The Race to the Base*

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Abstract

We study multi-district legislative elections between two office-seeking parties when the election pits a relatively strong party against a weaker party; when each party faces uncertainty about how voter preferences will evolve during the campaign; and, when each party cares not only about winning a majority, but also about its share of seats in the event that it holds majority or minority status. When the initial imbalance favoring one party is small, each party targets the median voter in the median district, in pursuit of a majority. When the imbalance is moderate, the advantaged party continues to hold the centre-ground, but the disadvantaged party retreats to target its core supporters; it does so to fortify its minority share of seats in the likely event that it fails to secure a majority. Finally, when the imbalance is large, the advantaged party advances toward its opponent, raiding its moderate supporters in pursuit of an outsized majority.

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1. Introduction

A near-axiomatic logic of two-party elections is that to win the contest, a party must carry the support of the median voter. To the extent that political parties care solely about winning the election, their platforms should therefore converge to the median voter's most-preferred policy (e.g., Hotelling 1929, Downs 1957). In legislative elections, however, winning is *not* everything. In fact, winning a majority of legislative seats may be neither necessary nor sufficient for a party to achieve its goals.

Two examples help illustrate this point. In 1992, John Major's Conservative party won a majority of seats in the House of Commons, and the largest number of votes of any party in British electoral history. Nonetheless, Major's overall majority fell from 102 to a mere 21 seats. Despite its victory, Major's government was persistently hampered by its small majority, which contributed to its first legislative defeat just over one year later.

In 2017, by contrast, Jeremy Corbyn's Labour party failed to win a majority of seats. Nonetheless, the party advanced its minority by 26 seats, and successfully denied the Conservative party its previously-held parliamentary majority. Because the Conservatives had enjoyed a 20-point lead in the polls at the moment Theresa May called the election, the press concluded that, despite its failure to achieve outright victory, Labour had triumphed over expectations of an electoral rout.

At the start of the 2017 election campaign, Theresa May enjoyed a 39 percentage point popularity advantage over Jeremy Corbyn.[1] May opted for "an aggressive strategy, influenced by her strong lead in the initial polls... parking her tank on Labour's lawn in heartlands such as the North East and the North West of England".[2] To the extent that "a party's electoral strategy is often betrayed by the pattern of seats visited by its leader",[3] May's campaign visits are instructive: she devoted 61 percent of her campaign stops to Labour-held constituencies, and only 27 percent to Conservative-held constituencies. May further targeted moderate Labour supporters with policy proposals that included a price cap on energy bills—a policy commitment that had been featured in Labour's 2015 election manifesto.[4]

By contrast, Labour's 2017 campaign opted for a defensive strategy, eschewing centrist vot-

[1] YouGov, 18-19 April 2018.

[2] "What Theresa May's campaign stops tell us about her failed strategy", *The Telegraph*, 13 June 2017.

[3] "Analysis shows Theresa May spent half of campaign targeting Labour seats", *The Guardian*, 8 June 2017.

[4] In that election cycle, David Cameron ridiculed energy price caps as evidence of Ed Miliband's desire to live in a 'Marxist universe'. See "Tories accused of stealing Labour's energy price cap promise", *The Guardian*, 23 April, 2017.

ers in favor of its core supporters. Jeremy Corbyn devoted 52 percent of his campaign visits to defending Labour-held seats, and only 42 percent to pursuing Conservative-held seats. Of the Labour seats he visited, the vast majority had been won in the previous 2015 election with a victory margin of more than 20 percentage points.[5] The party also opted for a radical manifesto that promised to nationalize public utilities, abolish university tuition fees, and levy new taxes on firms with highly-paid staff.[6] To observers who believed that a more moderate platform would maximize Labour's election performance, the party's strategy was "baffling".[7] Why did it forego the centrist—or even right-leaning—route that led Tony Blair's party to a majority of 179 seats in 1997?

Our paper asks: under what circumstances does an office-seeking party in a legislative election want to target its electoral platform toward its traditional supporters, rather than centrist voters? If that party targets its traditional supporters, should the opposing party try to maintain its hold on the centre-ground, cater to its own base, or instead try to raid its opponent's more moderate supporters? And, how do the answers to these questions depend on parties' expectations about their popularity, the extent of voters' partisan loyalties, and the relative marginal value that a party derives from winning additional seats below, at, or above the majority threshold?

Our Approach. To address these questions, we develop a model of two-party competition between two office-motivated parties in a multi-district legislative election. For example, the election could determine control of a legislative chamber such as the U.S. House of Representatives, or the British House of Commons. After the parties simultaneously choose platforms, an aggregate net valence payoff shock in favor of one party is realized. Each voter in every district then casts his or her ballot for one of the two parties. We assume that one of the parties holds an initial advantage, in that the valence shock is expected to favor that party. For example, its leadership may perceived as more competent; alternatively, its opponent may be dogged by scandal or simply worn out by a long period of incumbency.

We assume throughout that each party's payoff depends solely on its share of districts, or seats in the legislature. However, this does not imply that parties care solely about *winning* the election. If a party wins more than half of the total districts (seats), it not only derives a large fixed payoff from majority status, i.e., from winning the election, it also receives a strictly increasing

[5] "Analysis shows Theresa May spent half of campaign targeting Labour seats", *The Guardian*, 8 June 2017.

[6] "For the Many, Not the Few", *Labour 2017 Election Manifesto*, https://goo.gl/GZaTbk.

[7] "The baffling world of Labour's election strategy", *The Spectator*, 27 April 2017.

payoff from any additional seats that it wins beyond the majority threshold. The fixed office rent reflects the value of majority status *per se*: in a parliamentary democracy, majority status confers the right to form the government regardless of the size of a party's majority. Even in presidential systems, majority status grants a party control over crucial aspects of the legislative process, including scheduling bills and staffing committees. However, additional seats beyond the majority threshold are also valuable: they help insulate the majority party from the threat of confidence votes in a parliamentary context, insure against defections of a few party members on key votes, and mitigate the obstructionist legislative tactics that a minority party can employ.

If, instead, a party holds minority status, i.e., if its share of seats falls below one half, its payoff nonetheless strictly increases in its share of seats. This reflects that a stronger minority receives more committee positions, and can more effectively derail the majority party's agenda by use of parliamentary procedures that privilege a more numerous minority.

Results. We obtain a unique equilibrium, in pure strategies, for all levels of the initial popularity imbalance between the parties. The equilibrium characterization can be indexed according to whether the initial imbalance is small, moderate, or large.

If the advantage is *small*, both parties locate at the policy preferred by the median voter in the median district. The reason is that—with a small imbalance—both parties remain competitive for majority status, encouraging them to compete aggressively to win the election, outright. This reflects that while winning isn't everything, it certainly matters a lot.

If the advantage is *moderate*, the disadvantaged party assesses that its prospect of winning an outright majority is distant enough that it is no longer worthwhile to single-mindedly pursue outright victory. Instead, it reverts to moving its policy platform away from the median voter in the median district, and in the direction of its core supporters. This choice may initially seem paradoxical, because this shift renders the party's prospects of winning even more distant. However, the shift also *increases* its anticipated share of seats in the relatively more likely event that the election consigns the party to minority status. The reason is that the party raises its attractiveness to its core supporters by differentiating itself ideologically from the advantaged party. With further increases in the valence imbalance, the disadvantaged party retreats further toward its base, as the prospect of losing the election rises.

By contrast, with a moderate advantage, the advantaged party maintains its strategy of targeting its platform at the policy preferred by the median voter in the median district. Pursuing its weaker opponent makes the advantaged party more palatable to its opponent's core supporters, but weakens the party's policy appeal amongst its own core supporters. Because the party's advantage is only moderate, it continues to face a meaningful risk of winning only a minority of seats. It therefore holds back from pursuit of its weaker opponent.

Finally, if the imbalance is *large*, the disadvantaged party continues its retreat by locating its platform even further from the centre and toward its core supporters. But now the advantaged party gives chase, moving its platform beyond the median voter in the median district and into the disadvantaged party's ideological territory. This is strategically appealing for three reasons. First, the party's strong advantage makes it less concerned about the risk of losing the election—i.e., failing to win a majority of seats; instead, its focus shifts to generating a comfortable seat advantage *conditional on winning majority status*. Second, it reduces the policy wedge between the parties, which heightens the salience of the advantaged party's net valence advantage, raising its appeal amongst all voters. Third, it capitalizes on the opportunity created by the disadvantaged party's increasingly extreme lurch to raid its more moderate supporters.

While platforms fully converge when initial imbalances are small, we show how changes in political primitives in the context of either a moderate or large initial imbalance either exacerbate or mitigate the disadvantaged party's incentive to revert to its base.

The disadvantaged party increasingly retreats to its base whenever its initial disadvantage grows. The reason is that the party is less competitive for a majority, and its priority increasingly shifts to defending its anticipated minority. The disadvantaged party also retreats further whenever the parties' relative popularity becomes more volatile. More volatile preferences raise the prospect of a large swing on election day. One the one hand, this popularity swing may favor the disadvantaged party. On the other hand, if the popularity swing favors the advantaged party, the weaker party's core districts will be the front lines of the electoral contest. Under the assumption that a party places a premium on retaining its seats in the event of a legislative minority, the heightened risk of a swing in favor of its stronger opponent weighs most heavily on the disadvantaged party, encouraging it to adopt a more defensive strategy. Finally, the disadvantaged party further retreats when the partisan loyalty of its traditional supporters declines, since these voters are less easily taken for granted.

Because the advantaged party maintains its position in the centre, these changes trigger increased platform polarization. Once the imbalance is large enough, however, further increases in the popularity imbalance induce *both* parties to move toward the disadvantaged party's core supporters, with the stronger party advancing more quickly.

Contribution. Our premises and results contrast starkly with existing models of party positioning in elections. In the framework developed by Calvert (1985) and Wittman (1983), policy-motivated parties face uncertainty about the preferences of the electorate—specifically, the median voter's most preferred platform. In equilibrium, if a party becomes *more* advantaged, i.e., if the expected location of the median voter moves toward its most-preferred policy, *both* parties advance toward the advantaged party's ideal policy.

Our framework predicts the opposite. In particular, consistent with the campaigns of Tony Blair or Theresa May, when the advantaged party's net valence advantage is large enough, an increased electoral imbalance encourages both parties to move in the direction of the *disadvantaged* party's base. The advantaged party invades its opponent's ideological turf to pursue a strong majority, while the disadvantaged party retreats to its base to try to rally its core supporters. The first implication seems to describe well Tony Blair's electoral strategy in 1997 to transition his party to *New Labour*, at a time when the party enjoyed a clear preference advantage amongst voters. This advantage was so strong that even *The Sun* newspaper, which had supported the Conservatives in every election in the previous twenty years, endorsed Labour, condemning the Conservatives as "tired, divided and rudderless".[8] Our prediction also characterizes May's efforts to win over moderate Labour supporters in 2017. The second implication closely corresponds to Bogdanor's summary of the Conservative lurch to the right from 2001 to 2010, in which "three successive Conservative leaders... responded to defeat by seeking to mobilize the Tory 'core' vote".[9]

While our analysis focuses on legislative elections, our finding that an advantaged party advances on its weaker opponent—rather than cater to its own core voters—extends to the candidate-centered elections that are the focus of the Calvert-Wittman framework. Like both Bill Clinton and Tony Blair, Emanuel Macron—at one time a Socialist party minister—leveraged a large popularity advantage in his 2017 presidential campaign to adopt a '*Third Way*' manifesto that included reductions in corporate taxes and public spending, increased defense spending, and allowing firms to negotiate additional working hours beyond the country's 35-hour work week.

Groseclose (2001) augments the Calvert-Wittman framework by introducing a deterministic valence advantage for one party. However, Groseclose does not establish existence or uniqueness of an equilibrium. Moreover, his main theoretical results are limited to a context with a small valence advantage (specifically, moving from no advantage to an arbitrarily small advantage),

[8] See Butler and Kavanagh (1997).

[9] "The Conservative Party: From Thatcher to Cameron", *New Statesman*.

and his framework features a single (median) voter—precluding the question of whom to target that drives our framework. The predictions that he derives when an equilibrium exists differ substantially from our office-motivated context. For example, his framework predicts that if the advantaged party's net valence advantage is very large, then it always adopts more extreme policy positions in the direction of its ideal policy.

Our framework predicts the opposite: the advantaged candidate responds to a large advantage not by adopting extreme positions favored by its own core supporters, but instead by targeting its opponent's moderate supporters. Our analysis reconciles campaigning by the Australian Labor Party (ALP) during the first of several election victories, in 1983. The election came at a time of high unemployment, high inflation, industrial unrest and a prime minister (Malcolm Fraser) who had only recently survived an internal leadership challenge. The incumbent government was so unpopular that former ALP leader Bill Hayden quipped that "*a drover's dog could lead the Labor Party to victory, the way the country is and the way the opinion polls are showing up...*".[10] During the election and in government, the party—whose constitution still declares it to have "the objective of the democratic socialization of industry, production, distribution and exchange"—promoted tariff reductions, tax reforms, limits on union activity, transitioning from centralized bargaining to enterprise bargaining, privatization of government enterprises, and banking deregulation.

Aragones and Palfrey (2002) and Hummel (2010) characterize equilibria in a Downsian setup with purely office-motivated candidates and a deterministic net valence advantage. As in our setting, the advantaged candidate benefits by raising the salience of this valence advantage. This encourages the advantaged candidate to mimic the disadvantaged candidate, and the disadvantaged candidate to try to differentiate itself from the advantaged party. These "chase-and-evade" incentives yield equilibria in mixed strategies. Both papers are limited to characterizing a particular mixed strategy equilibrium, under the restriction either of a small (Aragones and Palfrey, 2002) or large (Hummel, 2010) initial valence advantage.[11]

In contrast with both Groseclose (2001) and Aragones and Palfrey (2002), our framework generates a unique pure strategy equilibrium. This stems from our distinct approach to representing the parties' electoral uncertainty. Existing work assumes that parties (or candidates) are *certain*

[10] "Statements from Hayden Bowen, Hawke". *The Canberra Times*, 4 February 1983.

[11] Aragones and Palfrey (2005) endow candidates with private information about their preferences, obtaining a pure strategy equilibrium. As the relative weights placed by candidates on policy outcomes as opposed to office rents converge to zero, the distribution over candidates' policies approaches the mixed strategy equilibrium in Aragones and Palfrey (2002).

about the net valence advantage on election day, but *uncertain* about voters' policy preferences. In particular, they are uncertain about the median voter's preferred policy. To see why this generates equilibria in mixed strategies in an office-motivated setting, notice that whenever the advantaged and disadvantaged parties adopt the same platform, the party with a known net valence advantage wins the support of *every* voter, *regardless* of policy preferences. As a result, chase-and-evade incentives overwhelm all other considerations.

In our setting, by contrast, parties face no uncertainty about voters' policy preferences, but are uncertain about whether an initially favored party's popularity advantage will increase, narrow, or even reverse during the election. So, even if the parties locate at the same platform, each has a chance of winning the election. While chase-and-evade incentives are present in our framework, they are tempered by incentives to target specific voters—such as the median voter, or a party's core voters—depending on the party's forecast of its popularity, and its relative value of winning additional seats below, at and above the majority threshold. For example, if parties only care winning a majority of seats, each locates at the median voter's ideal policy, regardless of the popularity imbalance.

Our framework offers an explanation for why parties may instrumentally choose relatively extreme policies. In Eguia and Giovannoni (2019), a party that is sufficiently disadvantaged today may give up on a mainstream policy, and instead invest in an extreme policy; it does so not to increase its office-motivated payoffs today, but instead to gamble on a shock to voters' preferences in a subsequent election. Our explanation emphasizes that the instrumental adoption of extreme policies in the face of a likely election defeat arises not only via dynamic office-holding incentives, but also via static office-holding incentives that emphasize the value of a strong minority position.

Our multi-district framework is closest to Callander (2005), in which two parties simultaneously choose national platforms, facing entry by local candidates, generating equilibrium platforms that differ greatly from ours. Other authors—for example, Austen-Smith (1984), Kittsteiner and Eyster (2007), and Krasa and Polborn (2018)—study multi-district competition in which party platforms are an aggregate of decentralized choices by local candidates. Our framework, like Callander's, instead reflects a context in which voters predominantly assess their view of the party on the basis of its national platform.[12]

[12] In Polborn and Snyder (2017), each party's platform is assumed to reflect the preferences of its median elected legislative candidate. Thus, in contrast to our approach, it is determined after the election, after net valence shocks are realized.

2. Model

Preliminaries. Two parties, L and R, simultaneously choose campaign platforms, z_L and z_R, prior to an election. The policy space is the one-dimensional continuum, $\mathbb{R}$. Competition involves multiple districts, with the winner of each individual district determined by a plurality rule. Each district features a continuum of voters, and each voter i is indexed by his or her preferred policy, x_i. There are a continuum of districts: each district is indexed by its median voter's preferred policy m, and district medians are uniformly distributed on the interval $[-1, 1]$.[13]

Voter Payoffs. If party L implements platform z_L, a voter i with preferred policy x_i derives payoff

$$u(x_i, z_L) = -|z_L - x_i| - \theta x_i. \tag{1}$$

If, instead, party R implements its platform z_R, the voter derives the payoff

$$u(x_i, z_R) = -|z_R - x_i| + \rho. \tag{2}$$

Here, ρ is a preference shock, uniformly distributed on the interval $[\rho_0 - \psi, \rho_0 + \psi]$. It captures developments that unfold over the course of an election campaign—right up to election day—including performances by party leaders in public debates or town hall meetings, or scandal revelations. If the legislative election coincides with a presidential election, ρ could also capture evaluations of a party arising from its presidential candidate's campaign. Its mean ρ_0 could reflect voters' relative assessment of the parties at the outset of a campaign—for example, evaluations of its leadership, or perceptions of competence that are inherited from a party's previous spell in government. Without loss of generality, we assume that $\rho_0 \geq 0$, so that R is the "advantaged" party.

The policy-related part of voters' preferences has two distinct components. The first component is a linear policy loss that increases with the distance between the party's policy platform and the voter's preferred policy. The second component, which states that voter i derives an additional net value $-\theta x_i$ from party L, has multiple interpretations. For example, it could reflect a fixed party policy position on another dimension of policy conflict (e.g., Xefteris, 2017). Our running interpretation is that $-\theta x_i$ reflects *partisanship*, i.e., a voter's "early-socialized, enduring, affective... identification with a specific political party" (Dalton, 2016, 1) that transcends short-

[13] Under Assumption 1, below, the distribution of voters' ideal policies within each district plays no role in our analysis, beyond the location of the median voter's ideal policy in that district.

term policy platforms that parties adopt from one election to the next, and which intensifies in a voter's ideological extremism. Our approach captures the fact that the average voter in Alabama perceives a different net value from a Republican versus a Democrat than the average voter in Rhode Island—above and beyond any evaluation of the parties' policies.

In sum, a voter with preferred policy x_i prefers party L if and only if:

$$\Delta(x_i; z_L, z_R, \rho) \equiv \underbrace{|z_R - x_i| - |z_L - x_i|}_{\text{Platform gap}} - \underbrace{\theta x_i}_{\text{Partisan gap}} - \underbrace{\rho}_{\text{Valence gap}} \geq 0 \tag{3}$$

Party Payoffs. Let $d_P \in [0, 1]$ denote the share of districts won by party $P \in \{L, R\}$, and let $M_P = \mathbb{I}[d_P > 1/2]$ denote the event that party P wins a majority of districts. Party P's payoff is

$$u_P(d_P) = M_P[r + \beta(d_P - 1/2)] + (1 - M_P)\alpha d_P. \tag{4}$$

A party receives a fixed payoff of $r > 0$ if it wins the election, i.e., if $d_P > 1/2$. Higher values of r reflect the majoritarian organization of a legislature: winning a majority gives a party agenda-setting authority, and control over committee assignments and leadership. And, in a parliamentary democracy, winning a majority yields formal control over the executive branch.

Parties also value winning additional seats both below and above the majoritarian threshold. Even if a party fails to achieve a majority, i.e., if $d_P < 1/2$, it still gains from winning more seats. And, if a party achieves a majority, it values increasing its share of seats above the majority threshold. To capture this idea, we let $\alpha > 0$ denote the marginal value of winning additional districts that nonetheless keep a party's total share of districts less than a majority; and we let $\beta > 0$ denote the marginal value of winning additional districts above and beyond the majority threshold of one half. This piecewise linear formulation facilitates tractable solutions, and, as we show in the Appendix, may be viewed as an approximation of more sophisticated payoff schedules.

Additional Assumptions. We impose two assumptions; the first assumption focuses on party preferences, while the second assumption relates to voters' preferences.

Assumption 1: $\alpha \geq \beta$ and $r \geq \frac{1}{2}\left(\alpha + \frac{\psi}{\theta}(\alpha - \beta)\right)$.

The first preference restriction $\alpha \geq \beta$ says that the marginal value of additional seats to a minority party exceeds that for a majority party, above and beyond the gains from winning majority status. We later describe properties of equilibrium policy platforms under the alternative assumption that $\beta > \alpha$. As we discuss below, we view $\alpha \geq \beta$ as inherently more plausible.

For majority status to convey a benefit, it *must* be that $r > \alpha/2$. The second preference restriction says that parties sufficiently value winning majority status. We make this assumption solely to streamline exposition, and relax it in Section 4.

Assumption 2: $\rho_0 - \psi < -1$, and $\theta > \rho_0 + \psi + 1$.

Recall that the preference shock ρ is uniformly distributed on $U[\rho_0 - \psi, \rho_0 + \psi]$, and that district medians are uniformly distributed on $[-1, 1]$. The first part of Assumption 2 ensures that, in equilibrium, each party wins with positive probability. The second part ensures that, in equilibrium, each party wins a strictly positive fraction of its core districts, thereby ensuring interior solutions.

Timing. The interaction proceeds as follows.

1. The parties simultaneously select platforms z_L and z_R.

2. The preference shock ρ is realized and observed by all agents.

3. Each voter chooses to vote for one of the two parties.

4. The party that wins a majority of districts implements its promised platform, and payoffs are realized.

Discussion. In our framework, parties know voters' policy preferences, but face uncertainty about whether party R's initial relative popularity advantage will increase, narrow, or even reverse during the election. Both Aragones and Palfrey (2002) and Groseclose (2001) adopt the opposite perspective that at the time parties choose platforms, they perfectly forecast their relative popularity on election day, but face uncertainty about voters' policy preferences—specifically, the median voter's preferred policy.

Our approach reflects the view that an individual's perceptions of a party or party leader's competence, honesty and charisma—arising from campaign rallies, public debates and town halls, and (social) media coverage—fluctuate far more over the course of a single election cycle than his or her views on policy issues such as taxation, health care or gay marriage. They therefore constitute the first-order source of uncertainty facing parties in an election. For example, while Theresa May started the 2017 election with a 39 percentage point popularity advantage, her popularity fluctuated throughout the campaign, and by polling day her margin had diminished to

10 percentage points.[14] In addition to its substantive motivation, our modeling framework yields a unique equilibrium in pure strategies, facilitating our goal of describing strategic behavior in real-world election campaigns.

Beyond the value that parties derive from winning a majority of districts, r, we assume that they derive an incremental value α from an additional seat below the majority threshold, and β from an additional seat in excess of a majority. We will verify that, under Assumption 2, districts can be ordered according to the location of their medians. Thus, the assumption that $\alpha \geq \beta$ implies that a party places a premium on successfully defending one of its core districts, versus winning one of its opponent's. We view this as natural for several reasons.

First, a party's incumbent legislators will typically be drawn disproportionately from its core districts. Defending these seats is likely to matter more than winning new seats (as in, for example, Snyder Jr, 1994). This would be true if existing incumbents are in a position to directly influence the party's platform, and "naturally value the seats the party already holds more than new ones it might win" (Cox and Katz, 2002, 36). Party leaders can also be expected to prioritize trusted friends and allies over bringing in freshmen whose loyalties are untested, and whose reputations are undeveloped.[15] Incumbents also possess seniority, which contributes to their legislative efficacy through formal rules as well as informally via their expertise and experience (Miquel and Snyder Jr, 2006).

Second, the assumption captures dynamic electoral considerations that extend beyond the immediate election cycle. While an outsized majority may build momentum in future elections, or in a party's other electoral arenas, a party whose showing is so disastrous that it cannot successfully retain its core constituencies can be expected to face a heightened vulnerability to splits, internal leadership challenges and factional conflict. This view is corroborated by Peabody (1967), who finds that: "Strong victories promote good will and generally reflect to the benefit of party leaders. Conversely, defeat results in pessimism, hostility and a search for scapegoats. If the net losses are particularly severe, as many as thirty to fifty seats, then the possibilities of minority leadership change through revolt are great enhanced." A severely weakened party may struggle to attract high quality candidates to contest subsequent elections, or even face the entry of rivalrous parties.

In a Supplemental Appendix, we provide explicit policy-motivated justifications, using alternative primitives. We prove that the property that parties care more about winning their core

[14] "Opinion Polling for the United Kingdom General Election, 2017", https://goo.gl/7mTYQW.

[15] We are grateful to an anonymous referee, who proposed this rationale.

constituencies emerges naturally when parties internalize the policy losses of their constituents—i.e., of the voters in districts that elect the party's candidates. This approach was first developed in Caplin and Nalebuff (1997), and subsequently applied to party formation and electoral competition by Baron (1993) and Roemer (2001). It also emerges when parties believe that larger electoral margins allow the winning party to more aggressively pursue its policy goals without making concessions to the losing party—as in Alesina and Rosenthal (1996).

3. Results

Preliminary Results. We begin by identifying the share of districts won by each party for any platform pair (z_L, z_R) and net valence advantage ρ—and thus each party's probability of winning the election. Under Assumption 2, preferences are single-peaked. Therefore, there is a unique voter who is indifferent between the candidates: there is some ideal policy $x^*(z_L, z_R, \rho)$ such that a voter weakly prefers party L if and only if her ideal policy lies weakly to the left of x^*. The voter's ideal policy solves $\Delta(x^*; z_L, z_R, \rho) = 0$, defined in expression (3). This implies that party L wins a district with median m if and only if $x^*(z_L, z_R, \rho) \geq m$.[16] Using the fact that district medians are uniformly distributed on $[-1, 1]$, party L's share of districts is given by

$$d_L = \frac{1}{2} + \frac{x^*(z_L, z_R, \rho)}{2}.$$

Party L wins the election if and only if $x^* \geq 0$, i.e., if and only if it is preferred by the median voter in the median district, who has ideal policy zero. We have:

$$x^*(z_L, z_R, \rho) \geq 0 \iff |z_R| - |z_L| \geq \rho.$$

Henceforth, we call the median voter in the median district the *median voter*. Substituting into the party payoff function in equation (4) yields party L's expected payoff:

$$\pi_L(z_L, z_R) = \frac{1}{2\psi} \int_{\rho_0 - \psi}^{|z_R| - |z_L|} \left(r + \beta \frac{x^*(z_L, z_R, \rho)}{2} \right) d\rho + \frac{1}{2\psi} \int_{|z_R| - |z_L|}^{\rho_0 + \psi} \alpha \left(\frac{1}{2} + \frac{x^*(z_L, z_R, \rho)}{2} \right) d\rho. \tag{5}$$

[16] We adopt the convention that if a voter is indifferent between the parties, she votes for party L. Since the set of indifferent voters has measure zero for any shock realization, this convention has no bearing on our results.

Party R's corresponding expected payoff is:

$$\pi_R(z_L, z_R) = \frac{1}{2\psi}\int_{\rho_0-\psi}^{|z_R|-|z_L|} \alpha\left(\frac{1}{2} - \frac{x^*(z_L, z_R, \rho)}{2}\right)d\rho + \frac{1}{2\psi}\int_{|z_R|-|z_L|}^{\rho_0+\psi}\left(r - \beta\frac{x^*(z_L, z_R, \rho)}{2}\right)d\rho. \quad (6)$$

Main Results. We now characterize equilibrium platforms choices and highlight how they depend on R's initial advantage (ρ_0), uncertainty about how preferences will evolve over the course of the election (i.e., uncertainty about ρ, captured by ψ), the relative value of seats to the minority party (α) versus the majority (β), and the value of winning a legislative majority (r). We first establish that our framework produces a unique equilibrium, in pure strategies.

Theorem 1. *Under Assumptions 1-2, there exists a unique pure strategy equilibrium.*

To understand why a pure strategy equilibrium obtains, recall that there exists a unique voter who is indifferent between the two parties, whose preferred policy $x^*(z_L, z_R, \rho)$ satisfies:

$$\Delta(x^*; z_L, z_R, \rho) = |z_R - x^*| - |z_L - x^*| - \theta x^* - \rho = 0. \quad (7)$$

We refer to this indifferent agent as the 'swing voter'. While parties know that the median district's *median voter* has preferred policy zero, they face uncertainty about the identity of the *swing voter* due to the valence shock, ρ.

Party L wins districts whose median voter's preferred policy lies to the left of the swing voter's preferred policy, $x^*(z_L, z_R, \rho)$. It therefore wins the support of the median voter, and thus a majority of seats, if $x^*(z_L, z_R, \rho) \geq 0$. In that event, the swing voter is a median voter in one of party R's core districts. Conversely, R wins a majority if $x^*(z_L, z_R, \rho) < 0$, in which case the swing voter is a median voter in one of L's core districts.

In Figure 1, the black line identifies the swing voter's preferred policy $x^*(0, 0, \rho)$ when the parties both locate at the median voter's preferred policy of zero. A district with median voter to the left of $x^*(0, 0, \rho)$ votes for L, and a district with median voter to the right of $x^*(0, 0, \rho)$ votes for R. Expression (3) reveals that party L secures the support of the median voter—and therefore wins the election—if and only if the shock resolves in its favor, i.e., if and only if $\rho \leq 0$.

What are the consequences of a shift by party L away from the median voter's preferred policy to the policy $a < 0$, given that R locates at zero? By moving away from the median voter, L differentiates itself from its stronger opponent, generating a new swing voter $x^*(a, 0, \rho)$ for each

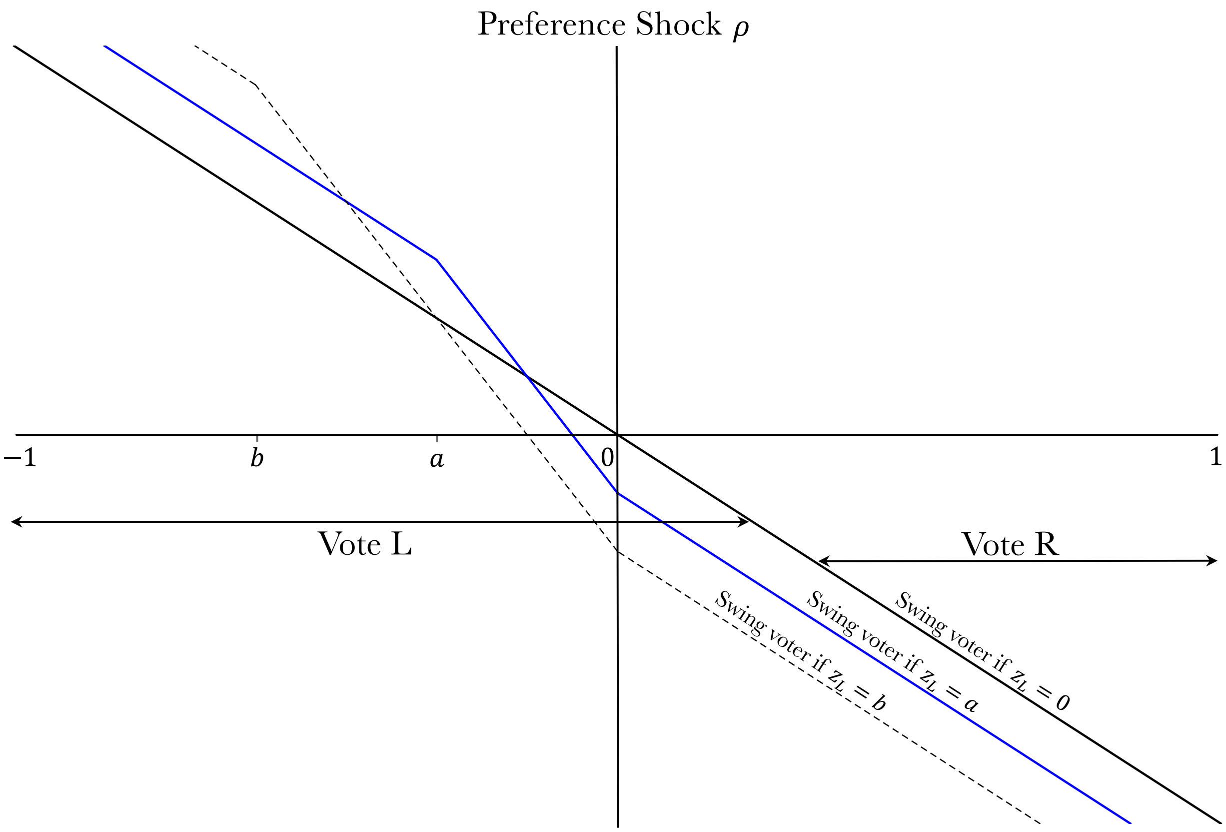

Figure 1 – Possible locations for the swing voter, depending on the valence shock ρ and party L's platform z_L, when party R locates at the median voter, i.e., $z_R = 0$.

realization of the valence shock ρ.

This new location is highlighted by the blue line in Figure 1.[17] L's policy differentiation makes it relatively *more* attractive to voters with ideal policies to the left of $\frac{a+0}{2}$. If $\rho \geq -\frac{2a}{\theta}$, i.e., if the net valence shock resolves sufficiently strongly in favor of party R—L's relocation moves the swing voter further to the *right*. Thus, despite losing the election, L nonetheless increases its share of legislative seats in the event that it is consigned to minority status.

However, L's policy differentiation also makes it relatively *less* attractive to voters with ideal policies to the right of $\frac{a+0}{2}$. If, despite $\rho_0 > 0$, the preference shock favors R less strongly, i.e., if $\rho < -\frac{2a}{\theta}$, then L's relocation moves the swing voter further to the *left*, reducing the share of districts that it wins in the event that party L either wins majority status, or loses a close election. Moreover, notice that L's move lowers its prospect of winning a majority, because the median voter now strictly prefers R on policy grounds.

[17] We are indebted to an anonymous referee, who proposed this figure.

Were L to locate at an even more extreme policy—such as b in Figure 1—then it would further buttress its minority seat share in the event of a very large swing ρ in favor of party R, i.e., when $\rho > -\frac{2b}{\theta}$. However, locating more extremely would also further cede support amongst its more moderate core districts, as well as R's core districts, and further diminish its prospects of a majority.

The parties choose platforms before they learn the net popularity advantage favoring party R, so they do not know the identity of the swing voter when they choose platforms. Thus, L's decision to target the median voter, or instead to abandon her in favor of its core supporters, turns on the prospect of a popularity swing in favor of party R in the run-up to Election Day. This prospect is determined by ρ_0, the mean of the net valence shock in favor of R. If ρ_0 is large, L anticipates that it is very unlikely to win a majority, and that the swing voter will be a median in one of its core districts. This encourages party L to move away from its stronger opponent in order to avoid an electoral rout.

Similar considerations guide party R. Suppose, for example, that L locates at a in Figure 1, and R is choosing between the median voter's preferred policy, versus co-locating at L's platform. In the event of a sufficiently favorable popularity shock of $\rho > -\frac{2a}{\theta}$, the decision to pursue L moves the swing voter to the *left*, further increasing R's share of districts. If, instead, $\rho < -\frac{2a}{\theta}$, then chasing L moves the swing voter to the *right*, lowering R's prospect of winning a majority *and* its share of districts in the event of an adverse popularity shock.

In sum: the parties' trade-offs depend on their forecast about the relative Election Day popularity of the parties. Together with platform choices, this popularity determines the likely location of the swing voter, and thus the front lines of the electoral battle. The next three propositions establish that how these trade-offs resolve, and thus the characterization of the unique equilibrium, can be indexed according to whether the initial imbalance is small, intermediate, or large.

Proposition 1. *If party R's advantage is **small** in the sense that*

$$0 \leq \rho_0 \leq \frac{\theta(2r - \alpha) - (\alpha - \beta)\psi}{\alpha + \beta} \equiv \underline{\rho}_0,$$

then both parties locate at the ideal policy of the median voter in the median district:

$$z_L^*(\rho_0) = 0, \quad z_R^*(\rho_0) = 0.$$

A party wins a majority of districts if and only if it is preferred by the median voter in the median district, with ideal policy zero. When the parties are initially balanced, i.e., when ρ_0 is zero, each party is equally competitive for a majority. Because parties place a premium r on majority status, each party aggressively pursues an outright victory.[18]

Starting from a position of initial symmetry, i.e., starting from $\rho_0 = 0$, increases in ρ_0 reduce L's chances of winning, but do *not* alter the policy platform that maximizes this probability. Thus—and to an extent that is proportional to the value r of winning majority status—L's electoral strategy continues to target a legislative majority by way of a centrist policy platform even as its prospects of winning deteriorate. Notice that as $(\alpha - \beta)\psi$ increases—implying a greater relative concern for incremental minority versus majority seat shares, $\alpha - \beta$, combined with the greater electoral risk encapsulated in ψ—the upper bound of initial imbalance for which the disadvantaged party wants to compete directly with the advantaged party ($\underline{\rho}_0$) *falls*.

When the imbalance between the parties is large enough, L no longer prefers unmitigated competition with R for outright victory.

Proposition 2. *If party R's advantage is* ***intermediate*** *in the sense that*

$$\underline{\rho}_0 \leq \rho_0 < \underline{\rho}_0 + (\alpha - \beta)\psi \frac{2\theta\alpha + \alpha - \beta}{(\alpha + \beta)(\alpha\theta + \alpha - \beta)} \equiv \overline{\rho}_0,$$

then party L retreats to its base,

$$z_L^*(\rho_0) = \frac{\theta(2r - \alpha) - \alpha(\rho_0 + \psi) + \beta(\psi - \rho_0)}{\alpha - \beta + 2\alpha\theta} < 0,$$

but R still locates at the ideal policy of the median voter in the median district, choosing $z_R^*(\rho_0) = 0$.

When the electoral imbalance in favor of party R surpasses an initial threshold $\underline{\rho}_0 > 0$, the disadvantaged party L's competitive environment shifts by enough to merit a change in electoral strategy. A sufficiently high ρ_0 implies that the prospect of winning a majority—even when targeting the *median* voter, directly—becomes a distant prospect. In essence: the party's core vote is likely to become its swing vote.

Anticipating a significant prospect that the swing voter will be a median in one of its core districts, L's best electoral strategy reverts to galvanizing its base by selecting a platform $z_L(\rho_0) < 0$.

[18] Under Assumption 1, $r > \frac{\alpha}{2} + \frac{\psi}{2\theta}(\alpha - \beta)$, which implies that $\underline{\rho}_0 > 0$.

By distancing itself from party R, it offers a meaningful programmatic alternative to R's centrist platform: policy differentiation partly mitigates L's valence disadvantage amongst voters who value more left-wing policies. While retreating from the political centre further reduces L's prospect of winning a majority of districts, $\rho_0 > \underline{\rho}_0$ implies that party L no longer finds it worthwhile to target an outright victory. That is, acknowledging that it is very likely to hold minority status, L's priority reverts from solely pursuing a majority to instead balancing this objective against the need to secure the most advantageous minority share of seats possible.

By contrast, the same primitives encourage party R to maintain its hold on the ideological centre-ground. Its prospect of winning the election is maximized by selecting the policy preferred by the median voter in the median district. Party R *could* chase L into its own ideological turf, in order to increase its seat advantage conditional on holding a majority. However, its initial electoral advantage is still small enough ($\rho_0 < \overline{\rho}_0$) that it does not want to risk its prospect of winning. Chasing disadvantaged party L makes advantaged party R more palatable to moderate left-wing districts, but harms R's standing with both the median voter and R's own core voters. And, in the event that R fails to win a majority, the swing voter will be one of R's core supporters. To the extent that R values insuring itself against an adverse popularity shock, it prefers not to give chase.

To see this point more clearly, notice that the size of the interval $[\underline{\rho}_0, \overline{\rho}_0]$ is proportional to $(\alpha - \beta)\psi$, and the interval is empty when $\alpha = \beta$. This reflects the advantaged party's incentive to hold back versus give chase. As it advances on its retreating opponent by shifting its platform to the left:

1. it raises its appeal amongst its opponent's core supporters and therefore—conditional on winning—raises its share of districts by moving the swing voter's preferred policy to the *right*. It values these districts at rate β; but,

2. it lowers its appeal amongst its own core supporters, and therefore—-conditional on losing—lowers its share of districts by shifting the swing voter's preferred policy to the *left*. It values these districts at rate $\alpha \geq \beta$.

As the wedge $\alpha - \beta$ increases—amplified by the magnitude of the election risk ψ—the advantaged party increasingly prefers to 'play it safe', holding back even as its initial advantage increases.

These channels generate natural effects of primitives on party L's platform, and thus the degree of policy divergence between the parties.

Corollary 1. *When Party R's advantage is immediate, Party L increasingly retreats to its base—and thus platform divergence* increases*—whenever*

1. *its initial disadvantage ρ_0* increases*,*
2. *the marginal value of minority seats α* increases*, or*
3. *uncertainty about voter preferences ψ* rises*.*

Conversely, L increasingly targets the median voter when

1. *the value of majority status r* increases*, or*
2. *party loyalty θ* increases*.*

If party loyalty θ amongst more ideologically polarized voters rises, party L grows less worried about losing support amongst its core districts—the rate at which higher valence shocks ρ_1 shift the identity of the swing voter further into its core districts slows. This encourages the party to target centrist districts whose support is crucial for the party to win.

When parties anticipate a more volatile electorate via higher ψ, then for any pair of platforms, there is a heightened prospect of a large post-election imbalance between the majority and minority party via more extreme realizations of $\rho \sim U[\rho_0 - \psi, \rho_0 + \psi]$. If the disadvantaged party competes more aggressively by moving its platform toward its opponent, it could win more seats in the event of a majority, but it may lose even more seats in the event of an unfavorable realization that consigns the party to minority status. Here, with $\alpha > \beta$, a concern for core districts encourages the weaker party to hasten its retreat. Thus, our framework predicts that platform polarization is greater when party loyalty is weaker (θ smaller) and attitudes toward the parties, or party leaders, are more volatile.

Finally, if the imbalance between the parties is very large, then party R becomes so emboldened by its initial advantage over L that it abandons the pursuit of mere victory, and instead chases its weaker opponent in an effort to plunder its moderate supporters.

Proposition 3. *If party R's advantage is **large**, i.e., $\rho_0 > \overline{\rho}_0$, then party L retreats by more to its base:*

$$z_L^*(\rho_0) = \frac{(\alpha - \beta + \beta\theta)(\theta(2r - \alpha) - (\alpha + \beta)\rho_0) - \beta\theta\psi(\alpha - \beta)}{\theta(\alpha^2 - \beta^2 + 2\alpha\beta\theta)} < 0, \tag{8}$$

and party R advances toward party L's base:

$$z_R^*(\rho_0) = z_L^*(\rho_0) + (\alpha - \beta)\frac{(\alpha + \beta)(\psi - \rho_0) + \theta(2r - \alpha)}{\alpha^2 - \beta^2 + 2\alpha\beta\theta} > z_L^*(\rho_0). \tag{9}$$

When the electoral imbalance in favor of party R is very large, party L overwhelmingly focuses on consolidating support amongst its base—the most likely location of the swing voter, and thus the most likely frontline of the political battle. In turn, party R also advances into L's ideological territory to win over centre-left districts that are increasingly ill-served by the more extreme L party. By reducing the policy differentiation between the parties, R intensifies the salience of its comparative valence advantage in the eyes of the likely swing voter, further increasing its support. If $\alpha = \beta$, then the parties locate at the same platform, reflecting the unmitigated chase-and-evade logic of Aragones and Palfrey (2002). As $\alpha - \beta$ increases, the advantaged party chases less quickly, reflecting a greater concern for an adverse valence shock that places the swing voter in one of its own core districts. Nonetheless, a sufficiently large advantage ($\rho_0 > \overline{\rho}_0$) makes party R less concerned about the risk of losing the election, and instead more focused on generating the largest possible legislative majority when it wins.

Corollary 2 summarizes the effect of primitives on the parties' platforms, and their consequences for platform divergence, when one party has a large valence advantage.

Corollary 2. *As R's initial advantage $\rho_0 \geq \overline{\rho}_0$ increases, both party L and party R move toward L's base, and platform divergence* decreases*.*

As party L grows more disadvantaged, it faces even greater incentives to target its base; by differentiating itself further from the advantaged party, it increases its appeal to its core supporters, consolidating its minority position. However, party R is also further emboldened to advance into its opponent's home turf. Its incentives are two-fold; a higher ρ_0 strengthens R's incentives to chase the increasingly weakened L *and*—independently—it wants to use its platform to turn centrist districts that L has abandoned, in pursuit of an outsized majority. The net effect is that platforms further converge, with the speed of convergence increasing in β, the marginal value of seats conditional on majority status.

Corollary 2 highlights that party R's platform moves to the left faster than party L's, so that the net effect is to reduce policy differentiation between the parties. Conversely, if ρ_0 *decreases*, both parties move their platforms toward the median voter in the median district, but party L moves more slowly than party R, *increasing* the degree of platform divergence.

Other changes in primitives may lead to different effects for the advantaged versus disadvantaged party, and may hinge on other features of the political environment.

Corollary 3. *When the marginal value of minority seats, α, increases, party L increasingly retreats to its base. By contrast, when α increases, party R moves towards L's base.*

As α rises, party L grows more concerned about not losing the election too badly, so it increasingly targets its core supporters. Party R, however, faces two conflicting incentives. First, as α increases, it too has a stronger incentive to consolidate its core support by reverting to the right, i.e., in the direction of its base. However, as party L increasingly moves toward its base, party R also faces a stronger incentive to advance toward party L's platform in order to reduce the policy differentiation between parties, and therefore heighten its comparative valence advantage. Because party R's initial advantage is large, it resolves in favor of chasing party L even more aggressively. The reason is that with a large advantage, the stronger party worries less about pleasing its core supporters, and instead prefers to reduce its platform differentiation with party L, in order to further press its advantage.

Corollary 4. *As the value of majority status r increases, both party L and party R revert toward the ideal policy of the median voter in the median district, but platform divergence* increases*.*

A party wins a majority if and only if it is preferred by the median voter. A higher value r of majority status encourages both parties to target this voter. Corollary 4 highlights that party R's platform moves faster than party L's. To see why, recall that party L remains at a competitive disadvantage; moving toward the centre raises its attractiveness to moderate voters, but dampens its relative appeal amongst its base. This represents a trade-off for party L. For party R, however, moving back toward the centre raises its appeal to both centrists and its core supporters.

Because both trade-offs are complementary to party R, but opposing for party L, the net effect is to increase platform divergence: L reluctantly abandons its base, while R's increased desire to win implies that its platform choice is governed less by the incentive to chase L, and more by the incentive to maximize its appeal to the median voter in a bid for outright victory.

Corollary 5. *As electoral volatility ψ increases, both party L and party R revert toward their respective core supporters, and platform divergence* increases*.*

When there is a large initial wedge in the parties' initial strength, more uncertainty *always* raises platform divergence. This reflects that both parties grow more concerned with insuring themselves against adverse popularity shocks by consolidating their core supporters. Greater volatility raises the prospect that the election will result in a larger imbalance in favor of one of the parties. Because $\alpha - \beta > 0$, each party resolves in favor of buttressing its seat share in the event that it is consigned to minority status.

4. Discussion

We pause to discuss some of our key assumptions, as well as alternative interpretations of our framework and results.

What about $\beta > \alpha$? In the less plausible context in which $\beta > \alpha$, the parties fully converge on the ideal policy of the median voter in the median district when R's advantage is not too large—as in our benchmark setting. However, with a sufficiently large initial advantage, the shape of preferences may induce parties to engage in implausible risk-taking behavior, generating platform separation in which party R gambles on a left-wing platform, leaving the centre-ground to its weaker opponent. Our benchmark presentation of $\alpha \geq \beta$, by contrast, reflects the more empirically relevant scenario in which relatively strong parties may court their opponent's core supporters (as detailed in Proposition 3), but never to the extent that their own core voters are better served by their opponent.

Core Supporters. Beyond policy- and valence-related considerations, we assume that a voter with preferred policy x_i derives a value from party L of $-\theta x_i$. The most natural interpretation is that a voter with preferred policy $x_i < 0$ is pre-disposed to prefer party L—for example, because the voter has a greater affinity for the policies that the party tends to offer. However, because ρ_0 is a primitive, Propositions 2 and 3 might appear to suggest that for some ρ_0, some voters with ideal policies $x_i < 0$ *always* prefer party R's policies.

To justify our interpretation, one could consider an initial stage in which ρ_0 is randomly drawn from a symmetric mean-zero distribution; the parties observe the realization of ρ_0, and then the interaction proceeds as in our benchmark model. In that extended model, it is indeed true that party L more often chooses policies that are preferred by voter types $x_i < 0$, while R more often chooses policies that are preferred by voter types $x_i > 0$.[19] The same logic also implies that a party's incumbent candidates are more likely to be drawn from its core districts, justifying the notion that protecting incumbents would rationalize a relative premium for maintaining a core district.

Returning to Base. Assumption 1 says that parties put a large premium $r \geq \frac{1}{2}\left(\alpha + \frac{\psi}{\theta}(\alpha - \beta)\right)$ on winning a majority. This reflects the majoritarian operation of legislative organization: the win-

[19] We are grateful to the Editor, who suggested this interpretation.

ning party enjoys control over the legislative timetable and the appointment of key positions such as committee chairs; in parliamentary democracies, the majority-winning party is also awarded control of the executive branch.

Parties may nonetheless face an election in which relative party popularity is especially volatile (high ψ), or long-standing party loyalties are in flux (low θ), or parties place an especially high premium on maintaining their core districts ($\alpha - \beta$ large), so that our assumption fails. In that case, if the initial advantage in favor of either party is not too large, we obtain a unique equilibrium in which *both* parties revert to their core districts.

Proposition 4. *Suppose that* $r < \frac{1}{2}\left(\alpha + \frac{\psi}{\theta}(\alpha - \beta)\right)$*, and party* R*'s advantage is not too large in the sense that*

$$\rho_0 < \frac{\alpha\theta(\alpha\theta + \psi(\alpha - \beta) - 2r\theta)}{(\alpha + \beta)(\alpha - \beta + \alpha\theta)}. \tag{10}$$

Then there exists a unique pure strategy equilibrium, in which party L *retreats to its base,*

$$z_L^*(\rho_0) = \frac{\alpha\theta(\theta(2r - \alpha) - \alpha(\psi + \rho_0) + \beta(\psi - \rho_0)) - (\alpha^2 - \beta^2)\rho_0}{2\alpha\theta(\alpha\theta + \alpha - \beta)} < 0, \tag{11}$$

and party R *also retreats to its base,*

$$z_R^*(\rho_0) = \; z_L^*(\rho_0) + \frac{\alpha\theta + \psi(\alpha - \beta) - 2\theta r}{\alpha\theta + \alpha - \beta} > 0. \tag{12}$$

The parties adopt differentiated platforms, with the advantaged party adopting a more moderate position than the disadvantaged party. The qualitative features of the equilibrium are therefore closely related to Groseclose (2001)'s Calvert-Wittman framework with policy-motivated parties, uncertainty about the median voter, and a deterministic advantage.

As each party's relative concern for its core districts α increases, both parties further retreat to their respective bases. Perhaps surprisingly, the stronger party retreats *more* quickly:

Corollary 6. *As* α *increases,* both *parties increasingly retreat to their respective bases, but the stronger party moves faster than the weaker party, and to an extent that increases in its initial advantage,* ρ_0*.*

When α increases, each party cares relatively more about catering to its core districts. On the one hand, this partly encourages a party to abandon centrist districts in favor of those whose medians are relatively more extreme than the party's platform—e.g., medians with preferred policies to the left of z_L^* for party L, or to the right of z_R^* for party R. On the other hand, each party also has core districts whose medians are relatively more moderate than the party's platform—e.g., medians with preferred policies between z_L^* and 0 for party L, or between 0 and z_R^* for party R. Increases inα also encourage each party to moderate further in order to increase its prospect of winning these districts. Critically, party R is initially positioned closer to the median voter than party L:

$$\frac{x_L^* + z_R^*}{2} = -\rho_0 \frac{\alpha + \beta}{2\alpha\theta} < 0. \tag{13}$$

Thus, as α increases, a relatively higher proportion of R's core district medians are more extreme than the party's platform, vis-à-vis party L's. This encourages a relatively greater retreat to the base by party R. Thus, the parties become more polarized, and the midpoint of the parties' platforms also moves in the direction of the stronger party's core districts.

5. Conclusion

We analyze two-party competition in multi-district legislative elections. We ask: how do initial electoral imbalances encourage an office-seeking party to target its traditional supporters, rather than the centrist voters that are crucial for outright victory? If a party targets its traditional supporters, when should the opposing party maintain its focus on courting centrist voters, and when instead should it chase its opponent, targeting voters who are more ideologically disposed toward its opponent? And, how do the answers to these questions depend on parties' expectations of how voters attitudes might change over the course of the campaign, the strength of pre-existing party loyalty, and the relative marginal value that a party derives from winning additional seats below, at, or even above the majority threshold?

A small initial imbalance does not deter a disadvantaged party from the sole pursuit of outright victory by way of a centrist policy agenda. However, a sufficiently large imbalance induces it to revert in favor of a strategy that consolidates its core supporters, in order to avoid a catastrophic defeat. Similarly, an advantaged party initially prefers to maintain uncontested control of the political centre to further fortify its prospects of a post-election majority. But, if the imbalance is large enough, it chases its opponent to plunder its increasingly ill-served moderate supporters; the advantaged party's goal evolves from seeking to win, to winning with a larger post-election majority. Thus we predict that a very advantaged party uses its strength as an opportunity to expand the frontier of its political support beyond the median voter; as illustrated by the campaigns of Tony Blair and Theresa May.

In ongoing work, we use our framework to study the dynamics of political campaigns in contexts where some voters cast ballots early, or make up their minds before a campaign concludes. That is, some voters cast their ballots after an initial valence shock that favors one of the parties, but before the parties have communicated their policy commitments, and prior to any other developments—such as leader debates, town hall meetings, or personal revelations—that occur over the course of a campaign. We interpret these voters as 'early deciders', who are insensitive or inattentive to the twists and turns of election campaigns.

If the initial valence shock favoring one of the parties is small, the parties converge on a platform that—rather than targeting the median voter in the median district, as in Proposition 1—moves toward the advantaged party's core districts, by an increment that grows with both the magnitude of the initial valence shock and the fraction of early deciders. It appears as if the parties believe that voters have shifted ideologically in favor of the advantaged party. In fact, voters' policy preferences have *not* changed. Instead, the strategies reflect that the initially more popular party enjoys a larger share of support amongst early deciders, and thus gains a starting lead in the polls. In order to win the election, the initially less popular party therefore needs to offset its disadvantage by carrying strictly more than a majority of supporters amongst the remaining voters. This leads it to move beyond the ideological centre-ground, targeting voters that are ideologically disposed toward its advantaged opponent. Thus, the disadvantaged party designs its policy to appeal to its rival's voters even though ideology is *not* the source of its disadvantage.

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6. Appendix: Proofs of Results

Let $x^*(z_L, z_R, \rho)$ denote the preferred policy of the swing voter, given party L's platform z_L, party R's platform z_R, and party R's net valence advantage, $\rho \in [\rho_0 - \psi, \rho_0 + \psi]$. Assumption 2 says that $\rho_0 - \psi < -1$ and $\theta > \rho_0 + \psi + 1$. Thus $\theta > 2$, implying that for any pair $(z_L, z_R) \in \mathbb{R}^2$, a voter with preferred policy $x_i > x^*(z_L, z_R, \rho)$ strictly prefers R, and a voter with preferred policy $x_i \leq x^*(z_L, z_R, \rho)$ weakly prefers L. We adopt the convention that if a voter is indifferent between the parties, she votes for party L. Since the set of indifferent voters has measure zero for any shock realization, this convention has no bearing on our results. Party L therefore wins a district with median m if and only if $x^*(\rho, z_L, z_R) \geq m$. Using the fact that district medians are uniformly distributed on $[-1, 1]$, party L's share of districts is given by $d_L = \frac{1+x^*(z_L,z_R,\rho)}{2}$, and party L therefore wins the election if and only if $d_L \geq \frac{1}{2}$, i.e., if and only if $x^*(z_L, z_R, \rho) \geq 0$. It is immediate that a platform $z_J < -1$ or $z_J > 1$ is strictly dominated, for either party $J \in \{L, R\}$. Thus, we focus on platform pairs $(z_L, z_R) \in [-1, 1]^2$ in the arguments that follow.

We first prove three intermediate results that streamline our proofs of existence and uniqueness.

Lemma 1. *For any $z_L \in [-1, 1]$, z_R is a best response to z_L only if $z_R \leq \max\{0, z_L\}$.*

Proof. We establish that party R's payoff strictly decreases in $z_R \geq \max\{0, z_L\}$. Because arguments used in the proof of this result are repeated throughout the Appendix, we provide some commentary, to guide the reader. Recall from expression (7) that the swing voter $x^*(z_L, z_R, \rho)$ solves $\Delta(x^*; z_L, z_R, \rho) = 0$, where

$$\Delta(x^*; z_L, z_R, \rho) = |z_R - x^*| - |z_L - x^*| - \theta x^* - \rho. \tag{14}$$

Whenever $z_R \geq z_L$, the swing voter $x^*(z_L, z_R, \rho)$ may be drawn from one of at most three intervals.

1. The swing voter's type is $x^*(z_L, z_R, \rho) \geq z_R$ if $\Delta(z_R; z_L, z_R, \rho) \geq 0$, i.e., if $z_L - z_R - \theta z_R - \rho \geq 0$, i.e., if $\rho \leq z_L - z_R - \theta z_R$. Thus, $x^*(z_L, z_R, \rho)$ solves $(x^* - z_R) - (x^* - z_L) - \theta x^* - \rho = 0$, i.e.,

$$x^*(z_L, z_R, \rho) = \frac{z_L - z_R - \rho}{\theta} \equiv x_1^*. \tag{15}$$

2. The swing voter's type is $x^*(z_L, z_R, \rho) \in (z_L, z_R)$ if both $\Delta(z_L; z_L, z_R, \rho) > 0$, i.e., $z_R - z_L - \theta z_L - \rho > 0$ and also $\Delta(z_R; z_L, z_R, \rho) < 0$, i.e., $z_L - z_R - \theta z_R - \rho < 0$. Thus, $x^*(z_L, z_R, \rho)$ solves

$(z_R - x^*) - (x^* - z_L) - \theta x^* - \rho = 0$, i.e.,

$$x^*(z_L, z_R, \rho) = \frac{z_L + z_R - \rho}{2 + \theta} \equiv {x_2}^*. \tag{16}$$

3. The swing voter's type is $x^*(z_L, z_R, \rho) \le z_L$ if $\Delta(z_L; z_L, z_R, \rho) \le 0$, i.e., $z_R - z_L - \theta z_L - \rho \le 0$, i.e., $\rho \ge z_R - z_L - \theta z_L$, i.e., $(z_R - x^*) - (z_L - x^*) - \theta x^* - \rho = 0$, i.e.,

$$x^*(z_L, z_R, \rho) = \frac{z_R - z_L - \rho}{\theta} \equiv {x_3}^*. \tag{17}$$

Assumption 2 that $\theta > \rho_0 + \psi + 1$ implies that $x_1^* < 1$ and $x_3^* > -1$ for all $\rho \in [\rho_0 - \psi, \rho_0 + \psi]$, whenever $z_L \le z_R$. In words: $x_1^* < 1$ states that even if the net valence shock in favor of party R is drawn most *unfavorably* to R, i.e., $\rho = \rho_0 - \psi$, the district median voter type $+1$ strictly prefers party R. This implies that R always wins a positive share of districts whenever $z_L \le z_R$. Likewise, $x_3^* > -1$ states that even if the net valence shock ρ in favor of party R is drawn most *favorably* to R, i.e., $\rho = \rho_0 + \psi$, the district median voter type -1 strictly prefers party L. This implies that L always wins a positive share of districts whenever $z_L \le z_R$.

We consider two possible cases for the location of party L's platform: weakly to the left of the median voter, i.e., $z_L \le 0$, or strictly to the right of the median voter, i.e., $z_L > 0$.

Case 1: $z_L \le 0$. If party R locates at $z_R \ge 0$, party R wins a majority if and only if ${x_2}^* < 0$, i.e., if and only if $\rho > z_L + z_R$. Party R's expected payoff from $z_R \ge 0$ is:

$$\begin{aligned}\pi_R(z_L, z_R) \;=\; & \frac{\alpha}{2\psi} \int_{\rho_0 - \psi}^{\max\{z_L - z_R - \theta z_R, \rho_0 - \psi\}} \left(\frac{1}{2} - \frac{x_1^*}{2}\right) d\rho + \frac{\alpha}{2\psi} \int_{\max\{z_L - z_R - \theta z_R, \rho_0 - \psi\}}^{z_L + z_R} \left(\frac{1}{2} - \frac{x_2^*}{2}\right) d\rho \\ & + \frac{1}{2\psi} \int_{z_L + z_R}^{\min\{z_R - z_L - \theta z_L, \rho_0 + \psi\}} \left(r - \beta \frac{x_2^*}{2}\right) d\rho + \frac{1}{2\psi} \int_{\min\{z_R - z_L - \theta z_L, \rho_0 + \psi\}}^{\rho_0 + \psi} \left(r - \beta \frac{x_3^*}{2}\right) d\rho.\end{aligned} \tag{18}$$

We explain how this expected payoff is constructed, taking each of the four integrals in turn.

First term. The first integral reflects R's share of districts for realizations of ρ such that the swing voter's type is $x^*(z_L, z_R, \rho) \in (z_R, 1]$. Since $x^*(z_L, z_R, \rho) > z_R \ge 0$, R wins a minority of districts. We have already shown that, with probability one, $x^*(z_L, z_R, \rho) < 1$, i.e., we have shown that with probability one the swing voter type $x^*(z_L, z_R, \rho)$ is realized strictly to the left of the district median with ideal policy 1. However, we have *not* shown that with positive probability the swing voter type $x^*(z_L, z_R, \rho)$ is realized strictly to the right of party R's platform, z_R. The net value that a voter with ideal policy x_i receives from party L is $\Delta(x_i; z_L, z_R, \rho)$, defined in (3). Thus, $x^*(z_L, z_R, \rho) > z_R$ with positive probability if and only

$$\begin{aligned}\Delta(z_R; z_L, z_R, \rho_0 - \psi) > 0 &\iff |z_R - z_R| - |z_R - z_L| - \theta z_R - (\rho_0 - \psi) > 0 \\ &\iff \rho_0 - \psi < z_L - z_R - \theta z_R. \end{aligned} \tag{19}$$

When (19) fails, the first integral in expression (18) is zero, since with probability one R wins every district with median $m \in [z_R, 1]$.

<u>*Second term*</u>. The second integral reflects party R's share of districts for realizations of ρ such that the swing voter's type is $x^*(z_L, z_R, \rho) \in [0, z_R]$, in which case party R wins a minority of districts. The median voter weakly prefers party L for some shock realization if and only if

$$\Delta(0; z_L, z_R, \rho_0 - \psi) \geq 0 \iff |z_R - 0| - |z_L - 0| - \theta \times 0 - (\rho_0 - \psi) \geq 0 \iff z_R + z_L \geq \rho_0 - \psi. \tag{20}$$

Likewise, the median voter strictly prefers party R for some shock realization if and only if

$$\Delta(0; z_L, z_R, \rho_0 + \psi) < 0 \iff |z_R - 0| - |z_L - 0| - \theta \times 0 - (\rho_0 + \psi) < 0 \iff z_R + z_L < \rho_0 + \psi, \tag{21}$$

Since $-1 < z_R + z_L < 1$ for any (z_L, z_R) such that $-1 \leq z_L \leq 0 \leq z_R \leq 1$, Assumption 2 that $\rho_0 - \psi < -1$ implies that $\rho_0 - \psi < z_R + z_L < \rho_0 + \psi$, implying that with positive probility each party wins a strict majority of districts. This yields the upper limit of integration in the second term of expression (18).

<u>*Third term*</u>. The third integral reflects party R's share of districts for realizations of ρ such that $x^*(z_L, z_R, \rho) \in [z_L, 0)$. Because $x^*(z_L, z_R, \rho) < 0$, party R wins a majority of districts. While we have shown that with positive probability $x^*(z_L, z_R, \rho) < 0$, we have *not* shown that with positive probability $x^*(z_L, z_R, \rho) < z_L$. The net value that a voter with ideal policy x_i receives from party L is $\Delta(x_i; z_L, z_R, \rho)$, defined in (3). Thus, with positive probability $x^*(z_L, z_R, \rho) < z_L$ if and only if $x^*(z_L, z_R, \rho_0 + \psi) < z_L$, i.e., if and only if

$$\begin{aligned}\Delta(z_L; z_L, z_R, \rho_0 + \psi) < 0 &\iff |z_R - z_L| - |z_L - z_L| - \theta z_L - (\rho_0 + \psi) < 0 \\ &\iff \rho_0 + \psi > z_R - z_L - \theta z_L. \end{aligned} \tag{22}$$

When this condition fails, the upper limit of integration in the third term of expression (18) is $\rho_0 + \psi$, the highest value taken by the preference shock.

Fourth term. The fourth integral reflects party R's share of districts for realizations of ρ such that $x^*(z_L, z_R, \rho) < z_L$. Since $z_L \le 0$, $x^*(z_L, z_R, \rho) < z_L$ implies that party R wins a majority of districts. As we highlighted in the previous paragraph, $x^*(z_L, z_R, \rho) < z_L$ occurs with positive probability if only if $\rho_0 + \psi > z_R - z_L - \theta z_L$. Otherwise, the fourth integral in (18) is zero.

We first argue that a platform z_R is not a best response if with probability one the swing voter's type $x^*(z_L, z_R, \rho)$ is realized weakly to the left of z_R. That is, we argue that z_R is not a best response if $z_L - z_R - \theta z_R \le \rho_0 - \psi$, i.e., if the first integral in expression (18) is zero. To prove this, we first observe that for any $z_L \le 0$, party R can select a platform $z_R \ge 0$ such that $z_L - z_R - \theta z_R > \rho_0 - \psi$. This follows from the fact that party R can select $z_R = 0$: Assumption 2 that $\rho_0 - \psi < -1$ implies that $z_L - 0 - \theta \times 0 = z_L \ge -1 > \rho_0 - \psi$.

Suppose, however, that party R locates at $z_R > 0$ such that $z_L - z_R - \theta z_R \le \rho_0 - \psi$, i.e., so that with probability one the swing voter's type $x^*(z_L, z_R, \rho)$ is realized weakly to the left of z_R. If, in addition, with positive probability $x^*(z_L, z_R, \rho)$ is realized strictly to the left of z_L, i.e., if $z_R - z_L - \theta z_L < \rho_0 + \psi$, then differentiation of (18) yields:

$$2\psi \frac{\partial \pi_R(z_L, z_R)}{\partial(-z_R)} = \frac{\alpha}{2} \int_{\rho_0-\psi}^{z_L+z_R} \frac{\partial x_2^*}{\partial z_R} d\rho + \frac{\beta}{2} \left[\int_{z_L+z_R}^{z_R-z_L-\theta z_L} \frac{\partial x_2^*}{\partial z_R} d\rho + \int_{z_R-z_L-\theta z_L}^{\rho_0+\psi} \frac{\partial x_3^*}{\partial z_R} d\rho \right] + \left(r - \frac{\alpha}{2} \right). \tag{23}$$

Because $\frac{\partial x_2^*}{\partial z_R} > 0$ and $\frac{\partial x_3^*}{\partial z_R} > 0$, (23) is strictly positive, and thus R's platform is not a best response. The argument if $z_L - z_R - \theta z_R \le \rho_0 - \psi$ and $z_R - z_L - \theta z_L \ge \rho_0 + \psi$ is the same. We conclude that R's expected payoff (18) strictly decreases in z_R whenever with probability one $x^*(z_L, z_R, \rho) \le z_R$.

We then verify via straightforward algebra (omitted) that for any $z_R \ge 0$ such that with positive probability $x^*(z_L, z_R, \rho) > z_R$ and with positive probability $x^*(z_L, z_R, \rho) < z_L$, party R's payoff strictly decreases in z_R under Assumption 1 that $r > \frac{\alpha}{2} + \frac{\psi}{2\theta}(\alpha - \beta)$. That is, for any z_R such that $z_L - z_R - \theta z_R > \rho_0 - \psi$ and $z_R - z_L - \theta z_L < \rho_0 + \psi$, (18) strictly decreases in z_R.

Finally, we argue that for any z_R such that with positive probability $x^*(z_L, z_R, \rho) > z_R$, and with probability one $x^*(z_L, z_R, \rho) \ge z_L$, party R's payoff strictly decreases in z_R. That is, we argue that for any z_R such that both $z_L - z_R - \theta z_R > \rho_0 - \psi$ and $z_R - z_L - \theta z_L \ge \rho_0 + \psi$, R's payoff strictly decreases in z_R. When $z_L - z_R - \theta z_R > \rho_0 - \psi$ and $z_R - z_L - \theta z_L \ge \rho_0 + \psi$, differentiation of (18) with respect to z_R yields:

$$2\psi\frac{\partial \pi_R(z_L,z_R)}{\partial z_R} = -\frac{\alpha}{2}\left[\int_{\rho_0-\psi}^{z_L-z_R-\theta z_R}\frac{\partial x_1^*}{\partial z_R}d\rho+\int_{z_L-z_R-\theta z_R}^{z_L+z_R}\frac{\partial x_2^*}{\partial z_R}d\rho\right]-\frac{\beta}{2}\int_{z_L+z_R}^{\rho_0+\psi}\frac{\partial x_2^*}{\partial z_R}d\rho-\left(r-\frac{\alpha}{2}\right). \tag{24}$$

Straightforward algebra reveals that (24) strictly *increases* in z_L. The restriction that $z_R-z_L-\theta z_L \geq \rho_0+\psi$ is equivalent to $z_L \leq \frac{z_R-(\rho_0+\psi)}{1+\theta} \equiv \hat{z}_L(z_R)$. Evaluated at $z_L=\hat{z}_L(z_R)$, straightforward algebra verifies that (24) is strictly negative evaluated for any $z_R \geq 0$, under Assumption 1 that $r > \frac{\alpha}{2}+\frac{\psi}{2\theta}(\alpha-\beta)$. We conclude that (18) strictly decreases in z_R such that with positive probability $x^*(z_L,z_R,\rho) > z_R$ and with probability one $x^*(z_L,z_R,\rho) \geq z_L$.

We have established that for any $z_L \leq 0$, party R's expected payoff strictly decreases in $z_R \geq 0$.

Case 2: $z_L > 0$. We consider $z_R \geq z_L$. Again, party R wins if and only if $x^*(z_L,z_R,\rho)<0$, i.e., if and only if $\rho > z_R - z_L$. Party R's expected payoff from $z_R \geq z_L$ is:

$$\begin{aligned}\pi_R(z_L,z_R) =& \frac{\alpha}{2\psi}\int_{\rho_0-\psi}^{\max\{z_L-z_R-\theta z_R,\rho_0-\psi\}}\left(\frac{1}{2}-\frac{x_1^*}{2}\right)d\rho+\frac{\alpha}{2\psi}\int_{\max\{z_L-z_R-\theta z_R,\rho_0-\psi\}}^{\min\{z_R-z_L-\theta z_L,\rho_0+\psi\}}\left(\frac{1}{2}-\frac{x_2^*}{2}\right)d\rho\\ &+\frac{\alpha}{2\psi}\int_{\min\{z_R-z_L-\theta z_L,\rho_0+\psi\}}^{z_R-z_L}\left(\frac{1}{2}-\frac{x_3^*}{2}\right)d\rho+\frac{1}{2\psi}\int_{z_R-z_L}^{\rho_0+\psi}\left(r-\beta\frac{x_3^*}{2}\right)d\rho.\end{aligned} \tag{25}$$

By similar arguments to those for Case 1, $z_R > z_L$ is a best response only if $z_L - z_R - \theta z_R > \rho_0 - \psi$. Assumption 2 that $\rho_0-\psi<-1$ and $0<z_L\leq z_R \leq 1$ implies that $z_R - z_L < \rho_0+\psi$, i.e., that with positive probability $x^*(z_L,z_R,\rho)<0$. This implies that with positive probability $x^*(z_L,z_R,\rho)<z_L$, i.e., $z_R-z_L-\theta z_L<\rho_0+\psi$. Without loss of generality, we may therefore re-write (25) as follows:

$$\begin{aligned}\pi_R(z_L,z_R) =& \frac{\alpha}{2\psi}\int_{\rho_0-\psi}^{z_L-z_R-\theta z_R}\left(\frac{1}{2}-\frac{x_1^*}{2}\right)d\rho+\frac{\alpha}{2\psi}\int_{z_L-z_R-\theta z_R}^{z_R-z_L-\theta z_L}\left(\frac{1}{2}-\frac{x_2^*}{2}\right)d\rho\\ &+\frac{\alpha}{2\psi}\int_{z_R-z_L-\theta z_L}^{z_R-z_L}\left(\frac{1}{2}-\frac{x_3^*}{2}\right)d\rho+\frac{1}{2\psi}\int_{z_R-z_L}^{\rho_0+\psi}\left(r-\beta\frac{x_3^*}{2}\right)d\rho.\end{aligned} \tag{26}$$

$$\frac{\partial \pi_R(z_L,z_R)}{\partial z_R} = \frac{\alpha(\theta-\rho_0+\psi)-\beta(\rho_0+\psi)-2\theta r+(z_L-z_R)(\alpha-\beta)-2\theta\alpha z_R}{4\theta\psi}. \tag{27}$$

For all $z_R \geq z_L > 0$, $(z_L-z_R)(\alpha-\beta)-2\theta\alpha z_R \leq 0$ under Assumption 1 that $\alpha \geq \beta$; the remainder of the numerator is strictly negative for all $\rho_0 \geq 0$ under Assumption 1 that $r>\frac{\alpha}{2}+\frac{\psi}{2\theta}(\alpha-\beta)$. $\square$

Lemma 2. *For any $z_R \in [-1,0]$ such that $z_R - \theta z_R \leq \rho_0+\psi$, z_L is a best response to z_R only if $z_L \leq 0$.*

For any $z_R > 0$, z_L is a best response to z_R only if $z_L \leq \max\{0, z_R\}$.

Proof. The net value that a voter with ideal policy x_i receives from party L is $\Delta(x_i; z_L, z_R, \rho)$, defined in (3). The swing voter $x^*(z_L, z_R, \rho)$ solves $\Delta(x^*; z_L, z_R, \rho) = 0$. When $z_L \geq z_R$, the swing voter $x^*(z_L, z_R, \rho)$ may therefore be drawn from one of at most three intervals.

1. The swing voter's type is $x^*(z_L, z_R, \rho) < z_R$ if $\Delta(z_R; z_L, z_R, \rho) < 0$, i.e., $z_R - z_L - \theta z_R - \rho < 0$, i.e., $\rho > z_R - z_L - \theta z_R$. Thus, $x^*(z_L, z_R, \rho)$ solves $(z_R - x^*) - (z_L - x^*) - \theta x^* - \rho = 0$, i.e.,

$$x^*(z_L, z_R, \rho) = \frac{z_R - z_L - \rho}{\theta} \equiv {x_4}^*. \tag{28}$$

2. The swing voter's type is $x^*(z_L, z_R, \rho) \in [z_R, z_L]$ if $\Delta(z_R; z_L, z_R, \rho) \geq 0 \geq \Delta(z_L; z_L, z_R, \rho)$, i.e. $z_R - z_L - \theta z_R - \rho \geq 0$, and $z_L - z_R - \theta z_L - \rho \leq 0$. Thus, $x^*(z_L, z_R, \rho)$ solves $(x^* - z_R) - (z_L - x^*) - \theta x^* - \rho = 0$, i.e.,

$$x^*(z_L, z_R, \rho) = \frac{-(z_L + z_R) - \rho}{\theta - 2} \equiv {x_5}^*. \tag{29}$$

3. The swing voter's type is $x^*(z_L, z_R, \rho) > z_L$ if $\Delta(z_L; z_L, z_R, \rho) > 0$, i.e., $z_L - z_R - \theta z_L - \rho > 0$. Thus, $x^*(z_L, z_R, \rho)$ solves $(x^* - z_R) - (x^* - z_L) - \theta x^* - \rho = 0$, i.e.,

$$x^*(z_L, z_R, \rho) = \frac{z_L - z_R - \rho}{\theta} \equiv {x_6}^*. \tag{30}$$

Notice that $x_6^* \leq 1$ if and only if $z_L - z_R - \rho \leq \theta$, i.e., $\rho \geq z_L - z_R - \theta$; similarly, $x_4^* \geq -1$ if and only if $z_R - z_L - \rho \geq -\theta$, i.e., $\rho \leq z_R - z_L + \theta$. We consider two possible cases for the location of party R's platform: weakly to the left of the median voter, i.e., $z_R \leq 0$, or strictly to the right of the median voter, i.e., $z_R > 0$.

Case 1: $z_R \leq 0$. Consider $z_L > 0$. Party L wins if and only if $x^*(z_L, z_R, \rho) \geq 0$, i.e., if and only if $\rho \leq -z_L - z_R$. Party L's expected payoff is therefore:

$$\begin{aligned}
\pi_L(z_L, z_R) \; &= \frac{1}{2\psi} \int_{\rho_0 - \psi}^{\max\{z_L - z_R - \theta, \rho_0 - \psi\}} \left(r + \frac{\beta}{2}\right) d\rho + \frac{1}{2\psi} \int_{\max\{z_L - z_R - \theta, \rho_0 - \psi\}}^{\max\{z_L - z_R - \theta z_L, \rho_0 - \psi\}} \left(r + \beta \frac{x_6^*}{2}\right) d\rho \\
&+ \frac{1}{2\psi} \int_{\max\{z_L - z_R - \theta z_L, \rho_0 - \psi\}}^{-z_L - z_R} \left(r + \beta \frac{x_5^*}{2}\right) d\rho + \frac{\alpha}{2\psi} \int_{-z_L - z_R}^{\min\{z_R - z_L - \theta z_R, \rho_0 + \psi\}} \left(\frac{1}{2} + \frac{x_5^*}{2}\right) d\rho \\
&+ \frac{\alpha}{2\psi} \int_{\min\{z_R - z_L - \theta z_R, \rho_0 + \psi\}}^{\min\{z_R - z_L + \theta, \rho_0 + \psi\}} \left(\frac{1}{2} + \frac{x_4^*}{2}\right) d\rho.
\end{aligned} \tag{31}$$

To understand the first term, note that Assumption 2 is not sufficient to ensure that either party wins a positive share of districts when $z_R < z_L$. Recalling that $\Delta(x_i; z_L, z_R, \rho)$ is voter type x_i's net value from party L, defined in (3), the voter type 1 weakly prefers party L for some ρ if and only if $\Delta(1; z_L, z_R, \rho_0 - \psi) \geq 0$, i.e., if and only if

$$|1 - z_R| - |1 - z_L| - \theta - (\rho_0 - \psi) \geq 0 \iff \rho_0 - \psi \leq z_L - z_R - \theta. \tag{32}$$

If (32) holds, then party L wins every district whenever $\rho \leq z_L - z_R - \theta$, in which case its payoff is $r + \beta/2$. The upper limit of integration in the final integration follows a similar derivation: if $z_R - z_L + \theta < \rho_0 + \psi$, then party R wins every district whenever the net preference shock in favor of R is $\rho > z_R - z_L + \theta$. If, instead, $z_R - z_L + \theta \geq \rho_0 + \psi$, L wins a positive share of districts for every realization of the preference shock.

The remaining terms in (31) follow a similar derivation to that for Lemma 1.

We first argue that $z_L \geq 0$ is not a best response if $z_L - z_R - \theta z_L \leq \rho_0 - \psi$. In words: we argue that z_L is not a best response if with probability one the swing voter's type $x^*(z_L, z_R, \rho)$ is realized weakly to the left of z_L. To prove this, we first observe that for any $z_R \leq 0$, there exists $z_L \geq 0$ such that $z_L - z_R - \theta z_L \geq \rho_0 - \psi$, since L may choose $z_L = 0$. The remainder of the argument is similar to that for Lemma 1.

Next, we claim that for any platform choice by party L strictly to the right of zero, with positive probability the swing voter's type $x^*(z_L, z_R, \rho)$ is drawn strictly to the left of z_R. In other words: for any $z_L > 0$, $\Delta(z_R; z_L, z_R, \rho + \psi) < 0$, where $\Delta(x_i; z_L, z_R, \rho)$ is voter type x_i's net value from party L, defined in (3). This is true if and only if

$$(z_R - z_R) - (z_L - z_R) - \theta z_R - (\rho_0 + \psi) < 0 \iff z_R - z_L - \theta z_R < \rho_0 + \psi. \tag{33}$$

This strict inequality follows from our assumption that $z_R - \theta z_R \leq \rho_0 + \psi$, which implies that for any $z_L > 0$, $z_R - z_L - \theta z_R < \rho_0 + \psi$. Thus, the objective function (31) becomes:

$$\begin{aligned} \pi_L(z_L, z_R) = {} & \frac{1}{2\psi} \int_{\rho_0 - \psi}^{\max\{z_L - z_R - \theta, \rho_0 - \psi\}} \left(r + \frac{\beta}{2}\right) d\rho + \frac{1}{2\psi} \int_{\max\{z_L - z_R - \theta, \rho_0 - \psi\}}^{z_L - z_R - \theta z_L} \left(r + \beta \frac{x_6^*}{2}\right) d\rho \\ & + \frac{1}{2\psi} \int_{z_L - z_R - \theta z_L}^{-z_L - z_R} \left(r + \beta \frac{x_5^*}{2}\right) d\rho + \frac{\alpha}{2\psi} \int_{-z_L - z_R}^{z_R - z_L - \theta z_R} \left(\frac{1}{2} + \frac{x_5^*}{2}\right) d\rho \\ & + \frac{\alpha}{2\psi} \int_{z_R - z_L - \theta z_R}^{\min\{z_R - z_L + \theta, \rho_0 + \psi\}} \left(\frac{1}{2} + \frac{x_4^*}{2}\right) d\rho. \end{aligned} \tag{34}$$

We are left to consider cases depending on the order of $z_L - z_R - \theta$ and $\rho_0 - \psi$ (the first term) and the order of $z_R - z_L + \theta$ and $\rho_0 + \psi$ (the last term). We show that there are three relevant intervals from which z_L can be drawn. Recall that, by supposition, $z_L > 0$.

[1.] Suppose, first, that $z_L \in (0, z_R + \theta - (\rho_0 + \psi))$. This implies $z_R - z_L + \theta > \rho_0 + \psi$, which further implies $z_L - z_R - \theta < \rho_0 - \psi$. On this domain, L's objective (34) is strictly concave, with first-order condition:

$$z_L'(z_R, \rho_0) = \frac{\alpha(\theta - \rho_0 - \psi) - \beta(\rho_0 - \psi) - 2\theta r + z_R(\alpha - \beta)}{\alpha + \beta(2\theta - 1)}, \tag{35}$$

which is strictly negative for all $\rho_0 \geq 0$ and $z_R \leq 0$, because $r > \frac{\alpha}{2}$ implies $r > \frac{\alpha}{2} - \frac{\psi}{2\theta}(\alpha - \beta)$. We conclude that (34) strictly decreases on this domain.

[2.] Suppose, second, that $z_L \in [z_R + \theta - (\rho_0 + \psi), z_R + \theta + (\rho_0 - \psi)]$. This implies $z_R - z_L + \theta \leq \rho_0 + \psi$ and $z_L - z_R - \theta \leq \rho_0 - \psi$. On this domain, L's objective (34) is strictly concave, with associated first-order condition:

$$z_L'(z_R, \rho_0) = \frac{2\theta r + \beta(\rho_0 - \psi + z_R)}{\beta(1 - 2\theta)}, \tag{36}$$

which is strictly negative by Assumption 1 that $2r > \alpha$, and $\alpha \geq \beta$, and Assumption 2 that $\theta > \rho_0 + \psi + 1$, which implies that $\theta > -\rho_0 + \psi - z_R$. We conclude that (34) strictly decreases on this domain.

[3.] Suppose, finally, that $z_L > z_R + \theta + (\rho_0 - \psi)$. This implies $z_R - z_L + \theta < \rho_0 + \psi$ and $z_L - z_R - \theta > \rho_0 - \psi$. We find that $\frac{\partial \pi_R(z_L, z_R)}{\partial z_R} = \frac{-2r + \beta - 2z_L\beta}{4\psi} < 0$, implying that (34) strictly decreases on this domain.

We have shown that L's payoff (34) strictly decreases in $z_L \geq 0$ whenever $z_R < 0$, verifying the claim.

Case 2: $z_R > 0$. Suppose $z_L > z_R$ is a best response. Party L wins if and only if $x^*(z_L, z_R, \rho) \geq 0$, i.e., if and only if $\rho \leq z_R - z_L$. Party L's expected payoff from a platform $z_L \geq z_R$ is therefore:

$$\begin{aligned} \pi_L(z_L, z_R) = &\frac{1}{2\psi} \int_{\rho_0 - \psi}^{\max\{z_L - z_R - \theta, \rho_0 - \psi\}} \left(r + \frac{\beta}{2}\right) d\rho + \frac{1}{2\psi} \int_{\max\{z_L - z_R - \theta, \rho_0 - \psi\}}^{\max\{z_L - z_R - \theta z_L, \rho_0 - \psi\}} \left(r + \beta \frac{x_6^*}{2}\right) d\rho \\ &+ \frac{1}{2\psi} \int_{\max\{z_L - z_R - \theta z_L, \rho_0 - \psi\}}^{\max\{z_R - z_L - \theta z_R, \rho_0 - \psi\}} \left(r + \beta \frac{x_5^*}{2}\right) d\rho + \frac{1}{2\psi} \int_{\max\{z_R - z_L - \theta z_R, \rho_0 - \psi\}}^{z_R - z_L} \left(r + \beta \frac{x_4^*}{2}\right) d\rho \\ &+ \frac{\alpha}{2\psi} \int_{z_R - z_L}^{\min\{z_R - z_L + \theta, \rho_0 + \psi\}} \left(\frac{1}{2} + \frac{x_4^*}{2}\right) d\rho. \end{aligned} \tag{37}$$

We first observe that for any $(z_L, z_R) \in [0, 1]^2$, Assumption 2 that $\theta > \rho_0 + \psi + 1$ implies that for any $(z_L, z_R) \in [0, 1]^2$, $z_R - z_L + \theta > \rho_0 + \psi$ and $z_L - z_R - \theta < \rho_0 - \psi$. This implies that the upper limit of integration in the final integral of (37) is $\rho_0 + \psi$, and that the first integral in (37) is zero.

Next, we observe that by a similar argument to that of Lemma 1, a pair $0 < z_R < z_L$ is not an equilibrium if with probability one the swing voter type $x^*(z_L, z_R, \rho)$ is drawn weakly to the left of z_L. This implies that (z_L, z_R) is an equilibrium only if $\Delta(z_L; z_L, z_R, \rho_0 - \psi) > 0$, i.e., only if

$$(z_L - z_R) - (z_L - z_L) - \theta z_L - (\rho_0 - \psi) > 0 \iff z_L - z_R - \theta z_L > \rho_0 - \psi. \tag{38}$$

Thus, the objective (37) becomes:

$$\begin{aligned}\pi_L(z_L, z_R) \; &= \frac{1}{2\psi}\int_{\rho_0-\psi}^{z_L-z_R-\theta z_L}\left(r+\beta\frac{x_6^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{z_L-z_R-\theta z_L}^{z_R-z_L-\theta z_R}\left(r+\beta\frac{x_5^*}{2}\right)d\rho \\ &+ \frac{1}{2\psi}\int_{z_R-z_L-\theta z_R}^{z_R-z_L}\left(r+\beta\frac{x_4^*}{2}\right)d\rho + \frac{\alpha}{2\psi}\int_{z_R-z_L}^{\rho_0+\psi}\left(\frac{1}{2}+\frac{x_4^*}{2}\right)d\rho.\end{aligned} \tag{39}$$

It is easily verified that $\frac{\partial \pi_L(z_L,z_R)}{\partial z_L}|_{z_L=z_R} < 0$ for all $z_R > 0$. Thus, a platform $z_L > z_R$ is not a best response by party L. $\square$

Lemma 3. *There does not exist an equilibrium in which $z_L > 0$, or in which $z_R > 0$.*

Proof. Suppose first that $z_L > 0$ in an equilibrium. We consider two possible cases for the location of party R's platform: weakly to the right of the median voter's ideal policy, i.e., $z_R \geq 0$, or strictly to the right of median voter's ideal policy, i.e., $z_R > 0$.

Case 1: $z_R \geq 0$. The previous lemmata imply that if $z_R \geq 0$ and $z_L > 0$, then $z_R = z_L \equiv \hat{z}$ in any equilibrium. Consider a deviation by party L to $z_L \in [0, \hat{z})$. This yields the following payoff to party L:

$$\begin{aligned}\pi_L(z_L, \hat{z}) \; &= \frac{1}{2\psi}\int_{\rho_0-\psi}^{\max\{z_L-\hat{z}-\theta\hat{z},\rho_0-\psi\}}\left(r+\beta\frac{x_1^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{\max\{z_L-\hat{z}-\theta\hat{z},\rho_0-\psi\}}^{\max\{\hat{z}-z_L-\theta z_L,\rho_0-\psi\}}\left(r+\beta\frac{x_2^*}{2}\right)d\rho \\ &+ \frac{1}{2\psi}\int_{\max\{\hat{z}-z_L-\theta z_L,\rho_0-\psi\}}^{\hat{z}-z_L}\left(r+\beta\frac{x_3^*}{2}\right)d\rho + \frac{\alpha}{2\psi}\int_{\hat{z}-z_L}^{\rho_0+\psi}\left(\frac{1}{2}+\frac{x_3^*}{2}\right)d\rho.\end{aligned} \tag{40}$$

By now standard arguments, if with probability one the swing voter type $x^*(\hat{z}, \hat{z}, \rho)$ is realized weakly to the left of $\hat{z}$, then L strictly prefers a platform strictly to the left of $\hat{z}$, and thus $z_L = \hat{z}$ is not a best response. Recalling that $\Delta(x_i; z_L, z_R, \rho)$ defined in (3) is voter type x_i's net value from party L, we observe that with probability one $x^*(\hat{z}, \hat{z}, \rho) \leq \hat{z}$ if and only if $\Delta(\hat{z}; \hat{z}, \hat{z}, \rho_0 - \psi) \leq 0$, i.e.,

$$(\hat{z} - \hat{z}) - (\hat{z} - \hat{z}) - \theta \hat{z} - (\rho_0 - \psi) \leq 0 \iff -\theta \hat{z} \leq \rho_0 - \psi. \tag{41}$$

We conclude that if (41) is satisfied, we cannot have an equilibrium. Suppose, instead, $-\theta \hat{z} > \rho_0 - \psi$. Straightforward algebra verifies that $\frac{\partial \pi_L(\hat{z}, \hat{z})}{\partial(-z_L)} > 0$ if $r > \frac{\alpha}{2} - \frac{\psi}{2\theta}(\alpha - \beta)$, which is true because $r > \frac{\alpha}{2}$. Thus a deviation by L to a platform $z_L < \hat{z}$ is profitable.

Case 2: $z_R < 0$. Party L wins with a platform $z_L > 0$ if and only if $x^*(z_L, z_R, \rho) \geq 0$, i.e., if and only if $\rho \leq -z_L - z_R$. Party L's expected payoff is therefore:

$$\begin{aligned}
\pi_L(z_L, z_R) &= \frac{1}{2\psi} \int_{\rho_0 - \psi}^{\max\{z_L - z_R - \theta, \rho_0 - \psi\}} \left(r + \frac{\beta}{2}\right) d\rho + \frac{1}{2\psi} \int_{\max\{z_L - z_R - \theta, \rho_0 - \psi\}}^{\max\{z_L - z_R - \theta z_L, \rho_0 - \psi\}} \left(r + \beta \frac{x_6^*}{2}\right) d\rho \\
&+ \frac{1}{2\psi} \int_{\max\{z_L - z_R - \theta z_L, \rho_0 - \psi\}}^{-z_L - z_R} \left(r + \beta \frac{x_5^*}{2}\right) d\rho + \frac{\alpha}{2\psi} \int_{-z_L - z_R}^{\min\{z_R - z_L - \theta z_R, \rho_0 + \psi\}} \left(\frac{1}{2} + \frac{x_5^*}{2}\right) d\rho \\
&+ \frac{\alpha}{2\psi} \int_{\min\{z_R - z_L - \theta z_R, \rho_0 + \psi\}}^{\min\{z_R - z_L + \theta, \rho_0 + \psi\}} \left(\frac{1}{2} + \frac{x_4^*}{2}\right) d\rho.
\end{aligned} \tag{42}$$

By now standard arguments, if $z_R < 0$ is a best response by party R to $z_L > 0$, we must have $z_R - z_L - \theta z_R < \rho_0 + \psi$, and if $z_L > 0$ is a best response by party L to $z_R < 0$, we must have $z_L - z_R - \theta z_L > \rho_0 - \psi$. Thus, the objective function (42) becomes

$$\begin{aligned}
\pi_L(z_L, z_R) &= \frac{1}{2\psi} \int_{\rho_0 - \psi}^{\max\{z_L - z_R - \theta, \rho_0 - \psi\}} \left(r + \frac{\beta}{2}\right) d\rho + \frac{1}{2\psi} \int_{\max\{z_L - z_R - \theta, \rho_0 - \psi\}}^{z_L - z_R - \theta z_L} \left(r + \beta \frac{x_6^*}{2}\right) d\rho \\
&+ \frac{1}{2\psi} \int_{z_L - z_R - \theta z_L}^{-z_L - z_R} \left(r + \beta \frac{x_5^*}{2}\right) d\rho + \frac{\alpha}{2\psi} \int_{-z_L - z_R}^{z_R - z_L - \theta z_R} \left(\frac{1}{2} + \frac{x_5^*}{2}\right) d\rho \\
&+ \frac{\alpha}{2\psi} \int_{z_R - z_L - \theta z_R}^{\min\{z_R - z_L + \theta, \rho_0 + \psi\}} \left(\frac{1}{2} + \frac{x_4^*}{2}\right) d\rho.
\end{aligned} \tag{43}$$

Since (43) replicates the objective function (34) in the proof of Lemma 2, we may follow the remaining steps in that proof to verify that $z_L > 0$ is not a best response to $z_R < 0$, and thus we cannot have an equilibrium.

We conclude that $z_L \leq 0$, in an equilibrium. This, together with Lemma 1, implies that $z_R \leq 0$ in an equilibrium. $\square$

Lemma 3 implies that to rule out the existence of equilibria that are not characterized in Propositions 1, 2, 3, it is sufficient to show that there is no equilibrium in which $z_R < z_L \leq 0$.

Lemma 4. *There does not exist an equilibrium in which $z_R < z_L \leq 0$.*

Proof. Party R's expected payoff from $z_R \leq z_L$ is:

$$\pi_R(z_L, z_R) = \frac{\alpha}{2\psi}\int_{\rho_0-\psi}^{z_L-z_R}\left(\frac{1}{2}-\frac{x_6^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{z_L-z_R}^{\min\{z_L-z_R-\theta z_L,\rho_0+\psi\}}\left(r-\beta\frac{x_6^*}{2}\right)d\rho$$
$$+\frac{1}{2\psi}\int_{\min\{z_L-z_R-\theta z_L,\rho_0+\psi\}}^{\min\{z_R-z_L-\theta z_R,\rho_0+\psi\}}\left(r-\beta\frac{x_5^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{\min\{z_R-z_L-\theta z_R,\rho_0+\psi\}}^{\rho_0+\psi}\left(r-\beta\frac{x_4^*}{2}\right)d\rho. \tag{44}$$

Similarly, party L's expected payoff from $z_L \in [z_R, 0]$ is:

$$\pi_L(z_L, z_R) = \frac{1}{2\psi}\int_{\rho_0-\psi}^{z_L-z_R}\left(r+\beta\frac{x_6^*}{2}\right)d\rho + \frac{\alpha}{2\psi}\int_{z_L-z_R}^{\min\{z_L-z_R-\theta z_L,\rho_0+\psi\}}\left(\frac{1}{2}+\frac{x_6^*}{2}\right)d\rho$$
$$+\frac{\alpha}{2\psi}\int_{\min\{z_L-z_R-\theta z_L,\rho_0+\psi\}}^{\min\{z_R-z_L-\theta z_R,\rho_0+\psi\}}\left(\frac{1}{2}+\frac{x_5^*}{2}\right)d\rho + \frac{\alpha}{2\psi}\int_{\min\{z_R-z_L-\theta z_R,\rho_0+\psi\}}^{\rho_0+\psi}\left(\frac{1}{2}+\frac{x_4^*}{2}\right)d\rho \tag{45}$$

By now standard arguments, party R's platform $z_R < z_L$ is a best response only if with positive probability the swing voter type $x^*(z_L, z_R, \rho)$ is realized strictly to the left of z_R. Recalling that $\Delta(x_i; z_L, z_R, \rho)$ defined in (3) is voter type x_i's net value from party L, with positive probability $x^*(z_L, z_R, \rho) < z_R$ if and only if $\Delta(z_R; z_L, z_R, \rho+\psi) < 0$ i.e., if and only if

$$(z_R - z_R) - (z_L - z_R) - \theta z_R - (\rho_0 + \psi) < 0 \iff z_R - z_L - \theta z_R < \rho_0 + \psi. \tag{46}$$

We therefore restrict attention to pairs (z_L, z_R) such that $z_R < z_L$ and that further satisfy $z_R - z_L - \theta z_R < \rho_0 + \psi$. Notice that since $z_R < z_L$, condition (46) further implies that with positive probability $x^*(z_L, z_R, \rho) < z_L$, i.e., $z_L - z_R - \theta z_L < \rho_0 + \psi$. Party R's first-order condition on this implied domain is therefore:

$$\hat{z}_R(z_L) = \frac{-\alpha(\theta+\rho_0-\psi) - \beta(\rho_0+\psi) + 2\theta r + z_L(\alpha-\beta)}{\alpha+\beta(2\theta-1)}. \tag{47}$$

Similarly, party L's first-order condition is:

$$\hat{z}_L(z_R) = \frac{-\alpha(\theta+\rho_0+\psi) + \beta(\psi-\rho_0) + 2\theta r + z_R(\alpha-\beta)}{2\alpha\theta+\alpha-\beta}. \tag{48}$$

We consider two possible cases for an equilibrium in which $z_R < z_L \leq 0$. In the first case, party

L's platform is strictly to the left of zero, i.e., $z_L < 0$. In the second case, party L's platform is zero, i.e., $z_L = 0$.

Case 1: $z_L < 0$. When $z_R < z_L < 0$, the platforms solve (47) and (48). This yields a unique pair (z_L^*, z_R^*), such that $z_L^* - z_R^* > 0$ if and only if $\rho_0 - \psi > \frac{(2r-\alpha)\theta}{\alpha+\beta}$, which contradicts Assumption 2 that $\rho_0 - \psi < -1$, and Assumption 1 that $2r > \alpha$.

Case 2: $z_L = 0$. We obtain party R's best response to party L's platform by substituting $z_L = 0$ into (47). This yields

$$\hat{z}_R(0) < 0 \iff \rho_0 > \frac{\theta(2r-\alpha) + \psi(\alpha-\beta)}{\alpha+\beta} \equiv \hat{\rho}_0. \tag{49}$$

Expression (48) reveals that $\hat{z}_L(z_R)$ strictly increases in $z_R < 0$. Thus, for any $z_R < 0$,

$$\hat{z}_L(z_R) < 0 \iff \rho_0 > \frac{\theta(2r-\alpha) - \psi(\alpha-\beta)}{\alpha+\beta} \equiv \underline{\rho}_0. \tag{50}$$

We have shown that $\hat{z}_R(0) < 0$ if and only if $\rho > \hat{\rho}_0$, and that for any $z_R < 0$, $\hat{z}_L(z_R) < 0$ if $\rho \geq \underline{\rho}_0$. Because $\underline{\rho}_0 \leq \hat{\rho}_0$ for all $\alpha \geq \beta$, we conclude that there does not exist an equilibrium in which $z_R < z_L = 0$. $\square$

Existence of equilibrium. We now verify that there exists an equilibrium in which $z_L \leq z_R \leq 0$. The (at most) three swing voter types are given by $x_1^* = \frac{z_L - z_R - \rho}{\theta}$, $x_2^* = \frac{z_L + z_R - \rho}{2+\theta}$ and $x_3^* = \frac{z_R - z_L - \rho}{\theta}$, defined in expressions (15), (16) and (17). Assumption 2 that $\theta > \rho_0 + \psi$ implies that $x_1^* \leq 1$ and $x_3^* \geq -1$ for all $\rho \in [\rho_0 - \psi, \rho_0 + \psi]$. Finally, party R wins if and only if $x^*(z_L, z_R, \rho) < 0$, i.e., if and only if $\rho > z_L - z_R$. Given $z_L \leq 0$, R's expected payoff from $z_R \in [z_L, 0]$ is therefore:

$$\begin{aligned}\pi_R(z_L, z_R) &= \frac{\alpha}{2\psi}\int_{\rho_0-\psi}^{z_L-z_R}\left(\frac{1}{2} - \frac{x_1^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{z_L-z_R}^{\min\{\rho_0+\psi, z_L-z_R-\theta z_R\}}\left(r - \beta\frac{x_1^*}{2}\right)d\rho \\ &+ \frac{1}{2\psi}\int_{\min\{\rho_0+\psi, z_L-z_R-\theta z_R\}}^{\min\{\rho_0+\psi, z_R-z_L-\theta z_L\}}\left(r - \beta\frac{x_2^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{\min\{\rho_0+\psi, z_R-z_L-\theta z_L\}}^{\rho_0+\psi}\left(r - \beta\frac{x_3^*}{2}\right)d\rho.\end{aligned} \tag{51}$$

Given $z_R \leq 0$, L's expected payoff from $z_L \leq z_R$ is:

$$\begin{aligned}\pi_L(z_L, z_R) &= \frac{1}{2\psi}\int_{\rho_0-\psi}^{z_L-z_R}\left(r + \beta\frac{x_1^*}{2}\right)d\rho + \frac{\alpha}{2\psi}\int_{z_L-z_R}^{\min\{\rho_0+\psi, z_L-z_R-\theta z_R\}}\left(\frac{1}{2} + \frac{x_1^*}{2}\right)d\rho \\ &+ \frac{\alpha}{2\psi}\int_{\min\{\rho_0+\psi, z_L-z_R-\theta z_R\}}^{\min\{\rho_0+\psi, z_R-z_L-\theta z_L\}}\left(\frac{1}{2} + \frac{x_2^*}{2}\right)d\rho + \frac{\alpha}{2\psi}\int_{\min\{\rho_0+\psi, z_R-z_L-\theta z_L\}}^{\rho_0+\psi}\left(\frac{1}{2} + \frac{x_3^*}{2}\right)d\rho.\end{aligned} \tag{52}$$

By now standard arguments, we observe that for any $z_L \leq 0$, $z_R \in [z_L, 0]$ is a best response only if with positive probability the swing voter type $x^*(z_L, z_R, \rho)$ is realized strictly to the left of party R's platform z_R, i.e., only if $z_L - z_R - \theta z_R \geq \rho_0 + \psi$. Similarly, for any $z_R \leq 0$, $z_L \leq z_R$ is a best response only if with positive probability the swing voter type $x^*(z_L, z_R, \rho)$ is realized strictly to the left of z_L, i.e., only if $z_R - z_L - \theta z_L < \rho_0 + \psi$.

We therefore focus on platforms satisfying $z_L - z_R - \theta z_R < \rho_0 + \psi$ and $z_R - z_L - \theta z_L < \rho_0 + \psi$, subsequently verifying that these conditions hold at the solutions we characterize, below. Assumption 2 then implies that R's objective (51) is strictly concave in z_R. Solving the first-order condition yields:

$$\hat{z}_R^{\text{int}}(z_L) = \frac{-\alpha(\theta + \rho_0 - \psi) - \beta(\rho_0 + \psi) + 2\theta r + z_L(\alpha - \beta)}{\alpha + \beta(2\theta - 1)}. \tag{53}$$

Similarly, L's objective (52) is strictly concave in z_L. Solving the first-order condition yields:

$$\hat{z}_L^{\text{int}}(z_R) = \frac{-\alpha(\theta + \rho_0 + \psi) + \beta(\psi - \rho_0) + 2\theta r + z_R(\alpha - \beta)}{2\alpha\theta + \alpha - \beta}. \tag{54}$$

Let (z_L^*, z_R^*) denote an equilibrium pair of platforms.

First, we identify conditions under which $z_L^* = z_R^* = 0$. We observe that $\hat{z}_L^{\text{int}}(0)$ strictly decreases in ρ_0, and also that $\hat{z}_R^{\text{int}}(0)$ strictly decreases in ρ_0. We find that:

$$\hat{z}_L^{\text{int}}(0) \geq 0 \iff \rho_0 \leq \frac{\theta(2r - \alpha) - (\alpha - \beta)\psi}{\alpha + \beta} \equiv \underline{\rho}_0, \tag{55}$$

and

$$\hat{z}_R^{\text{int}}(0) \geq 0 \iff \rho_0 \leq \frac{\theta(2r - \alpha) + (\alpha - \beta)\psi}{\alpha + \beta} = \rho_0', \tag{56}$$

where Assumption 2 that $\alpha \geq \beta$ implies that $\rho_0' \geq \underline{\rho}_0$. Thus, $z_L^* = z_R^* = 0$ if $\rho_0 \leq \underline{\rho}_0$.

Second, we identify conditions for $z_L^* < z_R^* = 0$. In that case, we have

$$z_L^* = \hat{z}_L^{\text{int}}(0) = \frac{-\alpha(\theta + \rho_0 + \psi) + \beta(\psi - \rho_0) + 2\theta r}{2\alpha\theta + \alpha - \beta}, \tag{57}$$

and further require that $\hat{z}_R^{\text{int}}(\hat{z}_L^{\text{int}}(0)) \geq 0$. We have already shown that $\hat{z}_L^{\text{int}}(0) < 0$ if and only if

$\rho_0 > \underline{\rho}_0$. We also have that

$$\hat{z}_R^{\text{int}}(\hat{z}_L^{\text{int}}(0)) \geq 0 \iff \rho_0 \leq \frac{\theta\left(\alpha\left(\frac{\psi(\alpha-\beta)}{\alpha\theta+\alpha-\beta}-1\right)+2r\right)}{\alpha+\beta} \equiv \overline{\rho}_0. \tag{58}$$

Therefore, $z_L^* < z_R^* = 0$ if $\rho \in (\underline{\rho}_0, \overline{\rho}_0]$.

Third, we identify conditions for $z_L^* < z_R^* < 0$. In that case, we may solve the system of interior solutions, directly, to obtain:

$$\begin{aligned} z_L^* &= \frac{\beta\theta\psi(\beta-\alpha)-(\alpha+\beta(\theta-1))(\alpha(\theta+\rho_0)+\beta\rho_0-2\theta r)}{\theta\left(\alpha^2+2\alpha\beta\theta-\beta^2\right)}, \\ z_R^* &= z_L^* + (\alpha-\beta)\frac{(\alpha+\beta)(\psi-\rho_0)+\theta(2r-\alpha)}{\alpha^2+2\alpha\beta\theta-\beta^2}. \end{aligned} \tag{59}$$

We now verify that for all $\rho_0 \geq 0$, the solution (z_L^*, z_R^*) is an equilibrium. To establish this, we proceed in two steps.

Step 1: verifying interior solutions. We verify that the pair (z_L^*, z_R^*) always satisfies the restrictions that $z_R^* - z_L^* - \theta z_L^* < \rho_0 + \psi$, which, in turn, implies $z_L^* - z_R^* - \theta z_R^* < \rho_0 + \psi$. We write $(z_L^*(\rho_0), z_R^*(\rho_0))$ to emphasize the dependence on R's advantage, ρ_0.

First, consider $\rho_0 \leq \underline{\rho}_0$, so that $z_L^*(\rho_0) = z_R^*(\rho_0) = 0$. In this case, the claim is immediate from $\rho_0 + \psi > 0$.

Second, consider $\rho_0 \in [\underline{\rho}_0, \overline{\rho}_0]$, so that $z_R^*(\rho_0) = 0$, and $z_L^*(\rho_0)$ is given by (57). We therefore want to verify that $\rho_0 + \psi - (-z_L^*(\rho_0) - \theta z_L^*(\rho_0)) > 0$. The left-hand side of this inequality is linear in ρ_0. Since $\rho_0 \in [\underline{\rho}_0, \overline{\rho}_0]$, it is sufficient to verify that

$$\underline{\rho}_0 + \psi - (0 - z_L^*(\underline{\rho}_0) - \theta z_L^*(\underline{\rho}_0)) = \frac{\theta(2r-\alpha)+2\beta\psi}{\alpha+\beta} > 0, \tag{60}$$

and that

$$\overline{\rho}_0 + \psi - (0 - z_L^*(\overline{\rho}_0) - \theta z_L^*(\overline{\rho}_0)) = \frac{\theta\left(\frac{\psi\left(\alpha^2+\beta^2\right)}{\alpha\theta+\alpha-\beta}+2r-\alpha\right)}{\alpha+\beta} > 0. \tag{61}$$

Third, we consider $\rho_0 > \overline{\rho}_0$, so that $z_L^*(\rho_0)$ and $z_R^*(\rho_0)$ are given by (59). Because $(\rho_0 + \psi) - (z_R^*(\rho_0) - z_L^*(\rho_0) - \theta z_L^*(\rho_0))$ strictly increases in ρ_0 it is sufficient to recall that, by expression (61), the difference is strictly positive evaluated at $\overline{\rho}_0$, and thus strictly positive for all $\rho_0 > \overline{\rho}_0$.

Step 2: verifying no "jump" deviations. First, we highlight that if $z_R^*(\rho_0) - \theta z_R^*(\rho_0) \leq \rho_0 + \psi$, then lemmata 1 and 2 imply that the only deviations to consider are by party R to $z_R < z_L^*(\rho_0)$, and by party L to $z_L \in (z_R^*(\rho_0), 0]$. If $\rho_0 \leq \overline{\rho}_0$, then $z_R^*(\rho_0) = 0$, and the inequality holds trivially. To verify that, indeed, $z_R^*(\rho_0) - \theta z_R^*(\rho_0) \leq \rho_0 + \psi$, for $\rho_0 > \overline{\rho}_0$, we observe that:

$$\rho_0 + \psi - (z_R^*(\rho_0) - \theta z_R^*(\rho_0)) = \frac{\theta\psi(\alpha\theta(\alpha+\beta) + \beta(\alpha-\beta)) + \rho_0\left(\alpha\left(\theta^2(\beta-\alpha) + \theta(\alpha+\beta) + \alpha\right) - \beta^2\right)}{\theta\left(\alpha^2 + 2\alpha\beta\theta - \beta^2\right)},$$

and thus the difference is linear in ρ_0. We have

$$\overline{\rho}_0 + \psi - (z_R^*(\overline{\rho}_0) - \theta z_R^*(\overline{\rho}_0)) = \overline{\rho}_0 + \psi > 0 \tag{62}$$

and since Assumption 2 that $\rho_0 - \psi < -1$ implies that $\rho_0 < \psi$, it is sufficient to verify that

$$\psi + \psi - (z_R^*(\psi) - \theta z_R^*(\psi)) = \left(\frac{1}{\theta} + 1\right)\psi > 0. \tag{63}$$

We may therefore invoke lemmata 1 and 2 and restrict attention to only two deviations: by party L to $z_L \in (z_R, 0]$, and by party R to $z_R < z_L$.

No profitable deviation by party L to $z_L \in (z_R^*, 0]$. We have $z_R^* < 0$ if and only if $\rho_0 > \overline{\rho}_0$. The (at most) three swing voter types are given by $x_4^* = \frac{z_R^* - z_L - \rho}{\theta}$, $x_5^* = \frac{-(z_L + z_R^*) - \rho}{\theta - 2}$, and $x_6^* = \frac{z_L - z_R^* - \rho}{\theta}$. Party R wins if $z_L - z_R^* - \rho < 0$, i.e., if $\rho > z_L - z_R^*$. Party L's expected payoff from this deviation is:

$$\begin{aligned}\pi_L(z_L, z_R) = &\frac{1}{2\psi}\int_{\rho_0-\psi}^{\max\{z_L - z_R^* - \theta, \rho_0 - \psi\}}\left(r + \frac{\beta}{2}\right)d\rho + \frac{1}{2\psi}\int_{\max\{z_L - z_R^* - \theta, \rho_0-\psi\}}^{z_L - z_R^*}\left(r + \beta\frac{x_6^*}{2}\right)d\rho \\ &+ \frac{\alpha}{2\psi}\int_{z_L - z_R^*}^{\min\{z_L - z_R^* - \theta z_L, \rho_0+\psi\}}\left(\frac{1}{2} + \frac{x_6^*}{2}\right)d\rho + \frac{\alpha}{2\psi}\int_{\min\{z_L - z_R^* - \theta z_L, \rho_0+\psi\}}^{\min\{z_R^* - z_L - \theta z_R^*, \rho_0+\psi\}}\left(\frac{1}{2} + \frac{x_5^*}{2}\right)d\rho \\ &+ \frac{\alpha}{2\psi}\int_{\min\{z_R^* - z_L - \theta z_R^*, \rho_0+\psi\}}^{\min\{z_R^* - z_L + \theta, \rho_0+\psi\}}\left(\frac{1}{2} + \frac{x_4^*}{2}\right)d\rho. \end{aligned} \tag{64}$$

We have already shown that $z_R^* - z_L^* - \theta z_L^* < \rho_0 + \psi$. Notice that $z_R^* - z_L^* - \theta z_R^* < z_R^* - z_L^* - \theta z_L^*$. And, for $z_L \in (z_R^*, 0]$, we have $z_R^* - z_L - \theta z_R^* < z_R^* - z_L^* - \theta z_R^*$, since $z_L > z_R^*$ implies $z_L > z_L^*$. We conclude that $z_R^* - z_L - \theta z_R^* < \rho_0 + \psi$. This, in turn, implies $z_L - z_R^* - \theta z_L < \rho_0 + \psi$. Assumption 2 that $\theta > \rho_0 + \psi + 1$ is equivalent to $\theta - 1 > \rho_0 + \psi$. This implies that for any $z_R^* < 0$ and $z_L \in (z_R^*, 0]$, $z_R^* - z_L + \theta > \rho_0 + \psi$. This implies that $z_L - z_R^* - \theta < -\rho_0 - \psi < \rho_0 - \psi$, for $\rho_0 > \underline{\rho}_0 > 0$. We then verify that (64) is strictly concave, and yields the same first-order condition as given in (54), which implies that $z_L > z_R^*$ cannot be optimal when $\rho_0 > \overline{\rho}_0$.

No profitable deviation by party R to $z_R < z_L^*$. Party R wins if and only if $\rho > z_L^* - z_R$. R's payoff from this deviation is

$$\begin{aligned}\pi_R(z_L^*, z_R) &= \frac{\alpha}{2\psi}\int_{\max\{\rho_0-\psi, z_L^*-z_R-\theta\}}^{z_L^*-z_R}\left(\frac{1}{2}-\frac{x_6^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{z_L^*-z_R}^{\min\{\rho_0+\psi, z_L^*-z_R-\theta z_L^*\}}\left(r-\beta\frac{x_6^*}{2}\right)d\rho \\ &+ \frac{1}{2\psi}\int_{\min\{\rho_0+\psi, z_L^*-z_R-\theta z_L^*\}}^{\min\{\rho_0+\psi, z_R-z_L^*-\theta z_R\}}\left(r-\beta\frac{x_5^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{\min\{\rho_0+\psi, z_R-z_L^*-\theta z_R\}}^{\min\{\rho_0+\psi, z_R-z_L^*+\theta\}}\left(r-\beta\frac{x_4^*}{2}\right)d\rho \\ &+ \frac{1}{2\psi}\int_{\min\{\rho_0+\psi, z_R-z_L^*+\theta\}}^{\rho_0+\psi}\left(r+\frac{\beta}{2}\right).\end{aligned} \tag{65}$$

By a similar argument to the previous paragraph, Assumption 2 implies that $z_L^* - z_R - \theta < \rho_0 - \psi$, and that $z_R - z_L^* + \theta > \rho_0 + \psi$. Also, by now familiar arguments, $z_R < z_L^*$ is not a best response if with probability one the swing voter's type $x^*(z_L^*, z_R, \rho)$ is realized weakly to the right of z_R. We therefore restrict attention to $z_R < z_L^*$ satisfying the restriction that $z_R - z_L^* - \theta z_R < \rho_0 + \psi$. Under these restrictions, (65) is strictly concave, with first-order condition that is equivalent to the first-order condition identified in expression (53), and which therefore implies that a deviation to $z_R < z_L^*$ is not profitable.

Proof of Corollary 1. In this case, we have $z_R^*(\rho_0) = 0$, so that

$$z_L^*(\rho_0) = \frac{-\alpha(\theta+\rho_0+\psi)+\beta(\psi-\rho_0)+2\theta r}{2\alpha\theta+\alpha-\beta}. \tag{66}$$

We obtain comparative statics for each of the primitives, in turn.

Higher ρ_0. We have $\frac{\partial z_L^*}{\rho_0} = -\frac{\alpha+\beta}{2\alpha\theta+\alpha-\beta} < 0$. Thus, z_L^* *decreases* in ρ_0.

Higher θ. We have

$$\frac{\partial z_L^*}{\partial\theta} = \frac{\alpha(\alpha(2\rho_0+2\psi-1)+2\beta(\rho_0-\psi)+\beta)+2r(\alpha-\beta)}{(2\alpha\theta+\alpha-\beta)^2}. \tag{67}$$

The numerator of this expression strictly increases in ρ_0, and is therefore positive if and only if $\rho_0 \geq -\frac{(\alpha-\beta)(\alpha(2\psi-1)+2r)}{2\alpha(\alpha+\beta)}$. This threshold is strictly negative and thus vacuously satisfied. We conclude that z_L^* *increases* in θ.

Higher α. We have

$$\frac{\partial z_L^*}{\partial\alpha} = \frac{\beta(2\theta\rho_0-2\theta\psi+\theta+2\rho_0)-2\theta(2\theta+1)r}{(2\alpha\theta+\alpha-\beta)^2}. \tag{68}$$

Calling $\nu(\rho_0)$ the numerator of this expression, we find that $\nu(\rho_0)$ strictly increases in ρ_0, and that $\nu(\overline{\rho}_0) < 0$. Thus, z_L^* strictly *decreases* in α.

Higher ψ. $\frac{\partial z_L^*}{\partial \psi} = \frac{\beta-\alpha}{2\alpha\theta+\alpha-\beta} < 0$.

Higher r. We have $\frac{\partial z_L^*}{\partial r} = \frac{2\theta}{2\alpha\theta+\alpha-\beta} > 0$. $\square$

Proof of Corollaries 2, 3 and 4 and 5.

$$z_L^* = \frac{\beta\theta\psi(\beta-\alpha) - (\alpha+\beta(\theta-1))(\alpha(\theta+\rho_0)+\beta\rho_0-2\theta r)}{\theta\left(\alpha^2+2\alpha\beta\theta-\beta^2\right)},$$
$$z_R^* = z_L^* + (\alpha-\beta)\frac{((\alpha+\beta)(\psi-\rho_0)+\theta(2r-\alpha))}{\alpha^2+2\alpha\beta\theta-\beta^2}. \tag{69}$$

We obtain comparative statics for each of the primitives, in turn.

Higher ρ_0. We find that $\frac{\partial z_L^*}{\partial \rho_0} = -\frac{(\alpha+\beta)(\alpha+\beta(\theta-1))}{\theta(\alpha^2+2\alpha\beta\theta-\beta^2)} < 0$. Moreover, $\frac{\partial [z_R^*-z_L^*]}{\partial \rho_0} = \frac{\beta^2-\alpha^2}{\alpha^2+2\alpha\beta\theta-\beta^2} < 0$, which implies that z_R^* also decreases in ρ_0, and faster than z_L^*. $\square$

Higher α. We start with the platform z_L^*. We find that $\frac{\partial z_L^*}{\partial \alpha}$ can be written as a quotient with a strictly positive denominator, and a numerator that we call $\nu(r,\rho_0)$, which strictly decreases in r. Recalling that Assumption 1 states $r > \frac{1}{2}\left(\alpha+\frac{\psi}{\theta}(\alpha-\beta)\right)$, we find that $\nu(\frac{1}{2}\left(\alpha+\frac{\psi}{\theta}(\alpha-\beta)\right), \rho_0)$ is linear in ρ_0. Straightforward algebra (omitted) verifies that $\nu(\frac{1}{2}\left(\alpha+\frac{\psi}{\theta}(\alpha-\beta)\right), \overline{\rho}_0) < 0$ and $\nu(\frac{1}{2}\left(\alpha+\frac{\psi}{\theta}(\alpha-\beta)\right), \psi) < 0$. Thus, $\frac{\partial z_L^*}{\partial \alpha} < 0$.

We next consider the platform z_R^*. We find that $\frac{\partial z_R^*}{\partial \alpha}$ can be written as a quotient with a strictly positive denominator, and a numerator that we call $\mu(\rho_0,\psi)$, which strictly decreases in ρ_0. Therefore there exists $\hat{\rho}_0$ such that $\mu(\rho_0,\psi) \geq 0$ if and only if $\rho_0 \leq \hat{\rho}_0$. Thus, $\rho > \hat{\rho}_0$ implies that z_R^* *decreases* in α, while $\rho \leq \hat{\rho}_0$ implies that z_R^* *increases* in α.

Next, we establish that $\hat{\rho}_0 < \overline{\rho}_0$. Straightforward substitution (omitted) verifies that $\overline{\rho}_0 - \hat{\rho}_0$ strictly increases in r, and that

$$\overline{\rho}_0 - \hat{\rho}_0 > 0 \iff r > \frac{1}{2}\beta\left(\frac{\psi\left(2\alpha^2\theta+(\alpha-\beta)^2\right)}{(\alpha\theta+\alpha-\beta)^2} - 1\right) \equiv \hat{r}. \tag{70}$$

Assumption 1 says that $r > \frac{1}{2}\left(\alpha+\frac{\psi}{\theta}(\alpha-\beta)\right) \equiv r^*$. We establish that $r^* > \hat{r}$. We observe that $r^* - \hat{r}$ is linear in ψ, and strictly positive positive evaluated at $\psi = 0$ and $\psi = \theta$. Because Assumption 2 that $\theta > \rho_0 + \psi + 1$ implies that $\psi < \theta$, we conclude that $r^* > \hat{r}$. Thus, $\overline{\rho}_0 > \hat{\rho}_0$, and z_R^* *decreases* in α.

Higher r. $\frac{\partial z_L^*}{\partial r} = \frac{2(\alpha+\beta(\theta-1))}{\alpha^2+2\alpha\beta\theta-\beta^2} > 0$, and $\frac{\partial z_R^*}{\partial r} = \frac{2(\alpha\theta+\alpha-\beta)}{\alpha^2+2\alpha\beta\theta-\beta^2} > 0$, and $\frac{\partial [z_R^*-z_L^*]}{\partial r} = \frac{2\theta(\alpha-\beta)}{\alpha^2+2\alpha\beta\theta-\beta^2} > 0$. $\square$

Higher ψ. $\frac{\partial z_L^*}{\partial \psi} = \frac{\beta(\alpha-\beta)}{\beta^2-\alpha(\alpha+2\beta\theta)} < 0$, and $\frac{\partial z_R^*}{\partial \psi} = \frac{\alpha(\alpha-\beta)}{\alpha^2+2\alpha\beta\theta-\beta^2} > 0$. $\square$

Supplementary Appendix to *The Race to the Base* (For Online Publication)

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Appendix A: Returning to Base.

In this Appendix, we focus on a setting in which $r < \frac{1}{2}\left(\alpha + \frac{\psi}{\theta}(\alpha - \beta)\right)$, i.e., in which Assumption 1 fails. We establish the following result.

Proposition 5. *If $r < \frac{1}{2}\left(\alpha + \frac{\psi}{\theta}(\alpha - \beta)\right)$, and party R's advantage is not too large in the sense that*

$$\rho_0 < \frac{\alpha\theta(\alpha\theta + \psi(\alpha - \beta) - 2r\theta)}{(\alpha + \beta)(\alpha - \beta + \alpha\theta)}, \tag{71}$$

then there exists a unique pure strategy equilibrium, in which party L retreats to its base:

$$z_L^*(\rho_0) = \frac{\alpha\theta(\theta(2r - \alpha) - \alpha(\psi + \rho_0) + \beta(\psi - \rho_0)) - (\alpha^2 - \beta^2)\rho_0}{2\alpha\theta(\alpha\theta + \alpha - \beta)} < 0, \tag{72}$$

and party R also retreats to its base:

$$z_R^*(\rho_0) = \ z_L^*(\rho_0) + \frac{\alpha\theta + \psi(\alpha - \beta) - 2\theta r}{\alpha\theta + \alpha - \beta} > 0. \tag{73}$$

Proof of Proposition 5. We characterize the unique equilibrium, which satisfies $z_L \leq 0 \leq z_R$. First, we rule out other possible equilibria.

Step 1: No equilibrium in which $z_R \leq z_L \leq 0$, with at least one strict inequality. The proof replicates verbatim the proof of Lemma 4.

Step 2: No equilibrium in which $z_L \leq z_R \leq 0$. Suppose, first, $z_L = z_R = 0$. Letting $\hat{z}_L^{\text{int}}(z_R)$ denote L's interior solution on $[-1, z_R]$ given $z_R \leq 0$, we showed in our benchmark proofs that:

$$\hat{z}_L^{\text{int}}(0) \geq 0 \iff \rho_0 \leq \frac{\theta(2r - \alpha) - (\alpha - \beta)\psi}{\alpha + \beta} \equiv \underline{\rho}_0. \tag{74}$$

If $r < \frac{1}{2}\left(\alpha + \frac{\psi}{\theta}(\alpha - \beta)\right)$, then $\underline{\rho}_0 < 0$, so that L strictly prefers to deviate to the left of zero, for all $\rho_0 \geq 0$. Suppose, next, that $z_L < z_R = 0$. Then, we showed in our benchmark proofs that L's best response (that we also showed is interior) is

$$\hat{z}_L^{\text{int}}(0) = \frac{-\alpha(\theta + \rho_0 + \psi) + \beta(\psi - \rho_0) + 2\theta r}{2\alpha\theta + \alpha - \beta}. \tag{75}$$

Consider, however, R's value from a platform $z_R > 0$. For $z_R > 0$ sufficiently close to zero, this payoff is:

$$\pi_R(z_L, z_R) = \frac{\alpha}{2\psi}\int_{\rho_0-\psi}^{\hat{z}_L^{\text{int}}(0)-z_R-\theta z_R}\left(\frac{1}{2}-\frac{x_1^*}{2}\right)d\rho + \frac{\alpha}{2\psi}\int_{\hat{z}_L^{\text{int}}(0)-z_R-\theta z_R}^{\hat{z}_L^{\text{int}}(0)+z_R}\left(\frac{1}{2}-\frac{x_2^*}{2}\right)d\rho$$
$$+ \frac{1}{2\psi}\int_{\hat{z}_L^{\text{int}}(0)+z_R}^{\min\{z_R-\hat{z}_L^{\text{int}}(0)-\theta\hat{z}_L^{\text{int}}(0),\rho_0+\psi\}}\left(r-\beta\frac{x_2^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{\min\{z_R-\hat{z}_L^{\text{int}}(0)-\theta\hat{z}_L^{\text{int}}(0),\rho_0+\psi\}}^{\rho_0+\psi}\left(r-\beta\frac{x_3^*}{2}\right)d\rho. \tag{76}$$

We show, first, that $-\hat{z}_L^{\text{int}}(0)-\theta\hat{z}_L^{\text{int}}(0) < \rho_0+\psi$. Straightforward substitution establishes:

$$-\hat{z}_L^{\text{int}}(0)-\theta\hat{z}_L^{\text{int}}(0)-(\rho_0+\psi) = \frac{2\beta\rho_0+\theta(-\psi(\alpha+\beta)+\alpha\theta-\alpha\rho_0+\alpha+\beta\rho_0-2(\theta+1)r)}{2\alpha\theta+\alpha-\beta}, \tag{77}$$

which is linear in ρ_0, and easily verified to be strictly negative evaluated at $\rho_0 = 0$ and $\rho_0 = \frac{\alpha\theta(\alpha\theta+\psi(\alpha-\beta)-2r\theta)}{(\alpha+\beta)(\alpha-\beta+\alpha\theta)}$. Using the appropriate limits of integration in (76), we have that

$$\frac{\partial\pi_R(\hat{z}_L^{\text{int}}(0),0)}{\partial z_R} = \frac{\alpha^2(\theta(\theta-\rho_0+\psi)-\rho_0)-\alpha\beta\theta(\rho_0+\psi)+\beta^2\rho_0-2\alpha\theta^2 r}{2\theta\psi(2\alpha\theta+\alpha-\beta)}, \tag{78}$$

which strictly decreases in ρ_0, and satisfies

$$\left.\frac{\partial\pi_R(\hat{z}_L^{\text{int}}(0),0)}{\partial z_R}\right|_{\rho_0=\frac{\alpha\theta(\alpha\theta+\psi(\alpha-\beta)-2r\theta)}{(\alpha+\beta)(\alpha-\beta+\alpha\theta)}} = 0. \tag{79}$$

Thus, for any $\rho_0 < \frac{\alpha\theta(\alpha\theta+\psi(\alpha-\beta)-2r\theta)}{(\alpha+\beta)(\alpha-\beta+\alpha\theta)}$, R strictly prefers to deviate to a platform strictly to the right of zero.

Suppose, finally, $z_L < z_R < 0$. We showed earlier that this implies $\rho_0 > \overline{\rho}_0$, where $\overline{\rho}_0$ is defined in (58). Straightforward algebra verifies that $\overline{\rho}_0 > \hat{\rho}_0$, implying that $\rho_0 > \overline{\rho}_0$ violating the parameter restriction that $\rho_0 < \hat{\rho}_0$.

Step 3: No equilibrium in which $0 \leq z_L \leq z_R$, with at least one strict inequality. Party L's payoff from $z_L \in [0, z_R]$ is

$$\pi_L(z_L, z_R) = \frac{1}{2\psi}\int_{\rho_0-\psi}^{\max\{z_L-z_R-\theta z_R,\rho_0-\psi\}}\left(r+\beta\frac{x_1^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{\max\{z_L-z_R-\theta z_R,\rho_0-\psi\}}^{\max\{z_R-z_L-\theta z_L,\rho_0-\psi\}}\left(r+\beta\frac{x_2^*}{2}\right)d\rho$$
$$+ \frac{1}{2\psi}\int_{\max\{z_R-z_L-\theta z_L,\rho_0-\psi\}}^{z_R-z_L}\left(r+\beta\frac{x_3^*}{2}\right)d\rho + \frac{\alpha}{2\psi}\int_{z_R-z_L}^{\rho_0+\psi}\left(\frac{1}{2}+\frac{x_3^*}{2}\right)d\rho. \tag{80}$$

Likewise, R's payoff from $z_R \geq z_L$ is:

$$\begin{aligned}\pi_R(z_L, z_R) &= \frac{\alpha}{2\psi}\int_{\rho_0-\psi}^{\max\{z_L-z_R-\theta z_R,\rho_0-\psi\}}\left(\frac{1}{2}-\frac{x_1^*}{2}\right)d\rho + \frac{\alpha}{2\psi}\int_{\max\{z_L-z_R-\theta z_R,\rho_0-\psi\}}^{\max\{z_R-z_L-\theta z_L,\rho_0-\psi\}}\left(\frac{1}{2}-\frac{x_2^*}{2}\right)d\rho \\ &+ \frac{\alpha}{2\psi}\int_{\max\{z_R-z_L-\theta z_L,\rho_0-\psi\}}^{z_R-z_L}\left(\frac{1}{2}-\frac{x_3^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{z_R-z_L}^{\rho_0+\psi}\left(r-\beta\frac{x_3^*}{2}\right)d\rho. \end{aligned} \tag{81}$$

Similar arguments to those used in benchmark proofs imply that we must have $z_L - z_R - \theta z_R > \rho_0 - \psi$, and thus $z_R - z_L - \theta z_L > \rho_0 - \psi$, in an equilibrium. Both objectives are concave on their implied domains. We first argue that we cannot have $0 < z_L = z_R$. To see this, observe that

$$\frac{\partial \pi_L(z_R, z_R)}{\partial z_L} = -\frac{\alpha(-\theta+\rho_0+\psi)+2\theta r+\beta(\rho_0-\psi+2\theta z_R)}{4\theta\psi}, \tag{82}$$

which strictly decreases in z_R and ρ_0, and satisfies $\frac{\partial \pi_L(z_R,z_R)}{\partial z_L}|_{\rho_0=z_R=0} = \frac{\alpha(\theta-\psi)+\beta\psi-2\theta r}{4\theta\psi}$, which is strictly negative if and only if $r > \frac{\alpha}{2} - \frac{\psi}{2\theta}(\alpha-\beta)$, which holds under $\alpha > \beta$ and $r > \frac{\alpha}{2}$.

We next argue that we cannot have $0 < z_L < z_R$. To show this, we characterize unique interior best responses, using (80) and (81):

$$\hat{z}_L(z_R) = \frac{\alpha(\theta-\rho_0-\psi)-\beta(\rho_0-\psi)-2\theta r+z_R(\alpha-\beta)}{\alpha+\beta(2\theta-1)}, \tag{83}$$

and

$$\hat{z}_R(z_L) = \frac{\alpha(\theta-\rho_0+\psi)-\beta(\rho_0+\psi)-2\theta r+z_L(\alpha-\beta)}{2\alpha\theta+\alpha-\beta}. \tag{84}$$

Solving the pair of best responses, we obtain:

$$z_L^* = \frac{\alpha\theta\psi(\beta-\alpha)-(\alpha\theta+\alpha-\beta)(\rho_0(\alpha+\beta)+\theta(2r-\alpha))}{\theta\left(\alpha^2+2\alpha\beta\theta-\beta^2\right)} < 0, \tag{85}$$

a contradiction. Suppose, finally, that $0 = z_L < z_R$. R's interior best response to $z_L = 0$ is:

$$\hat{z}_R(0) = \frac{\alpha(\theta-\rho_0+\psi)-\beta(\rho_0+\psi)-2\theta r}{2\alpha\theta+\alpha-\beta}. \tag{86}$$

It is straightforward to verify that L's payoff strictly decreases in $z_L \in [0, z_R]$. We further show that a platform $z_L < 0$ is strictly preferred to $z_L = 0$. To see this, note that L's payoff from a platform

$z_L < 0$ when $z_R > 0$ is:

$$\pi_L(z_L, z_R) = \frac{1}{2\psi}\int_{\rho_0-\psi}^{\max\{z_L-z_R-\theta z_R,\rho_0-\psi\}}\left(r+\beta\frac{x_1^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{\max\{z_L-z_R-\theta z_R,\rho_0-\psi\}}^{z_L+z_R}\left(r+\beta\frac{x_2^*}{2}\right)d\rho$$
$$+\frac{\alpha}{2\psi}\int_{z_L+z_R}^{\min\{z_R-z_L-\theta z_L,\rho_0+\psi\}}\left(\frac{1}{2}+\frac{x_2^*}{2}\right)d\rho + \frac{\alpha}{2\psi}\int_{\min\{z_R-z_L-\theta z_L,\rho_0+\psi\}}^{\rho_0+\psi}\left(\frac{1}{2}+\frac{x_3^*}{2}\right)d\rho. \quad (87)$$

For $z_L < 0$ sufficiently close to zero, we have $z_R - z_L - \theta z_L < \rho_0 + \psi$. If $-\theta z_R \geq \rho_0 - \psi$, then R has a profitable deviation to $z_R' < z_R$. Suppose, instead, $-\theta z_R < \rho_0 - \psi$. We find that

$$\frac{\partial \pi_L(0, \frac{\alpha(\theta-\rho_0+\psi)-\beta(\rho_0+\psi)-2\theta r}{2\alpha\theta+\alpha-\beta})}{\partial z_L} = \frac{\alpha^2(-(\theta(\theta+\rho_0+\psi)+\rho_0)) + \alpha\beta\theta(\psi-\rho_0) + \beta^2\rho_0 + 2\alpha\theta^2 r}{2\theta\psi(2\alpha\theta+\alpha-\beta)}, \quad (88)$$

which strictly decreases in $\rho_0 \geq 0$, and is strictly negative when evaluated at $\rho_0 = 0$, under the parameter restriction $r < \frac{\alpha}{2} + \frac{\psi}{2\theta}(\alpha - \beta)$.

Step 4: No equilibrium in which $0 \leq z_R \leq z_L$, with at least one strict inequality. To rule out $z_L > z_R \geq 0$, we may replicate the argument of Lemma 2, Case 2. Similarly, to rule out $z_L = z_R > 0$, we may replicate the argument of Lemma 3, Case 1.

Step 5: No equilibrium in which $z_R \leq 0 \leq z_L$, with at least one strict inequality. If $z_R \leq 0$ is a best response to $z_L \geq 0$, then we must have $z_R - z_L - \theta z_R \leq \rho_0 + \psi$. It is then straightforward to extend the argument of Lemma 2 in our benchmark model to verify that L's payoff strictly decreases in $z_L \geq 0$. To rule out $z_R < z_L = 0$, it suffices to replicate verbatim the proof of Lemma 4.

Characterizing the equilibrium. We now verify that there exists an equilibrium in which $z_L \leq 0 \leq z_R$. The (at most) three swing voter types are given by $x_1^* = \frac{z_L - z_R - \rho}{\theta}$, $x_2^* = \frac{z_L + z_R - \rho}{2+\theta}$ and $x_3^* = \frac{z_R - z_L - \rho}{\theta}$. Assumption 2 that $\theta > \rho_0 + \psi$ implies that $x_1^* \leq 1$ and $x_3^* \geq -1$ for all $\rho \in [\rho_0 - \psi, \rho_0 + \psi]$. Finally, party R wins if and only if $\rho > z_L - z_R$. Given $z_L \leq 0$, R's expected payoff from $z_R \in [z_L, 0]$ is therefore:

$$\pi_R(z_L, z_R) = \frac{\alpha}{2\psi}\int_{\rho_0-\psi}^{\max\{z_L-z_R-\theta z_R,\rho_0-\psi\}}\left(\frac{1}{2}-\frac{x_1^*}{2}\right)d\rho + \frac{\alpha}{2\psi}\int_{\max\{z_L-z_R-\theta z_R,\rho_0-\psi\}}^{z_L+z_R}\left(\frac{1}{2}-\frac{x_2^*}{2}\right)d\rho$$
$$+\frac{1}{2\psi}\int_{z_L+z_R}^{\min\{z_R-z_L-\theta z_L,\rho_0+\psi\}}\left(r-\beta\frac{x_2^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{\min\{z_R-z_L-\theta z_L,\rho_0+\psi\}}^{\rho_0+\psi}\left(r-\beta\frac{x_3^*}{2}\right)d\rho. \quad (89)$$

L's corresponding payoff is

$$\pi_L(z_L, z_R) = \frac{1}{2\psi}\int_{\rho_0-\psi}^{\max\{z_L-z_R-\theta z_R,\rho_0-\psi\}} \left(r+\beta\frac{x_1^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{\max\{z_L-z_R-\theta z_R,\rho_0-\psi\}}^{z_L+z_R} \left(r+\beta\frac{x_2^*}{2}\right)d\rho$$
$$+ \frac{\alpha}{2\psi}\int_{z_L+z_R}^{\min\{z_R-z_L-\theta z_L,\rho_0+\psi\}} \left(\frac{1}{2}+\frac{x_2^*}{2}\right)d\rho + \frac{\alpha}{2\psi}\int_{\min\{z_R-z_L-\theta z_L,\rho_0+\psi\}}^{\rho_0+\psi} \left(\frac{1}{2}+\frac{x_3^*}{2}\right)d\rho. \quad (90)$$

By now standard arguments, we must have $z_L - z_R - \theta z_R > \rho_0 - \psi$ and $z_R - z_L - \theta z_L < \rho_0 + \psi$ in any equilibrium. We therefore solve for equilibrium under the presumption that both strict inequalities hold, and then verify that they indeed hold at the solutions we derive, below. Both objectives are strictly concave, and the corresponding system of first-order conditions yields a unique solution (z_L^*, z_R^*) as given in the statement of the Proposition. We observe that $z_R^*(\rho_0)$ decreases in ρ_0, and is strictly positive so long as ρ_0 is strictly less than the cut-off in the Proposition. Similarly, we observe that $z_L^*(\rho_0)$ strictly decreases in ρ_0, and $z_L^*(0) < 0$ so long as $r < \frac{\alpha}{2} + \frac{\psi}{2\theta}(\alpha - \beta)$.

Verifying interior solutions. We first verify that $z_L^*(\rho_0) - z_R^*(\rho_0) - \theta z_R^*(\rho_0) - (\rho_0 - \psi) > 0$. Straightforward algebra yields that this difference strictly *decreases* in ρ_0, and because $\rho_0 < \frac{\alpha\theta(\alpha(\theta+\psi)-\beta\psi-2\theta r)}{(\alpha+\beta)(\alpha\theta+\alpha-\beta)} \equiv \rho_0^*$, it is sufficient to observe that

$$z_L^*(\rho_0^*) - z_R^*(\rho_0^*) - \theta z_R^*(\rho_0^*) - (\rho_0^* - \psi) = \frac{2\alpha\beta\theta\psi + \theta(2r-\alpha)(\alpha\theta+\alpha+\beta)}{(\alpha+\beta)(\alpha\theta+\alpha-\beta)} > 0. \quad (91)$$

We next verify that $\rho_0 + \psi - (z_R^*(\rho_0) - z_L^*(\rho_0) - \theta z_L^*(\rho_0)) > 0$. Straightforward algebra yields that this difference strictly *increases* in ρ_0, and because $\rho_0 \geq 0$, it is sufficient to observe that

$$0 + \psi - (z_R^*(0) - z_L^*(0) - \theta z_L^*(0)) = \frac{\theta\psi(\alpha+\beta) + \theta(\theta+2)(2r-\alpha)}{2(\alpha\theta+\alpha-\beta)} > 0. \quad (92)$$

Verifying no "jump" deviations. We consider four possible deviations: to $z_L \in (0, z_R^*]$, to $z_L \in (z_R^*, 1]$, to $z_R \in [-1, z_L^*)$, and to $z_R \in [z_L^*, 0)$.

Case 1: $z_R \in [z_L^*, 0)$. R's payoff from this platform is:

$$\pi_R(z_L^*, z_R) = \frac{\alpha}{2\psi}\int_{\rho_0-\psi}^{z_L^*-z_R} \left(\frac{1}{2}-\frac{x_1^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{z_L^*-z_R}^{\min\{\rho_0+\psi,z_L^*-z_R-\theta z_R\}} \left(r-\beta\frac{x_1^*}{2}\right)d\rho$$
$$+ \frac{1}{2\psi}\int_{\min\{\rho_0+\psi,z_L^*-z_R-\theta z_R\}}^{\min\{\rho_0+\psi,z_R-z_L^*-\theta z_L^*\}} \left(r-\beta\frac{x_2^*}{2}\right)d\rho + \frac{1}{2\psi}\int_{\min\{\rho_0+\psi,z_R-z_L^*-\theta z_L^*\}}^{\rho_0+\psi} \left(r-\beta\frac{x_3^*}{2}\right)d\rho. \quad (93)$$

Recall that $z_R^* - z_L^* - \theta z_L^* < \rho_0 + \psi$, and $z_R^* > 0$, implying that for any $z_R < 0$, $z_R - z_L^* - \theta z_L^* < \rho_0 + \psi$. In turn, this implies $z_L^* - z_R - \theta z_R < \rho_0 + \psi$. R's optimal interior platform on this interval is therefore:

$$z_R(\rho_0) = \frac{-\alpha(\theta + \rho_0 - \psi) - \beta(\rho_0 + \psi) + 2\theta r + z_L^*(\rho_0)(\alpha - \beta)}{\alpha + \beta(2\theta - 1)}. \tag{94}$$

It is straightforward to verify that $z_R(\rho_0)$ strictly *decreases* in ρ_0, and recalling that $\rho_0 < \frac{\alpha\theta(\alpha(\theta+\psi)-\beta\psi-2\theta r)}{(\alpha+\beta)(\alpha\theta+\alpha-\beta)} \equiv \rho_0^*$, we verify $z_R(\rho_0^*) = \frac{2\theta(2r-\alpha)}{\alpha+\beta(2\theta-1)} > 0$, so that a deviation to $z_R \in [z_L, 0)$ cannot be optimal.

Case 2: $z_R \in [-1, z_L^*]$. Party R wins if and only if $\rho > z_L^* - z_R$. R's payoff from this deviation is

$$\begin{aligned}
\pi_R(z_L^*, z_R) = & \frac{\alpha}{2\psi}\int_{\max\{\rho_0-\psi, z_L^*-z_R-\theta\}}^{z_L^*-z_R} \left(\frac{1}{2} - \frac{x_6^*}{2}\right) d\rho + \frac{1}{2\psi}\int_{z_L^*-z_R}^{\min\{\rho_0+\psi, z_L^*-z_R-\theta z_L^*\}} \left(r - \beta\frac{x_6^*}{2}\right) d\rho \\
& + \frac{1}{2\psi}\int_{\min\{\rho_0+\psi, z_L^*-z_R-\theta z_L^*\}}^{\min\{\rho_0+\psi, z_R-z_L^*-\theta z_R\}} \left(r - \beta\frac{x_5^*}{2}\right) d\rho + \frac{1}{2\psi}\int_{\min\{\rho_0+\psi, z_R-z_L^*-\theta z_R\}}^{\min\{\rho_0+\psi, z_R-z_L^*+\theta\}} \left(r - \beta\frac{x_4^*}{2}\right) d\rho \\
& + \frac{1}{2\psi}\int_{\min\{\rho_0+\psi, z_R-z_L^*+\theta\}}^{\rho_0+\psi} \left(r + \frac{\beta}{2}\right).
\end{aligned} \tag{95}$$

We first claim $z_L^* - z_R - \theta < \rho_0 - \psi$, i.e., $\theta > z_L^* - z_R - \rho_0 + \psi$. To see this, note that $\rho_0 - \psi < -1$ and $\theta > \rho_0 + \psi + 1$, we have that $\theta > \rho_0 + \psi + 1 > -\rho_0 + \psi + (z_L^* - z_R)$. Similarly, $\theta - 1 > \rho_0 + \psi$ implies $z_R - z_L^* + \theta > \rho_0 + \psi$. If $-\theta z_L^* \geq \rho_0 + \psi$, the non-profitability of $z_R < z_L^*$ is immediate. Suppose, instead, $-\theta z_L^* > \rho_0 + \psi$. Then we may restrict attention to z_R such that $z_R - z_L^* - \theta z_R < \rho_0 + \psi$, i.e., $z_R \in [\min\{-1, \frac{-(z_L+\rho_0+\psi)}{\theta-1}\}, z_L]$. This implies that (95) is strictly concave, with a first-order condition that is equivalent to the first-order condition identified in expression (94), and which therefore implies that a deviation to $z_R < z_L^*$ is not profitable.

Case 3: $z_L \in (0, z_R^*]$. Party L's payoff from this deviation is

$$\begin{aligned}
\pi_L(z_L, z_R^*) = & \frac{1}{2\psi}\int_{\rho_0-\psi}^{\max\{z_L-z_R^*-\theta z_R^*, \rho_0-\psi\}} \left(r + \beta\frac{x_1^*}{2}\right) d\rho + \frac{1}{2\psi}\int_{\max\{z_L-z_R^*-\theta z_R^*, \rho_0-\psi\}}^{\max\{z_R^*-z_L-\theta z_L, \rho_0-\psi\}} \left(r + \beta\frac{x_2^*}{2}\right) d\rho \\
& + \frac{1}{2\psi}\int_{\max\{z_R^*-z_L-\theta z_L, \rho_0-\psi\}}^{z_R^*-z_L} \left(r + \beta\frac{x_3^*}{2}\right) d\rho + \frac{\alpha}{2\psi}\int_{z_R^*-z_L}^{\rho_0+\psi} \left(\frac{1}{2} + \frac{x_3^*}{2}\right) d\rho.
\end{aligned} \tag{96}$$

Because $z_L^* - z_R^* - \theta z_R^* > \rho_0 - \psi$, we have $z_L - z_R^* - \theta z_R^* > \rho_0 - \psi$, since $z_L^* < 0 < z_L$. This, in turn, yields $z_R^* - z_L - \theta z_L > \rho_0 - \psi$. We obtain L's optimal platform on this domain:

$$z_L(\rho_0) = \frac{\alpha(\theta - \rho_0 - \psi) - \beta(\rho_0 - \psi) - 2\theta r + z_R^*(\rho_0)(\alpha - \beta)}{\alpha + \beta(2\theta - 1)}, \tag{97}$$

It is straightforward to verify that $z_L(\rho_0)$ strictly *decreases* in $\rho_0 \geq 0$, and that $z_L(0) < 0$, so that $z_L > 0$ cannot be optimal.

Case 4: $z_L \in [z_R^*, 1]$. The argument ruling out a deviation to $z_L > z_R^*$ replicates that for Lemma 2, Case 2. Similarly, to rule out $z_L = z_R^*$, we may replicate the argument of Lemma 3, Case 1.

Comparative Statics. We provide comparative statics on a subset of primitives.

Higher ρ_0. We have $\frac{\partial z_L^*}{\partial \rho_0} = \frac{\partial z_R^*}{\partial \rho_0} = -\frac{\alpha+\beta}{2\alpha\theta} < 0$.

Higher α. We have

$$\frac{\partial z_R^*}{\partial \alpha} = \frac{\alpha^2\theta^2(\beta(\psi-1)+2(\theta+1)r)+\beta\rho_0(\alpha\theta+\alpha-\beta)^2}{2\alpha^2\theta(\alpha\theta+\alpha-\beta)^2}, \tag{98}$$

which is strictly positive. However:

$$\frac{\partial z_L^*}{\partial \alpha} = \frac{\alpha^2\theta^2(\beta(-\psi)+\beta-2(\theta+1)r)+\beta\rho_0(\alpha\theta+\alpha-\beta)^2}{2\alpha^2\theta(\alpha\theta+\alpha-\beta)^2}. \tag{99}$$

It is straightforward to observe that (99) *increases* in ρ_0. We evaluate (99) at $\rho_0 = \frac{\alpha\theta(\alpha(\theta+\psi)-\beta\psi-2\theta r)}{(\alpha+\beta)(\alpha\theta+\alpha-\beta)}$, its highest value under the Proposition, obtaining:

$$\left.\frac{\partial z_L^*}{\partial \alpha}\right|_{\rho_0=\frac{\alpha\theta(\alpha(\theta+\psi)-\beta\psi-2\theta r)}{(\alpha+\beta)(\alpha\theta+\alpha-\beta)}} = \frac{\frac{\beta(\alpha\theta+\alpha-\beta)(\alpha(\theta+\psi)-\beta\psi-2\theta r)}{(\alpha+\beta)}+\alpha\theta(\beta(-\psi)+\beta-2(\theta+1)r)}{2(\alpha\theta+\alpha-\beta)^2}. \tag{100}$$

Noting that (100) *decreases* in r, we substitute $r = \frac{\alpha}{2}$, and after re-arranging, find that (100) is strictly negative if and only

$$\beta\psi(\beta-\alpha)(\beta-\alpha(\theta+1))-\alpha\theta(\alpha+\beta)(\alpha(\theta+1)+\beta(\psi-1)) < 0. \tag{101}$$

Expanding the LHS of (101) yields:

$$\begin{aligned}&-\alpha^2\theta(\alpha(\theta+1)+\beta(\psi-1))-\alpha\beta\psi(\beta-\alpha(\theta+1))\\ &\quad+\underbrace{\alpha(-\beta)\theta(\alpha(\theta+1)+\beta(\psi-1))+\beta^2\psi(\beta-\alpha(\theta+1))}_{<0},\end{aligned} \tag{102}$$

so that it is sufficient to verify that the first term of (102) is strictly negative. Rearranging the first term of (102)

$$- \alpha \left(\alpha^2 \theta^2 + \alpha^2 \theta - \alpha \beta (\theta + \psi) + \beta^2 \psi \right), \tag{103}$$

which is indeed strictly negative under $\alpha \geq \beta$ and $\theta > \psi$. We conclude that z_L^* strictly *decreases* α. Finally, we have:

$$\frac{\partial(.5(z_L^* + z_R^*))}{\partial \alpha} = \frac{\rho_0 \beta}{2\alpha^2 \theta} > 0, \quad \frac{\partial^2(.5(z_L^* + z_R^*))}{\partial \alpha \partial \rho_0} = \frac{\beta}{2\alpha^2 \theta} > 0. \tag{104}$$

Appendix B: Policy-Motivated Justifications of $\alpha \geq \beta$

B1: Parties Represent Their Constituents. Our benchmark model treats parties as a single, decisive agent. In practice, parties may consist of factions that are differentiated by their political goals. To illustrate how the reduced form payoff function in our main presentation can be justified, we recognize that a party's electoral strategy partly determines which voters support the party, but the set of voters that are expected to support a party also determine the party's electoral strategy. It is a perspective—and model formulation—that was originally introduced in theories of party formation and electoral competition with endogenous parties by Baron (1993) and Roemer (2001).[20]

We assume that whichever party wins the election implements its platform. Recalling that $d_L \in [0,1]$ is the share of districts (i.e., legislative seats) won by party L, we denote the winning policy

$$z^*(d_L) = \begin{cases} z_R & \text{if } d_L < \frac{1}{2} \\ z_L & \text{if } d_L \geq \frac{1}{2}. \end{cases} \tag{105}$$

For any district with median $m \in [-1,1]$, the total welfare of voters in that district is:

$$v(m, z_L, z_R, d_L) = \frac{1}{2Z} \int_{m-Z}^{m+Z} -|x - z^*(d_L)| \, dx. \tag{106}$$

For any $x^* \in [-1,1]$, a district with median type $m \in [-1, x^*]$ is subsequently represented by a member of party L, while a district with median type $m \in (x^*, 1]$ is represented by a member of party R. Because $d_L = \frac{1+x^*}{2}$, we may define the welfare of constituents served by L's representatives as:

$$W_L(d_L, z_L, z_R) = \frac{1}{x^* - (-1)} \int_{-1}^{x^*} v(m, z_L, z_R, d_L) \, dm = \frac{1}{2d_L} \int_{-1}^{2d_L - 1} v(m, z_L, z_R, d_L) \, dm. \tag{107}$$

Likewise, the welfare of R's constituents is:

$$W_R(d_L, z_L, z_R) = \frac{1}{1 - x^*} \int_{x^*}^{1} v(m, z_L, z_R, d_L) \, dm = \frac{1}{2(1 - d_L)} \int_{2d_L - 1}^{1} v(m, z_L, z_R, d_L) \, dm. \tag{108}$$

We assume that each party $P \in \{L, R\}$ balances a concern for its constituents with a desire to

[20] Recent work includes Gomberg, Marhuenda, and Ortuño-Ortín (2016). See also Caplin and Nalebuff (1997).

increase its share of a fixed office rent normalized to one, that depends on its share of seats, $d_P \in [0,1]$, according to the following specification:

$$\pi_p(d_P) = \mathbf{1}[d_P > .5]\eta + (1-\eta)d_P, \tag{109}$$

where $\eta \in [0,1)$ reflects the extent to which legislative power is concentrated in the hands of the majority. Critically, we do not assume that the marginal contribution of a seat above the majority threshold exceeds the marginal value of a seat below the majority threshold. To see this, observe that this formulation is a special case of our benchmark setting in which $\alpha = \beta = 1-\eta$, and $r = \eta + \frac{1}{2}(1-\eta)$.

We assume that party J trades off the desire of party leaders to capture a share of office rents with the pressure to reflect the preferences of the party's electoral constituency:

$$u_P(d_P, z_L, z_R) = \pi_P(d_P) + \gamma W_P(d_P, z_L, z_R), \tag{110}$$

where $\gamma > 0$. Finally, to simplify the analysis, we assume that voter preferences within each district are also uniformly distributed around their medians.

Assumption 3. In a district with median $m \in [-1,1]$, voter types are uniformly distributed on $[m-Z, m+Z]$, where $Z > 2$.

The restriction that $Z > 2$ implies that there is more heterogeneity within districts than there is across district medians.

Introducing a seat motivation together with a policy-motivated component ensures that the marginal value of an additional seat is always positive. The subtlety in this formulation—that is absent in our benchmark—is that the party's trade-off over seats is partly a function of its platform and its opponent's platform.

Analysis. Define $\alpha_P(d_P) = \frac{\partial u_P}{\partial d_P}$ for $d_P \in [0, \frac{1}{2})$, and $\beta(d_P) = \frac{\partial u_P}{\partial d_P}$ for $d_P \in (\frac{1}{2}, 1]$.

Proposition 6. *Whenever $z_L \le z_R$, and for any $\eta \in [0,1]$ and $\gamma > 0$: $d < \frac{1}{2} < d'$ implies $\alpha(d) > \beta(d')$.*

Proof. We prove the result for party L, because the extension to party R is immediate. By Assumption 3 that in any district with median m, voter types are distributed uniformly on $[m-Z, m+Z]$

for $Z > 2$, we obtain:

$$v(m, z_L, z_R, d_L) = \frac{1}{2Z} \int_{m-Z}^{m+Z} -|x - z^*(d_L)|\, dx = -\frac{Z}{2} - \frac{(m-Z)^2}{2Z}. \tag{111}$$

It is therefore sufficient to observe that for any $d < \frac{1}{2} < d'$ and $z_L \leq z_R$,

$$\alpha(d) - \beta(d') = \frac{4\gamma(d' - d)}{3Z} + \frac{\gamma(z_R - z_L)}{Z} > 0. \tag{112}$$

Finally, w sufficient condition for the marginal value of an additional seat always to be positive is that $\beta(1) > 0$. Straightforward substitution yields that

$$\beta(1) = 1 - \eta + \gamma \frac{z_L - \frac{1}{3}}{Z} > 1 - \eta - \gamma \frac{4}{3Z}, \tag{113}$$

which is strictly positive if (1) districts are sufficiently heterogeneous (Z large enough) or if parties are predominantly concerned with winning more seats (i.e., $\gamma < 1 - \eta$). □

Figure 2 illustrates L's value from winning additional districts. The discontinuous increase in L's value from winning a majority arises for any $\eta > 0$ and $\gamma > 0$, so long as $z_L \leq z_R$. Our piece-wise linear formulation can be interpreted as an approximation of u_P, and captures its key property that for the marginal of winning a core district exceeds the marginal value of winning an opponent's core district.

To understand the real-world interpretation of this property, suppose that the R party offers a centrist platform, and wins the election. As its legislative majority advances from small to large, it represent districts whose voters increasingly dislike the party's platform. Party leaders may internalize this consequence for both non-instrumental and instrumental reasons. For instance, rank-and-file legislators from these districts may be more difficult to corral, and may require a larger share of side payments and transfers in exchange for their cooperation on other aspects of the party's legislative agenda. This idea is reflected in former Democratic House Minority Whip and Majority Leader Steny Hoyer's claim that "...the larger your majority, the harder it is to maintain your unity" (quoted in Poole, 2004)

Note that Proposition 6 also applies if the average welfare of the party's constituents is evaluated at the party's platform rather than the winning platform, i.e., if we replace $v(m, z^*(d_L))$ in expression (106) with $v(m, z_J)$. This could reflect the preferences of a party faction that cares solely

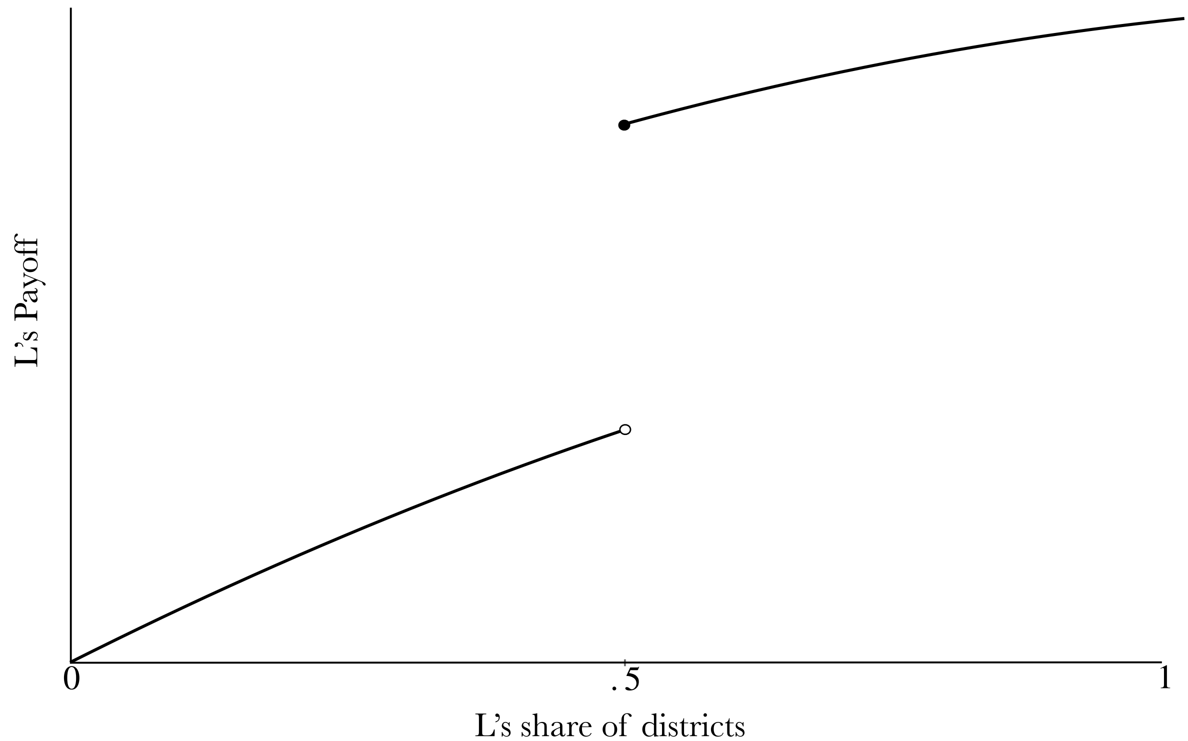

Figure 2 – L's induced preferences over seats when $z_L = -.25$ and $z_R = 0$. Primitives: $Z = 3$, $\gamma = 1.5$, $\eta = .5$

about the congruence of the party's platform with the preferences of the party's constituents—regardless of whether the party wins power and implements the platform. This is analogous to the preferences of the 'militant' party faction in Roemer (1999).

Equilibrium in Extended Model: An Example. We close by highlighting an example of equilibrium under our extended model for a set of parameters. The main qualitative properties of the equilibrium platforms replicate those of our benchmark setting. All approximations are to three decimal places.

Example 1. *Set* $\eta = \frac{1}{5}$, $\gamma = \frac{3}{10}$, $Z = 2$, $\psi = 3$, $\theta = 6$. *Then, there exist thresholds* $\underline{\rho}_0 = 1.307$ *and* $\overline{\rho}_0 = 1.547$, *such that:*

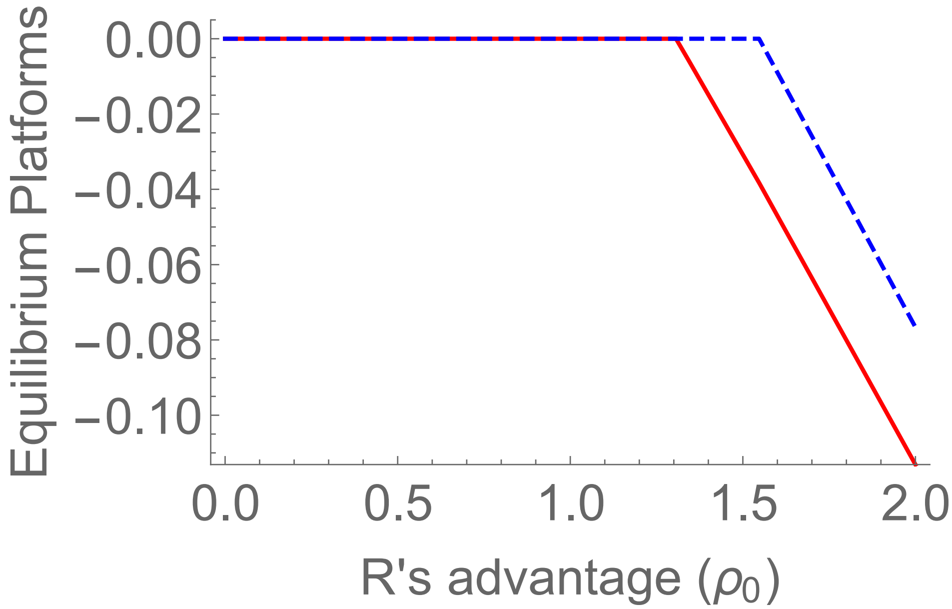

Figure 3 – Equilibrium platforms z_L^* (*red*) and z_R^* (*blue*) in the extended model when $\eta = \frac{1}{5}$, $\gamma = \frac{3}{10}$, $Z = 2$, $\psi = 3$, $\theta = 6$.

*[1.] if R's advantage is **small**, i.e., $\rho_0 \leq \underline{\rho}_0$, both parties locate at the ideal policy of the median voter in the median district: $z_L^* = z_R^* = 0$,*

*[2.] if R's advantage is **intermediate**, i.e., $\rho_0 \in (\underline{\rho}_0, \overline{\rho}_0]$, $z_L^* < z_R^* = 0$, then party L retreats to its base but R still locates at the ideal policy of the median voter in the median district: $z_L^* < z_R^* = 0$; and*

*[3.] if R's advantage is **large**, i.e., $\rho_0 > \overline{\rho}_0$, then party L retreats by more to its base, and party R advances towards L's base: $z_L^* < z_R^* < 0$.*

The platforms are highlighted in Figure 3.

B2: A Policy Outcome Function, and Policy-Motivated Parties. We now provide an alternative justification based on post-election legislative policymaking employed in Grossman and Helpman (1996), Alesina and Rosenthal (1996), and Lizzeri and Persico (2001), in which the final policy outcome depends on both the parties' platforms, and the winner's margin of victory. The idea is that a party's vote share exerts marginal effects on the final policy outcome. Controlling a slight majority may give a party formal agenda-setting power, but winning more seats gives the party leadership a buffer to protect against defections and to weaken the negotiating leverage of the party's marginal legislators in shaping the final policy outcome.[21] Because this is a property of

[21] For example, the Democratic leadership was forced to make many concessions to the Blue Dog Democrats, in

legislatures, not of election systems, the logic may hold in both majoritarian and proportional election contexts: parties with only small margins of victory will find it more challenging to implement their platforms than a party with an outsized victory. A larger majority may also be perceived as granting the majority party a greater electoral mandate to pursue its agenda, rather than mandating compromise with the minority party.

Party Platforms. Recall that, in addition to her policy payoffs from a party's platform choice z_L or z_R, a voter type x_i also derives a net value $-\theta x_i$ from party L. One interpretation is that the parties have fixed platforms y_L and y_R on a second policy. For example, let party R's fixed platform on this second policy be $y_R = 1$ and party L's fixed platform be $y_L = -1$. If a voter type x_i's relative value from party L on this policy dimension is $\theta|y_R - x_i| - \theta|y_L - x_i|$, then each district median voter's net value from L is $-2\theta x_i$.

In the analysis that follows, we adopt the interpretation that each party's platform is a vector with two components: party L's platform is $p_L = (y_L, z_L)$, and party R's platform is $p_R = (y_R, z_R)$. We admit any $(y_L, y_R) \in \mathbb{R}^2$, such that $y_L \neq y_R$.

Policy Outcome Function. To capture the reality that a party's margin of victory affects its ability to implement its campaign promise, we assume that if the winning party's share of districts is $d \in (1/2, 1)$, the majority-winning party's platform is p^M and the minority party's platform is p^m, the final policy outcome is:

$$p^*(p^M, p^m, d) = \eta p^M + (1-\eta)(dp^M + (1-d)p^m). \tag{114}$$

The parameter $\eta \in [0, 1)$ reflects the majoritarian organization of the legislature: higher values imply that the majority party increasingly dominates the policy outcome, regardless of its margin.

Party Goals. We assume that parties have both policy and office goals. Specifically, they divide a fixed office rent that's normalized to one—e.g., committee chairs, funding for districts—where the division is determined by the same rule specified in (114).[22] Second, they aim to represent the

shaping the final form of the Affordable Care Act. "Blue Dogs Delay, Water Down House Health Care Bill", *Huffington Post*, August 29 2009. https://www.huffpost.com/entry/blue-dogs-delay-water-dow_n_247177.

[22] We adopt the same sharing rule on rents as for policy only for parsimony—we could allow for any division rule that increases with a party's share of seats.

entire polity, but may prioritize some districts over others. Specifically, party J's payoff is:

$$u_J(p_L, p_R, d_J) = M_J\eta + (1-\eta)d_J + \frac{1}{2}\int_{-1}^{1} w_J(m)v(m, p^*)\, dm, \tag{115}$$

where, as before, M_J is an indicator taking the value 1 if J wins a majority, and for any policy outcome $p = (y, z)$:

$$v(m, p) = \int_{m-Z}^{m+Z} -|x-z| - \theta|x-y|\, f(x)dx \tag{116}$$

is the welfare of citizens in a district with median m, and $w_J(m)$ is the weight that party J places on the welfare of voters in a district with median $m \in [-1, 1]$, satisfying $w_J(m) \geq 0$ for all m, and $\int_{-1}^{1} w_J(m)\, dm = 1$. We maintain Assumption 3 that voter types are uniformly distributed around their district medians.

Analysis. We analyze the model from party L's perspective, nothing that the analysis for party R's is symmetric. Define $\alpha(d_L) = \frac{\partial u_L}{\partial d_L}$ for $d_L \in [0, \frac{1}{2})$, and $\beta(d_L) = \frac{\partial u_L}{\partial d_L}$ for $d_L \in (\frac{1}{2}, 1]$.

Proposition 7. *For any $(z_L, z_R) \in \mathbb{R}^2$, any $(y_L, z_R) \in \mathbb{R}^2$ such that $y_L \neq y_R$, and any $d < \frac{1}{2} < d'$, $\alpha(d) > \beta(d')$.*

Proof. Under Assumption 3 that in a district with median type $m \in [-1, 1]$, voter ideal points are uniformly distributed on $[m - Z, m + Z]$, we have that for any $d_L \in [0, 1]$:

$$u_L(p_L, p_R, d_J) = \eta\mathbf{1}[d_L \geq 1/2] + (1-\eta)d_L - \frac{1}{4Z}\int_{-1}^{-1} w_L(m)\Big[(Z^2 + (m - z^*)^2) + \theta(Z^2 + (m - y^*)^2)\Big]\, dm \tag{117}$$

where

$$z^* = \begin{cases} \eta z_R + (1-\eta)[d_L z_L + (1-d_L)z_R] & \text{if } d_L < \frac{1}{2} \\ \eta z_L + (1-\eta)[d_L z_L + (1-d_L)z_R] & \text{if } d_L \geq \frac{1}{2} \end{cases} \tag{118}$$

and

$$y^* = \begin{cases} \eta y_R + (1-\eta)[d_L y_L + (1-d_L)y_R] & \text{if } d_L < \frac{1}{2} \\ \eta y_L + (1-\eta)[d_L y_L + (1-d_L)y_R] & \text{if } d_L \geq \frac{1}{2}. \end{cases} \tag{119}$$

Thus, for any $d_L \in [0,1/2) \cup (1/2,1]$:

$$\frac{\partial u_L}{\partial d_L} = 1-\eta+\frac{1}{2Z}\int_{-1}^{1} w_L(m)(m-z^*)(z_L-z_R)(1-\eta)\,dm+\frac{\theta}{2Z}\int_{-1}^{1} w_L(m)(m-y^*)(y_L-y_R)(1-\eta)\,dm, \tag{120}$$

and

$$\frac{\partial^2 u_L}{\partial d_L^2} = -\frac{1}{2Z}\int_{-1}^{1} w_L(m)(z_L-z_R)^2(1-\eta)^2\,dm - \frac{\theta}{2Z}\int_{-1}^{1} w_L(m)(y_L-y_R)^2(1-\eta)^2\,dm < 0, \tag{121}$$

First, inspection of (121) reveals that for any pair $(z_L, z_R) \in [0,1]^2$, the party's payoff is strictly concave in $d_L \in [0, \frac{1}{2})$ and strictly concave in $d_L \in (\frac{1}{2}, 1]$, for any pair $(z_L, z_R) \in [-1,1]^2$.

Second, for any pair $(z_L, z_R) \in [0,1]^2$, we verify that for any $d_L^m < \frac{1}{2} < d_L^M$:

$$\left.\frac{\partial u_L}{\partial d_L}\right|_{d_L^m} > \left.\frac{\partial u_L}{\partial d_L}\right|_{d_L^M}. \tag{122}$$

We prove this by observing that, by strict concavity of u_J in d_L^m and in d_L^M, it is sufficient to verify:

$$\lim_{d_L^m \uparrow \frac{1}{2}} \frac{\partial u_L}{\partial d_L} - \lim_{d_L^M \downarrow \frac{1}{2}} \frac{\partial u_L}{\partial d_L} = \frac{(z_L-z_R)^2\eta(1-\eta)}{2Z}\int_{-1}^{1} w_L(m)\,dm + \theta\frac{(y_L-y_R)^2\eta(1-\eta)}{2Z}\int_{-1}^{1} w_L(m)\,dm > 0. \tag{123}$$

This yields the result. □

Note that we have not relied on any parameter or weighting function restrictions. Parameter restrictions do, however, ensure that the marginal value of an additional district is positive, i.e., that (120) is positive. For example, this holds whenever districts are sufficiently heterogeneous (Z is large enough).

Figure 4 illustrates L's value from winning additional districts. Our piece-wise linear formulation can be interpreted as an approximation of u_P in this context, that again captures its key property that the marginal of winning a core district exceeds the marginal value of winning an opponent's core district.

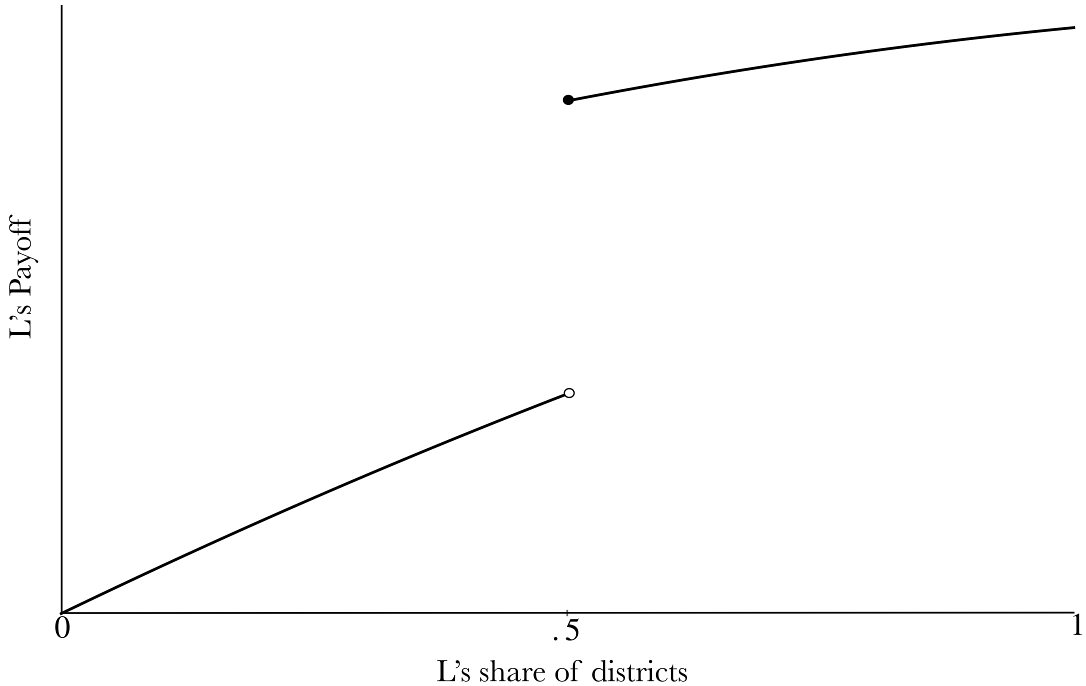

Figure 4 – L's induced preferences over seats when $z_L = 0$ and $z_R = 0$. Primitives: $Z = 3$, $\gamma = .5$, $\eta = .5$, $y_L = -1$, $y_R = 1$, $\theta = 6$, $w_L(m) = \frac{3}{8}(1-m)^2$.

Equilibrium in Extended Model: An Example. We close by highlighting an example of equilibrium under our extended model for a set of parameters. The main qualitative properties of the equilibrium platforms replicate those of our benchmark setting. All approximations are to three decimal places.

Example 2. *Set* $\eta = \frac{1}{5}$, $\gamma = \frac{3}{10}$, $Z = 2$, $\psi = 3$, $\theta = 6.$, $y_L = -1$, $y_R = 1$, *and* $w_L(m) = \frac{1-m}{2}$, *and* $w_R(m) = \frac{1+m}{2}$. *Then, there exist thresholds* $\underline{\rho}_0 = .650$ *and* $\overline{\rho}_0 = 1.211$, *such that:*

*[1.] if R's advantage is **small**, i.e., $\rho_0 \leq \underline{\rho}_0$, both parties locate at the ideal policy of the median voter in the median district: $z_L^* = z_R^* = 0$,*

*[2.] if R's advantage is **intermediate**, i.e., $\rho_0 \in (\underline{\rho}_0, \overline{\rho}_0]$, $z_L^* < z_R^* = 0$, then party L retreats to its base but R still locates at the ideal policy of the median voter in the median district: $z_L^* < z_R^* = 0$; and*

*[3.] if R's advantage is **large**, i.e., $\rho_0 > \overline{\rho}_0$, then party L retreats by more to its base, and party R advances towards L's base: $z_L^* < z_R^* < 0$.*

The platforms are highlighted in figure 5.

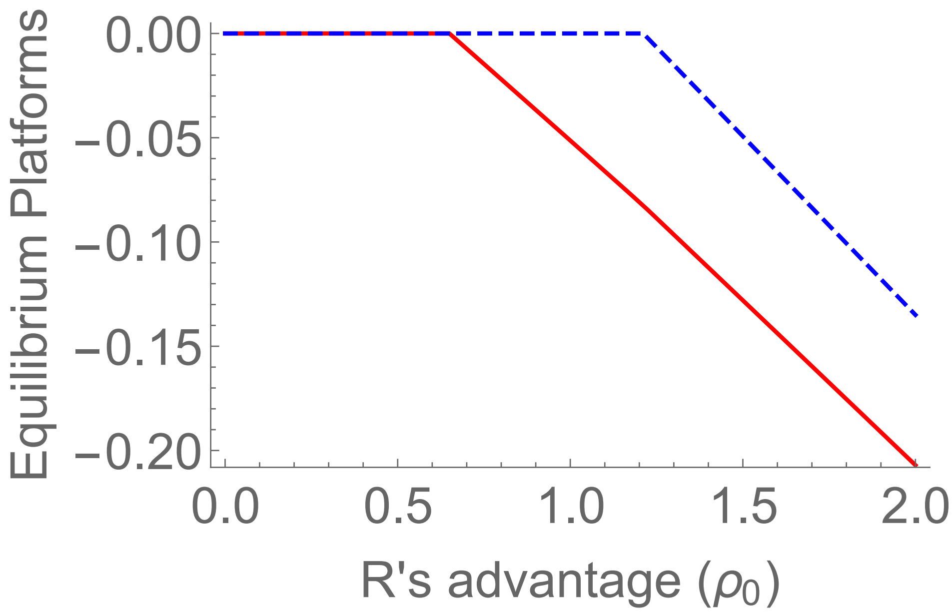

Figure 5 – Equilibrium platforms z_L^* (*red*) and z_R^* (*blue*) in the extended model when $\eta = \frac{1}{5}$, $\gamma = \frac{3}{10}$, $Z = 2$, $\psi = 3$, $\theta = 6$, $y_L = -1$, $y_R = 1$, $w_L(m) = \frac{1-m}{2}$, $w_R(m) = \frac{1+m}{2}$.