PCA with missing data using the missMDA R package

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Using missMDA to deal with missing data

```
> library(missMDA)
> data(orange)
```

	Color intensity	Odor intensity	Attack intensity	Sweet	Acid	Bitter	Pulp	Typicity
1	4.79	5.29	NA	NA	NA	2.83	NA	5.21
2	4.58	6.04	4.42	5.46	4.13	3.54	4.62	4.46
3	4.71	5.33	NA	NA	4.29	3.17	6.25	5.17
4	6.58	6.00	7.42	4.17	6.75	NA	1.42	3.42
5	NA	6.17	5.33	4.08	NA	4.38	3.42	4.42
6	6.33	5.00	5.38	5.00	5.50	3.63	4.21	4.88
7	4.29	4.92	5.29	5.54	5.25	NA	1.29	4.33
8	NA	4.54	4.83	NA	4.96	2.92	1.54	3.96
9	4.42	NA	5.17	4.62	5.04	3.67	1.54	3.96
10	4.54	4.29	NA	5.79	4.38	NA	NA	5.00
11	4.08	5.13	3.92	NA	NA	NA	7.33	5.25
12	6.50	5.88	6.13	4.88	5.29	4.17	1.50	3.50

Some (bad) easy methods

- Delete individuals or variables with missing data : usually not a good idea
- Replace missing data with the mean (default in several packages including FactoMineR)

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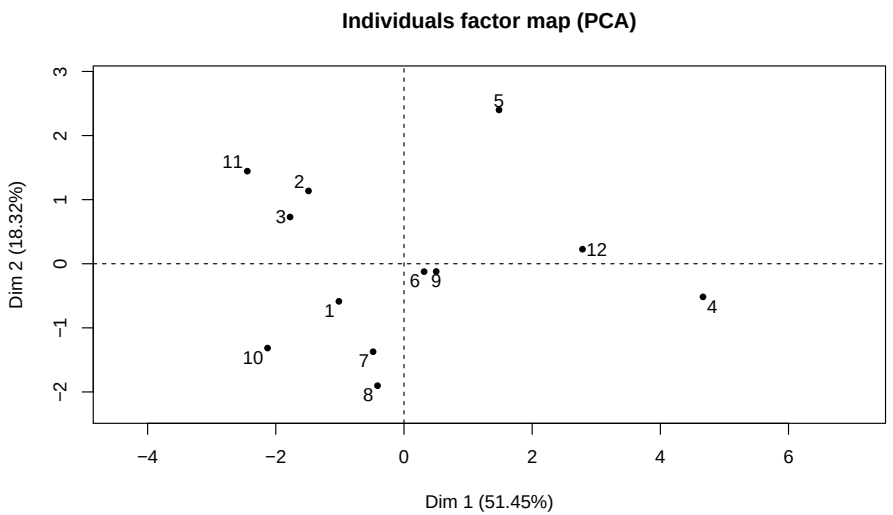
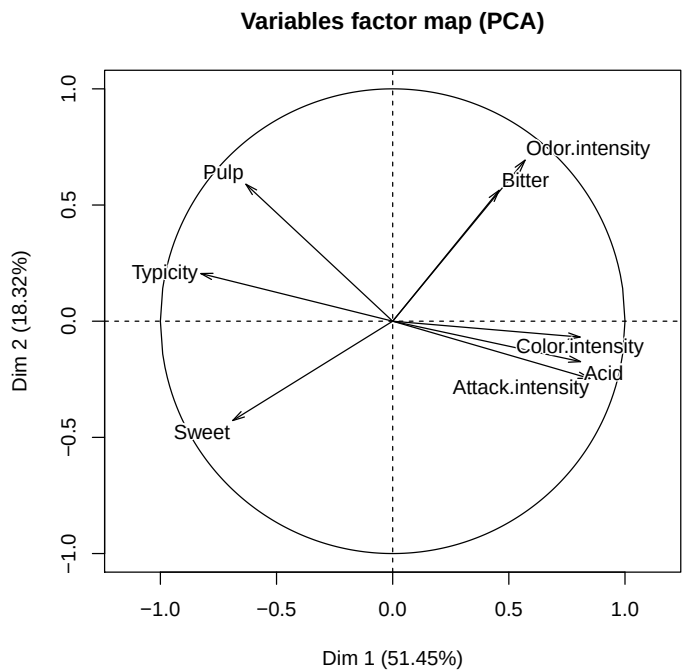
```
> res.pca <- PCA(orange)
```





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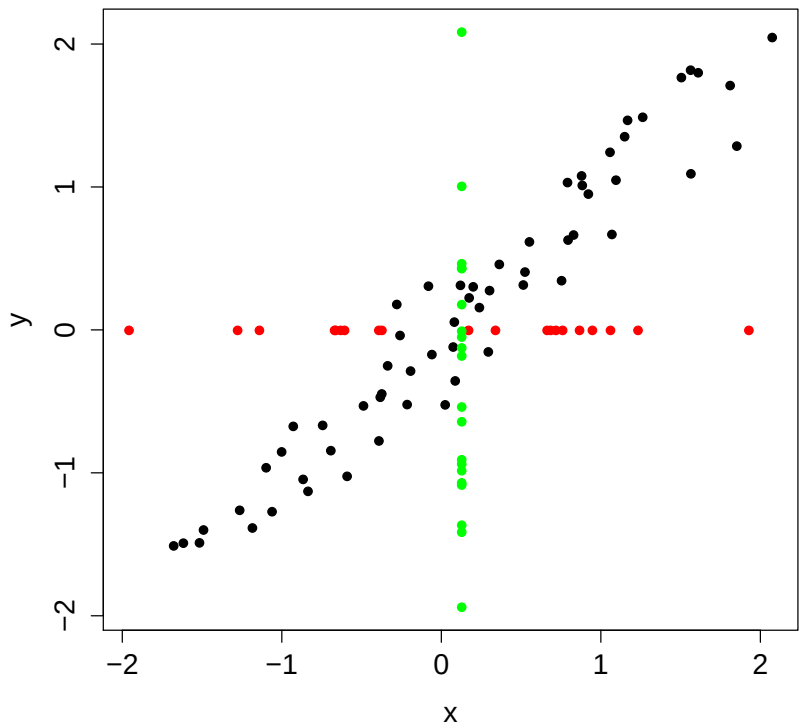


Big distortion of links between variables

Iterative PCA

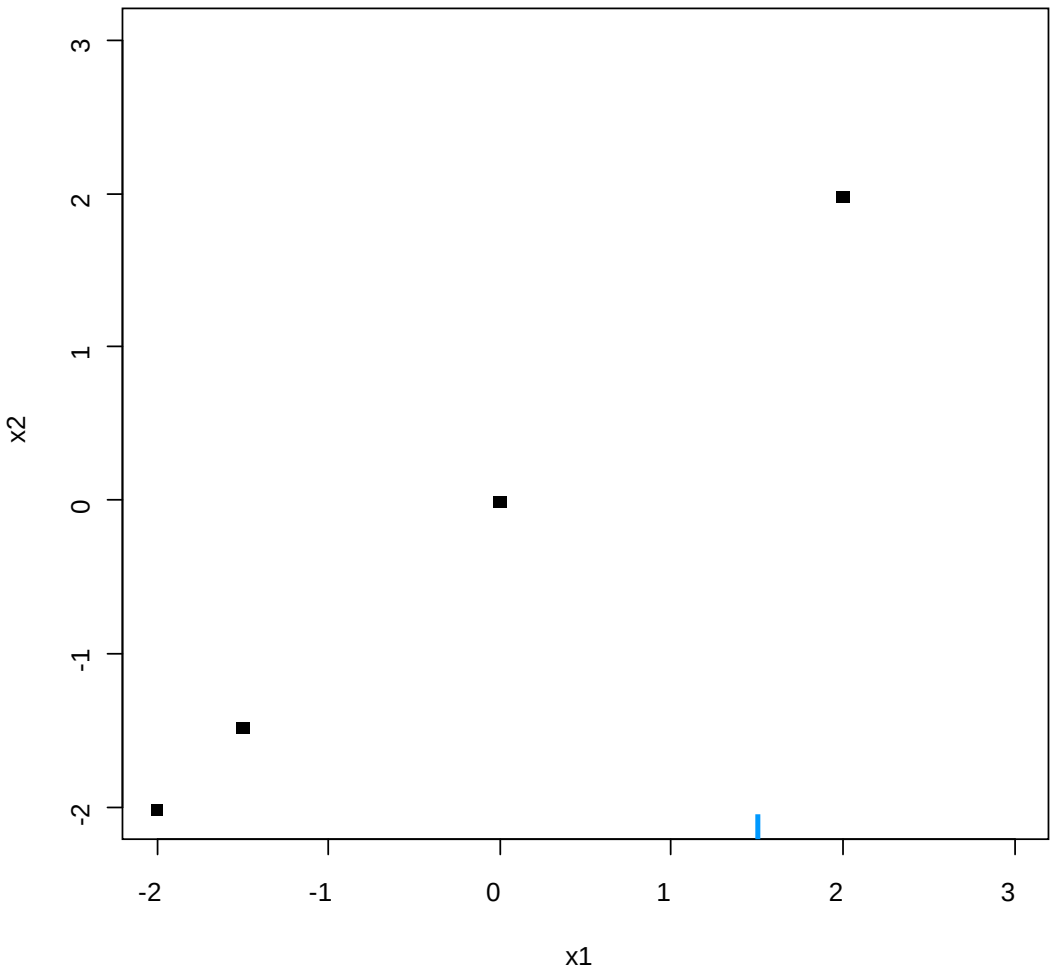
Ideas :

- As x and y strongly correlated : impute missing y value using x value
- if individuals i and j have similar values for all variables, impute missing i value using j value for that variable

$\Rightarrow$ takes into account global similarity between individuals and links between variables

Iterative PCA

x1	x2
-2.0	-2.01
-1.5	-1.48
0.0	-0.01
1.5	NA
2.0	1.98



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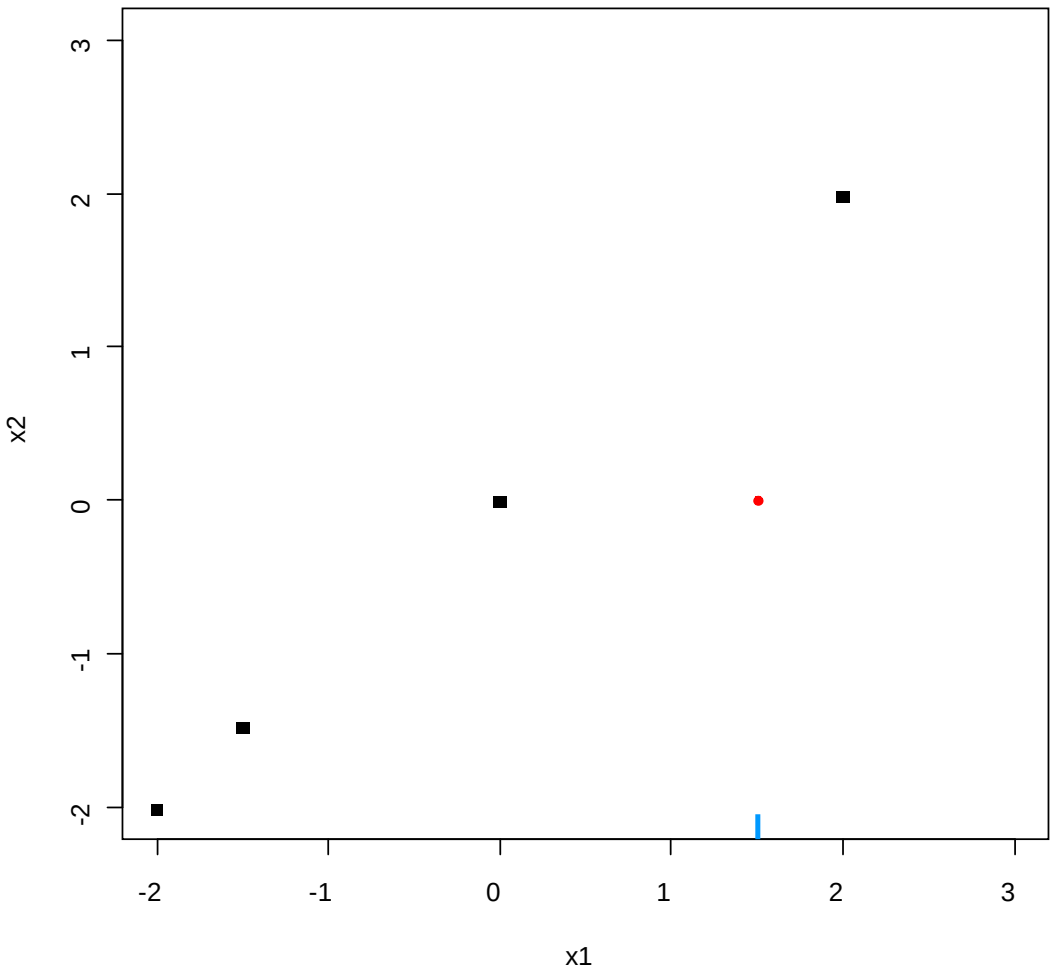

Initialize : impute the mean

Iterative PCA

x1	x2
-2.0	-2.01
-1.5	-1.48
0.0	-0.01
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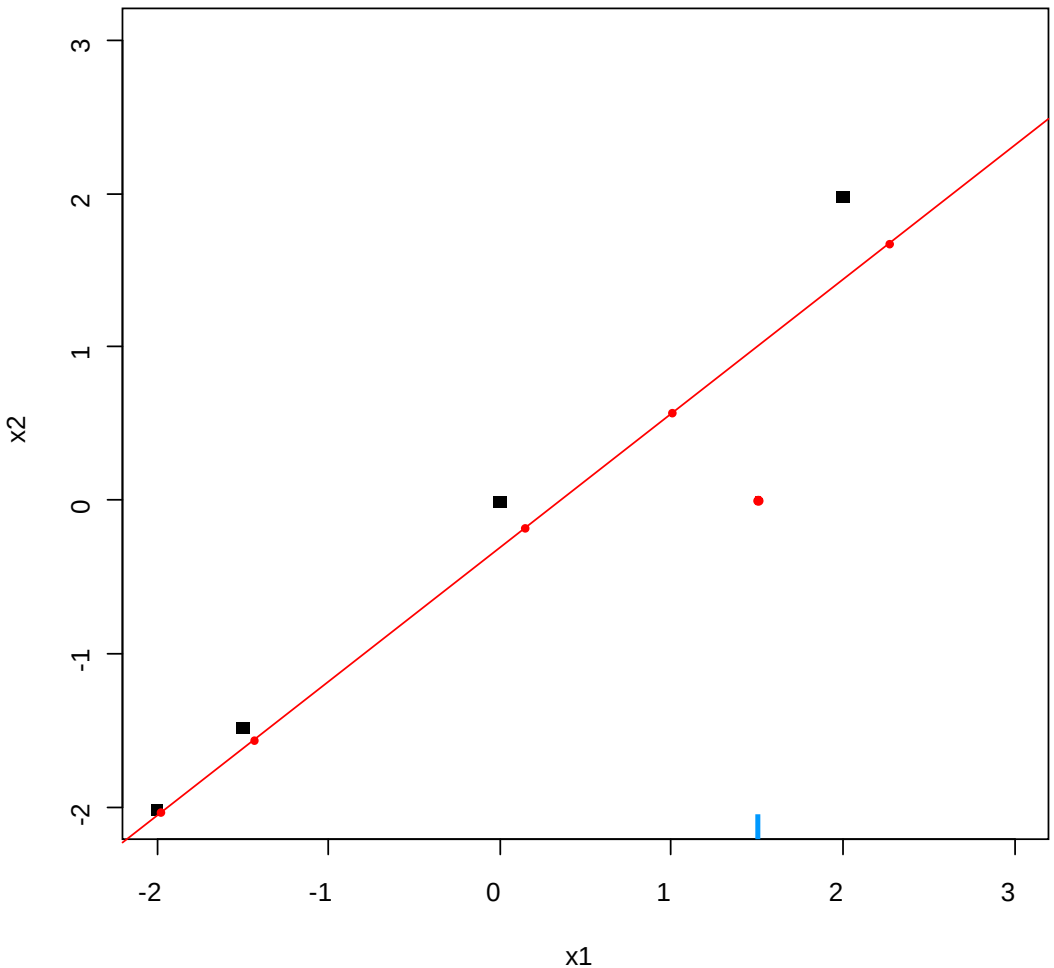
$\widehat{x1}$	$\widehat{x2}$
-1.98	-2.04
-1.44	-1.56
0.15	-0.18
1.00	0.57
2.27	1.67



Do PCA on imputed table $\rightarrow$ axes and components ;

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$\hat{x}1$	$\hat{x}2$
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-1.44	-1.56
0.15	-0.18
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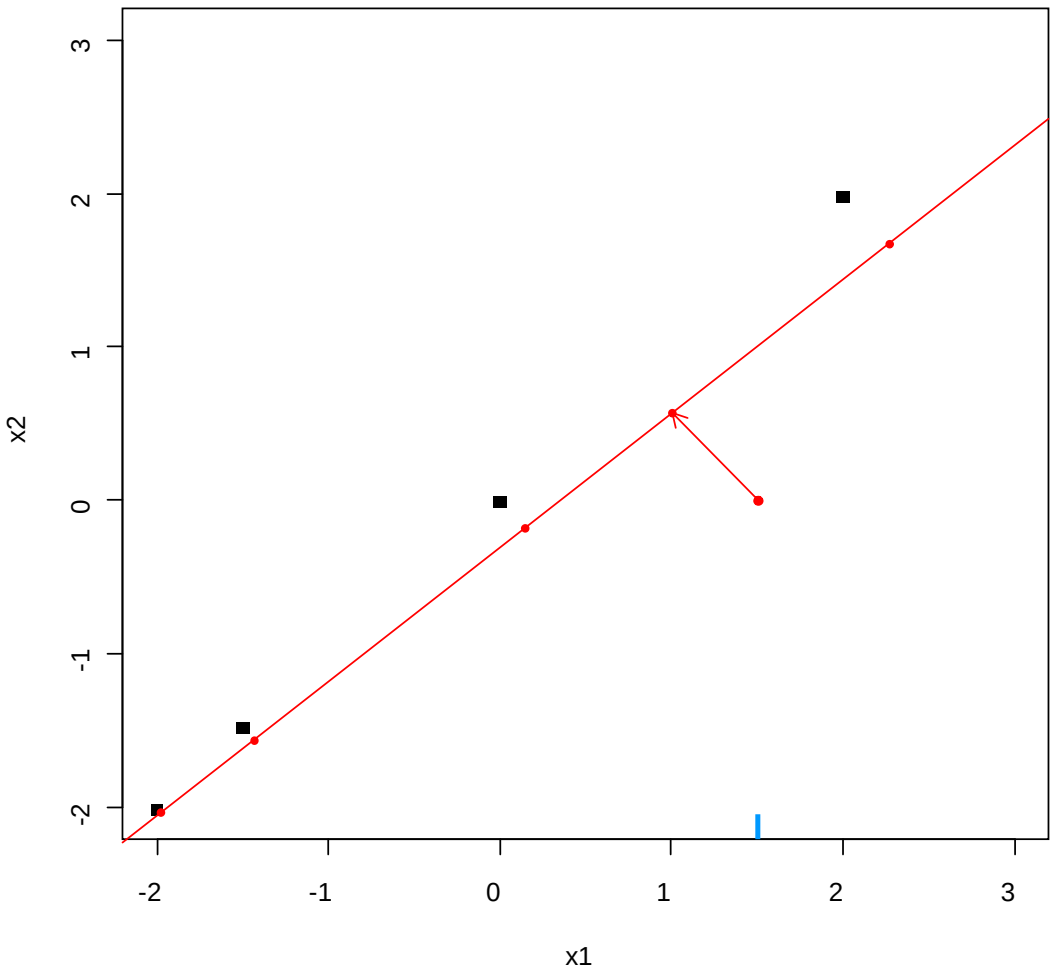

Missing data imputed using PCA

Iterative PCA

x1	x2
-2.0	-2.01
-1.5	-1.48
0.0	-0.01
1.5	**NA**
2.0	1.98

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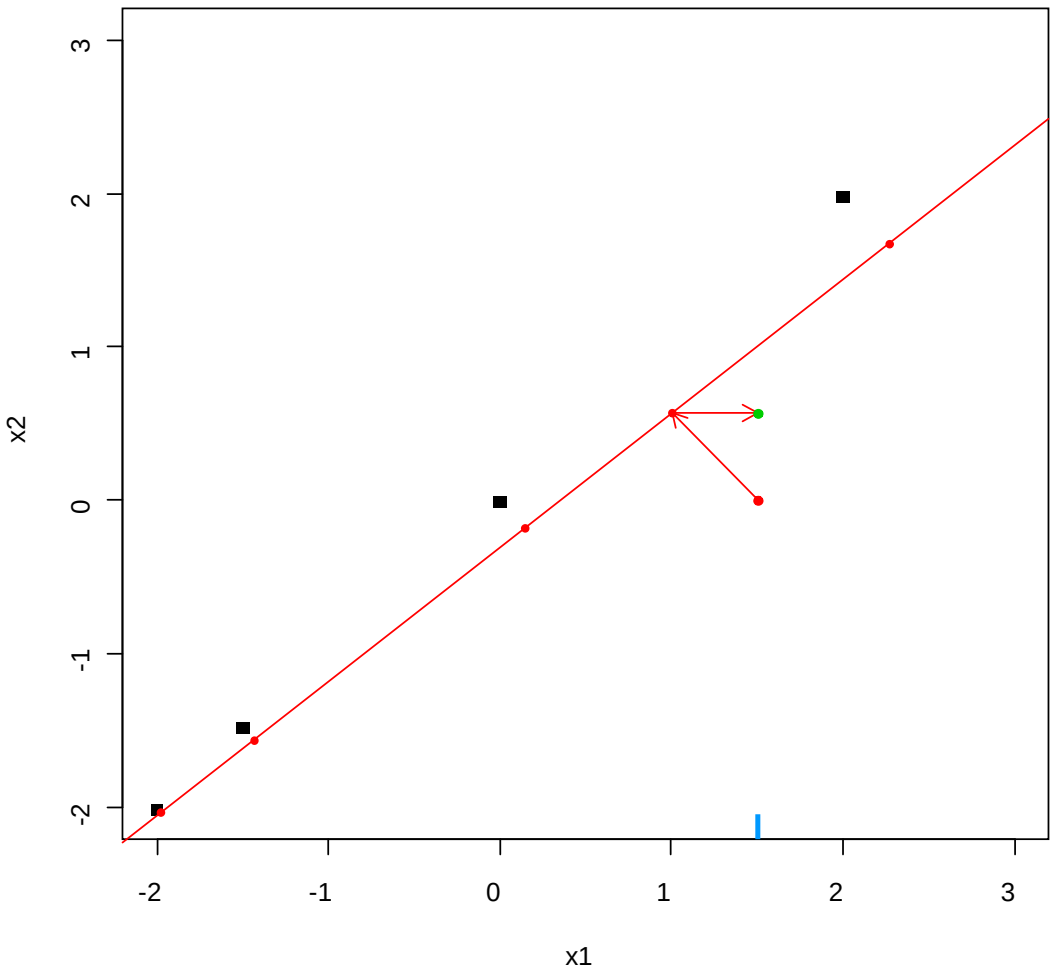

New imputed data table

Iterative PCA

x1	x2
-2.0	-2.01
-1.5	-1.48
0.0	-0.01
1.5	NA
2.0	1.98

x1	x2
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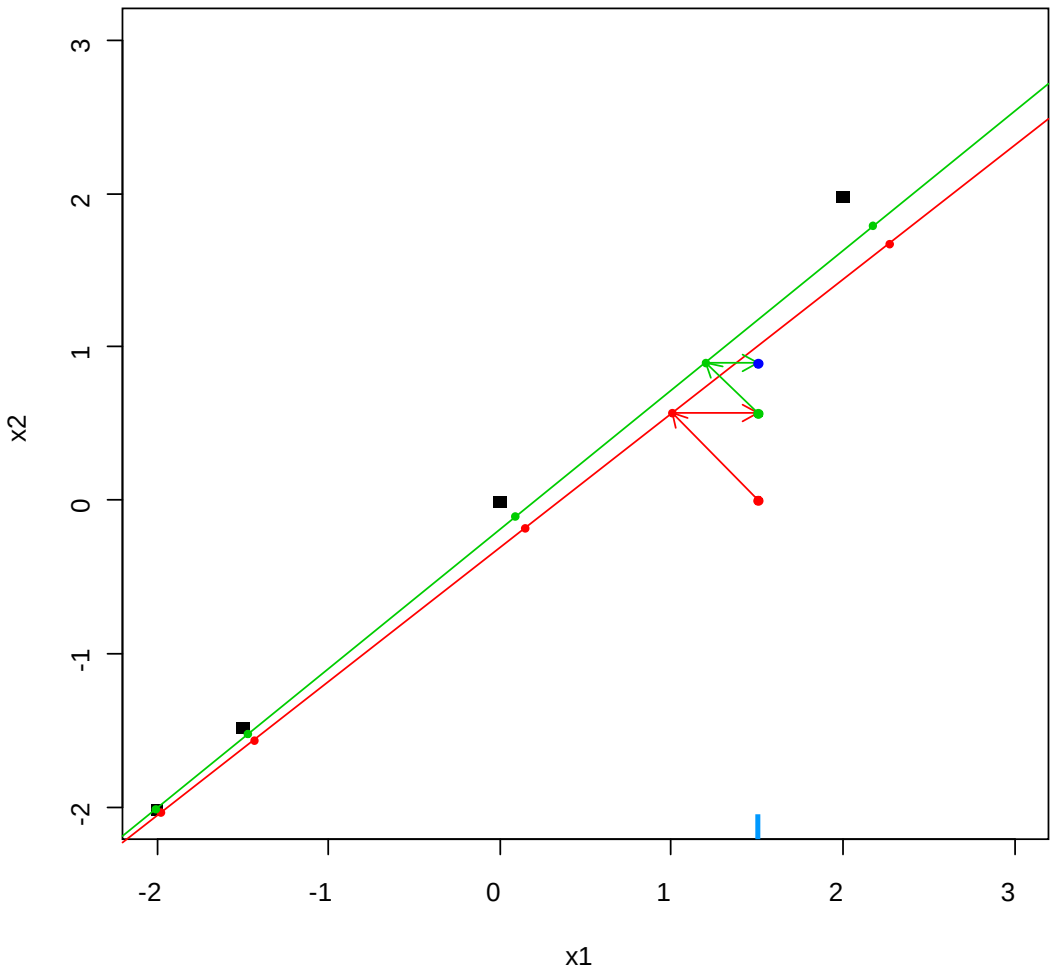

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x1	x2
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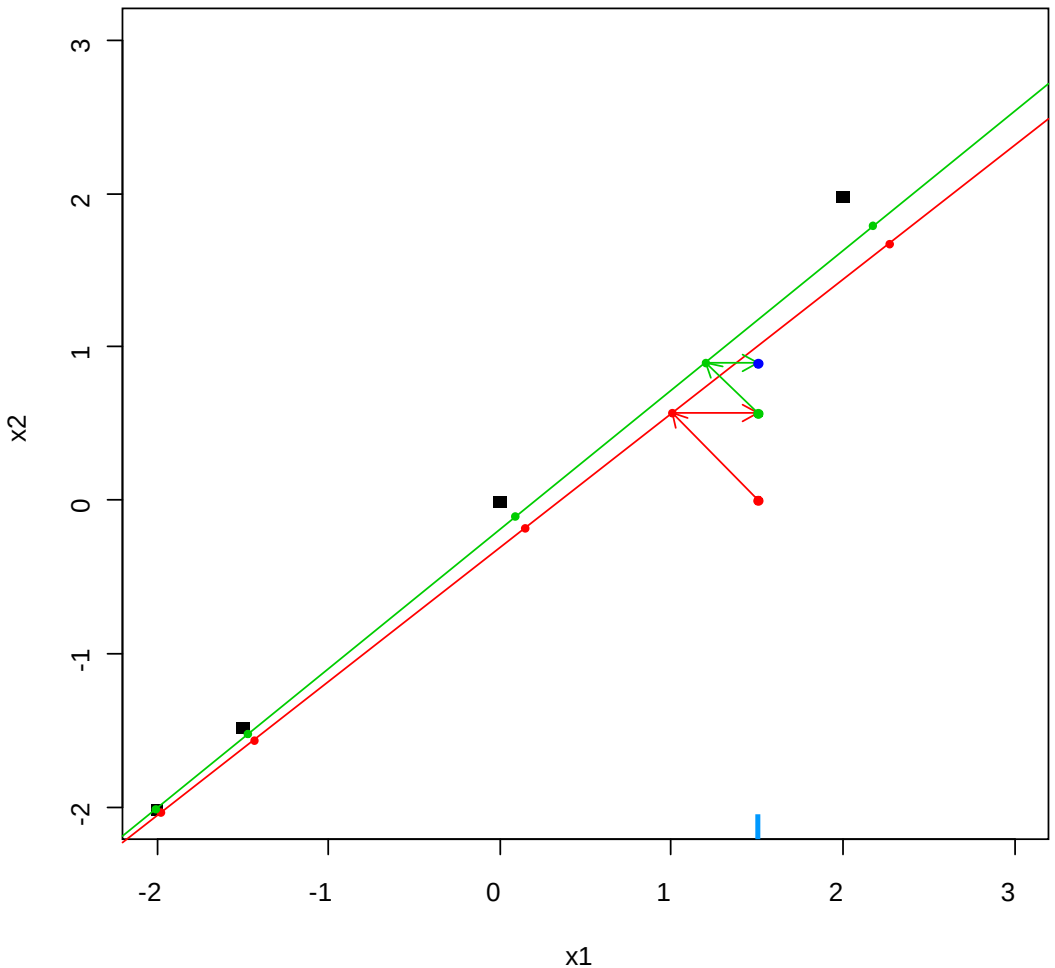
-1.5	-1.48
0.0	-0.01
1.5	**0.57**
2.0	1.98

$\hat{x}1$	$\hat{x}2$
-2.00	-2.01
-1.47	-1.52
0.09	-0.11
1.20	**0.90**
2.18	1.78

x1	x2
-2.0	-2.01
-1.5	-1.48
0.0	-0.01
1.5	**0.90**
2.0	1.98



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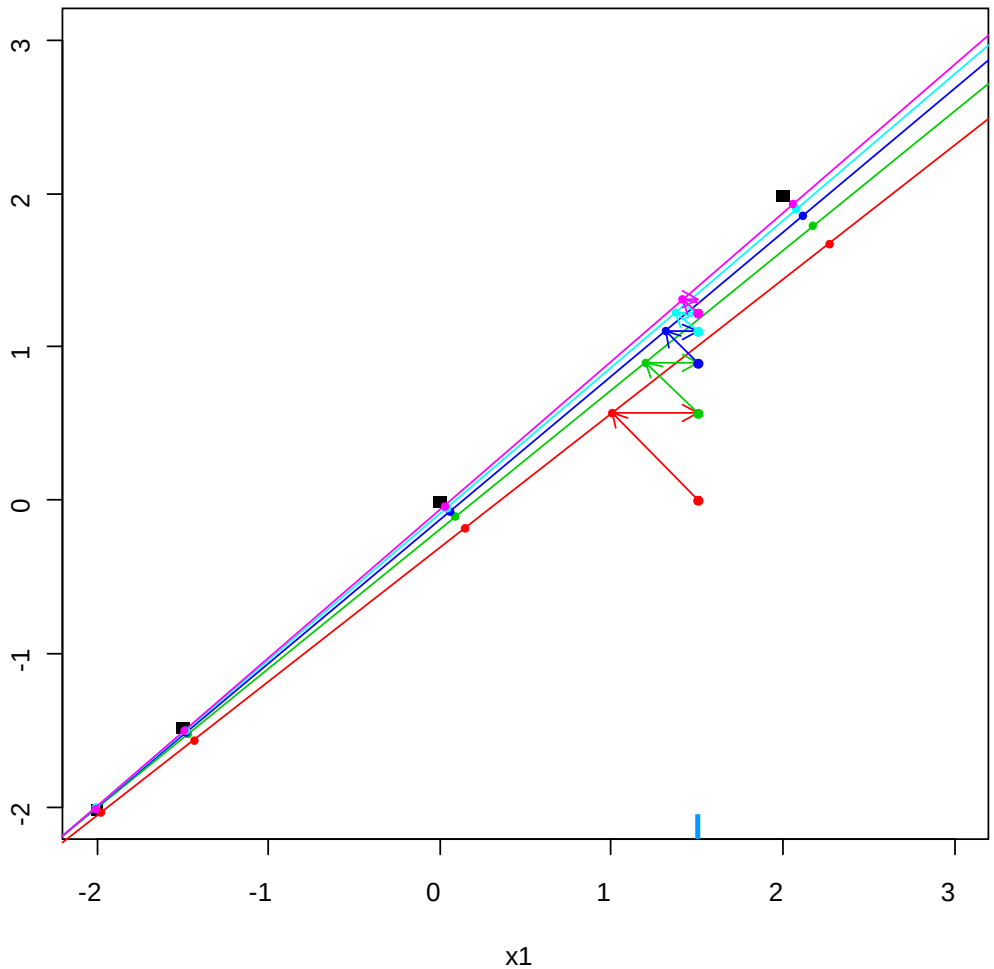
x2



Repeat these steps until convergence

Iterative PCA

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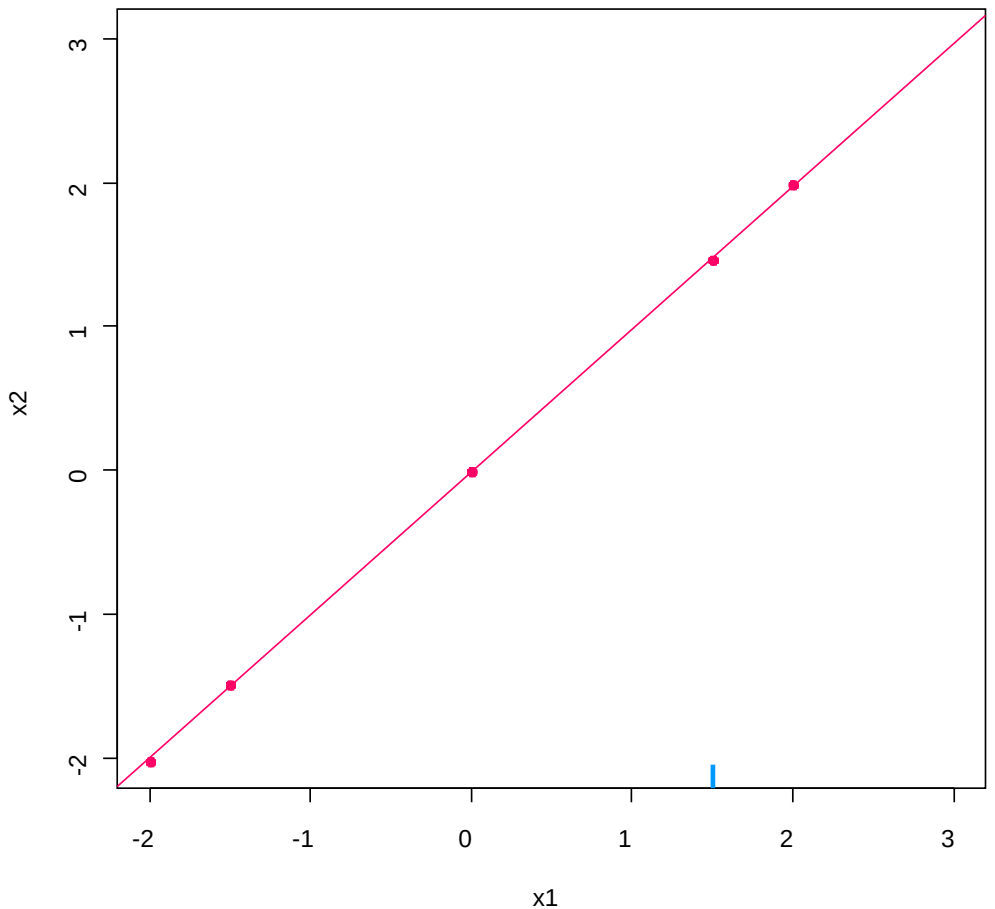

x1	x2
-2.0	-2.01
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Do PCA on imputed data table

Iterative PCA

1. initialization : impute using the mean
2. Step ℓ :
   (a) do PCA on imputed data table
   S dimensions retained
   (b) missing data imputed using PCA
   (c) means (and standard deviations) updated
3. iterate the estimation and imputation steps

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3. iterate the estimation and imputation steps

Overfitting problem due to believing too much in links between variables

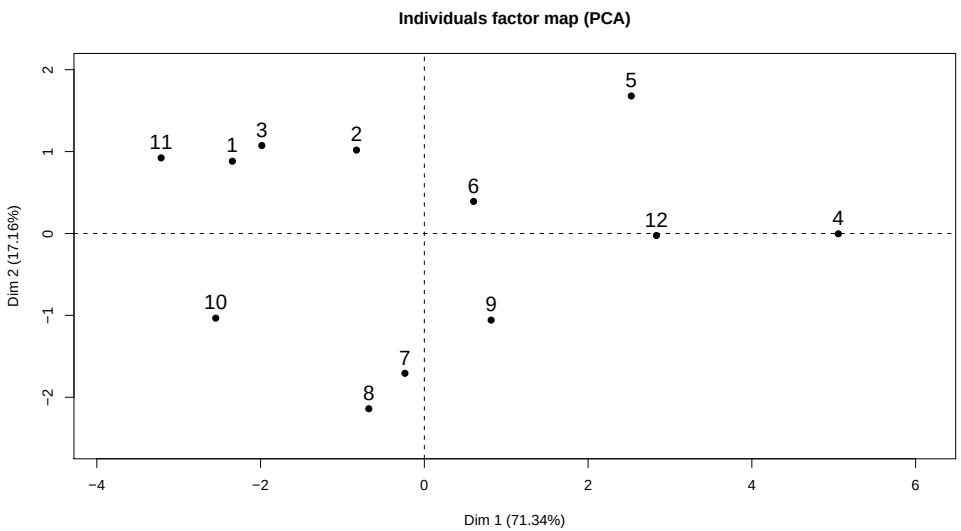
$\Rightarrow$ regularized iterative PCA

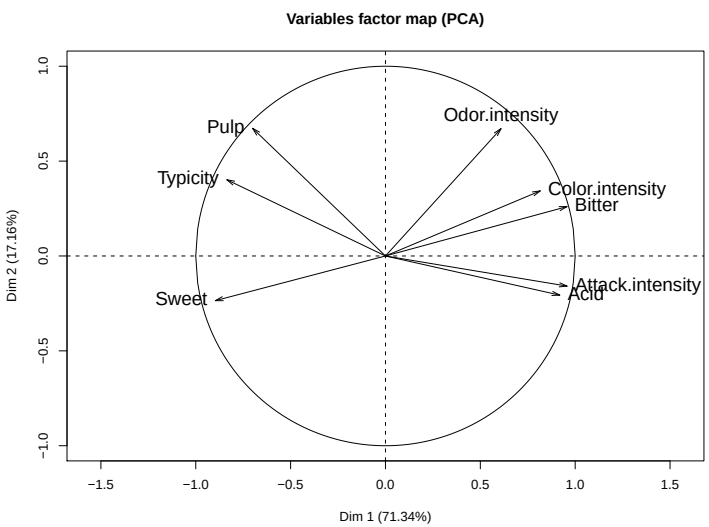
Running missMDA in R

```
> library(missMDA)
> data(orange)
> nb <- estim_ncpPCA(orange, scale=TRUE)          ## Estimate no. of dimensions
> comp <- imputePCA(orange, ncp=2, scale=TRUE)    ## Impute the table
> res.pca <- PCA(comp$completeObs)                ## Do the PCA
```

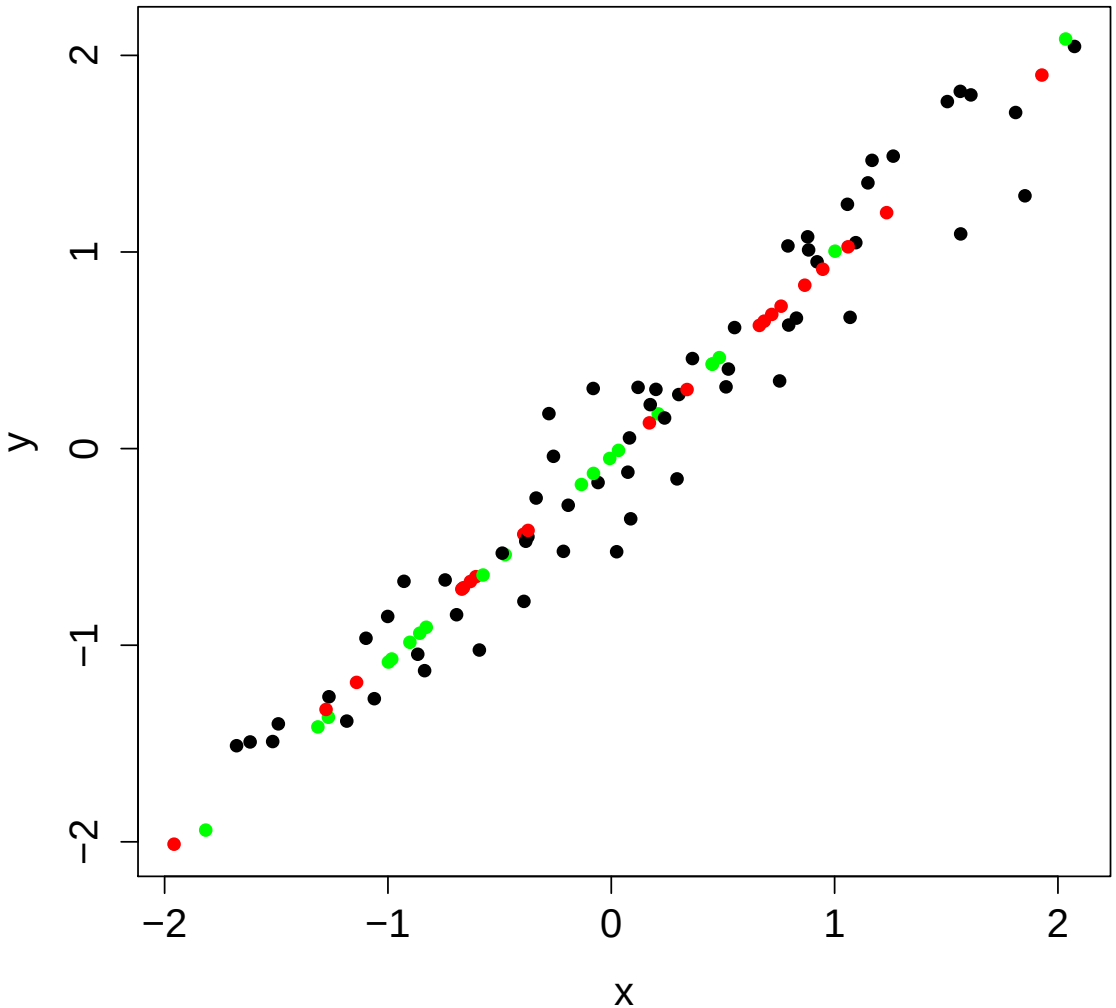
```
> orange
 Sweet Acid Bitter Pulp Typicity
  NA   NA  2.83   NA      5.21
5.46 4.13  3.54 4.62      4.46
  NA 4.29  3.17 6.25      5.17
...
4.88 5.29  4.17 1.50      3.50
```

```
> comp$completeObs
Sweet Acid Bitter Pulp Typicity
 5.54 4.13   2.83 5.89     5.21
 5.46 4.13   3.54 4.62     4.46
 5.45 4.29   3.17 6.25     5.17
...
 4.88 5.29   4.17 1.50     3.50
```





Is running the imputation algorithm once sufficient ?

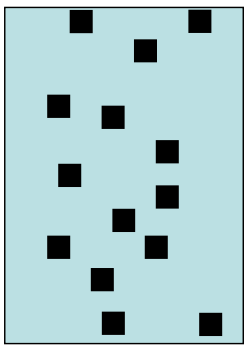


$\Rightarrow$ Reinforces links between variables

Visualizing uncertainty due to missing data

What confidence can we give to the results ? Idea of variance ?

$\Rightarrow$ A single value cannot show variability in the predicted value



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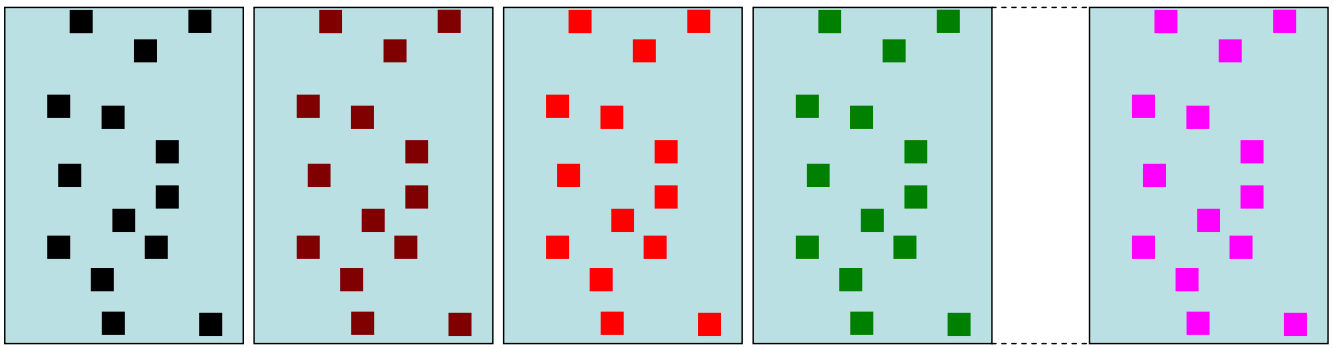

$\Rightarrow$ Multiple imputation : generate several plausible values for each missing data point

Visualizing uncertainty due to missing data

```
> mi <- MIPCA(orange, scale = TRUE, ncp=2)
> plot(mi)
```

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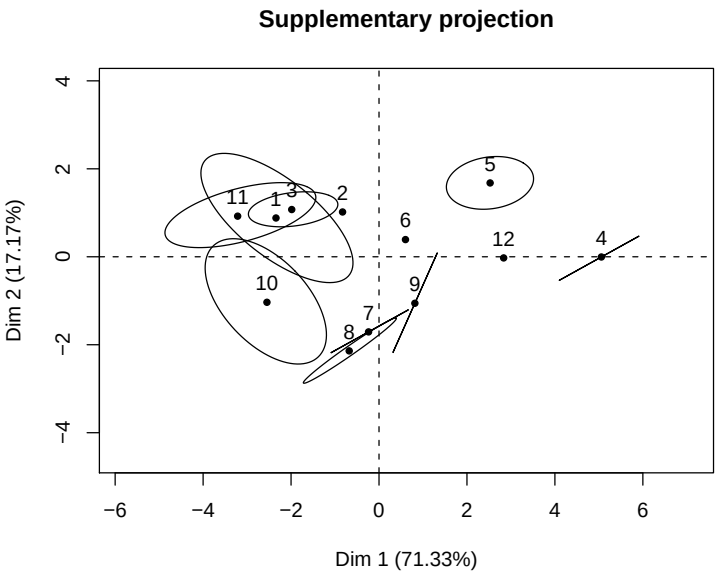

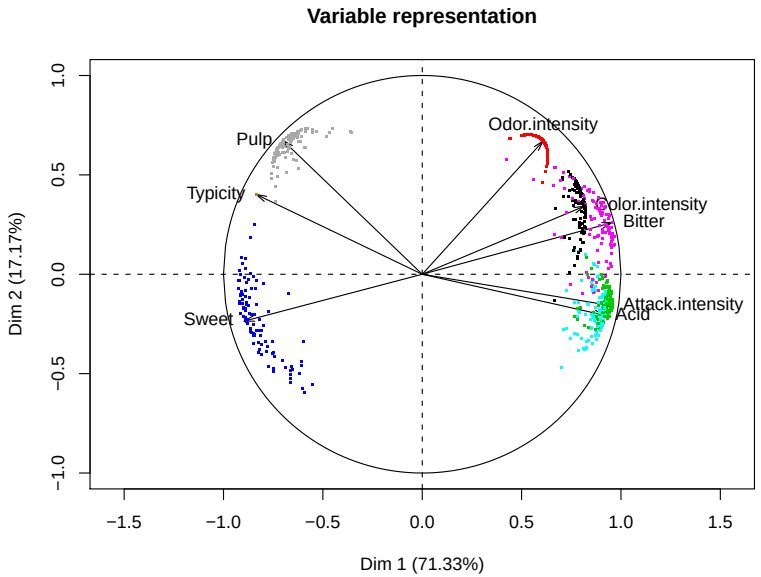

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