Stable Homotopy Theory

Spring 2025

Course Description

This course is an introduction to stable homotopy theory. The basic objects of stable homotopy theory are called *spectra*, and the exact definition of a spectrum depends on who you ask, what they care about, and even when you ask them. Nevertheless, concepts from stable homotopy theory have found many applications both within algebraic topology and in other fields like geometric topology, algebraic geometry, number theory, algebra, etc. The goal of this course is that you'll be able to work with spectra in a practical manner and perform computations – you can choose your favorite definition later.

Learning Outcomes

After taking this course, students will be able to work with spectra in a practical manner, perform computations, engage in research projects in stable homotopy theory or related fields (such as algebraic geometry or geometric topology) that use stable homotopy techniques. Students will also gain experience communicating mathematics by giving talks, writing, and discussing mathematics with peers.

Topics

A tentative list of topics includes:

- Homotopical Categories
- Homotopy Limits and Colimits
- Stable Phenomena in Spaces
- Generalized Cohomology Theories
- Spectra as Sequences of Spaces
- The Stable Homotopy Category
- The Smash Product
- Symmetric and Orthogonal Spectra
- Ring and Module Spectra
- Operads
- Localization
- Chromatic Homotopy

The function between topics covered in the course and the list above may be neither injective nor surjective.

References

We will draw from a variety of sources, including but not limited to the following. Students are not required to purchase any of these references.

[Dug22] Daniel Dugger. Stable categories and spectra via model categories. In *Stable categories and structured ring spectra*, pages 75–150. Cambridge: Cambridge University Press, 2022.

[Mal23] Cary Malkiewich. Spectra and stable homotopy theory. http://people.math.binghamton.edu/malkiewich/spectra_book_draft.pdf, October 2023.

[Ric22] Birgit Richter. Commutative ring spectra. In *Stable categories and structured ring spectra*, volume 69 of *Math. Sci. Res. Inst. Publ.*, pages 249–299. Cambridge Univ. Press, Cambridge, 2022.

[Rie14] Emily Riehl. *Categorical homotopy theory*, volume 24 of *New Mathematical Monographs*. Cambridge University Press, Cambridge, 2014.

[Sch07] Stefan Schwede. An untitled book project about symmetric spectra. http://www.math.uni-bonn.de/people/schwede/SymSpec.pdf, 2007.

[Sto22] Bruno Stonek. Introduction to stable homotopy theory. https://bruno.stonek.com/stable-homotopy-2022/stable-online.pdf, July 2022.

Prerequisites

- Students should have taken the equivalent of two semesters of algebraic topology at the graduate level, including homology, cohomology, and homotopy groups π_n for $n > 1$. Exposure to K-theory, characteristic classes, cobordism, and other topics in algebraic topology is not required, although it may be helpful.
- Students should have a working knowledge of category theory, including but not limited to: natural transformations, limits, colimits, adjunctions, abelian categories, and symmetric monoidal categories.

Please contact the instructor if you do not meet the prerequisites but would still like to enroll in the course.

Logistics

This course will be taught in a flipped-classroom format, with all course meetings online. We will cover one topic each week. On Tuesdays, a student will give a talk on the topic of the week. On Thursdays, we will work in small groups on problem sets. These problem sets will be due for a grade at the beginning of the following week.

Contact Information

Instructor: David Mehrle

Email: davidm@uky.edu

Office Hours: TBA

Teaching Assistants: TBA

Class Time and Location

Dates: 28 January 2025 to 6 May 2025

Break: No class meetings on 1 April 2025 and 3 April 2025

Meeting Time: Tuesdays and Thursdays, 12pm to 12:50pm Eastern U.S. time

Classroom Meeting Link: TBA

Technology Requirements:

- Hardware to access and participate in Zoom meetings, such as a computer, phone, or tablet with speakers, microphone, and webcam.
- A tablet and pen with the capability of sharing digital handwriting.
- A reliable connection to the internet that does not interrupt your participation in the course.

If you anticipate having any trouble meeting the technology requirements, please contact the instructor.

Grades

As a general rule, students are expected to obtain 2-3 independent study credits from their home universities for their participation in the course. At the end of the semester, a grade of A/B/C/Fail will be reported to your home institution. Grades will be based on a combination of participation, problem sets, presentations, and a take-home final exam.