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PHYS 331, HW05

1b.) The solution for x has residuals [0,0,0], thus it is confirmed to be an accurate solution.

1c.) Given the form $Ax = b$, x can be found using the inverse of A as such:

$$Ax = b$$

$$A^{-1}Ax = A^{-1}b$$

$$Ix = A^{-1}b$$

$$x = A^{-1}b$$

1d.) The system of equations can be written as a matrix of the form:

$$\begin{pmatrix} 50+R & -1 & -30 \\ -R & 65+R & -15 \\ -30 & -15 & 45 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 120 \end{pmatrix}$$

The input vector space, V, represents the current I in the given loop. The output vector space, W, represents the voltage V in the given loop.

1e.) All non-zero residuals are on the order of magnitude of less than or equal to 10^{-14}, so it is sufficiently close to zero within machine precision. Thus residuals are effectively zero and a valid solution has been found.

1f.) All residuals are zero, thus valid solutions have been found. As voltage decreases, the found currents decrease as well. This is expected as it follows directly from Ohm's Law. V=IR implies I is proportional to V, so a decrease in V naturally causes a decrease in I as well.

2c.) The residuals for matrix equations 1 and 2 are both zero vectors, thus the found solutions are valid.

2d.) Initially, there is zero error for most iterations (although the random nature of the numbers will occasionally spit out very small, 10^{-14} or below residuals. Increasing the matrix size increases the error. For n = 3, residuals were always zero. For n = 10, there are more non-zero residuals than before (roughly one in three are non-zero), but still on the same order of magnitude. For n = 30, there were still about 1 in three non-zero residuals, but the order of magnitude varied from 10^{-14} to 10^{-12}. Finally, at n = 100, residuals were on the order of magnitude of up to 10^{-2}, thus residuals were much higher.