

$$\begin{aligned}
f(n) &= n^2! \prod_{j=0}^{n-1} \frac{j!}{(j+n)!} \\
f(n+1) &= (n+1)^2! \prod_{j=0}^n \frac{j!}{((j+n)!(j+n+1))} \\
&= (n+1)^2! \frac{n!}{(2n+1)!} \prod_{j=0}^n \frac{j!}{(j+n)!} \\
&= (n+1)^2! \frac{n!}{(2n+1)!} \left(\prod_{j=0}^{n-1} \frac{j!}{(j+n)!} \right) \frac{n!}{2n!} \\
&= 36! \frac{0!1!2!3!4!5!}{6!7!8!9!10!11!}
\end{aligned}$$