Paul Buttles

09/01/2020

PHYS 331, HW03

1a.) There are 3 real roots, whose values are approximately x = -0.5, -1.1, -2.3.

1b.) The Newton-Raphson equation for this function is: $x_{n+1} = f(x_n) = x_n - \frac{x_n{}^5 - 3x_n{}^3 + 15x_n{}^2 + 27x_n + 9}{5x_n{}^4 - 9x_n{}^2 + 30x_n + 27}$

2a.)

$$f(x) = x^5 - 3x^3 + 15x^2 + 27x + 9$$

$$g1(x) = x^5 - 3x^3 + 15x^2 + 28x + 9 = x$$

$$dg1(x) = 5x^4 - 9x^2 + 30x + 28$$

$$g2(x) = \sqrt{x^5 - 3x^3 + 16x^2 + 27x + 9} = \sqrt{x^2}$$

$$dg2(x) = \frac{1}{2}(x^5 - 3x^3 + 16x^2 + 27x + 9)^{-\frac{1}{2}}(5x^4 - 9x^2 + 32x + 27)$$

$$g3(x) = \sqrt[3]{x^5 - 2x^3 + 15x^2 + 27x + 9} = \sqrt[3]{x^3}$$

$$dg3(x) = \frac{1}{3}(x^5 - 2x^3 + 15x^2 + 27x + 9)^{-2/3}(5x^4 - 6x^2 + 30x + 27)$$

2b.) None of the three methods with converge on any roots. Setting the absolute value of dg1(x), dg2(x), and dg3(x) less than 1 reveals that the roots are visually not within the basin of convergence of any of these methods.

2d.)

Method:	X_start	Prediction:	Result:	Iterations to Root/Failure
g1(x)	X=-1.84 (from basin)	Failure	Failure	2
g1(x)	X=-0.8 (from basin)	Failure	Failure	2
g1(x)	X=-1.1 (from root)	Failure	Failure	3
g1(x)	X=-2.3 (from root)	Failure	Failure	2
g1(x)	X=-0.5 (from root)	Failure	Failure	3
g2(x)	X=-1.9 (from basin)	Failure	Failure	3
g2(x)	X=-1.1 (from root)	Failure	Failure	3
g2(x)	X=-2.3 (from root)	Failure	Failure	3
g2(x)	X=-0.5 (from root)	Failure	Failure	4

g3(x)	X=-1.7	Failure	Failure	5
g3(x)	X=-0.8	Failure	Failure	6
g3(x)	X=-1.1	Failure	Failure	6
g3(x)	X=-2.3	Failure	Failure	5
g3(x)	X=-0.5	Failure	Failure	6

4b.) The error is effectively zero within machine precision after 9 iterations.

4c.) The value m=2 returns the most consistent constants, outputting:

[0.11270298606826683, 0.14681716126587527, 0.17806337267289848, 0.18635333835279233, 0.17345219417683244, 0.16693720683834357, 0.16650375628777025]

Whereas m =1 returned:

[0.6987446082579157, 0.6360310816374742, 0.49063039653650586, 0.2519251258189481, 0.05907253239919829, 0.0033584935840104864, 1.1250192115066117e-05]

And m =1.5 returned:

[0.2806253799102434, 0.305581867725947, 0.2955728389807355, 0.21667276758122653, 0.10122381320728323, 0.02367820808463361, 0.0013686486934628489]

Thus m=2 is the clear choice. This was expected, as this is the order of convergence we discussed in class (Lecture 5) for the Newton-Raphson method, so since we used this method, this was to be expected.