HW06 Polynomial Interpolation

Problem 1 (5 points)

(Analytical) Find by hand the quadratic interpolating polynomial to $\ln(x)$ through $x = 1, 2, 3$. Typeset your results in a Markdown cell.

Solution

$x_1 = 1, y_1 = \ln(1) = 0$

$L_1(x) = \frac{(x-2)(x-3)}{(1-2)(1-3)} = \frac{(x-2)(x-3)}{2}$

$x_2 = 2, y_2 = \ln(2)$

$L_2(x) = \frac{(x-1)(x-3)}{(2-1)(2-3)} = -(x-1)(x-3)$

$x_3 = 3, y_3 = \ln(3)$

$L_3(x) = \frac{(x-1)(x-2)}{(3-1)(3-2)} = \frac{(x-1)(x-2)}{2}$

$p_2(x) = 0L_1 + \ln(2)L_2 + \ln(3)L_3$

$= -\ln(2)(x-1)(x-3) + \frac{\ln(3)}{2}(x-1)(x-2)$

Problem 2 (5 points)

(Julia) Find the quadratic interpolating polynomial to $\ln(x)$ through $x = 1, 2, 3$ using the `Polynomials.jl` package in Julia and without writing a function to do it.

Solution

In [1]:
```
using Polynomials
xs = 1:3
```

Out[1]:
```
1:3
```

In [2]:
```
y = log.(xs)
```

Out[2]:
```
3-element Vector{Float64}:
 0.0
 0.6931471805599453
 1.0986122886681098
```

In [3]:
```
L1 = fromroots( xs[1:end .!=1] )/( prod( xs[1] .- xs[1:end .!=1] ) )
```

Out[3]: $3.0 - 2.5 \cdot x + 0.5 \cdot x^2$

In [4]:
```
L2 = fromroots( xs[1:end .!=2] )/( prod( xs[2] .- xs[1:end .!=2] ) )
```

Out[4]: $-3.0 + 4.0 \cdot x - 1.0 \cdot x^2$

In [5]:
```
L3 = fromroots( xs[1:end .!=3] )/( prod( xs[3] .- xs[1:end .!=3] ) )
```

Out[5]: $1.0 - 1.5 \cdot x + 0.5 \cdot x^2$

In [6]:
```
p = y[1]*L1 + y[2]*L2 + y[3]*L3
```

Out[6]: $-0.980829253011726 + 1.1246702892376166 \cdot x - 0.1438410362258904 \cdot x^2$