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REVIEWS

Numerical cross references are to previous reviews in this JOURNAL or to *A bibliography of symbolic logic* (this JOURNAL, vol. 1, pp. 121–218), or to *Additions and corrections* to the latter (this JOURNAL, vol. 3, pp. 178–212).

References beginning with a Roman numeral are by volume and page to the place at which a publication has previously been reviewed or listed. When necessary in connection with such references, a third number will be added in parentheses, to indicate position on the page. Such a reference is ordinarily to the publication itself, but when so indicated the reference may be to the review or to both the publication and its review. Thus "XXVII 363" will refer to the review beginning on page 363 of volume 27 of this JOURNAL, or to the publication which is there reviewed; "XXVII 368" will refer to one of the reviews or one of the publications reviewed or listed on page 368 of volume 27, with reliance on the context to show which one is meant; "XXVIII 283(4)" will refer to the fourth item listed on page 283 of volume 28, i.e., to Scholz's *Der Anselmische Gottesbeweis*; and "XIV 184(5)" will refer to the fifth item listed on page 184 of volume 14, i.e., to Frege's *On sense and nominatum.*

References such as 49***16***, 125***3*** are to the entries so numbered in the *Bibliography*. Similar references preceded by the letter A or containing the fraction $\frac{1}{2}$ or a decimal point (as A17***1***, 70$\frac{1}{2}$***1***, 3827.***1***) are to the *Additions and corrections*. A reference followed by the letter A is a double reference to an entry of the same number in the *Bibliography* and in the *Additions and corrections*.

NOAM CHOMSKY. ***Syntactic structures.*** Janua linguarum, Studia memoriae Nicolai van Wijk dedicata, series minor no. 4. Mouton & Co., 's-Gravenhage 1957, 116 pp.

NOAM CHOMSKY. *Three models for the description of language.* A reprint of XXIII 71. ***Readings in mathematical psychology,*** Volume II, edited by R. Duncan Luce, Robert R. Bush, and Eugene Galanter, John Wiley and Sons, Inc., New York, London, and Sydney, 1965, pp. 105–124.

NOAM CHOMSKY. *Logical structures in language.* ***American documentation,*** vol. 8 (1957), pp. 284–291.

NOAM CHOMSKY and GEORGE A. MILLER. *Finite state languages.* ***Information and control,*** vol. 1 (1958), pp. 91–112. Reprinted in ***Readings in mathematical psychology,*** Volume II, edited by R. Duncan Luce, Robert R. Bush, and Eugene Galanter, John Wiley and Sons, Inc., New York, London, and Sydney, 1965, pp. 156–171.

NOAM CHOMSKY. *On certain formal properties of grammars.* Ibid., vol. 2 (1959), pp. 137–167. Reprinted ibid., pp. 125–155.

NOAM CHOMSKY. *A note on phrase structure grammars.* Ibid., pp. 393–395.

NOAM CHOMSKY. *On the notion "rule of grammar."* ***Structure of language and its mathematical aspects,*** Proceedings of symposia in applied mathematics, vol. 12, American Mathematical Society, Providence 1961, pp. 6–24.

ARTHUR SARD, NOAM CHOMSKY, W. P. LIVANT, A. G. OETTINGER, L. M. COURT. *Comments.* Ibid., pp. 255–257.

The eight items listed in the heading of this review will be referred to by number, as (1), (2), (3), and so on, in the order in which they are listed—which, with the exception of (1), is the same as the chronological order of their first publication.

The author proposes and investigates several new conceptions for the description of natural languages. Notwithstanding the alleged complexity of natural languages, these papers show that mathematical tools can be used with benefit in linguistics by means of idealizations apparently less drastic than some that are customary in natural science. Since readers of this JOURNAL may be primarily interested in the logical content of these conceptions, this reviewer will discuss their first exposition in XXIII 71 in somewhat greater detail. The same ideas recur in the other publications, variously phrased and sometimes improved or extended in several directions. At the same time this reviewer will pay some attention to the relevance of these conceptions to linguistics which, as the study of natural languages, is in some ways related to the study of artificial languages. Moreover the author's ideas have provoked discussions among linguists which are not dissimilar to some early reactions to the rise of modern logic.

In XXIII 71 a language L is defined as a set of finite strings, formed by concatenating the symbols of a finite alphabet A (concatenation will be expressed by $\frown$). A *grammar* of L is a device that generates all strings of L and only these. Grammars are constructed by establishing *grammatical rules*, which project the finite set of observed strings (the linguist's corpus) into an infinite set of grammatical strings (the strings of L). *Linguistic theory* can be viewed as a metatheory which is concerned with the problem of how to choose a grammar for each L. The author investigates these concepts of grammar with a view to determining whether they can provide simple and "revealing" grammars of English.

A *finite-state grammar* (FSG) is a system with a finite number of *states* $S_0, \ldots, S_q$, a set $A = \{a_{ijk} \mid 0 \leqq i,\ j \leqq q;\ 1 \leqq k \leqq N_{ij}$ for each $i, j\}$ of *transition symbols* and a set $C = \{(S_i, S_j)\}$ of *pairs* of states. If G moves from S_i to S_j, a symbol a_{ijk} is generated (alternatively, recognized). A sequence of states $S\alpha_1, \ldots, S\alpha_m$ ($\alpha_1 = \alpha_m = 0$; $\alpha_i \neq 0$ for $1 \neq i \neq m$ and $(S\alpha_i, S\alpha_{i+1}) \epsilon C$ for each $i < m$) generates all strings:

$$a_{\alpha_1\alpha_2k_1} \frown a_{\alpha_2\alpha_3k_2} \frown \ldots \frown a_{\alpha_{m-1}\alpha_m k_{m-1}} \qquad (k_j \leqq N_{\alpha_i\alpha_{i+1}}).$$

A language containing all and only such strings is called a *finite-state language* (FSL). For example, English is a FSL if a FSG can be constructed with words as transition symbols such that all and only the generated strings are English sentences.

In order to study the limitations of FS grammars the author introduces the notion of *m-dependency with respect to L*. The definition of this notion given in XXIII 71 contains a defect which was pointed out by E. Assmuss and which is corrected in (2). For example, a string $a_1 \frown \ldots \frown a_{2m+1}$ has an m-dependency if the choice of each symbol of $a_{m+2} \frown \ldots \frown a_{2m+1}$ depends on $a_1 \frown \ldots \frown a_m$ (e.g., "mirror-image" strings, where $a_{i+1} = a_{2m-i+1}$ for $0 \leqq i < m$). The author shows that for each FSL $\epsilon\ L$, there is an n such that for $m > n$ no string of L has an m-dependency with respect to L. Since English contains strings with arbitrary large dependencies (e.g., all sentences "if A, then B," "if if A, then B, then B," "if if if A, then B, then B, then B," ...) English is not a FSL. The author argues that also sequences of FS grammars which generate statistical approximations to English meet with the same difficulty. It might of course be objected that there is a finite upper limit to sentence length in English, and some linguists have expressed something like this. The reviewer agrees with the author that a FSG is not sufficiently "revealing" since it intrinsically fails to describe certain processes of sentence formation. The "idealizing" assumption that languages are infinite simplifies the description (cf. Henkin in XIX 289, page 27).

A *phrase-structure grammar* (PSG) consists of a finite alphabet V_P, a finite set of initial strings $\Sigma = \{\Sigma_1, \ldots, \Sigma_n\}$ and a finite set of rewriting rules $F = \{X_1 \rightarrow Y_1, \ldots, X_m \rightarrow Y_m\}$, where Y_i is formed from X_i by the replacement of a single symbol of X_i by some string (Σ_i, X_i, and Y_i are strings in V_P). In each such grammar G, a string β *follows from* a string α if for some $i \leqq m$, $\alpha = Z \frown X_i \frown W$ and $\beta = Z \frown Y_i \frown W$. A string S_t is *derivable* from G if there is a *derivation* of S_t or a sequence $(S_1, \ldots, S_t)$ of strings such that $S_1 \epsilon \Sigma$ and for each $i < t$, S_{i+1} follows from S_i. A derivable string S_t is *terminal* if there is no string which follows from it. A *derivable language* is the set of derivable strings from some PSG, and a *terminal language* is the set of terminal strings from some PSG. For example, the language consisting of the strings $a^\frown b$, $a^\frown a^\frown b^\frown b$, $a^\frown a^\frown a^\frown b^\frown b^\frown b, \ldots$ is derivable with $\Sigma = \{a^\frown b\}$ and $F = \{a^\frown b \rightarrow a^\frown a^\frown b^\frown b\}$. The author proves: every FSL is terminal, but not conversely; every derivable language is terminal, but not conversely; there are derivable, non-finite-state languages and finite-state, non-derivable languages. E. Shamir has pointed out that the example given for the proof of the last part is incorrect.

The author generally considers a PSG in which the first rewriting rule of F is:

$$\textit{Sentence} \rightarrow \mathrm{NP}^\frown\mathrm{VP}$$

where NP and VP denote *noun phrase* and *verb phrase*, respectively. Several linguists have failed to note that this apparently traditional binary division of the sentence, despite its obvious applicability to many English sentences, is not necessitated by the definition of PSG and is nowhere required as a "linguistic universal" (cf. below (7)).

A PSG for English provides at least two non-equivalent derivations for *constructional homonyms* (e.g., "they are flying planes"). The relative adequacy of the grammar is thereby increased, since this formal property can be taken to explain the semantic ambiguity of such sentences.

The problem arises whether a PSG can contain rules which would, when applied to strings like "the man ate the food" (a) produce "the food was eaten by the man" (b). This is decided in the negative, since such rules involve inversion of symbols, discontinuous symbols, or both, which is excluded by the definition of PSG, and apply only to terminal strings if earlier lines of their derivation are also known (compare "the food was eaten by the seashore"). Though the author does not show that a PSG is inadequate for the generation of some strings (e.g., (a) and (b)), he succeeds in showing that a PSG is inadequate for the generation of certain strings in a simple and natural way (e.g., (b) from (a)). He proposes to construct a *transformational grammar* which transforms the terminal strings of a PSG into new strings with derived phrase structure. Sentences derived by the use of only obligatory transformations are called *kernel sentences*. Some formal properties of such grammars are studied, but not their generative capacity (this continues to pose unsolved problems). The possibility is not excluded that for a given transformational grammar there should exist a weakly equivalent (i.e., generating not the same structures but the same strings) PSG (see XXX 383(5)). The need for transformational grammars might also be relativized if PS grammars could be strengthened in a simple manner, e.g., by allowing inversion of symbols (see XXXI 289(2)). However, this extension is not simple from the point of view of generation of structures and leads to counterintuitive results, as Chomsky points out in (5), pages 144–148.

Since the derivation of a derivable string S can be represented by a tree, a substring s of S can be defined if it is traceable back to a single node labelled X. A *phrase marker* K of a terminal string S can be defined as the set of strings occurring in all equivalent derivations of S. (S, K) is *analyzable* into $(X_1, \ldots, X_n)$ iff there are strings $s_1, \ldots, s_n$ such that (i) $S = s_1{}^\frown \ldots {}^\frown s_n$ and (ij) for each $i \leq n$, K contains the string $s_1{}^\frown \ldots {}^\frown s_{i-1}{}^\frown X_i{}^\frown s_{i+1}{}^\frown \ldots {}^\frown s_n$. In this case s_i is an X_i in S with respect to K. For example, *the*$^\frown$*man* is a NP. With each transformation T a *restricting class* R is associated, which specifies to which strings T can be applied. R is a set of sequences of strings $\{(X_1^1, \ldots, X_r^1), \ldots, (X_1^m, \ldots, X_r^m)\}$. The terminal string S with phrase marker K belongs to the domain of T, if R contains a sequence $(X_i^j, \ldots, X_r^j)$ into which (S, K) is analyzable. An *elementary transformation* is defined which converts the occurrence of one symbol Y_n of a string $Y_1{}^\frown \ldots {}^\frown Y_r$ into a string W_n. This is written as: $t(Y_1, \ldots, Y_n; Y_n, \ldots, Y_r) = W_n$. Then t^* is the *derived transformation* of t iff for all $Y_1, \ldots, Y_r$, $t^*(Y_1, \ldots, Y_r) = W_1{}^\frown \ldots {}^\frown W_r$. For example, a passive transformation can be applied to "the man ate the food" (substrings *the*$^\frown$

man — *past* — *eat* — *the⌢food* or $Y_1 - Y_2 - Y_3 - Y_4$) as follows:

$$t_p(Y_1;\ Y_1, Y_2, Y_3, Y_4) = Y_4$$
$$t_p(Y_1, Y_2;\ Y_2, Y_3, Y_4) = Y_2{}^\frown be^\frown en$$
$$t_p(Y_1, Y_2, Y_3;\ Y_3, Y_4) = Y_3$$
$$t_p(Y_1, Y_2, Y_3, Y_4;\ Y_4) = by^\frown Y_1$$
$$t_p^*(Y_1, Y_2, Y_3, Y_4) = Y_4 - Y_2{}^\frown be^\frown en - Y_3 - by^\frown Y_1$$

or *the⌢food* — *past⌢be⌢en* — *eat* — *by⌢the⌢man*. Other rules convert this into: "the food was eaten by the man."

Corrections to XXIII 71 other than those mentioned: page 114, column 1, line 4, and page 116, line 1 below (15), for "falling," read "phrase-final"; page 119, line 4 of the final paragraph, after "terminal language" insert a footnote "This is not true if the grammar contains rules rewriting a symbol in a non-null context" (author's communications; cf. (5) below); page 122, (48) (i), replace the second occurrence of Y_1 by Y_4.

In (1) the author gives a general exposition of the ideas of XXIII 71 in a wider linguistic context and with less emphasis on their mathematical content. Since the exposition is mainly concerned with syntax, strings generally consist of morphemes or words. The syntactic description of English is given in greater detail and numerous illustrations of English phrase structure and transformational rules are provided. Among transformational rules the distinction made in XXIII 71 between obligatory and optional rules is carried through. The goal of linguistic theory is described as a practical evaluation procedure for grammars. There are numerous oversimplifications, partly due to the fact that (1) was intended to be a brief and informal introduction to a longer and much more comprehensive book — i.e., "The logical structure of linguistic theory"—which, however, has never appeared. The repeated reference to a grammar as a "sentence-generating device," instead of a "structure-generating device," is misleading. Apart from misprints, the only serious error is the statement on page 31 that the language (10iii) is not a PSL. This error, the same as that on page 119 of XXIII 71, is corrected in the second edition of (1) as well as in (2), since the contrary was proved in (5).

A final chapter, dealing with syntax and semantics, may be of special interest to readers of this JOURNAL. In the author's opinion, objections to the aim of constructing grammars without appeal to meaning seem to imply that it is possible to construct grammars *with* appeal to meaning. It seems likely, on the contrary, that semantic features of language can be studied only when formal features are known (for a significant step in this direction see Katz and Fodor, *The structure of a semantic theory*, ***Language,*** vol. 39 (1963), pp. 170–210, and Katz and Postal, ***An integrated theory of linguistic descriptions,*** reviewed in ***Foundations of language,*** vol. 1 (1965), pp. 133–154). Semantics is not excluded from linguistics, but is assigned a place among other approaches to the study of language.

In (3) any appeal to meaning in the study of linguistic form is again rejected. The study of meaning is arrived at on the basis of a formal study. The term "formal" would of course require further specification. Semantic ambiguity is considered in connection with constructional homonymity and the understanding of linguistic expression is related to transformational analysis. Attention is drawn to somewhat similar methods of semantic analysis employed by certain British philosophers, e.g., J. L. Austin.

Though a FSG is not adequate for most natural languages, its mathematical properties are well suited to the needs of communication engineers. Some of these properties are studied in (4) by Chomsky and Miller (the authors' notion of FSL corresponds to "regular event" in Kleene's XXIII 59 and their notion of FSG to "finite automaton" in Rabin and Scott's XXV 163; equivalences of the latter's formulations with those of (4) are established in XXX 383(4)).

If F_i is a set of FS grammars, $L(F_i)$ denotes the set of languages generated by grammars of F_i. F_1 is the set of unrestricted FS grammars with S_0 as initial state and W_0 as identity element among its transition symbols. Each state S_i is considered a set of pairs $\{(j, k)\}$, where (j, k) is a grammatical rule which indicates that W_j is produced when the grammar moves from S_i to S_k. F_2 is the set of FS grammars such that if $(i, 0) \in S_j$, then $i = 0$ (i.e., W_0 plays the role of a period ending a sentence). F_3 is the set of FS grammars such that if $(i, j) \in S_k$, then $i = 0$ iff $j = 0$ (i.e., W_0 occurs only at the end of a sentence). F_4 is the set of FS grammars such that, in addition, if $(i, h) \in S_j$ and $(i, k) \in S_j$ then $h = k$ (i.e., one string is produced by only one state sequence). F_5 is the set of such "deterministic" FS grammars without W_0. The authors prove that $L(F_1)$, $L(F_2)$, $L(F_3)$, and $L(F_4)$ are identical and properly contain $L(F_5)$ (similar results were obtained by Rabin and Scott).

The main result of (4) is a characterization theorem for FS languages suggested by the loops which occur in state diagrams. A FSL can be represented by a finite number of finite notations of the form $a_1(a_2, \ldots, a_m)a_{m+1}$ (*), where each a_i is either a string or again of the form (*). If all a_i are strings, (*) represents the set of all sequences of strings $(b_1, \ldots, b_{n+1})$ such that $b_1 = a_1$, $b_{n+1} = a_{m+1}$, and each b_i $(2 \leqq i \leqq n)$ is one of $a_2, \ldots, a_m$. If an a_i is of the form (*) it is expanded in the same manner. Thus every notation (*) represents a sequence of strings. For example, $a_1(a_2, a_3)a_4$ represents $a_1{}^\frown a_4$, $a_1{}^\frown a_2{}^\frown a_4$, $a_1{}^\frown a_2{}^\frown a_2{}^\frown a_4$, $a_1{}^\frown a_3{}^\frown a_4$, $a_1{}^\frown a_3{}^\frown a_3{}^\frown a_4$, $a_1{}^\frown a_2{}^\frown a_3{}^\frown a_4$, $a_1{}^\frown a_3{}^\frown a_2{}^\frown a_4$, etc.

The authors further prove that the set of FS grammars in a given alphabet is a Boolean algebra (cf. XXV 163(6)) and provide procedures for calculating the number of sentences of length λ, the number of sentences of length λ or less, the number of strings of sentences, and the number of sentences in the complementary language.

Authors' corrections to the 1958 printing: on page 97, delete the section in lines 10–17 beginning with "Perform ..." and ending with "... revised," which is unnecessary for the proof; page 97, line 18, for "T_m", read "T_n".

In (5) a hierarchy of grammars is studied by imposing increasingly heavy restrictions on the class F of functions from which grammars may be drawn. It is proved that F is thereby first specifiable as a Turing machine (type 0 grammar), then restricted to PS (type 1 and 2) grammars, and finally to FS (type 3) grammars. Though transformational grammars are not considered, some grammars of this hierarchy function as essential components in adequate grammars for natural languages.

Unrestricted grammars contain strings in a finite alphabet (including an identity element I) and a binary relation $\rightarrow$ (read "can be rewritten as") between strings, satisfying several conditions including the following (Greek letters denote arbitrary strings): there is a finite set of pairs $\{(\chi_1, \omega_1), \ldots, (\chi_n, \omega_n)\}$ such that for all φ, ψ, $\varphi \rightarrow \psi$ iff there are φ_1, φ_2, and $j \leqq n$ such that $\varphi = \varphi_1\chi_j\varphi_2$ and $\psi = \varphi_1\omega_j\varphi_2$. The ancestral of $\rightarrow$ is written $\Rightarrow$. A type i language is one with a type i grammar meeting the following restrictions for $i = 1, 2, 3$, respectively (A, B are single non-terminal symbols and a is a single terminal symbol): *Restriction* 1: If $\varphi \rightarrow \psi$, then there are $A, \varphi_1, \varphi_2, \omega$ such that $\varphi = \varphi_1 A \varphi_2$, $\psi = \varphi_1\omega\varphi_2$, and $\omega \neq I$ (i.e., the grammar contains rules $A \rightarrow \omega$ in the context $\varphi_1 - \varphi_2$ which may be null); *Restriction* 2: The same, but $A \rightarrow \omega$ (i.e., the limiting context must be null); *Restriction* 3: If $\varphi \rightarrow \psi$, then there are A, $\varphi_1, \varphi_2, \omega, a, B$ such that $\varphi = \varphi_1 A\varphi_2$, $\psi = \varphi_1\omega\varphi_2$, $\omega \neq I$, $A \rightarrow \omega$, but $\omega = aB$ or $\omega = a$ (i.e., rules are limited to the form $A \rightarrow aB$ or $A \rightarrow a$). The author proves that in this terminology, for both grammars and languages, type 0 $\supset$ type 1 $\supset$ type 2 $\supset$ type 3.

A grammar is *self-embedding* (s.e.) if it contains a non-terminal symbol A such that for some φ, ψ ($\varphi \neq I \neq \psi$), $A \Rightarrow \varphi A \psi$. The main theorem of (5) states that a type 2 language is not a type 3 language iff all of its grammars are s.e. In other words, the extra power of PS grammars over FS grammars as language generators lies in the fact that the former may be s.e. This interesting result is established by means of a very long and cumbersome proof, which provides a method not only for constructing a FSG equivalent to a non-s.e. PSG, but also for showing that this FSG can assign structural descriptions to strings of the PSG.

On page 151 it is shown that the language L_3 discussed in (1) and XXIII 71 is a type 1 language but not a type 2 language.

Author's corrections: page 141 (***Readings,*** page 129), Axiom 3, add the condition that ψ and ω are non-null; page 147 (***Readings,*** page 135), line 10 from the bottom, for "$m + n$", read "m, n". Furthermore, in ***Readings,*** page 125, footnote, for "**1**, 91–112", read "**2**, 137–67".

In (6) the author provides a short and simple proof for the main theorem of (5) by *reductio ad absurdum* and induction. Direct use is made of two results from Kleene's XXIII 59. This proof does not establish that the equivalent FSG can assign structural descriptions to strings of the PSG. The type 2 grammars are now called *context-free* PS grammars.

In (7), which gives a survey of some of the results obtained so far, the significance of the main theorem of (5) and (6) is brought out. The author lists a series of requirements which an adequate grammar should meet. General linguistic theory should provide, at least, complete specifications of sentences s_i and of a function f such that $f(i, j)$ is the set of structural descriptions of s_i provided by the grammar G_j. Part of the structural description of a terminal string is a *phrase-marker* (P-marker), which can be represented as a labelled tree. The type 2 and type 1 grammars of (5) are now called *context-free* and *context-restricted constituent structure* (CS) grammars, respectively. In context-free CS grammars, each non-terminal symbol A is of one or more of four types: (i) A is *non-recursive* if for no non-null φ, ψ, $A \Rightarrow \varphi A \psi$; (ij) A is *left-recursive* if there is a non-null φ such that $A \Rightarrow A\varphi$ (in this case the grammar generates P-markers that branch indefinitely far to the left); (iij) A is *right-recursive*, correspondingly with $A \Rightarrow \varphi A$; (iv) A is s.e. (in this case P-markers contain *nested dependencies*). Nesting of dependencies is common in natural languages, as illustrated earlier. In some languages (e.g., English) right-recursive structures are abundant, in others (e.g., Japanese) the opposite appears to be the case. Next *branches*, *s.e. nodes*, and *degrees* of self-embedding are defined for P-markers. The result of (5) can now be stated as follows. There is a mechanical procedure g such that where G_i belongs to a certain class of context-free CS grammars, $g(i, n)$ is the description of a finite automaton which, given a string s as input, will give as output all P-markers of degree $< n$ assigned to s by G_i. If a grammar G generates P-markers of arbitrary degree, there will not, in general, exist a finite device that will accept (alternatively, produce) just the sentences of the language specified by G. We may think of the automaton $g(i, n)$ as being a model for a speaker or hearer with a finite memory permitting the degree of self-embedding n.

This paper ends with a description of transformational grammars largely corresponding to XXIII 71. The notion of kernel is dropped and transformations are described as mappings of P-markers into P-markers. It is noted that although binary divisions are characteristic of the simple structural descriptions generated by CS grammars, they are rare in P-markers associated with actual sentences. In footnote 24 some publications are listed in which transformations are applied to material from such languages as English, Turkish, German, Russian, and Hidatsa, to which Mohawk and Sanskrit may now be added. This paper also contains some comments on Yngve's "depth hypothesis" (see, e.g., pages 130–138 in the volume in which (7) occurs) in which *depth* is a measure for structural complexity to which self-embedding and left-branching contribute equally. Examples such as given in footnote 10 of (7) appear to refute Yngve's hypothesis that a memory restriction prevents structures with depth > 7.

The comments of (8) add little that may be of immediate interest to readers of this JOURNAL. The author repeats that his concept of grammar is neutral as between speaker and hearer.

Concatenation is the basic relation presupposed in all grammars studied in these publications. To the present reviewer there seems to be one feature of natural language which can be described, but not in a simple or natural fashion, on this basis: the optional word order in some highly inflectional languages such as Latin or Sanskrit. The string *puer puellam amat* (cf. Curry (XXV 341) page 66 and Hiż page 265 in the volume in which (7) occurs) is grammatically equivalent to the five strings obtained from it by permutation of its three elements. These strings can be incorporated by simple permutational transformations in a transformational grammar, but would thereby increase its complexity in a non-revealing manner. Of course this hardly affects the value of the impressive array of methods and techniques marshalled by the author in his effort to bring a respectable branch of scholarship into the forefront of contemporary scientific disciplines.

J. F. STAAL

TADEUSZ BATÓG. *A contribution to axiomatic phonology.* English, with Polish and Russian summaries. ***Studia logica,*** vol. 13 (1962), pp. 67–80.

This paper, according to the author, presents some modifications of the system of axiomatic phonology developed in his earlier article, XXXI 288(5). The system is based on Leśniewski's mereology, supplemented and formalized by A. Tarski (Appendix E in III 42). The system has four primitive terms of its own: **I** the class of all idiolects, **D** and **O** the class of all segments and pauses that can be recognized in the whole of human speech, **B** the relation of phonetic similarity. Twelve axioms are introduced concerning these terms. According to one of them, the relation **B** is an equivalence relation and *sounds* are defined as its equivalence classes. The notion of *environment* of a sound and the relation of two sounds' being in *free variation* and in *complementary distribution* are given formal definitions. A class of sounds of an idiolect that are in free variation or in complementary distribution with each other is defined as an *allophone*. But, as the relation of complementary distribution is not an equivalence relation, it may be the case that two allophones are not disjoint. The sounds are said to be *phonologically* equivalent if they belong to the same allophones, and equivalence classes defined by this relation are called *phonemes*. If an idiolect satisfies the condition that two sounds Y and Z which are both in complementary distribution with one sound X are necessarily either in free variation or in complementary distribution with each other, the idiolect is said to satisfy the axiom of *distribution*. That the concept of phoneme coincides with that of allophone in such an idiolect is given as a theorem.

As the author notes, what is formalized here is a "somewhat simplified version of theoretical phonology, as developed by some American structuralists." From the linguistic point of view, however, it should be added that the concept of phoneme based on the notions of phonetic similarity and complementary distribution has been shown to be of no linguistic significance; see N. Chomsky, ***Current issues in linguistic theory,*** The Hague 1964.

S.-Y. KURODA

ROMAN SIKORSKI. ***Boolean algebras.*** Ergebnisse der Mathematik und ihrer Grenzgebiete, n.s. no. 25. Springer-Verlag, Berlin-Göttingen-Heidelberg 1960, IX + 176 pp.

ROMAN SIKORSKI. ***Boolean algebras.*** Second edition. Ergebnisse der Mathe-