Spectral Methods in MATLAB

On infinite domains

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September 16, 2016

Outline

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A function may be expressed as
a linear combination of cardinal functions.

$$f(x) = \sum_{j=1}^{n} f(x_j) C_j(x)$$

$$C_j(x_i) = \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

The nodes correspond to the zeros of some special polynomial.

$$P_n(x_i) = 0$$

$$C_j(x) = \frac{P_n(x)}{(x - x_j)P_n'(x_j)}$$

The derivate of a function is the combination of the derivates of the cardinal functions.

$$\frac{d}{dx} f(x)|_{x_i} = \sum_{j=1}^{n} f(x_j) C_j'(x_i)$$

Outline

Ortogonal polynomials form great bases.

$$\int_R P_m(x)P_n(x)w(x)dx = c_m\delta_{mn}$$

Hermite polynomials are ideal for infinite domains.

$$H_n(x)$$

$$x \in (-\infty, \infty)$$

$$w(x) = e^{-x^2}$$

Laguerre polynomials are ideal for the radial coordinate.

$$L_n^1(x)$$

$$r \in [0, \infty)$$

$$w(r) = re^{-r}$$

Outline

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Summary

- The first main message of your talk in one or two lines.
- The second main message of your talk in one or two lines.
- Perhaps a third message, but not more than that.

- Outlook
  - Something you haven't solved.
  - Something else you haven't solved.

For Further Reading I

A. Author.
Handbook of Everything.
Some Press, 1990.

S. Someone.
On this and that.
Journal of This and That, 2(1):50–100, 2000.