Intermediate Microeconomics: Advanced Notes

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Contents

1 Mathematics of Optimization

1.1 Unconstrained Optimization

- First order conditions: $\frac{\delta f}{\delta x} = 0, \frac{\delta f}{\delta y} = 0$
- Second order conditions: $\frac{\delta^2 f}{\delta x^2} < 0, \frac{\delta^2 f}{\delta y^2} < 0$
- Additionally, $(\frac{\delta^2 f}{\delta x^2} \times \frac{\delta^2 f}{\delta y^2}) - (\frac{\delta^2 f}{\delta x \delta y})^2 > 0$

Example: $G(T, C) = 50 + 10T + 16C - T^2 - 2TC - 2C^2$

$\frac{\delta G}{\delta T} = 10 - 2T - 2C$

$\frac{\delta G}{\delta C} = 16 - 2T - 4C$

$$\begin{bmatrix} -2 & -2 & -10 \\ -2 & -4 & -16 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & -2 & -10 \\ 0 & -2 & -6 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 0 & -4 \\ 0 & -2 & -6 \end{bmatrix}$$

$\Rightarrow T = 2, C = 3$

$\frac{\delta^2 G}{\delta T^2} = -2 < 0$

$\frac{\delta^2 G}{\delta C^2} = -4 < 0$

$(\frac{\delta^2 G}{\delta T^2} \times \frac{\delta^2 G}{\delta C^2}) - (\frac{\delta^2 G}{\delta T \delta C})^2 = (-2 \times -4) - (-2)^2 = 8 - 4 = 4 > 0$

$\Rightarrow G(2, 3)$ is a maximum $\Rightarrow G(2, 3) = 50 + 20 + 48 - 4 - 12 - 18 = 84$

1.2 Constrained Optimization

To find the critical points of $f(x, y)$ subject to some constraints $c - g(x, y) = 0$ we have

Lagrangian: $L = f(x, y) - \lambda(c - g(x, y))$

We take the derivative wrt x, y, and λ to get

$$\begin{bmatrix} \frac{\delta f}{\delta x} & = \lambda \frac{\delta g}{\delta x} \\ \frac{\delta f}{\delta y} & = \lambda \frac{\delta g}{\delta y} \\ g(x) & = c \end{bmatrix}$$

Example: $T = 19.25 - 6\ln(R) - 4\ln(W)$, constraint is $R + W = 5$

$$\begin{bmatrix} 19.25 - \frac{6}{R} & = \lambda \\ 19.25 - \frac{4}{W} & = \lambda \\ R + W & = 5 \end{bmatrix}$$

$19.25 - \frac{6}{R} = 19.25 - \frac{4}{W} \Rightarrow 6W = 4R$

$W = 5 - R \Rightarrow 6(5 - R) = 4R \Rightarrow 30 - 6R = 4R \Rightarrow 10R = 30 \Rightarrow R = 3, W = 2$

$T(3, 2) = 19.25 - 6\ln(3) - 4\ln(2) \approx 9.89$

Example: $I = \frac{P^2}{250} + \frac{PA}{100} + \frac{A^2}{1000}$, constraint is $P + A = 110$

$$\begin{bmatrix} \frac{P}{125} + \frac{A}{100} & = \lambda \\ \frac{P}{100} + \frac{A}{500} & = \lambda \\ P + A & = 110 \end{bmatrix}$$

$\frac{P}{125} + \frac{A}{100} = \frac{P}{100} + \frac{A}{500}$

$\frac{P}{125} - \frac{P}{100} = \frac{-4A}{500}$

$\frac{-P}{500} = \frac{-4A}{500}$

$P = 4A \Rightarrow 5A = 110 \Rightarrow A = 22 \Rightarrow P = 88$

$I(88, 22) = 50.82$

2 Consumer Theory

Defining preferences:

- Preferences reflect choices
  - Defined over bundles of goods
  - Based upon all properties of a bundle
- Notation
  - Strictly Preferred $\succ$
  - Weakly Preferred $\succeq$
  - Indifferent $\sim$
- Preferences are assumed to be well-bheaved
  - Complete: An individual knows hiw or her preferences over any two bundles
  - Reflexive: An individual must be indifferent between two identical bundles of goods
  - Transitivity: No cycles

Anchoring and Adjustment:

In multi-dimensional problems, people tend to anchor on one dimension and adjust for the other.

2.1 Utility Functions

A utility function is a function from bundles of goods into real numbers.

- $x \succ y \Rightarrow u(x) > u(y)$
- More preferred bundles must be assigned larger numbers
- If preferences are well-behaved, we can write down a utility function that captures the preferences

Ordinal utility function: the only thing that matters is the order

Cardinal utility function: the numbers have significance and meaning

Assumptions about utility functions:

- Monotonicity: $x > x' \Rightarrow u(x, y) \geq u(x', y)$
- Convexity: Let α s.t. $0 < \alpha < 1$. If $u(x, y) = u(x', y'), u(\alpha x + (1 - \alpha)x', \alpha y + (1 - \alpha)y') \geq u(x, y)$

Indifference curve: a curve along which $u(x, y) = c$ for some constant c

Marginal Utility: the marginal utility of x (MU_x) is defined as the increase in utility from consuming one additional good of x

$MU_x = \frac{\delta u}{\delta x}$

The marginal rate of substitution of x, y is the amount of good y needed to make up for the loss of one of good x

$MRS_{xy} = \frac{MU_x}{MU_y}$

2.2 Budget Constraints

The budget line is the line along which the amount spent exactly equals the available budget.

Let B be the budget, and p_x, p_y be the price of good x or good y. Thus, we have the budget line as $p_x x + p_y y = B$

Solving for y, we see have $y = \frac{B}{p_y} - \frac{p_x}{p_y} x$

The slope of the budget line is $\frac{p_x}{p_y}$, which can be interpreted as the opportunity cost of good x in terms of good y

2.2.1 Non-linear Constraints

Complexities

- Charity with restrictions
- Taxes with brackets
- Programs with income eligibility

2.3 Conditions for Utility Maximization

Consider an individual choosing how much they want to consume, they are choosing optimally, with a positive amount of all goods.

Then, the following must be true at their optimal consumption bundle: $MRS_{xy} = \frac{p_x}{p_y}$

GARP states that people will always have consistent preferences

If x is revealed preferred to y, then y cannot be directly revealed preferred to x.

2.4 Incentive Systems

Incentive systems are good (maybe?) (commissions or other incentives)

Risk-averse subjects are averse to incentives

3 Choice Under Uncertainty

3.1 Stuff I haven't written down yet

3.2 Prospect Theory

Kahneman and Tversky (1979)

Expected utility theory embodies strong assumptions:

- Perceived are equal to true probabilities
- Independence of gambles
- Preferences are over wealth (not presentation)

3.2.1 Probability Weighting Function

Small probability events are over-weighted.

Lies above a 45-degree line for low values

3.2.2 Loss Aversion

Losses are weighted more heavily than gains

Losing x money is worse than gaining x money is good

Work harder when incentives are framed as a fine than a bonus

4 Theory of the Firm

We will maintain the assumption that firms are profit maximizing

This is a weird assumption, but useful

Entrepeneurs make a number of mistakes, lack of planning/marketing etc.

Non-profits are obviously not profit maximizing.

They engage in behavior opposite to the assumption we made.

Profit maximizing is simple, and clearly relevant.

4.1 The Production Function

The production function for a firm, y = f($\vec{x}$) gives the max. amount of output (y) that can be made from the bundle $\vec{x}$

An isoquant is a set of bundles that all give the same amount of output

Production functions have similar assumptions as utility functions, monotonicity and convexity

The technical rate of substitution is the marginal rate of substitution

Production functions are cardinal

4.2 Profit Maximization

Maximize $py - (w_1x_1 + w_2x_2)$ where $f(x_1, x_2) = y$

$L = py - (w_1x_1 + w_2x_2) + \lambda(f(x_1, x_2) - y)$

$c(y) = w_1x_1(y) + w_2x_2(y)$

5 Equilibrium and Efficiency

Market for goods are interlinked, cars and steel, houses and furniture

Consumers optimize simultaneously

The same inputs are used in many different goods

This is called the general equilibrium model

Partial equilibrium models look at goods in isolation

The demand schedule for a good is a function telling us how much of a good consumers want at any price

5.1 Market Equilibrium

If the quantity demand is equal to the quantity supplied, the market is in equilibrium

Market forces tend to push to equilibrium

Taxes are a huge issue to all market forces

Subsidies also influence forces a lot

Total surplus is the total benefit to all parties generated by a market. It is the sum of consumer, producer, and tax

When tax is introduced, inefficiencies arise such as deadweight loss

We require a general equilibrium model

5.2 Welfare Theorems

Stated in terms of general equilibrium, hold in partial

First Welfare Theorem: The production and allocation of goods resulting from the equilibrium of a perfectly competitive market must be Pareto efficient

Second Welfare Theorem: For any Pareto efficient allocation, there must exist prices and initial allocation to consumers that makes this the equilibrium output of a perfectly competitive market

Prove the FWT:

Consider a two person, two good exchange model

Let ω_i^1, ω_i^2 be the i-th individual's intiial allocation

Let $u_i(x_i^1, x_i^2)$ be the i-th individual's utility function

Let p^1, p^2 be the prices

Let $(x_i^1(p^1, p^2), x_i^2(p^1, p^2))$ be the utility maximizing bundle

Walrasian equilibrium:

$x_1^1(p^1, p^2) + x_2^1(p^1, p^2) = \omega_1^1 \omega_2^1$

Same with good 2