

Intermediate Microeconomics: Advanced

Notes

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1 Mathematics of Optimization

1.1 Unconstrained Optimization

- First order conditions: $\frac{\delta f}{\delta x} = 0, \frac{\delta f}{\delta y} = 0$
- Second order conditions: $\frac{\delta^2 f}{\delta x^2} < 0, \frac{\delta^2 f}{\delta y^2} < 0$
- Additionally, $(\frac{\delta^2 f}{\delta x^2} \times \frac{\delta^2 f}{\delta y^2}) - (\frac{\delta^2 f}{\delta x \delta y})^2 > 0$

Example: $G(T, C) = 50 + 10T + 16C - T^2 - 2TC - 2C^2$

$$\frac{\delta G}{\delta T} = 10 - 2T - 2C$$

$$\frac{\delta G}{\delta C} = 16 - 2T - 4C$$

$$\begin{bmatrix} -2 & -2 & -10 \\ -2 & -4 & -16 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & -2 & -10 \\ 0 & -2 & -6 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 0 & -4 \\ 0 & -2 & -6 \end{bmatrix}$$
$$\Rightarrow T = 2, C = 3$$

$$\frac{\delta^2 G}{\delta T^2} = -2 < 0$$

$$\frac{\delta^2 G}{\delta C^2} = -4 < 0$$

$$(\frac{\delta^2 G}{\delta T^2} \times \frac{\delta^2 G}{\delta C^2}) - (\frac{\delta^2 G}{\delta T \delta C})^2 = (-2 \times -4) - (-2)^2 = 8 - 4 = 4 > 0$$

$$\Rightarrow G(2, 3) \text{ is a maximum} \Rightarrow G(2, 3) = 50 + 20 + 48 - 4 - 12 - 18 = 84$$

1.2 Constrained Optimization

To find the critical points of $f(x, y)$ subject to some constraints $c - g(x, y) = 0$

we have

$$\text{Lagrangian: } L = f(x, y) - \lambda(c - g(x, y))$$

We take the derivative wrt x , y , and λ to get

$$\begin{bmatrix} \frac{\delta f}{\delta x} & = \lambda \frac{\delta g}{\delta x} \\ \frac{\delta f}{\delta y} & = \lambda \frac{\delta g}{\delta y} \\ g(x) & = c \end{bmatrix}$$

Example: $T = 19.25 - 6 \ln(R) - 4 \ln(W)$, constraint is $R + W = 5$

$$\begin{bmatrix} 19.25 - \frac{6}{R} & = \lambda \\ 19.25 - \frac{4}{W} & = \lambda \\ R + W & = 5 \end{bmatrix}$$

$$19.25 - \frac{6}{R} = 19.25 - \frac{4}{W} \Rightarrow 6W = 4R$$

$$W = 5 - R \Rightarrow 6(5 - R) = 4R \Rightarrow 30 - 6R = 4R \Rightarrow 10R = 30 \Rightarrow R = 3, W = 2$$

$$T(3, 2) = 19.25 - 6 \ln(3) - 4 \ln(2) \approx 9.89$$

Example: $I = \frac{P^2}{250} + \frac{PA}{100} + \frac{A^2}{1000}$, constraint is $P + A = 110$

$$\begin{bmatrix} \frac{P}{125} + \frac{A}{100} & = \lambda \\ \frac{P}{100} + \frac{A}{500} & = \lambda \\ P + A & = 110 \end{bmatrix}$$

$$\frac{P}{125} + \frac{A}{100} = \frac{P}{100} + \frac{A}{500}$$

$$\frac{P}{125} - \frac{P}{100} = \frac{-4A}{500}$$

$$\frac{-P}{500} = \frac{-4A}{500}$$

$$P = 4A \Rightarrow 5A = 110 \Rightarrow A = 22 \Rightarrow P = 88$$

$$I(88, 22) = 50.82$$

2 Consumer Theory

Defining preferences:

- Preferences reflect choices

- Defined over bundles of goods
- Based upon all properties of a bundle
- Notation
 - Strictly Preferred \succ
 - Weakly Preferred \succeq
 - Indifferent \sim
- Preferences are assumed to be well-behaved
 - Complete: An individual knows his or her preferences over any two bundles
 - Reflexive: An individual must be indifferent between two identical bundles of goods
 - Transitivity: No cycles

Anchoring and Adjustment:

In multi-dimensional problems, people tend to anchor on one dimension and adjust for the other.

2.1 Utility Functions

A utility function is a function from bundles of goods into real numbers.

- $x \succ y \Rightarrow u(x) > u(y)$

- More preferred bundles must be assigned larger numbers
- If preferences are well-behaved, we can write down a utility function that captures the preferences

Ordinal utility function: the only thing that matters is the order

Cardinal utility function: the numbers have significance and meaning

Assumptions about utility functions:

- Monotonicity: $x > x' \Rightarrow u(x, y) \geq u(x', y)$
- Convexity: Let α s.t. $0 < \alpha < 1$. If $u(x, y) = u(x', y')$, $u(\alpha x + (1 - \alpha)x', \alpha y + (1 - \alpha)y') \geq u(x, y)$

Indifference curve: a curve along which $u(x, y) = c$ for some constant c

Marginal Utility: the marginal utility of x (MU_x) is defined as the increase in utility from consuming one additional good of x

$$MU_x = \frac{\delta u}{\delta x}$$

The marginal rate of substitution of x , y is the amount of good y needed to make up for the loss of one of good x

$$MRS_{xy} = \frac{MU_x}{MU_y}$$

2.2 Budget Constraints

The budget line is the line along which the amount spent exactly equals the available budget.

Let B be the budget, and p_x, p_y be the price of good x or good y . Thus, we have the budget line as $p_x x + p_y y = B$

Solving for y , we see have $y = \frac{B}{p_y} - \frac{p_x}{p_y} x$

The slope of the budget line is $\frac{p_x}{p_y}$, which can be interpreted as the opportunity cost of good x in terms of good y

2.2.1 Non-linear Constraints

Complexities

- Charity with restrictions
- Taxes with brackets
- Programs with income eligibility

2.3 Conditions for Utility Maximization

Consider an individual choosing how much they want to consume, they are choosing optimally, with a positive amount of all goods.

Then, the following must be true at their optimal consumption bundle:

$$MRS_{xy} = \frac{p_x}{p_y}$$

GARP states that people will always have consistent preferences

If x is revealed preferred to y , then y cannot be directly revealed preferred to x .

2.4 Incentive Systems

Incentive systems are good (commissions or other incentives)

Model an employee as consuming two goods: leisure and consumption

8 hours per day can go between these two goods