1a.) $f(z) = 2z^3 - 1 + 3i = 0$

$f(x+iy) = 2(x+iy)^3 - 1 + 3i = 0$

$f(x+iy) = 2x^3 + 6x^2yi - 6xy^2 - 2y^3i - 1 + 3i = 0$

$f(x+iy) = (2x^3 - 6xy^2 - 1) + (6x^2y - 2y^3 + 3)i$

$f_1(x,y) = 2x^3 - 6xy^2 - 1$

$f_2(x,y) = 6x^2y - 2y^3 + 3$

1b.) J = $\begin{pmatrix} 6x^2 - 6y^2 & -12xy \\ 12xy & 6x^2 - 6y^2 \end{pmatrix}$

J(inverse) = $\frac{1}{\det(J)}\begin{pmatrix} 6x^2 - 6y^2 & 12xy \\ -12xy & 6x^2 - 6y^2 \end{pmatrix}$

$$= \frac{1}{36x^4 + 72x^2y^2 + 36y^4}\begin{pmatrix} 6x^2 - 6y^2 & 12xy \\ -12xy & 6x^2 - 6y^2 \end{pmatrix}$$

1c.) There should be three complex roots, as it is a complex polynomial equation of degree three. All three roots were found, which are:

z = 1.06546973x + -0.47115078iy

multiplicity 2, found at (1,0) and (1,-1)

z = -0.94076341x + -0.68714847iy

multiplicity 4, found at (-1,0), (0,-1), (-1,-1), and (1,1).

z = -0.12470632x + 1.15829925iy

multiplicity 2, found at (0,1) and (-1,1)

The starting point (0,0) did not converge, as the resultant Jacobian is not invertible (the determinant of the Jacobian is zero).

2a.) System of equations:

f1(x) = 6870 + 6870e*sin(-30° + α) - C

f2(x) = 6728 + 6728e*sin(0° + α) - C

f3(x) = 6615 + 6615e*sin(30° + α) – C

2b.) The function numpy.sin takes angles in radians, not degrees.

2d.) The smallest trajectory R is 6580.454017137386km, or rounded to the appropriate significant digits is 6580km. The smallest corresponding θ is 70.47451519126884°, or rounded to the appropriate significant digits 70°.

3a.) The resultant array has a height of 41 and width of 2, thus a size of 82.

3f.) The function scipy.optimize.leastsq() outputs a tuple of size 2. The first element is the array of c1, c2, v01, v02, gamma1, and gamma2, but now optimized by minimizing their sum of squares. The second element is the inverse of the Hessian, and can be ignored.

3g.) The data type of x1 is an array of float64, and the size of x1 is 6, or as stated in the variable explorer, is (6,). The graph of residuals is very close to zero, with its peak on the order of magnitude of 10^(-6).

3h.) The fitted line matches the scatter plot quite well. The best-fit curve does not go precisely through every point, but visually, it appears to cross through every point. There are no points that lie near but off the line. The best-fit wavenumbers are v01 = 20237.000016006663 and v02 = 20696.00008814348, or adjusted for significant digits, v01 = 20237m^{-1} and v02 = 20696m^{-1}.