

Solutions to Robin Hartshorne's
Algebraic Geometry

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ABSTRACT. This document contains solutions to the exercises of Robin Hartshorne's *Algebraic Geometry*.

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7. The Serre Duality Theorem

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8. Higher Direct Images of Sheaves

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9. Flat Morphisms

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