Modelo Multifásico

$$\frac{\partial}{\partial t}\left(\varepsilon_k \rho_k \left\langle \phi_k \right\rangle^k\right)+\frac{\partial}{\partial x_i}\left(\varepsilon_k \rho_k \left\langle \phi_k \right\rangle^k \left\langle v_k \right\rangle_i^k\right)=\frac{\partial}{\partial x_i}\left(\varepsilon_k \Lambda_k^* \frac{\partial \left\langle \Phi_k \right\rangle^k}{\partial x_i}\right)+\sum_{\substack{j=1\\ j\neq k}}^{n}\Theta_{kj}$$

Continuidade:

$$\frac{\partial}{\partial t}\left(\varepsilon_k \rho_k\right)+\frac{\partial}{\partial x_i}\left(\varepsilon_k \rho_k \left\langle v_k \right\rangle_i^k\right)=\sum_{\substack{j=1\\ j\neq k}}^{n}\Gamma_{kj}$$

Energia:

$$\frac{\partial}{\partial t}\left(\varepsilon_k \rho_k \left\langle h_k \right\rangle^k\right)+\frac{\partial}{\partial x_i}\left(\varepsilon_k \rho_k \left\langle h_k \right\rangle^k \left\langle v_k \right\rangle_i^k\right)=\frac{\partial}{\partial x_i}\left(\varepsilon_k K_k^* \frac{\partial \left\langle T_k \right\rangle^k}{\partial x_i}\right)+\sum_{\substack{j=1\\ j\neq k}}^{n}Q_{kj}$$

Espécies Químicas:

$$\frac{\partial}{\partial t}\left(\varepsilon_k \rho_k \left\langle C_k \right\rangle^k\right)+\frac{\partial}{\partial x_i}\left(\varepsilon_k \rho_k \left\langle C_k \right\rangle^k \left\langle v_k \right\rangle_i^k\right)=\frac{\partial}{\partial x_i}\left(\varepsilon_k D_k^* \frac{\partial \left\langle C_k \right\rangle^k}{\partial x_i}\right)+\sum_{\substack{j=1\\ j\neq k}}^{n}J_{kj}$$

Discretização:

$$\frac{1}{\Delta t}\left[\left(\varepsilon_k \rho_k \left\langle \phi_k \right\rangle^k\right)_{x_i}\right]_t^{t+\Delta t}+\frac{1}{\Delta x_i}\left[\left(\varepsilon_k \rho_k \left\langle \phi_k \right\rangle^k \left\langle v_k \right\rangle_i^k\right)^t\right]_{x_i-\frac{\Delta x_i}{2}}^{x_i+\frac{\Delta x_i}{2}}=\frac{1}{\Delta x_i}\left[\left(\varepsilon_k \Lambda_k^* \frac{\partial \left\langle \Phi_k \right\rangle^k}{\partial x_i}\right)^t\right]_{x_i-\frac{\Delta x_i}{2}}^{x_i+\frac{\Delta x_i}{2}}+\sum_{\substack{j=1\\ j\neq k}}^{n}\left(\Theta_{kj}\right)_{\vec{x}_i}^t$$

Continuidade:

$$\frac{1}{\Delta t}\left[\left(\varepsilon_k \rho_k\right)_{\vec{x}}\right]_t^{t+\Delta t}+\frac{1}{\Delta x_i}\left[\left(\varepsilon_k \rho_k \left\langle v_k \right\rangle_i^k\right)^t\right]_{x_i-\delta x_i^-}^{x_i+\delta x_i^+}=\sum_{\substack{j=1\\ j\neq k}}^{n}\left(\Gamma_{kj}\right)_{\vec{x}}^t$$

$$\frac{\Delta\left(\varepsilon_k \rho_k\right)_{\vec{x}}^t}{\Delta t}=\sum_{\substack{j=1\\ j\neq k}}^{n}\left(\Gamma_{kj}\right)_{\vec{x}}^t-\frac{1}{\Delta x_i}\left[\left(\varepsilon_k \rho_k \left\langle v_k \right\rangle_i^k\right)^t\right]_{x_i-\delta x_i^-}^{x_i+\delta x_i^+}$$

$$\frac{1}{\Delta t}\left[\left(\varepsilon_k\rho_k\left\langle\phi_k\right\rangle^k\right)_{\vec{x}}\right]_t^{t+\Delta t}=\frac{1}{\Delta t}\left[\left(\varepsilon_k\rho_k\right)_{x_i}^{t+\Delta t}\left(\left(\left\langle\phi_k\right\rangle^k\right)_{x_i}^t+\left(\Delta\left\langle\phi_k\right\rangle^k\right)_{x_i}^t\right)-\left(\varepsilon_k\rho_k\right)_{x_i}^t\left(\left\langle\phi_k\right\rangle^k\right)_{x_i}^t\right]=$$

$$=\frac{1}{\Delta t}\left[\left(\left(\varepsilon_k\rho_k\right)_{\vec{x}}^t+\Delta\left(\varepsilon_k\rho_k\right)_{\vec{x}}^t\right)\left(\left(\left\langle\phi_k\right\rangle^k\right)_{\vec{x}}^t+\left(\Delta\left\langle\phi_k\right\rangle^k\right)_{\vec{x}}^t\right)-\left(\varepsilon_k\rho_k\right)_{\vec{x}}^t\left(\left\langle\phi_k\right\rangle^k\right)_{\vec{x}}^t\right]=$$

$$=\frac{1}{\Delta t}\left[\left(\varepsilon_k\rho_k\right)_{\vec{x}}^t\left(\Delta\left\langle\phi_k\right\rangle^k\right)_{\vec{x}}^t+\Delta\left(\varepsilon_k\rho_k\right)_{\vec{x}}^t\left(\left(\left\langle\phi_k\right\rangle^k\right)_{\vec{x}}^t+\left(\Delta\left\langle\phi_k\right\rangle^k\right)_{\vec{x}}^t\right)\right]=$$

$$=\frac{1}{\Delta t}\left[\left(\varepsilon_k\rho_k\right)_{\vec{x}}^t\left(\Delta\left\langle\phi_k\right\rangle^k\right)_{\vec{x}}^t+\Delta t\left(\sum_{\substack{j=1\\j\neq k}}^n\left(\Gamma_{kj}\right)_{\vec{x}}^t-\frac{1}{\Delta x_i}\left[\left(\varepsilon_k\rho_k\left\langle v_k\right\rangle_i^k\right)^t\right]_{x_i-\delta x_i^-}^{x_i+\delta x_i^+}\right)\left(\left(\left\langle\phi_k\right\rangle^k\right)_{\vec{x}}^t+\left(\Delta\left\langle\phi_k\right\rangle^k\right)_{\vec{x}}^t\right)\right]$$

$$\left(\Delta\left\langle\phi_k\right\rangle^k\right)_{x_i}^t\left[\frac{\left(\varepsilon_k\rho_k\right)_{x_i}^t}{\Delta t}+\sum_{\substack{j=1\\j\neq k}}^n\left(\Gamma_{kj}\right)_{\vec{x}}^t-\frac{1}{\Delta x_i}\left[\left(\varepsilon_k\rho_k\left\langle v_k\right\rangle_i^k\right)^t\right]_{x_i-\delta x_i^-}^{x_i+\delta x_i^+}\right]+\left(\sum_{\substack{j=1\\j\neq k}}^n\left(\Gamma_{kj}\right)_{\vec{x}}^t-\frac{1}{\Delta x_i}\left[\left(\varepsilon_k\rho_k\left\langle v_k\right\rangle_i^k\right)^t\right]_{x_i-\delta x_i^-}^{x_i+\delta x_i^+}\right)\left(\left\langle\phi_k\right\rangle^k\right)_{x_i}^t$$

$$\left[\left(\left\langle\phi_k\right\rangle^k\right)_{x_i}^{t+\Delta t}-\left(\left\langle\phi_k\right\rangle^k\right)_{x_i}^t\right]\left[\frac{\left(\varepsilon_k\rho_k\right)_{x_i}^t}{\Delta t}+\sum_{\substack{j=1\\j\neq k}}^n\left(\Gamma_{kj}\right)_{\vec{x}}^t-\frac{1}{\Delta x_i}\left[\left(\varepsilon_k\rho_k\left\langle v_k\right\rangle_i^k\right)^t\right]_{x_i-\delta x_i^-}^{x_i+\delta x_i^+}\right]+\left(\sum_{\substack{j=1\\j\neq k}}^n\left(\Gamma_{kj}\right)_{\vec{x}}^t-\frac{1}{\Delta x_i}\left[\left(\varepsilon_k\rho_k\left\langle v_k\right\rangle_i^k\right)^t\right]_{x_i-\delta x_i^-}^{x_i+\delta x_i^+}\right)\left(\left\langle\phi_k\right\rangle^k\right)_{x_i}^t=$$

$$=\left(\left\langle\phi_k\right\rangle^k\right)_{x_i}^{t+\Delta t}\left[\frac{\left(\varepsilon_k\rho_k\right)_{x_i}^t}{\Delta t}+\sum_{\substack{j=1\\j\neq k}}^n\left(\Gamma_{kj}\right)_{\vec{x}}^t-\frac{1}{\Delta x_i}\left[\left(\varepsilon_k\rho_k\left\langle v_k\right\rangle_i^k\right)^t\right]_{x_i-\delta x_i^-}^{x_i+\delta x_i^+}\right]-\frac{\left(\varepsilon_k\rho_k\right)_{x_i}^t}{\Delta t}\left(\left\langle\phi_k\right\rangle^k\right)_{x_i}^t$$

$$\frac{1}{\Delta x_i}\left[\left(\varepsilon_k\rho_k\left\langle\phi_k\right\rangle^k\left\langle v_k\right\rangle_i^k\right)^t\right]_{x_i-\delta x_i^-}^{x_i+\delta x_i^+}=\frac{1}{\Delta x_i}\left[\left(\varepsilon_k\rho_k\left\langle\phi_k\right\rangle^k\left\langle v_k\right\rangle_i^k\right)_{x_i+\delta x_i^+}^t-\left(\varepsilon_k\rho_k\left\langle\phi_k\right\rangle^k\left\langle v_k\right\rangle_i^k\right)_{x_i-\delta x_i^-}^t\right]$$

$$\frac{1}{\Delta x_i}\left[\left(\varepsilon_k\rho_k\left\langle v_k\right\rangle_i^k\right)_{x_i+\delta x_i^+}^t\left(\left\langle\phi_k\right\rangle^k\right)_{x_i+\delta x_i^+}^t-\left(\varepsilon_k\rho_k\left\langle v_k\right\rangle_i^k\right)_{x_i-\delta x_i^-}^t\left(\left\langle\phi_k\right\rangle^k\right)_{x_i-\delta x_i^-}^t\right]$$