Intermediate Microeconomics: Advanced Notes

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1 Mathematics of Optimization

1.1 Unconstrained Optimization

- First order conditions: $\frac{\delta f}{\delta x} = 0, \frac{\delta f}{\delta y} = 0$
- Second order conditions: $\frac{\delta^2 f}{\delta x^2} < 0, \frac{\delta^2 f}{\delta y^2} < 0$
- Additionally, $(\frac{\delta^2 f}{\delta x^2} \times \frac{\delta^2 f}{\delta y^2}) - (\frac{\delta^2 f}{\delta x \delta y})^2 > 0$

Example: $G(T, C) = 50 + 10T + 16C - T^2 - 2TC - 2C^2$

$\frac{\delta G}{\delta T} = 10 - 2T - 2C$

$\frac{\delta G}{\delta C} = 16 - 2T - 4C$

$$\begin{bmatrix} -2 & -2 & -10 \\ -2 & -4 & -16 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & -2 & -10 \\ 0 & -2 & -6 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 0 & -4 \\ 0 & -2 & -6 \end{bmatrix}$$

$\Rightarrow T = 2, C = 3$

$\frac{\delta^2 G}{\delta T^2} = -2 < 0$

$\frac{\delta^2 G}{\delta C^2} = -4 < 0$

$(\frac{\delta^2 G}{\delta T^2} \times \frac{\delta^2 G}{\delta C^2}) - (\frac{\delta^2 G}{\delta T \delta C})^2 = (-2 \times -4) - (-2)^2 = 8 - 4 = 4 > 0$

$\Rightarrow G(2, 3)$ is a maximum $\Rightarrow G(2, 3) = 50 + 20 + 48 - 4 - 12 - 18 = 84$

1.2 Constrained Optimization

To find the critical points of $f(x, y)$ subject to some constraints $c - g(x, y) = 0$ we have

Lagrangian: $L = f(x, y) - \lambda(c - g(x, y))$

We take the derivative wrt x, y, and λ to get

$$\begin{bmatrix} \frac{\delta f}{\delta x} & = \lambda \frac{\delta g}{\delta x} \\ \frac{\delta f}{\delta y} & = \lambda \frac{\delta g}{\delta y} \\ g(x) & = c \end{bmatrix}$$

Example: $T = 19.25 - 6\ln(R) - 4\ln(W)$, constraint is $R + W = 5$

$$\begin{bmatrix} 19.25 - \frac{6}{R} & = \lambda \\ 19.25 - \frac{4}{W} & = \lambda \\ R + W & = 5 \end{bmatrix}$$

$19.25 - \frac{6}{R} = 19.25 - \frac{4}{W} \Rightarrow 6W = 4R$

$W = 5 - R \Rightarrow 6(5-R) = 4R \Rightarrow 30 - 6R = 4R \Rightarrow 10R = 30 \Rightarrow R = 3, W = 2$

$T(3,2) = 19.25 - 6\ln(3) - 4\ln(2) \approx 9.89$

Example: $I = \frac{P^2}{250} + \frac{PA}{100} + \frac{A^2}{1000}$, constraint is $P + A = 110$

$$\begin{bmatrix} \frac{P}{125} + \frac{A}{100} & = \lambda \\ \frac{P}{100} + \frac{A}{500} & = \lambda \\ P + A & = 110 \end{bmatrix}$$

$\frac{P}{125} + \frac{A}{100} = \frac{P}{100} + \frac{A}{500}$

$\frac{P}{125} - \frac{P}{100} = \frac{-4A}{500}$

$\frac{-P}{500} = \frac{-4A}{500}$

$P = 4A \Rightarrow 5A = 110 \Rightarrow A = 22 \Rightarrow P = 88$

$I(88, 22) = 50.82$

2 Consumer Theory

Defining preferences:

- Preferences reflect choices

- Defined over bundles of goods
- Based upon all properties of a bundle

- Notation
  - Strictly Preferred $\succ$
  - Weakly Preferred $\succeq$
  - Indifferent $\sim$

- Preferences are assumed to be well-bheaved
  - Complete: An individual knows hiw or her preferences over any two bundles
  - Reflexive: An individual must be indifferent between two identical bundles of goods
  - Transitivity: No cycles

Anchoring and Adjustment:
In multi-dimensional problems, people tend to anchor on one dimension and adjust for the other.

2.1 Utility Functions

A utility function is a function from bundles of goods into real numbers.

- $x \succ y \Rightarrow u(x) > u(y)$

- More preferred bundles must be assigned larger numbers

- If preferences are well-behaved, we can write down a utility function that captures the preferences

Ordinal utility function: the only thing that matters is the order

Cardinal utility function: the numbers have significance and meaning

Assumptions about utility functions:

- Monotonicity: $x > x' \Rightarrow u(x, y) \geq u(x', y)$

- Convexity: Let α s.t. $0 < \alpha < 1$. If $u(x, y) = u(x', y'), u(\alpha x + (1-\alpha)x', \alpha y + (1-\alpha)y') \geq u(x, y)$

Indifference curve: a curve along which $u(x, y) = c$ for some constant c

Marginal Utility: the marginal utility of x (MU_x) is defined as the increase in utility from consuming one additional good of x

$MU_x = \frac{\delta u}{\delta x}$

The marginal rate of substitution of x, y is the amount of good y needed to make up for the loss of one of good x

$MRS_{xy} = \frac{MU_x}{MU_y}$

2.2 Budget Constraints

The budget line is the line along which the amount spent exactly equals the available budget.

Let B be the budget, and p_x, p_y be the price of good x or good y. Thus, we have the budget line as $p_x x + p_y y = B$

Solving for y, we see have $y = \frac{B}{p_y} - \frac{p_x}{p_y}x$

The slope of the budget line is $\frac{p_x}{p_y}$, which can be interpreted as the opportunity cost of good x in terms of good y

2.2.1 Non-linear Constraints

Complexities

- Charity with restrictions
- Taxes with brackets
- Programs with income eligibility