# Solutions to Manfredo do Carmo's *Riemannian Geometry*

Patrick Borse

Abstract. This document contains solutions to the exercises of Manfredo do Carmo's *Riemannian Geometry*.

# Contents

CHAPTER 0

# Differentiable Manifolds

**Exercise 0.1.** a.
b.

**Exercise 0.2.**

**Exercise 0.3.** a.
b.

**Exercise 0.4.**

**Exercise 0.5.** a.
b.

**Exercise 0.6.**

**Exercise 0.7.** a.
b.

**Exercise 0.8.**

**Exercise 0.9.** a.
b.
c.

**Exercise 0.10.**

**Exercise 0.11.** a.
b.

**Exercise 0.12.** a.
b.
c.

CHAPTER 1

# Riemannian Metrics

**Exercise 1.1.**

**Exercise 1.2.**

**Exercise 1.3.**

**Exercise 1.4.** a.
b.

**Exercise 1.5.**

**Exercise 1.6.**

**Exercise 1.7.** a.
b.
c.

CHAPTER 2

# Affine Connections; Riemannian Connections

**Exercise 2.1.**

**Exercise 2.2.**

**Exercise 2.3.**

**Exercise 2.4.** a.
b.

**Exercise 2.5.**

**Exercise 2.6.**

**Exercise 2.7.**

**Exercise 2.8.** a.
b.

**Exercise 2.9.** a.
b.

CHAPTER 3

# Geodesics; Convex Neighborhoods

**Exercise 3.1.** a.
b.
c.
d.

**Exercise 3.2.** a.
b.
c.
d.
e.

**Exercise 3.3.** a.
b.

**Exercise 3.4.** a.
b.

**Exercise 3.5.** a.
b.
c.
d.
e.

**Exercise 3.6.**

**Exercise 3.7.**

**Exercise 3.8.** a.
b.

**Exercise 3.9.** a.
b.

**Exercise 3.10.**

**Exercise 3.11.**

**Exercise 3.12.**

**Exercise 3.13.**

**Exercise 3.14.**

CHAPTER 4

# Curvature

**Exercise 4.1.** a.
b.
c.

**Exercise 4.2.** a.
b.

**Exercise 4.3.**

**Exercise 4.4.**

**Exercise 4.5.**

**Exercise 4.6.** a.
b.
c.

**Exercise 4.7.**

**Exercise 4.8.**

**Exercise 4.9.**

**Exercise 4.10.** a.
b.

CHAPTER 5

# Jacobi Fields

**Exercise 5.1.**

**Exercise 5.2.**

**Exercise 5.3.**

**Exercise 5.4.**

**Exercise 5.5.** a.
b.
c.
d.

**Exercise 5.6.** a.
b.
c.
d.

**Exercise 5.7.**

**Exercise 5.8.** a.
b.

CHAPTER 6

# Isometric Immersions

**Exercise 6.1.** a.
b.
c.

**Exercise 6.2.**

**Exercise 6.3.**

**Exercise 6.4.**

**Exercise 6.5.**

**Exercise 6.6.**

**Exercise 6.7.**

**Exercise 6.8.** a.
b.
c.

**Exercise 6.9.**

**Exercise 6.10.**

**Exercise 6.11.** a.
b.
c.
d.

**Exercise 6.12.** a.
b.
c.

CHAPTER 7

# Complete Manifolds; Hopf-Rinow and Hadamard Theorems

**Exercise 7.1.**

**Exercise 7.2.**

**Exercise 7.3.**

**Exercise 7.4.**

**Exercise 7.5.**

**Exercise 7.6.**

**Exercise 7.7.**

**Exercise 7.8.**

**Exercise 7.9.**

**Exercise 7.10.**

**Exercise 7.11.**

**Exercise 7.12.**

**Exercise 7.13.**

CHAPTER 8

# Spaces of Constant Curvature

**Exercise 8.1.** a.
b.
c.
d.

**Exercise 8.2.**

**Exercise 8.3.** a.
b.
c.
d.
e.
f.

**Exercise 8.4.** a.
b.

**Exercise 8.5.**

**Exercise 8.6.** a.
b.
c.
d.
e.

**Exercise 8.7.** a.
b.
c.

**Exercise 8.8.** a.
b.

**Exercise 8.9.** a.
b.
c.

**Exercise 8.10.** a.
b.

**Exercise 8.11.** a.
b.
c.

d.

**Exercise 8.12.** a.
b.

**Exercise 8.13.**

**Exercise 8.14.**

CHAPTER 9

# Variations of Energy

**Exercise 9.1.**

**Exercise 9.2.**

**Exercise 9.3.**

**Exercise 9.4.**

**Exercise 9.5.** a.
b.

**Exercise 9.6.** a.
b.
c.

CHAPTER 10

# The Rauch Comparison Theorem

**Exercise 10.1.**

**Exercise 10.2.**

**Exercise 10.3.**

**Exercise 10.4.** a.
b.
c.

**Exercise 10.5.** a.
b.

**Exercise 10.6.**

**Exercise 10.7.** a.
b.
c.

CHAPTER 11

# The Morse Index Theorem

**Exercise 11.1.**

**Exercise 11.2.**

**Exercise 11.3.**

**Exercise 11.4.**

**Exercise 11.5.**

**Exercise 11.6.**