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PHYS 331, HW06

1a.) The matrix equation, in the form of Ax=b, is:

$$\begin{pmatrix} 1 & t_1 & \frac{1}{2}(t_1)^2 \\ 1 & t_2 & \frac{1}{2}(t_2)^2 \\ 1 & t_3 & \frac{1}{2}(t_3)^2 \end{pmatrix} \begin{pmatrix} x_0 \\ v_0 \\ a \end{pmatrix} = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

This system is linear. If it were nonlinear, there would be some terms that could not be substituted with a new variable to form a linear system. This is not the case, thus it is linear. This is what allows the problem to be written and solved in this way.

1c.) Because delta t is seen to be proportional to R, and a higher R is a more desirable R, the higher delta t is the more desirable delta t. Over the range delta t = 0.1s to delta t = 5s, this implies that delta t = 5s is the optimal delta t. This is to be expected. Mathematically, the closer delta t gets, the more precise measurements need to be in order to detect relatively small changes. Error due to precision thus propagates more with a smaller delta t, and less with a larger delta t as they are more distinct measurements. This also makes sense intuitively. Experimentally, it is much easier to record values at well-spaced intervals than close together if using something such as a stopwatch, due to things like reaction time (although this is obviously not reflected in the model we coded).

1d.) This parabolic curve suggests the optimal t2 is at t=5 seconds. Of course, given a 10 second time interval, this implies an even spacing of delta t = 5s, which was found to be the optimal spacing in part c. Thus, delta t = 5s is not only the optimal even time spacing, but choosing t2 such that measurements are evenly spaced is shown to be the optimal method of data collection.

1e.) The best-fit parameters for strategy 1 is x0 = 0.3, v0 = 0.24999999999999994, a = 0.23, while the best-fit parameters for strategy 2 is x0 = 0.3, v0 = 0.24999999999999983, a = 0.23000000000000004. These are effectively the same. Both strategies are inputting valid values of t and their correlated valid values of x(t), so they are expected to find equivalent values for x0, v0, and a.

1f.) Upper-Lower Bound Error comparison between Strategy 1 and Strategy 2

	v_0	a
Strategy 1 Upper Bound	0.2515	0.2298
Strategy 1 Lower Bound	0.2485	0.2302
Strategy 1 Error	0.003	-0.0004
Strategy 2 Upper Bound	0.2555	0.229
Strategy 2 Lower Bound	0.2445	0.231
Strategy 2 Error	0.011	-0.002

Looking at this table, the error in both v0 and a is lower for strategy 1 than strategy 2. This is to be expected, as stated in parts c and d, as evenly spacing is implied to be the optimal method by the graphs generated. These numerical results confirm this, thus evenly spacing time intervals is most accurate in both theory and practice.

2a.) Gaussian Elimination is the process of converting a matrix into upper triangular form by row operations so that it may be more easily solved. After Gaussian Elimination, the left hand side of the matrix equation effectively represents a system of linear equations that gradually decrease the number of variables, until it directly solves for a single value x_n = b_n. This value of x_n can be plugged into the prior equation of the form ax_(n-1) + bx_n = b_(n-1), which effectively directly solves for x_(n-1). This process is repeated for all values of x. Gaussian elimination converts the matrix into a form such that all x values can be directly, and hence efficiently, solved.

2c.) When inputted as written, thus returns the error “cannot convert float NaN to integer.” This is because the multiplicative factor is defined by a(21)/a(11), and a(11) is zero in this instance, thus it divides by zero and returns infinity, which cannot be used in calculations. This is a potential issue for any value of zero on the diagonal. To mitigate this, use row operations to add to the rows in question until there are no zeros on the diagonal. In this case, Row 1 -> Row 1 + Row 2, and Row 3 -> Row 3 + Row 2. Thus:

$$A_2 = \begin{pmatrix} 2 & 4 & 3 & 3 \\ 2 & 2 & 3 & 2 \\ 6 & -1 & 3 & 3 \\ 6 & 1 & -6 & -5 \end{pmatrix}, b_2 = \begin{pmatrix} -2 \\ -2 \\ -9 \\ 6 \end{pmatrix}$$

3.)

Prob 3.) $A = LU$ $\quad L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix}$ $\quad U = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & -2 \\ 0 & 0 & -5 \end{pmatrix}$

$Ax = b$, where $b = \begin{pmatrix} 7 \\ -12 \\ -8 \end{pmatrix}$ $\qquad Ax = b \Rightarrow LUx = b$

Let $y = Ux$. $Ly = b$

$Ly = b$:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 7 \\ -12 \\ -8 \end{pmatrix}$$

Thus $y_1 = 7$, $y_2 = -12$.

$2y_1 + y_2 + y_3 = -8$

$2(7) + (-12) + y_3 = -8$

$y_3 = -10$ $\qquad$ so $y = \begin{pmatrix} 7 \\ -12 \\ -10 \end{pmatrix}$

$Ux = y$:

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & -2 \\ 0 & 0 & -5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ -12 \\ -10 \end{pmatrix}$$

Thus $x_3 = 2$.

$4x_2 - 2x_3 = -12$

$4x_2 - 2(2) = -12$

$x_2 = -2$

$x_1 + 2x_2 + 3x_3 = 7$

$x_1 + 2(-2) + 3(2) = 7$

$x_1 = 5$ $\qquad$ so $\boxed{x = \begin{pmatrix} 5 \\ -2 \\ 2 \end{pmatrix}}$

4d.) There are no differences between the input A and the dot product of LU, as the residuals for the 3x3 and 4x4 matrices were a 3x3 matrix of zeros and a 4x4 matrix of zeros respectively.