

Solutions to David Eisenbud's
Commutative Algebra with a View Toward Algebraic Geometry

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ABSTRACT. Solutions to the exercises of David Eisenbud's *Commutative Algebra: With a View Toward Algebraic Geometry*.

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Part I

Basic Constructions

CHAPTER 1

Roots of Commutative Algebra

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CHAPTER 2

Localization

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CHAPTER 3

Associated Primes and Primary Decomposition

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CHAPTER 4

Integral Dependence and the Nullstellensatz

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CHAPTER 5

Filtrations and the Artin-Rees Lemma

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CHAPTER 6

Flat Families

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CHAPTER 7

Completions and Hensel's Lemma

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Part II

Dimension Theory

CHAPTER 9

Fundamental Definitions of Dimension Theory

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CHAPTER 10

The Principal Ideal Theorem and Systems of Parameters

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CHAPTER 11

Dimension and Codimension One

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CHAPTER 12

Dimension and Hilbert-Samuel Polynomials

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CHAPTER 13

The Dimension of Affine Rings

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CHAPTER 14

Elimination Theory, Generic Freeness, and the Dimension of Fibers

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CHAPTER 15

Gröbner Bases

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CHAPTER 16

Modules of Differentials

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Part III

Homological Methods

CHAPTER 17

Regular Sequences and the Koszul Complex

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CHAPTER 18

Depth, Codimension, and Cohen-Macaulay Rings

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CHAPTER 19

Homological Theory of Regular Local Rings

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CHAPTER 20

Free Resolutions and Fitting Invariants

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CHAPTER 21

Duality, Canonical Modules, and Gorenstein Rings

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