puted by Z relative to ψ. Let Z_0 be a sequential Turing machine, i.e., one which moves only one way, always moves after printing and never enters an "ask" state after being in the corresponding "answered" state. A sequence of Turing machines $Z_0, Z_1, \cdots, Z_n$ is then a predicting sequence over a class Ψ of partial functions provided that for every $\psi \in \Psi$ and $i = 0, 1, \cdots, n-1$, if there is a computation of $Z_i[\psi](\bar{x})$, then $Z_{i-1}[\psi](\bar{x})$ is defined and is a bound on the length of tape required. Functional F is predictably computable over Ψ if it is the Z_n for some predicting sequence over Ψ. Some indications are given of how the property of predictable computability restricts F. For example, given any F which is predictably computable over the class $\mathcal{T}$ of total functions, if F maps $\mathcal{T}$ into $\mathcal{T}$ then F is primitive recursive and maps the class of elementary functions into itself; but the converse fails. However, every partial recursive function is an explicit transformation of a minimal fixed point of a predictably computable functional over the class of partial functions. PAUL AXT

NOAM CHOMSKY and GEORGE A. MILLER. *Introduction to the formal analysis of natural languages.* ***Handbook of mathematical psychology,*** Volume II, edited by R. Duncan Luce, Robert R. Bush, and Eugene Galanter, John Wiley and Sons, Inc., New York and London 1963, pp. 269–321.

NOAM CHOMSKY. *Formal properties of grammars.* Ibid., pp. 323–418.

GEORGE A. MILLER and NOAM CHOMSKY. *Finitary models of language users.* Ibid., pp. 419–491.

These three interrelated chapters provide a remarkably extensive summary of recent thought and research involving the application of formal models to several dimensions of linguistics. They also speculate on what may lie ahead. The authors focus on problems about the acquisition and use of natural languages—problems of behavioral science. They show how these behavioral problems have helped to spawn formal models and to motivate their investigation. (See Staal XXXI 245.) Such investigation is pursued in some detail, especially in the Chomsky paper. Prominent among the models studied are various sorts of automata and rewriting systems. These, having their heritage in the work of Turing and Post, are natural candidates for the logician's interest.

The Chomsky-Miller paper cites this "fundamental fact" as its point of departure: "A native speaker of a language has the ability to comprehend an immense number of sentences that he has never previously heard and to produce . . . novel utterances that are similarly understandable to other native speakers" (p. 271). "What," they ask, "is the precise nature of this ability?" and "How is it put to use?" Both questions demand exploration of the concept of *grammar*, a concept that readily lends itself to study in abstract terms. A grammar is taken to have a *syntactic* component and a *phonological* one and to be specifiable in terms of *rules*. To introduce study of the syntactic part, brief discussions of concatenation systems and algebraic aspects of coding are included. A *generative grammar* that is productive of all and only the sentences of a language is seen as but a first step toward the syntactic component. One wants in addition to provide a *structural description* (or *P-marker*) for each sentence, revealing the sentence's underlying structure. And one also seeks, ideally, a theory of *degree of grammaticalness* to account for the intelligibility of some deviant sentences. *Phrase-structure* (or *constituent-structure*) grammars are presented as models useful in the first step, but their limitations are emphasized. It is argued (somewhat loosely) that these limitations urge *transformational* grammars upon us. Rudimentary properties of these several kinds of syntactic models are discussed and illustrated with examples from English. The treatment of the phonological component is brief, but a small sample of phonological rules for English is given. It is suggested that phonology and syntax are connected in that "identifying an observed acoustic event as such-and-such a particular phonetic sequence is, in part, a matter of determining its syntactic structure" (p. 318).

As readers of Chomsky would expect, the view of language learning is anti-inductivist. One finds reference, for example, to "*innate* data-processing and concept-forming capacities that a child brings to bear in language learning" (p. 307, italics the reviewer's). Again on page 330: "For acquisition of language to be possible at all . . . the organism must, necessarily, be preset to search for and identify certain kinds of structural regularities." But such philosophical leanings play no consequential role in these chapters.

The Chomsky paper surveys much of what is known about phrase-structure grammars and the related kinds of automata. On the former: "This seems to be the area in which the study of mathematical models is most likely to provide significant insight into linguistic structure and the capacities of the language user" (p. 325); the motivations for the investigations undertaken remain near the surface throughout the paper. Various representations are provided for the families of *regular*, *context-free*, and *context-sensitive* languages. They are seen to be specifiable in terms of *restricted rewriting systems* on the one hand and in terms of *finite*, *pushdown storage*, and *linear-bounded automata*, respectively, on the other. A large number of theorems about these families and their properties are cited. Much more is covered here than in Bar-Hillel, Perles, and Shamir XXX 383(6), a paper on which Chomsky draws heavily. While most proofs are omitted, enough are given or sketched to expose the reader to a reasonable sample of the relevant techniques and to familiarize him with the concepts under study. Definitions are all set down carefully. Among the proofs, it is established in detail that the context-free languages are exactly the class accepted by pushdown storage automata, though the pieces needed to fill out the argument are widely scattered in the paper. *Finite transduction* is studied, and results concerning such subjects as *ambiguity*, *linear grammars*, and *categorial grammars* are recorded. With the aid of the Post correspondence problem (XII 55(3)) a number of problems about context-free languages are shown to be undecidable. The selection of results included is on the whole fortunate; certainly a large variety of topics receives treatment. And as far as possible, the presentation seems designed to be informative and palatable to those who shun mathematical detail as well as to those who savor it.

There are several small slips. The empty word is unevenly treated; Figure 4 on page 335 suggests that it is to be accorded membership in the set represented by the expression defined in (iii) of Definition 3 on that page, but the definition of "context-free language" on page 366 and the discussion following it indicate otherwise. Clause (iii) can be understood either way, depending on whether or not one takes $m = 0$ to be permitted therein. In any case the parenthesized expressions beginning on line 6 of page 367 and eleven lines up page 392 are both mistaken, since one can also generate $L \cup \{e\}$ for context-free L. And similarly in Theorem 31. The explanation of *k-limited automaton* on page 336 is rendered opaque by an unannounced shift in the sense of "accepted" from that of earlier pages. The asterisk serves in three different capacities in the short stretch from page 335 to page 342, and in a fourth on page 378; in a book so rich in symbols this would seem to have been avoidable. The last 'y' seven lines from the bottom of page 348 should be 'x'. The subscript just before the arrow on line 3 of page 366 should be '2'. The inequality on line 6 of page 368 points the wrong way. The sentence beginning fifteen lines up page 378 is false. The last 'a' on line 14 of page 381 should be 'c'. On line 14 of page 397 the reference should be to Theorem 34, and fourteen lines up that page the commas should be deleted from inside the square brackets.

The Miller-Chomsky paper discusses both stochastic and algebraic models of users of language. Much of the treatment is information-theoretic and derives from work of Shannon. *Markov sources* are studied in an approach to the problem of how to assign probabilities for the next letter or word in a message in natural language on the basis of the part of the message just received. Measures of *amount of information* and *redundancy* are defined, and there is a section on *minimum-redundancy codes*. Other problems in the statistical analysis of language are touched upon, notably those regarding word frequencies. Models incorporating rewriting systems and transformational grammars are briefly examined. The subject of this paper is acknowledged to be one about which little has been settled.

The three papers here reviewed constitute nearly half of the volume containing them. Each contains many helpful diagrams and an excellent bibliography. The volume is very well produced.

JOSEPH S. ULLIAN

SEYMOUR GINSBURG. ***The mathematical theory of context free languages.*** McGraw-Hill Book Company, New York, San Francisco, St. Louis, Toronto, London, and Sydney, 1966, xii + 232 pp.

Mathematical linguistics—a new branch of mathematics—has been developed in the last ten years. One of the most important directions of this domain bases on some ideas given by N. Chomsky in 1959 and known today as the theory of phrase-structure grammars. A phrase-