercise 12.23.

Exercise 12.25.

Exercise 12.26.

Exercise 12.27.

LECTURE 13

Hilbert Polynomials

Exercise 13.1.

Exercise 13.5.

Exercise 13.6.

Exercise 13.8.

Exercise 13.9.

Exercise 13.13.

Exercise 13.14.

Exercise 13.15.

Exercise 13.17.

Exercise 13.18.

LECTURE 14

Smoothness and Tangent Spaces

Exercise 14.1.

Exercise 14.2.

Exercise 14.3.

Exercise 14.5.

Exercise 14.6.

Exercise 14.7.

Exercise 14.11.

Exercise 14.12.

Exercise 14.13.

Exercise 14.14.

Exercise 14.15.

LECTURE 15

Gauss Maps, Tangential and Dual Varieties

Exercise 15.5.

Exercise 15.6.

Exercise 15.8.

Exercise 15.9.

Exercise 15.11.

Exercise 15.12.

Exercise 15.15.

Exercise 15.16.

Exercise 15.18.

Exercise 15.19.

Exercise 15.23.

Exercise 15.25.

LECTURE 16

Tangent Spaces to Grassmannians

Exercise 16.3.

Exercise 16.4.

Exercise 16.5.

Exercise 16.7.

Exercise 16.9.

Exercise 16.11.

Exercise 16.14.

Exercise 16.15.

Exercise 16.16.

Exercise 16.17.

Exercise 16.19.

Exercise 16.22.

Exercise 16.23.

LECTURE 17

Further Topics Involving Smoothness and Tangent Spaces

Exercise 17.3.

Exercise 17.4.

Exercise 17.6.

Exercise 17.7.

Exercise 17.9.

Exercise 17.10.

Exercise 17.12.

Exercise 17.13.

Exercise 17.15.

Exercise 17.17.

Exercise 17.19.

Exercise 17.20.

Exercise 17.21.

LECTURE 18

Degree

Exercise 18.2.

Exercise 18.7.

Exercise 18.11.

Exercise 18.14.

Exercise 18.18.

Exercise 18.21.

Exercise 18.22.

Exercise 18.23.

LECTURE 19

Further Examples and Applications of Degree

Exercise 19.2.

Exercise 19.3.

Exercise 19.6.

Exercise 19.7.

Exercise 19.12.

Exercise 19.13.

Exercise 19.15.

Exercise 19.16.

Exercise 19.19.

LECTURE 20

Singular Points and Tangent Cones

Exercise 20.3.

Exercise 20.4.

Exercise 20.6.

Exercise 20.7.

Exercise 20.8.

Exercise 20.9.

Exercise 20.10.

Exercise 20.11.

Exercise 20.12.

Exercise 20.13.

Exercise 20.14.

Exercise 20.15.

Exercise 20.16.

Exercise 20.17.

LECTURE 21

Parameter Spaces and Moduli Spaces

Exercise 21.3.

Exercise 21.4.

Exercise 21.5.

Exercise 21.6.

Exercise 21.9.

Exercise 21.10.

Exercise 21.11.

Exercise 21.12.

LECTURE 22

Quadrics

Exercise 22.1.

Exercise 22.2.

Exercise 22.3.

Exercise 22.4.

Exercise 22.6.

Exercise 22.9.

Exercise 22.10.

Exercise 22.11.

Exercise 22.15.

Exercise 22.16.

Exercise 22.17.

Exercise 22.18.

Exercise 22.19.

Exercise 22.20.

Exercise 22.23.

Exercise 22.24.

Exercise 22.25.

Exercise 22.26.

Exercise 22.29.

Exercise 22.30.

Exercise 22.35.

Exercise 22.36.

Exercise 22.37.

Exercise 22.39.

Exercise 22.40.