# trab4

November 28, 2020

## 1 Neste trabalho (Trabalho 4), a idéia é ter uma revisão de PCA, passando por seus conceitos e construindo um pouco de código

É essencial ver os códigos e comentários de cada célula, além das 3 questões...

[4]:
```
import numpy as np
import pandas as pd
from sklearn import linear_model
from sklearn.metrics import accuracy_score
import matplotlib.pyplot as plt
from sklearn.preprocessing import StandardScaler
```

[5]:
```
df=pd.read_csv('iris.txt',names=['m1','m2','m3','m4','esp']) #acertar path para␣
 ↪o dataset
```

[9]:
```
df.head()
```

[9]:

| | m1 | m2 | m3 | m4 | esp |
|---|---|---|---|---|---|
| 0 | 5.1 | 3.5 | 1.4 | 0.2 | Iris-setosa |
| 1 | 4.9 | 3.0 | 1.4 | 0.2 | Iris-setosa |
| 2 | 4.7 | 3.2 | 1.3 | 0.2 | Iris-setosa |
| 3 | 4.6 | 3.1 | 1.5 | 0.2 | Iris-setosa |
| 4 | 5.0 | 3.6 | 1.4 | 0.2 | Iris-setosa |

Passo 1: Criar uma matriz com as features numéricas

Standardizar as Features: importante para PCA, KNN,K-means...na dúvida, standardizar!

[22]:
```
X=df.to_numpy()[ : , :4]
scaler=StandardScaler()
scaler.fit(X)
XS=scaler.transform(X)
XS[:]
```

[22]:
```
array([[-9.00681170e-01,  1.03205722e+00, -1.34127240e+00,
        -1.31297673e+00],
       [-1.14301691e+00, -1.24957601e-01, -1.34127240e+00,
        -1.31297673e+00],
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```

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         7.90590793e-01]])
```

Geraremos a Matriz de Covariânvcia Na diagonal principal aparecem as variâncias das features. Na Standardização, a média já foi para 0 e o desvio padrão para 1. Desvio padrão 1 implica variância 1. Fora da diagonal principal aparecem as covariâncias

[23]:
```
COV=np.cov(XS.T)
COV
```

[23]:
```
array([[ 1.00671141, -0.11010327,  0.87760486,  0.82344326],
       [-0.11010327,  1.00671141, -0.42333835, -0.358937  ],
       [ 0.87760486, -0.42333835,  1.00671141,  0.96921855],
       [ 0.82344326, -0.358937  ,  0.96921855,  1.00671141]])
```

Vamos procurar um novo sistema de eixos que maximizará as variâncias de cada feature E as covariâncias serão anuladas Esse novo sistema de eixos é dado pelos autovetores da matriz de covariância

Utilizaremos Sklearn

[35]:
```
from sklearn.decomposition import PCA #fazendo PCA inicialmente com todas as 4␣
 ↪features
p=PCA(4)
p.fit(XS)
XS4=p.transform(XS)#muda XS para o novo sistema de eixos composto pelos␣
 ↪autovetores
XS4[:7]
```

[35]:
```
array([[-2.26454173,  0.5057039 , -0.12194335, -0.02307332],
       [-2.0864255 , -0.65540473, -0.22725083, -0.10320824],
       [-2.36795045, -0.31847731,  0.05147962, -0.02782523],
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       [-2.38877749,  0.6747674 ,  0.02142785,  0.03739729],
       [-2.07053681,  1.51854856,  0.03068426, -0.00439877],
       [-2.44571134,  0.07456268,  0.34219764,  0.03809657]])
```

[53]:
```python
p
```

[53]:
```
PCA(n_components=4)
```

[26]:
```python
p.explained_variance_#os autovalores
```

[26]:
```
array([2.93035378, 0.92740362, 0.14834223, 0.02074601])
```

[27]:
```python
p.components_
#os autovetores aparecem nas linhas
```

[27]:
```
array([[ 0.52237162, -0.26335492,  0.58125401,  0.56561105],
       [ 0.37231836,  0.92555649,  0.02109478,  0.06541577],
       [-0.72101681,  0.24203288,  0.14089226,  0.6338014 ],
       [-0.26199559,  0.12413481,  0.80115427, -0.52354627]])
```

Questão 1) Mostre que p.explained_variance[0] e p.components_[0] são um par autovalor e autovetor da matriz de covariância Obviamente isso vale para os 4 autovalores e autovetores de COV

[58]:
```python
p.explained_variance_[0], p.components_[0]
```

[58]:
```
(2.9303537755893116,
 array([ 0.52237162, -0.26335492,  0.58125401,  0.56561105]))
```

[67]:
```python
# Demonstração de que p.explained_variance[0] e p.components_[0] são auto valor␣
↪e
# auto vetor da matriz COV
auto_valores, auto_vetores = np.linalg.eig(COV)
auto_valores[0], auto_vetores[0][0], auto_vetores[1][0], auto_vetores[2][0],␣
↪auto_vetores[3][0]
```

[67]:
```
(2.9303537755893183,
 0.5223716204076599,
 -0.26335491531394034,
 0.5812540055976478,
 0.5656110498826492)
```

[55]:
```python
p.explained_variance_ratio_ #observe que com duas das novas features, já temos␣
↪mais de 95% da variância
```

[55]:
```
array([0.72770452, 0.23030523, 0.03683832, 0.00515193])
```

As features são agora ortogonais…nada fora da diagonal principal Vamos observar a nova matriz de covariância COVB (nos novos eixos) as features têm alta variância (diagonal principal) e nenhuma covariância (fora da diagonal) Mais variância, mais informação…assim, com as duas primeiras features (duas primeiras colunas de XS4) já teremos informação suficiente para a visualização, por exemplo. O PCA concentra a variância nas primeiras features

[56]:
```
COVB=np.cov(XS4.T)
COVB
```

[56]:
```
array([[ 2.93035378e+00,  1.16657243e-16,  9.18728181e-16,
        -1.02453467e-17],
       [ 1.16657243e-16,  9.27403622e-01, -1.42480174e-16,
         3.87373066e-17],
       [ 9.18728181e-16, -1.42480174e-16,  1.48342226e-01,
        -4.91776642e-17],
       [-1.02453467e-17,  3.87373066e-17, -4.91776642e-17,
         2.07460140e-02]])
```

Questão 2) Teoricamente, como a Matriz de Covariância é Hermitiana, então os autovetores são ortogonais. Mostre que os 4 autovetores dela são ortogonais entre si (dica..produto interno...).

[77]:
```
p.explained_variance_
ortog = []
ortog.append(np.inner(p.components_[0], p.components_[1]))
ortog.append(np.inner(p.components_[0], p.components_[2]))
ortog.append(np.inner(p.components_[0], p.components_[3]))
ortog.append(np.inner(p.components_[1], p.components_[2]))
ortog.append(np.inner(p.components_[1], p.components_[3]))
ortog.append(np.inner(p.components_[2], p.components_[3]))
```

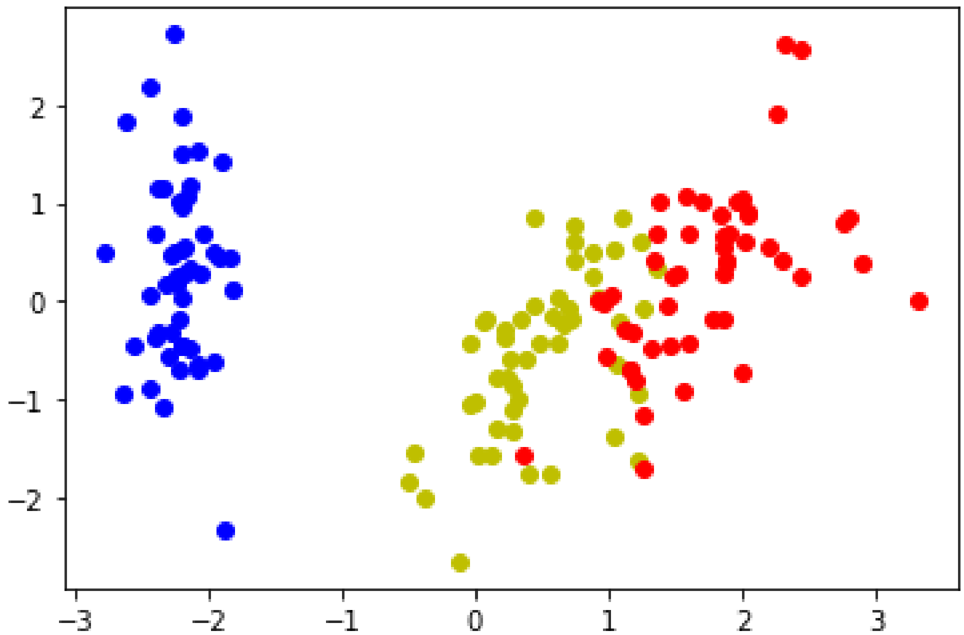
[78]:
```
ortog
```

[78]:
```
[-6.938893903907228e-18,
 5.551115123125783e-17,
 1.6653345369377348e-16,
 2.0816681711721685e-17,
 -4.163336342344337e-17,
 -5.551115123125783e-17]
```

Questão 3) Com as duas primeiras colunas de XS4, faça a visualização do dataset (cada espécie com uma cor)

[79]:
```
esp = df['esp'].copy()
esp = esp.map({'Iris-setosa':0, 'Iris-virginica':2, 'Iris-versicolor':1})
```

[82]:
```
cor = ['bo', 'yo', 'ro']
for i in range(len(XS4)):
    plt.plot(XS4[i,0], XS4[i,1], cor[esp[i]])
plt.show()
```



2
1
0
−1
−2
−3
−2
−1
0
1
2
3