# Algebraic Topology I

MA 654, Fall 2024

## Contact Information

**Instructor:** David Mehrle
**Email:** davidm@uky.edu (Emails should expect a reply within one business day.)
**Office:** Patterson Office Tower 707
**Office Hours:** After class or by appointment. Appointments made by email are preferred.

## Class Time and Location

**Dates:** August 26, 2024 to December 20, 2024 (see the University of Kentucky academic calendar)
**Meeting Time:** Monday, Wednesday, and Friday 11:00 to 11:50
**Location:** Chemistry Physics Building 183

## Course Description

This course is an introduction to algebraic topology. In the quest to understand topological spaces, it is often difficult to determine if two topological spaces are homeomorphic, or even whether or not they are homotopy equivalent. So we turn to algebra to help us determine whether or not spaces are the same. The idea of algebraic topology is to transform topological questions to questions in algebra that may be easier to answer. Our slogan is: *topology is hard, but algebra is easier*.

**Official catalogue description:** Homotopy and homology theories, complexes and applications.

### Learning Outcomes

After taking this course, students will be able to:

- compute homology groups and cohomology rings of spaces built as cell complexes or simplicial complexes
- apply homological algebra (long exact sequences, Universal Coefficients, Künneth Theorem) to find the (co)homology of spaces from other spaces whose (co)homology is already known

In the process, students will also:

- strengthen proof-writing skills
- practice communicating mathematics through writing and presentations

### Topics

A partial list of topics includes:

- Simplicial and singular homology
- Simplicial and singular cohomology
- The Mayer–Vietoris sequence
- Cup products and cohomology rings
- Universal coefficients theorem and the Künneth theorem
- Poincaré duality

We will also cover selected topics from category theory and homological algebra as required, in service of the topology topics listed above.

### Prerequisites

In addition to the formal prerequisites of MA 551 and MA 651, students should be familiar with the following topics:

- Algebra: groups, rings, modules, and chain complexes
- Category theory: categories, functors, natural transformations, and some basic limits and colimits (such pullbacks and pushouts)

Please contact the instructor if you do not meet the prerequisites but would still like to enroll in the course.

### References

We will draw from a variety of sources, including but not limited to those listed below. Students are not required to obtain any of these references.


[AGP02] Marcelo Aguilar, Samuel Gitler, and Carlos Prieto. *Algebraic topology from a homotopical viewpoint*. Universitext. New York, NY: Springer, 2002.

[Awo10] Steve Awodey. *Category theory.*, volume 52 of *Oxf. Logic Guides*. Oxford: Oxford University Press, 2nd ed. edition, 2010.

[Hat02] Allen Hatcher. *Algebraic topology*. Cambridge: Cambridge University Press, 2002. https://pi.math.cornell.edu/~hatcher/AT/ATpage.html.

[Rie16] Emily Riehl. *Category theory in context*. Mineola, NY: Dover Publications, 2016. https://emilyriehl.github.io/files/context.pdf.

[Spa95] Edwin H. Spanier. *Algebraic topology*. Berlin: Springer-Verlag, 1995.

[tD08] Tammo tom Dieck. *Algebraic topology*. EMS Textb. Math. Zürich: European Mathematical Society (EMS), 2008.

[Wei94] Charles A. Weibel. *An introduction to homological algebra*, volume 38 of *Camb. Stud. Adv. Math.* Cambridge: Cambridge University Press, 1994.


## Logistics

Each week, this class will have two days of lecture and one day of group problem solving. In the final weeks of the semester, we will transition to student presentations.

### Attendance

Students are expected to attend all classes, unless exempted in advance by the instructor. Please email the instructor if you are unable to attend class for whatever reason. Emergencies, illnesses, religious holidays, etc. may also be excused retroactively, at the instructor's discretion.

### Problem Sessions

One class each week will be reserved for a group problem-solving session. Students will be asked to carefully write and turn in their solutions to four problems over the course of the semester, approximately one per month. The deadlines to turn in problems are listed on canvas. Late work will be accepted at the instructor's discretion.

## Final Presentations

At the end of the semester, each student will be expected to give a 45 minute talk on an additional topic related to the theme of the course. Prior to their presentation, each student is required to schedule a one-on-one practice with the instructor. A list of suggested presentation topics and a grading rubric will be made available closer to midterm.

## Grades

Your grade will be based on your final presentation, four individual problems that you turn in over the course of the semester, and attendance in class. Students who engage with the course material and participate in class should expect to earn a good grade for attendance.

## Official University Policies

- Academic Policies
- Academic Standards
- Academic Offenses