

Exiton-Drift diffusion equations (complete form): 3 Equations of drift diffusion equation are as following:

The first equation is:

$$\nabla \cdot (\lambda^2 \varphi) - n + p = 0 \quad (1)$$

One can find derivative of equation 1 respects to  $n, p$  and  $\varphi$  in equation 2. This is an important step for finding Jacobian.

$$Der \Rightarrow \nabla \cdot (\lambda^2 \delta \varphi) - \delta n + \delta p = 0 \quad (2)$$

The weak form of equation 2 is shown in equation 3:

$$Weak form \Rightarrow -(\nabla \omega, \lambda^2 \delta \varphi) - (\omega, \delta n) + (\omega, \delta p) = 0 \quad (3)$$

By using Galerkin method, we can find the first row of Jacobian matrix as following:

$$\underbrace{-(dN(i, k), \lambda^2 dN(j, k))}_{A[11]} - \underbrace{(N(i), N(j))}_{A[12]} + \underbrace{(N(i), N(j))}_{A[13]} \quad (4)$$

Residual of the equation 1 can be written as:

$$\underbrace{(dN(i, k), \lambda^2 d\varphi_{,k}) - (N(i), n_{pre}) - (N(i), p_{pre})}_{b[1]} \quad (5)$$

The second equation of Drift Diffusion problem is:

$$\nabla \cdot (J_n) - U = 0 \quad (6)$$

Which  $J_n = -\mu_n n \nabla \varphi + \mu_n \nabla n$ , So equation 5 will be in form of:

$$\nabla \cdot (-\mu_n n \nabla \varphi + \mu_n \nabla n) - U \quad (7)$$

By considering that  $U = -K_{diss} X + \gamma np$ , derivative of equation 6 will be in form of:

$$Der \Rightarrow \nabla \cdot (-\mu_n \delta n \nabla \varphi - \mu_n n \nabla \delta \varphi + \mu_n \nabla \delta n) - \gamma \delta np - \gamma n \delta p \quad (8)$$

and the weak form is:

$$(\nabla \omega, \mu_n n \nabla \delta \varphi) + (\nabla \omega, \mu_n \nabla \varphi \delta n - \mu_n \nabla \delta n) + (\omega, -\gamma \delta np) + (\omega, -\gamma n \delta p) \quad (9)$$

Equation 8 shows the second row of Jacobian matrix:

$$\underbrace{(dN(i, k), \mu_n n dN(j, k))}_{A[21]} + \underbrace{(dN(i, k), \mu_n \varphi_{,k} N(j) - \mu_n dN(j, k)) + (N(i), -\gamma p N(j))}_{A[22]}$$

$$\underbrace{(N(i), -\gamma n N(j))}_{A[23]} \quad (10)$$

Residual of the equation 6 can be written as:

$$\underbrace{-(dN(i, k), -\mu_n n^{pre} d\varphi_{,k} + \mu_n dn_{,k}^{pre}) + (N(i), K_{diss} X^{pre}) - (N(i), \gamma n^{pre} p^{pre})}_{b[2]} \quad (11)$$

Third equation is similar to the second equation and  $J_p = -\mu_p p \nabla \varphi - \mu_p \nabla p$ , So by doing the same procedure the weak form will be in form of:

$$Weak\ form \Rightarrow -(\nabla \omega, \mu_p p \nabla \delta \varphi) - (\nabla \omega, \mu_p \nabla \varphi \delta p + \mu_p \nabla \delta p) + (\omega, -\gamma \delta n p) + (\omega, -\gamma n \delta p) \quad (12)$$

Equation 11 represents the third row of Jacobian matrix:

$$\begin{aligned} & \underbrace{(dN(i, k), \mu_p p dN(j, k))}_{A[31]} + \\ & \underbrace{(N(i), \gamma p N(j))}_{A[32]} + \\ & \underbrace{(dN(i, k), \mu_p \varphi_{,k} N(j) + \mu_p dN(j, k)) - (N(i), n \gamma N(j))}_{A[33]} \end{aligned} \quad (13)$$

Residual of the third equation can be written as:

$$\underbrace{(dN(i, k), \mu_p p^{pre} d\varphi_{,k} + \mu_p dp_{,k}^{pre}) - (N(i), K_{diss} X^{pre}) + (N(i), \gamma n^{pre} p^{pre})}_{b[3]} \quad (14)$$

The last equation of drift diffusion equations can be written as:

$$-\nabla \cdot (\mu_x \nabla X) = G - K_{diss} X - \tau X + \gamma n p = 0 \quad (15)$$

The Equation LHS of Exciton (in DDExciton) with assumption of constant G will be as following:

$$\underbrace{(\tau + K_{diss})(dN(i, k), dN(j, k)) + \mu_x (N(i), N(j))}_{Ae} \quad (16)$$

The RHS for Exciton equation is:

$$\underbrace{(G + \gamma n p) N(i)}_{be} \quad (17)$$